RONALDO DE ALBUQERQUE E ARRAES
Alternative Evaluations of Economically Optimal Rations For Broilers
(Under the direction of Bill R. Miller)

Least cost per pound of broiler production from finishing rations was investigated through application of an optimizing model to quadratic production surface estimated from feed experiments with protein and energy. Broiler's carcass fat and time were restrictions in the least cost framework. Results of feed formulation derived in the study were compared with those from linear programming.

There was a small trade-off in economic levels of protein and metabolizable energy as size of the bird increased. Birds marketed at different ages, and thus size, should be fed on different rations to achieve least cost per pound.

It was possible to estimate least cost broiler output as a function of time and to relate this to the trade-off between protein and metabolizable energy. Likewise, least cost output could be related to the level of bird's carcass fat. Lean or fat birds are produced according to the level of protein and metabolizable energy which change with the production of a desired fat level.

Application of the levels of protein and metabolizable energy from industry linear programming formulation of feed into the quadratic programming model demonstrated that essentially no difference in cost per pound of broiler could be observed between the results of the two formulations. However, the least cost broiler model projected the changing ratios of protein and energy in response to feed price which was not found in linear programming of least cost feed.

A sensitivity analysis on the prices of the two main feed ingredients in the ration, corn and soybean meal, indicated how to set the right hand side for protein and metabolizable energy to attain an economically efficient solution.

Relaxation of all nutrients constraints (protein, metabolizable energy, methionine, cystine, lysine, etc.) applied to the production response surface for protein and energy gave estimates of economic efficiency where feed cost/pound broiler was minimum. This effect was potentially important in that about a one-half pound increase in liveweight could be achieved at the common level of cost now incurred by industry. Future research is needed to determine if the level of nutrient constraints associated with economic optimum levels of protein and metabolizable energy are biologically efficient.

INDEX WORDS: Broilers, Least Cost, Protein, Metabolizable Energy.

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OPTIMAL RATIONS FOR BROILERS

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TABLE OF CONTENTS

LIST OF TABLES

Table | Page

Table | Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Statement of the Problem

In general, the main objective of a firm is to maximize profits, which implies cost minimization. This means that firms are always trying to improve technical and/or economic efficiency in production. The broiler industry is not an exception to this general rule as seen by the evolutionary improvement of efficiency in broiler production (Table 1-1). Yet, there are continuing problems that hinder improvements in efficiency of broiler production.

The proportion of feed costs to total cost of broiler production has increased over time. Percentage distribution of production cost items, Table 1-2, demonstrate that feed has accounted for a major part of production costs. Hence, given a stable price for broilers, minimizing feed costs per pound of broiler is of primary concern to broiler growers seeking improvement in economic efficiency. However, how to derive the array of feedstuffs (feed formula, diet or ration) by feed cost minimization while maintaining the minimum nutrient requirements for the required

TABLE 1-1. Source Indicators of Efficiency in Broiler Production for Selected Years, U.S., 1952-1982

Year	Time required to reach 3.5 lb.	Market Age	Market Weight	Technical Feed Efficiency
	(days)	(days)	(days)	lbs. of feed lbs. of broiler)
1952	74	80	3.35	3.17
1955	65	74	3.35	2.87
1958	60	68	3.44	2.56
1961	53	67	3.80	2.24
1965	49	58	3.75	2.05
1982[1/]	46	46-49	3.6-4.0	2.02

SOURCES: (28, p. 375).

[1/]Table 5-1.

TABLE 2-1. Percentage Distribution of Broiler Production Costs for Selected Years, U.S., 1967-1979

Item	1967	1973	1979
		(Percent)	
Feed	62.4	74.1	72.9
Chicks	19.2	12.5	11.9
Grower Payment[1/]	12.0	10.2	11.0
Fuel	2.0	.6	
Medications	1.6	1.1	
Vaccination	1.2		
Litter	.8	.1	
Miscellaneous	.8	1.4	
TOTAL	100.0	100.0	95.8[2/]

SOURCES: Poultry and Egg Situation, June, 1975.

[1/] Includes housing.

[2/] (35, p. 43). The remaining 4.2% is distributed among the other items listed.

level of growth continues to be one of the main problems of economic efficiency in poultry nutrition. There are a large variety of feedstuffs that can be used as source of protein and metabolizable energy which are the fundamental nutrients needed for chicken growth. Therefore, an appropriate choice of the feedstuffs will improve efficiency in production. For instance, corn and soybean meal have been the two principal feedstuffs used in feed-mix formulas due to their high nutrient contents, relative low price and availability. In general they usually represent over 80% of the ration composition as currently derived by the industry. However, prices of corn and soybean meal have recently shown increased variation within short periods of time (20). Feedstuffs prices are crucial determinants of least cost rations and high percentages of corn and soybean meal in the ration might not lead to an optimum solution.

Specification of a unique ration for the entire growing period is now regarded by nutritionists as a misguided procedure. The biological nature of livestock seems to indicate that at marketable weight, gain in weight increases at a decreasing rate. That is, additional units of feed input added result in less and less gain in weight. Broilers appear to behave in this manner. As they increase in size their body composition changes and the nature of this biological change requires that during the early growing period, rations of a higher protein and a lower energy content must be supplied. As the birds develop their body weight, less protein and more energy are required in the ration. For these reasons, ration formulations are now changed once or twice during the birds' growing period.

Another area of interest to feed formulators and animal nutritionists concerns the "quality" of the final product as measured by the carcass fat content of the animal. To date, the assumption made for least cost ration problems is that one pound of meat has the same value regardless of the diet formulation. This can hardly be true. It is observed in practice that some enterprises require birds weighing X pounds with not more than X% carcass fat. Some excess fat is lost with the offal and decreases the percent yield of the carcass. However, the relationship between carcass quality and diet composition is not well defined. The production of carcass fat is one variable to be investigated in this work.

It is an hypothesis of this study that the broiler industry does not produce broilers as efficiently as it could. Allison et. al., (2), Chao (8) and others, using different techniques have found ration formulas with lower costs than the ones employed by the broiler industry. The reason for that might be closely associated with the techniques used to derive a minimum cost or maximum profit broiler ration. There has not been a unique way for the broiler industry to define the least cost ration formulation. Present use of linear programming in the industry determines the least-cost formulas for a given set of specifications. Whether these specifications lead to the least cost production of broilers is not known. The choice and proper use of technique that relates specifications to production is very fundamental to deduce such a ration; otherwise the supposed optimum ration is not an optimum one.

The traditional technique of linear programming, the most widely used practice today, deals with minimizing a feed cost function subject to nutrient requirements that can be provided by the various feedstuffs available. Certainly this technique succeeds in getting lower feed cost, but it fails by not taking into account the performance of the bird. Moreover, the right hand side of the constraints set (specification) from requirement tables seems to be inadequate as pointed out by Dent (11). Furthermore, Brown and Arscott (6, p. 69) state that marginal analysis of production economics theory would seem to afford a better approach than the linear programming least cost model. The way marginal analysis has been applied on livestock raises major problems by the type of feedstuffs that were pre-specified for the analysis. For instance, the early work done by Heady et. al., (23) using this technique on broilers, specifies corn and soybean as the feedstuffs. Consequently, the optimum solution is a function of the feedstuffs and their prices only.

This method is inaccurate in that it does not consider the level of the fundamental nutrients required in broiler growing.

More recently, Townsley (38) has used quadratic programming to specify optimum livestock rations. He demonstrated that this technique is more efficient than linear programming in defining least cost or maximum profit ration specifications.

The economic literature on broilers is extensive. Heady, Balloun and McAlexander (25) were the first to develop a marginal analysis model of broiler production to explain least-cost ration formulation and optimum marketing weight. Their analysis was carried out by assuming that the rate of substitution between nutrients affects the bird's body composition in such a way that high protein content ration is required in the early stage of growth. As the fattening period approaches, less protein is required and is substituted for by increased metabolizable energy (M.E.). They also assumed that the ration is unchanged or changed only once throughout the production period. Production functions were estimated and least-cost rations were determined by minimizing linear feed costs subject to a weight gain level. The second step was to determine the optimum level of feeding and the most profitable marketing weight. This was done by maximizing profits above feed costs and taking into account the results from the least-cost rations. All results in their study were based only on the protein level of the ration, which varied with different input-output prices. This approach of marginal analysis to determine economic optimum in production was applied to other agricultural commodities. Heady, Catron, Mckee, Ashton and Speer (23) studied economic efficiency in pork production by estimating production functions and finding rate of gain, least-cost ration and contrasting least-cost ration with least time ration. Brown, Heady, Pesek and Stritzel (7), in an experiment on corn, estimated optimum levels of fertilization and optimum ration of nutrients through estimation of production functions. Profit maximization was the criterion selected to find those optimum situations in production.

Brown and Arscott (6) used data from an experiment on broilers to predict the most profitable weights (above feed costs) for a range of ration specifications. The experimental design included four protein levels (16, 22, 26 and 32%) and three metabolizable energy (M.E.) levels (1200, 1400 and 1600 kcal/lb.), giving twelve different protein-energy combinations. Data on growth and feed consumption were obtained and pooled for estimation of weight and feed consumption equations. Significant statistical results were obtained. They conjugated a linear programming study to the above model to determine the most profitable ration based on nine and ten week feeding periods. For ten-weeks the best ration was 20% protein and 1500 kcal/lb. of M.E.

The findings were based on given prices which, for the authors, were crucial factors for determining the best ration.

By using a different approach from the studies reported above, Allison and Baird (1) elaborated an experimental design used with swine to contract two common procedures in animal nutrition. The first minimized feed cost per animal to produce a specified amount of weight gain. The second procedure minimized feel ingredient cost for pre-established protein and energy levels. For adding 170 lbs. to finishing swine, the former procedure reduced the cost per head $6.35 from the cost obtained by using the latter procedure which adopted current feeding recommendations for swine. For this type of feeding operation, minimizing feed ingredient cost for pre-established rations may not be a sufficient criterion for minimizing cost and maximizing profits.

Pesti (36) conducted an experiment to test the hypothesis that broiler weight gain, feed consumption and feed conversion for three weeks of age broilers, depends on the concentration of protein and energy in the feed ration. He combined seven protein levels (from 17.3 to 25.9%) with five energy levels (from 2800 to 3600 kcal/kg) in fourteen different combinations. By estimating a quadratic function, having protein and energy as arguments to explain weight gain, feed consumption and feed conversion (Model A), he did not find any relevant statistical results. However, as initial chick weight was introduced in the equations (Model B), the weight gain equation was improved remarkably.

Waldroup, Mitchell, Payne and Johnson (39) conducted a study to test the effects of dietary nutrients density levels on broiler performance. Broilers were fed until nine weeks of age with six dietary energy levels which were similar to those in practical usage by broiler producers: 2970, 3080, 3190, 3300 and 3520 M.E. kcal/kg. Running simple linear equations for each energy level, they predicted the amount of feed needed and days of age required to attain a desired bird weight. For instance, a 1.9kg. broiler can be produced in the least average time of 61 days by using either 3300 or 3520 M.E. kcal/kg in the ration, according to the estimated equations.

The studies summarized above dealt with regression analysis and marginal analysis of production economics. Another technique much used in developing least cost rations for animal nutrition is linear programming analysis. Hutton, King and Boucher (29), in an early study, developed a broiler least-cost ration formula by applying linear programming. From this study, selection of broiler feeds, optimum level of nutrients and compositions of the feed can be determined. Extensive explanations of the final solution, price changes and feed and nutrient specification are outlined. Given a price set and pre-determined minimum nutrients requirements, the least-cost feed formula consisted of, among other results, fourteen feed ingredients (soybean meal and corn products accounted for 90% of a pound of feed), 22.6% protein and 950 kcal/lb. of energy.

An application of Farrell efficiency analysis to determine broiler least-cost rations was done by Chao (8). Chao used weekly data from an experiment that applied eight different treatments where each treatment varied diets from the second to the eight week. For a given set of feed ingredient prices the optimum ration or least feed cost was determined among the observations in the experiment, and the optimum diet was indicated. Chao's results provided a total feed cost per pound of gain below that observed in the broiler industry.

Allison, Ely and Amato (2) elaborated on an economic analysis of broiler production to specify the most profitable rations and the most profitable final live-weight. Utilizing nineteen feed ingredients previously established for five different price situations, five energy levels (2976, 3086, 3197, 3307 and 3417 kcal/kg) and three growth periods, the following main results were obtained. There were insignificant cost differences between minimizing cost per unit of gain and minimizing cost per unit of feed. The energy level of feed for the second and third growth. period was the same for any of the five price levels. Given the price structure in 1977, returns were optimized by producing a bird weighting 1.91kg, in opposition to the normal market weight bird of 1.72kg.

Dent (11) pointed out the need to include the performance of the animal as a way to improve the results from applying mathematical programming to livestock ration formulation. He also introduced the time factor into the analysis of livestock feed response. His analysis on bacon pigs was based on an experiment that involved four levels of energy each combined with four levels of protein to give a factorial design with 16 dietary treatments. A response surface representing the daily rate of growth was estimated as a function of energy and protein intakes; quadratic form was the best fit. Two levels of growth rate were selected, for pigs between 60 and 120 pounds liveweight, for defining the least cost rations, namely, 1.00 and 1.25 pounds of daily gain per pig. The weight range was divided into two other ranges: 60-80 and 100-120 pounds. By applying linear programming, economic results were obtained, but the optimum ration presented a higher protein/energy proportion for younger pigs than for older pigs. This seems to contradict the biological nature of a more efficient growth. With Dent's procedure, a particular growth rate is specified and the ration that meets that rate at the lowest possible cost is then selected. Apparently, there exists a very large set of rations to be searched.

Due to the inefficiency problem in attaining an optimum ration at many steps, Townsley (38) applied quadratic programming to determine optimal livestock rations for a quadratic polynomial response. He demonstrated that through this procedure rations are found more efficiently since fewer steps are required to reach the optimum ration.

His conclusions stem from comparing the results with Dent's based on the same source of data. He also shows how quadratic programming can be handled to find the optimum ration by maximizing average profit per period of time.

Objectives

The primary objectives of the study will be accomplished for a finishing ration. It is the practice of the broiler industry to change the ration once during the growing period so that most growth occurs with the finishing ration. Thus, the specific objectives are:

1. To determine an optimal finishing ration for broilers that produces the least feed cost per pound of broiler;

2. To estimate the effect of bird quality, as measured by carcass fat content, on least cost of production;

3. To estimate least cost of production per given period of time;

4. To compare the results from objectives 1, 2 and 3 with the results derived from linear programming feed formulation.

Methodology

Basic data used for estimation of response functions were collected from experiments (Chapter IV has a detailed description of these experiments). The experiments generated data on broiler growth at several levels of protein and metabolizable energy. Data on percentage of body fat were collected at the end of 6, 7 and 8 weeks. Also, time rate of consumption response was calculated from the experiment data. Based on these data, estimations of production, fat and time rate of consumption responses were performed by appropriate econometric models.

Data on linear programming formulation results, as well as data on broiler growth, technical feed efficiency, time elasped to reach a certain liveweight level and fat content of birds based on observed feed formulation, were provided by a leading broiler firm.

Given that the best response function for broilers was of the quadratic type, a quadratic programming model was chosen as the basic methodological procedure to provide results on broiler growth, time rate of feed consumption, fat content and cost to compare with the results found by the broiler firm.

Scope of the Thesis

The present chapter has described important problems in broiler feed formulation, what has been done in the area and what and how this thesis has proposed to investigate the problems.

In order to provide a better understanding of the methodology used in this research, the next chapter (Chapter II) presents the basic concepts of the theoretical framework of production economics. Chapter III discusses how broiler rations are formulated by linear and quadratic programming and what information can be drawn from the solution of each formulation. The content of Chapter IV is a description of three experiments that will be used in the analysis; two of which will serve for empirical estimations of the production, fat and consumption-time responses. The estimated equations are presented in the chapter. The results of linear and quadratic programming are discussed in Chapter V. It shows actual performance of broilers fed on rations derived from linear programming. These results are the referrential points for comparison with the quadratic programming solutions. Conclusions, implications and suggestions are presented in Chapter VI.

CHAPTER II[1/]

ECONOMIC THEORY OF PRODUCTION

Production economics theory provides the theoretical sturcture for determining an optimum marketable weight or least feed cost for livestock. Hence, this chapter will be concerned with a brief review of the main topics of production economics theory that will be used in the following analysis of broiler production.

Some Concepts

A production relationship is a process where physical inputs are converted into output. If there is a known technology this relationship can be expressed as a single valued continuous mathematical function such as;

$$Y=f(x_1, x_2, \ldots, x_n)$$

where,

$x_1, x_2, \ldots, x_n$ are physical inputs (fixed and variable),
Y is the maximum output attainable from the specified set of inputs through a technological process which is implicit in the function f.

[1/]Most of this chapter is based on Ferguson (18), Chiang (9) and Henderson and Quandt (26).

The technological process of a production function relates to the important concept of efficient production. Further, a distinction must be made between technical and economic efficiency. Initially, technical efficiency is considered and later in the analysis economic efficiency will be emphasized.

For simplicity, assume only one of the inputs is variable input, say x_1, all others being fixed. Assume also that there are two firms facing the same type of production function but with different technologies. The firms' production functions are:

$$Y_1=f_1(x) \qquad \text{firm 1}$$

$$Y_2=f_2(x) \qquad \text{firm 2}$$

f_1 and f_2 could be, for instance, quadratic functions. In this case, the explicit form of the production functions would be:

$$Y_1 = a_0 + a_1x + a_2x^2$$

$$Y_2 = b_0 + b_1x + b_2x^2$$

where,

a_0, a_1, a_2 and b_0, b_1, b_2 are the known technological coefficients of each firm's production function. If a_0, b_0, a_1, b_1 are positive and a_2, b_2 are negative coefficients, then the functions are plotted as shown in Figure 2-1.

From the following graph, it may be said that firm 2 is more technically efficient then firm 1. There are two simple ways to explain this. Suppose each firm is to

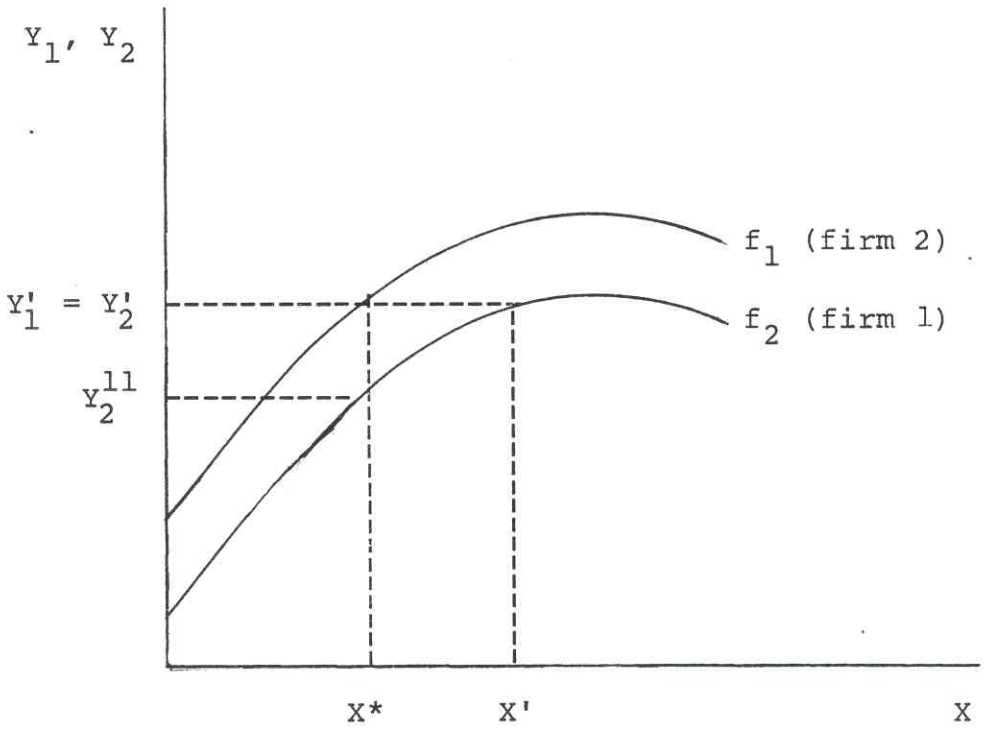

Figure 2-1. Single Input Quadratic Production Function for Two Firms.

use x* amount of the input x to produce Y. It is then clear that firm 2 can produce more output than firm 1 can since $Y_1' > Y_2''$. Similarly, if each firm is to produce $Y_1' = Y_2'$ amount of output, then firm 2 would require only x* amount of the input x, while firm 1 would need as much as x' of x to produce that same amount of output. Before discussing economic efficiency further definitions are required.

Whatever method is employed to find the best optimal solution for input usage, a particular point or level of production must be selected. So, given any point on a production function (or response surface), the marginal productivity of input at that point is an important descriptor of technology. Consider a production function given by, $Y=f(x_1, x_2, \ldots, x_n)$. The marginal product of the ith input (MP_i) is defined as the partial derivative of the production function with respect to the input x_i, i.e., $MP_{x_i} = \frac{\partial Y}{\partial x_i} = \frac{\partial}{\partial x_i} f_i(x_1, x_2, \ldots, x_n)$. As it can be in the right-most expression, the marginal product of any input is, in general, a function of all inputs in the production process which, incidently, is not obvious in the simple graph of Figure 2-1.

The average product of the ith input (APx_i) is the quantity of output per unit of the input used. Taking the production function stated above, then

$$AP_{x_i} = \frac{Y}{x_i} = \frac{f(x_1, x_2, \ldots, x_n)}{x_i}$$

Again, AP_{x_i} is expected to be a function of all inputs in f. In poultry production this concept is useful to define production. Obviously, the change in input levels affects marginal productivity of each input.

Another concept that helps the understanding of economic efficiency is the isocost curve. It is defined as the locus of all input combinations that exhaust a given total cost. When input prices are fixed, isocost curves are straight lines (two inputs case). Mathematically, an isocost line may be expressed as,

$$\bar{C} = r_1x_1 + r_2x_2$$

$$x_1 = \frac{-r_2x_2}{r_1} + \frac{\bar{C}}{r_1}$$

where,

r_1, r_2 are fixed prices of inputs X_1 and x_2 respectively

$\bar{C}$ is a given total cost.

The above equation represents a negatively sloped line where its slope is given by the negative of the input-price ratio, i.e.,

$$\frac{dx_1}{dx_2} = - \frac{r_2}{r_1}$$

A graphical representation of three different isocost lines is shown in Figure 2-2. Changes in an isocost line are only possible if either input price-ratio or given total cost or both vary. Assume initially that AB is an isocost defined from a given input price ratio (r_2/r_1) and a given

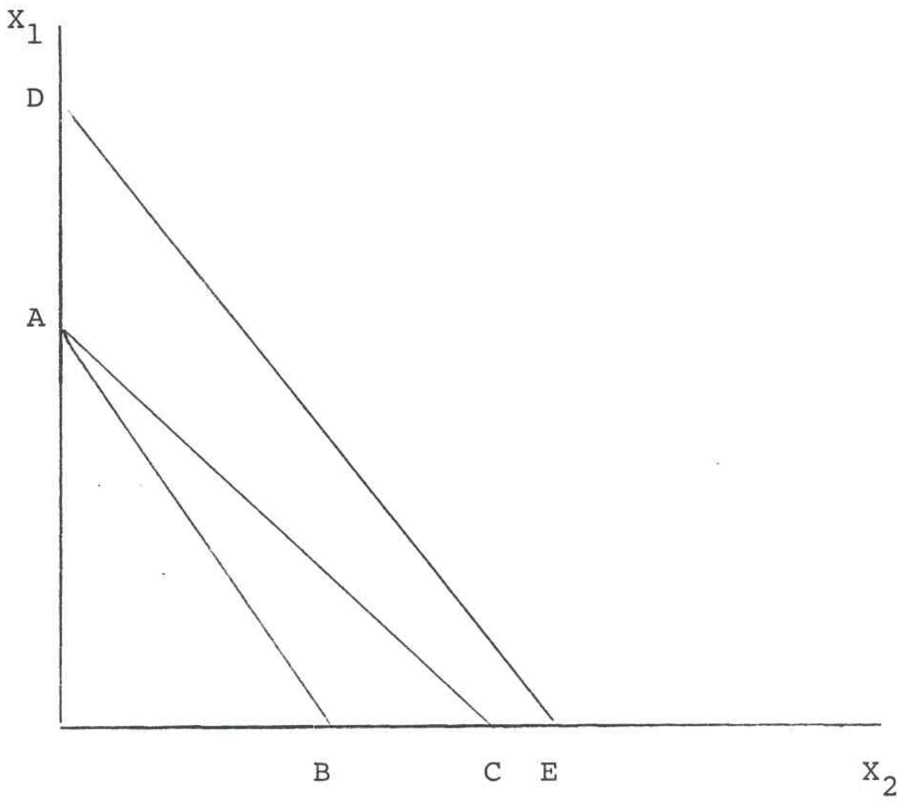

Figure 2-2. Representation of Isocost Lines.

total cost ($\bar{C}$). Suppose that only the price of x_2 decreases, then the new isocost line is defined by the line AC. It is worth noting that if prices of both inputs change propertionally, the slope of the isocost line is not affected, but it will shift to a new level in the input space.

Suppose now that instead of changing the price of an input, the given total cost increases to $\bar{C}'$. This will make the isocost line shift up to DE parallel to AB keeping the original slope. With isocost lines and isoquants thus defined, it is now appropriate to introduce the concept of economic efficiency between two firms. Assume that the two firms employ the same two inputs (x_1, x_2) to produce a homogeneous product (Y) through the same type of production function, but, marginal productivities of the production function are different.

Figure 2-3 shows isocost line and isoquants of two different production responses combined on the same graph, i.e., firm 1 and firm 2 isoquants are not on the same production surface in the same manner as shown in Figure 2-1. The two firms are faced initially with an isocost line AB. I_1 and I_2 represent isoquants of firm 1 and firm 2 respectively. In order to exhaust the same total cost, firm 1 could produce at a higher level of output if it equated $\frac{dx_1}{dx_2} = \frac{-r_2}{r_1}$. This condition is fulfilled by firm 2

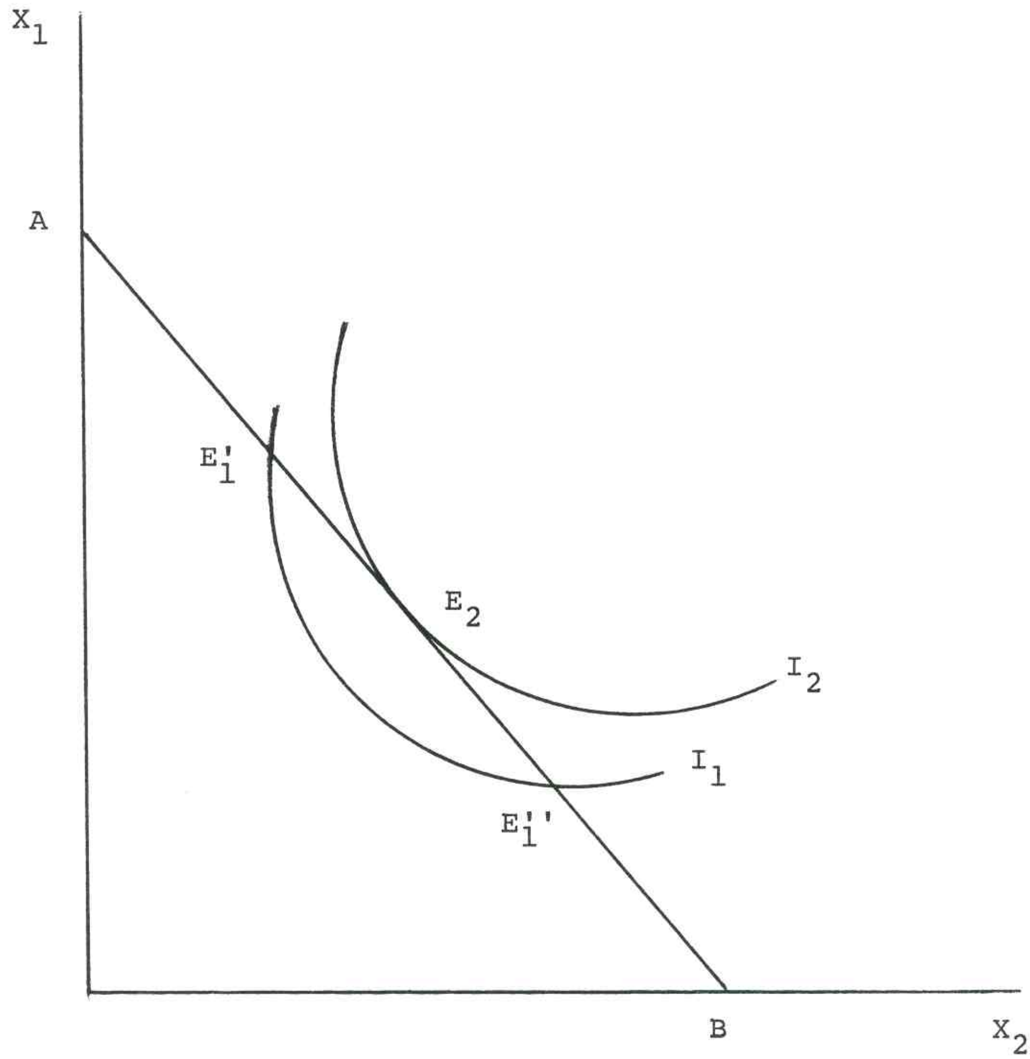

Figure 2-3. Isocost Line and Isoquants Measuring Economic Efficiency For Two Firms.

at point E_2. By producing either at point E_1' or at point E_1'', firm 1 is said to be less economic efficient than firm 2.

In sum, one criterion of economic efficiency is that each level of output is produced at the least possible cost i.e., marginal rate of substitution equal to the price ratio. Alternative techniques are available to find the least cost of production and usage of inputs and these techniques are discussed in the next section.

Alternative Techniques to Find Optimum Level of Production

To begin with, a simple case of one input production is investigated. For example, broiler weight as a function of total feed consumption only. It is plausible to assume a quadratic response for this case, since it is expected that feed input is converted into broiler output at a decreasing rate. The input output relationship could be represented by the following equation[1/]:

$$y = a_0 + a_1x + a_2x^2 \qquad (1)$$

where,

y is broiler liveweight, (kilogram)

x is feed consumption, (kilogram)

a_0, a_1, a_2 are the technological coefficients with the following conditions: $a_0, a_1 > 0$, $a_2 < 0$.

[1/] Using data from experiment three (See Chapter IV) the cubic specification was rejected. The coefficient of x^3 was near zero (.00038) with standard error of .0034.

Data from experiment three (See Chapter IV) were used to estimate equation (1), and the estimates were:

$$Y = .051 + .6334x - .0406x^2 \qquad (2)$$
$$\qquad\qquad (.0047)^* \quad (.0007)^*$$

$$R^2 = .99$$

Standard errors are in parentheses,

(*) Significant at 1%.

A graphical representation of equation (2) is depicted in Figure 2-4.

The hypothesized relationship between feed input and broiler output is that, the higher the feed consumption level, the lower the rate at which broiler weight increases, i.e., the marginal physical product of feed is decreasing throughout. However, total broiler weight should not decrease.

Assume a perfectly competitive broiler producer attempts to maximize profit. Let b and r be the price per pound of broiler and feed respectively. The profit function is then given by, $\Pi = b \cdot y - rx$ where the first order condition for a maximum is:

$$\frac{d\Pi}{dx} = b \cdot \frac{dy}{dx} - r = 0$$

or, $$\frac{dy}{dx} = \frac{r}{b} \qquad (3)$$

$$\frac{dy}{dx} = \frac{d(a_0 + a_1x + a_2x^2)}{dx}$$

[1/] The second order condition for a maximum is attained since $d^2\Pi/dx^2 = bd^2y/dx^2 = 2ba_2 \simeq -.0812b < 0$, given $b > 0$.

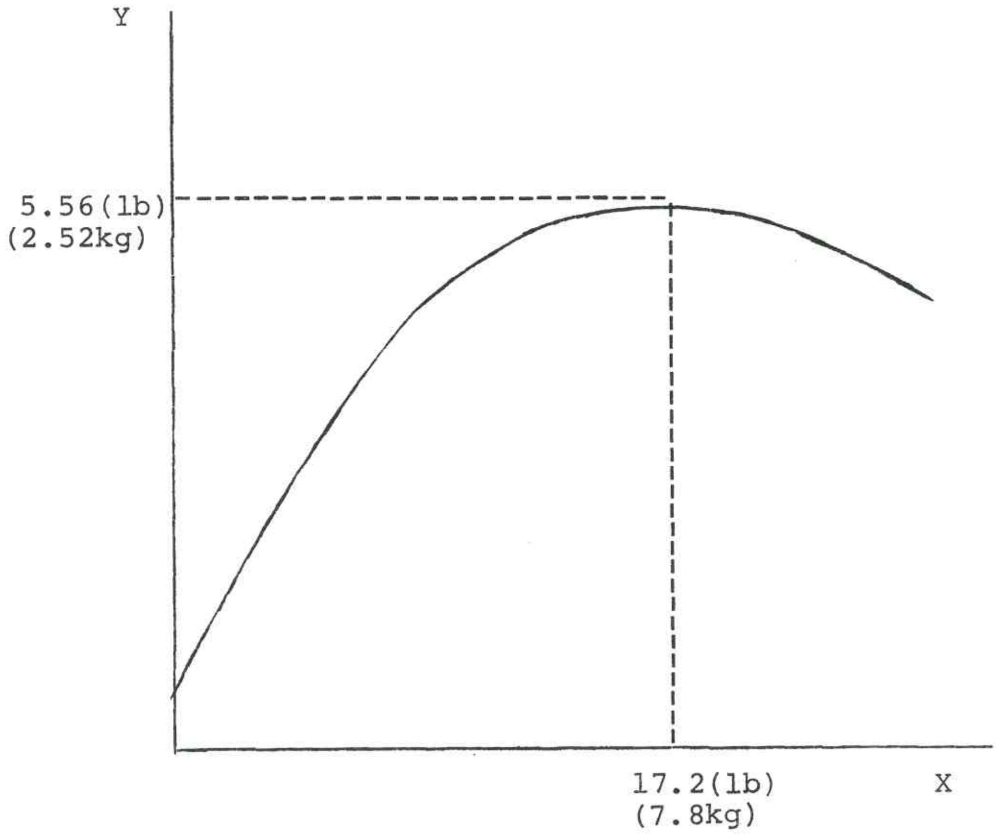

Figure 2-4. Estimated Quadratic Function: Broiler Weight--Feed Consumption.

$$= a_1 + 2a_2x \simeq .6334 - .0812x \tag{4}$$

By substituting (4) into (3) it comes,

$$(.6334 - .0812x^*) = \frac{r}{b} \tag{5}$$

It follows from (5) that,

$$x^* = 7.8 - r/.0812b. \tag{6}$$

By substituting x* into equation (2), an optimum selling weight of broiler, say $\hat{y}^*$, is determined. Note from (6) that the absolute maximum of production response can never be optimum.

The one input production function is therefore optimized at the point where the marginal product of feed input equals to the feed input--broiler output price ratio (equation (3)) according to economic theory.

For sake of this example, it was assumed that a common mix of feed is available in the market. While there are common feed mixes available each feed mix may give a different production response. This is because there are n-feed ingredients available that once combined form a unique mix of basic nutrients. A more fundamental approach is to find production response to nutrients in feed ingredients.

It might be worth pointing out that minimization of feed conversion (total feed intake per pound of broiler) may be misleading with respect to a correct economic decision. It is believed that the quadratic function is a good description of broiler response, then, the lower the broiler weight, the higher the feed conversion will be. On the other hand better feed conversion would be a correct principle if, for instance, total feed consumption is fixed at a certain level and a search for a heavier broiler is pursued. This latter procedure would simply imply improvements in technical efficiency as discussed earlier. Since increased technical efficiency always improves economic efficiency the additional steps in searching for economic efficiency are not always pursued. But, for any level of technical efficiency, economic efficiency can always be obtained through a best selection of feedstuffs available by considering their nutrient content, their prices their marginal productivity, and value of products they produce.

All empirical applications of this study will be derived from a two-input quadratic production function. Heady and Dillon (24), Brown and Arscott (6), the pioneers in using marginal analysis of production responses to crops and livestocks, define a typical two-inputs quadratic production function as: $W=f(P, E)$

$$W = f(P, E) = a_0 + a_1P + a_2E + a_3P^2 + a_4E^2 + a_5PE \qquad (1)$$

where,

W is output, say broiler liveweight, P and E are nutrient inputs, say protein intake and metabolizable energy intake respectively.

The linear and quadratic part of this equation would account for the diminishing marginal productivity of each input. Also, an interaction term (PE) appears in the equation to incorporate the effect of the marginal physical product of an input being a function of the level of the other input. The marginal product in broiler weight from a small increment in protein may depend on the level of energy that the broiler is consuming. That is:

$$(MPP)_p = f_p(P, E) = a_1 + 2a_3) + a_5E$$

$$(MPP)_e = f_e(P, E) = a_2 + 2a_4E + a_5P$$

The conditions for concavity of this quadratic function, as outlined in Appendix A, are given by

$$f_{pp} = 2a_3 < 0 \qquad (2)$$

$$f_{ee} = 2a_4 < 0 \qquad (3)$$

$$f_{pp} \cdot f_{ee} - f_{pe}^2 = 4a_3a_4 - a_5^2 > 0 \qquad (4)$$

Equations (2) and (3) imply that the coefficients a_3 and a_4 must be negative. From equation (4) no expectation can be infered concerning the sign of a_5, unless prior information is provided. Mathematically, it would depend on the magnitude of a_3 and a_4. For the case of a broiler response though, the expectation is to have a positive signal for a_5 for protein and energy within a feasible biological range. Given the expectations, it is also reasonable to expect that a_1 and a_2 are positive for the expectation of positive marginal productivities.

By fixing output level at, say W_0, and rearranging equation (1), the isoquant equation for W_0 is derived,

$$a_3P^2 + (a_1 + a_5E)P + (a_2E + a_4E^2 - W_0 + a_0) = 0 \qquad (4)$$

Equation (4) is a simple quadratic equation in P. By solving (4):

$$P = \frac{-(a_1 + a_5E) \pm \left[a_1 + a_5E)^2 - 4a_3(a_2E + a_4E^2 - W_0 + a_0)\right]^{1/2}}{2a_3} \qquad (5)$$

An immediate characteristic of this isoquant is that it intersects the axes. As a proof, let E = 0. Then,

$$P = \frac{\left[a_1 \pm a_1^2 + 4a_3(W_0 + a_0)\right]^{1/2}}{2a_3} \neq 0$$

To reemphasize, the isoquant in equation (5) is convex to the origin as it is derived from a strict concave production function. This means that a point of economic efficiency can be found on the isoquant for a given total feed cost. Since there are a number of possible levels of feed cost, and investigation is required on the nature of the expansion path for the quadratic function.

As proved in the last section, economic efficiency occurs at the point where the marginal rate of technical substitution equals to the input price ratio, i.e.,

$$(MRTS)_{PE} = \frac{(MPP)_P}{(MPP)_E} = \frac{r_p}{r_e} \qquad (6)$$

where r_p and r_e are the prices of protein and metabolizable energy respectively. By substituting the marginal products of P and E into (6), it follows that,

$$\frac{a_1 + 2a_3P + a_5E}{a_2 + a_5P + 2a_4E} = \frac{r_p}{r_e} \tag{7}$$

Solving (7) for P:

$$P = \frac{(a_2r_p - a_1r_e)}{(2a_3r_e - a_5r_p)} + \frac{(2a_4r_p - a_5r_e)}{(2a_3r_e - a_5r_p)} \cdot E$$

or simply,

$$P = K_1 + K_2E \tag{8}$$

Equation (8) shows all combinations of P and E to achieve economic efficiency for any given level of total fixed feed cost. In other words, the expansion path from a quadratic production function is a positive sloped straight line not passing through the origin ($K_1 \neq 0$, $K_2 > 0$)[1/]. This indicates that, in fact, there must be a trade off between protein and energy as the birds get heavier (higher isoquants). It is worthwhile noting that the common practice of linear programming technique for feed formulating for broilers assumes fixed protein-energy ratios, regardless of the weight which the birds will be slaughted.

Finding market prices for protein intake (r_p) and metabolizable energy (r_e) is a difficult, if not impossible, task. This reduces the precision of using such an analysis straight forwardly. The model needs then to be transformed. It is suitable to write the quadratic

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[1/] As $a_5 > 0$, $a_3 < 0$ and $a_4 > 0$, then K_2 must be positive.

equation (1) as a function of all available feedstuffs that may be used to provide protein and metabolizable energy for broilers.

In a matrix format, the quadratic form of equation (1) becomes,

$$W' = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} + \begin{bmatrix} P & E \end{bmatrix} \begin{bmatrix} a_3 & a_5/2 \\ a_5/2 & a_4 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} \qquad (9)$$

or: $A_1 P + P' A_2 P$

The first expression on the right side consists of the linear part and the second expression is the quadratic, including interaction term, from equation (1)[1/].

The transformation from the nutrient space (P, E) into the feed ingredient space is made through the coefficients (content) of protein and energy in each feed input. Assuming that these coefficients are fixed, the $[P\ E]'$ vector can be expressed as function of the n available feeds and equation (9) is written out as[2/],

$$W' = \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} \sum_{i=1}^{n} c_{ij} x_i x_j \qquad (10)$$

[1/] W' includes intercept term a_0 of equation (1), or, if Liveweight = W then $W' = W + a_0$.

[2/] See Chapter III and Appendix D for details on this transformation.

The production function is then expressed as a function of the n-feeds. Since the price of all x_i's are well defined, equation (1) can be analyzed to find an exact point of economic efficiency.

When more than two inputs (n-feeds) are used in the production process, graphical analysis to find optimal combination of inputs is of no help. However, the problem can be solved by means of the lagrange technique. Then mathematical problem is formulated as follows:

Maximize production $W=f(x_1, x_2, \ldots, x_n)$ (10a)

Subject to a given total feed cost, $\bar{C} = \sum_{i=1}^{n} r_i x_i$ (10b)

The lagrange function is:

$$L = f(x_1, x_2, \ldots, x_n) + \lambda(\bar{C} - \sum_{i=1}^{n} r_i x_i)$$

The first order conditions for an optimal are[1/]

$$\frac{\partial L}{\partial x_i} = f_i - \lambda r_i = 0 \qquad (i=1, 2, \ldots, n) \tag{11}$$

$$\frac{\partial L}{\partial \lambda} = \bar{C} - \sum_{i=1}^{n} r_i x_i = 0 \tag{12}$$

Equations (11) and (12) embody implicity (n + 1) equations to solve for the (n + 1) unknowns $(x_1, \ldots, x_n, \lambda)$ terms of the (n + 1) parameters $(r_1, r_2, \ldots, r_n, \bar{C})$.

For any two feed inputs, say x_i and x_j, equation (11) allows the following relationship: $\frac{f_i}{f_j} = \frac{r_i}{r_j}$

[1/] The second order conditions are outlined in Appendix A.

which defines the optimal point of production where the isocost line is tangent to the isoquant in the (x_i, x_j) space. Extending this relationship to the n feed inputs:

$$\frac{f_i}{f_j} = \frac{r_i}{r_j} = \ldots\ldots = \frac{f_n}{f_n} = \frac{r_{n-1}}{r_n} = \frac{1}{\lambda}$$

or equivalently,

$$\frac{f_1}{r_1} = \frac{f_2}{r_2} = \ldots\ldots = \frac{f_{n-1}}{r_{n-1}} = \frac{f_n}{r_n} = \lambda \qquad (13)$$

Equalities in (13) say that the optimal input combinations occurs at the production level where the marginal productivity per dollar spent on input i (f_i/r_i) is the same for all inputs.

In other words, the above approach is formulated to answer the question: If a broiler producing firm has available only $\bar{C}$ dollars to spend on feed/broiler, what is the best combination of feed inputs to formulate a ration and what is the expected optimum broiler liveweight. A different variation of the problem occurs when the broiler grower is imposed, by market demand, to produce an "exact" W_0 pounds broiler.

The prior discussion brings up an important theorem in economic theory, namely the duality theorem which may be stated for this problem as: The principle of optimal input combination is obtained whether through maximizing output (broiler weight) for a given total resources (feed inputs) cost or minimizing the total resource cost of producing a given level of output. The second part of the theorem is formulated as:

$$\text{Minimize } C \sum_{i=1}^{n} r_i x_i \tag{14}$$

$$\text{Subject to } W_0 = f(x_1, x_2, \ldots, x_n). \tag{14a}$$

The lagrange function is:

$$L = \sum_{i=1}^{n} r_i x_i + \lambda^*(W_0 - f(x_1, x_2, \ldots, x_n))$$

where the first order conditions are:

$$\frac{\partial L}{\partial x_i} = r_i - \lambda^* f_i = 0 \qquad (i=1, 2, \ldots, n) \tag{14b}$$

$$\frac{\partial L}{\partial \lambda} = W_0 - f(x_1, x_2, \ldots, x_n) = 0.$$

From the equations in (14), it follows,

$$\frac{f_1}{r_1} = \frac{f_2}{r_2} = \ldots = \frac{f_{n-1}}{r_{n-1}} = \frac{f_n}{r} = \frac{1}{\lambda^*} \tag{15}$$

which is exactly the same as expression (13), where,

$$\lambda = \frac{1}{\lambda^*}$$

The duality theorem is then proved. As specification of the problem via equations 10a and 10b yield the same solution as solving 14 and 14a. Would it matter if the price of output (broiler) is not invoked in either approach of the duality theorem? To answer this question, a third approach, namely the profit maximization approach may be used.

A profit function can be set up as follows:

$$\Pi = p.w - \sum_{i=1}^{n} r_i x_i \tag{16}$$

Substituting w in (16),

$$\Pi = p.f(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} r_i x_i$$

where p is a constant price of output, so $p.f(x_1, x_2, \ldots, x_n)$ and $\sum_{i=1}^{n} r_i x_i$ define total revenue and total cost,

The first order condition for an optimum is,

$$\frac{\partial \Pi}{\partial x_i} = p.f_i - r_i = 0$$ [1]

$$p = \frac{r_i}{f_i} \tag{17}$$

The cost minimization first order conditions equations permit expression of x_i optimal solution as a function of the input prices (r_i's) and output (w). Since r_i's are fixed it can be written,

$$\sum_{i=1}^{n} r_i x_i = g(w)$$

And a profit function in terms of output is equivalently set up as,

$$\Pi = p.w - g(w)$$

First order condition,

$$\frac{\partial \Pi}{\partial w} = p - \frac{dg(w)}{dw} = 0 \tag{18}$$

---

[1] Second order condition is satisfied since $\partial^2\Pi/\partial x_i^2 = f_{ii} < 0$ as shown in Appendix A.

But, by definition $\frac{dg(w)}{dw}$ = marginal cost (MC)

Then, (18) becomes,

$$p = MC \tag{19}$$

Recalling equation (14) in the cost minimization approach it was found that,

$$\lambda^* = \frac{r_i}{f_i} \tag{20}$$

It is claimed that λ^* equals marginal cost[1/], i.e.,

$$\lambda^* = MC$$

Therefore, from (17), (19) and (20),

$$p = MC = \frac{r_i}{f_i} = \lambda^*$$

This result leads to the conclusions that cost minimization and profit maximization are equivalents to find optimal usage of inputs when output price is a constant. And by the duality theorem, the overall conclusion is that output maximization, cost minimization and profit maximization are equivalent approaches to determine the optimal combination of inputs.

[1/]Total cost of optimum level of inputs, $C = r_i x_i^*$ and $dC/dx_i = r_i$.

Marginal cost is defined as, $MC = \frac{dC}{dw} \cdot \frac{dx_i^*}{dx_i^*} = \frac{dC_i/dx_i^*}{dw/dx_i^*}$.

Numerator is just r_i. Denominator is marginal physical product of x_i, which is f_i^*. Hence, $MC = \frac{r_i}{f_i^*}$.

The decision of which approach is to be used in an applied real world problem is based on the suitability of the approach in being incorporated into a mathematical programming technique if inequality restraints are added to the optimization problem.

CHAPTER III

FEED FORMULATION

Feed Formulation by Linear Programming

Linear programming has been a practical and effective technique for solving some optimization problems. Its application is in optimizing (minimizing or maximizing) a linear mathematical function (objective function) subject to a set of linear inequality restraints. A typical linear programming minimization problem can be formulated as:

$$\text{Minimize} \quad \sum_{i=1}^{n} c_i x_i \qquad (i=1, 2, \ldots, n)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} a_{ij} x_i \gtrless b_j \qquad (j=1, 2, \ldots, m).$$

In feed formulation for livestock, linear programming is used to minimize feed cost subject to a set of nutrients requirements where c_i in the objective function of the above formulation is the price per unit of the i^{th} feed (x_i); a_{ij} is the feed nutrient coefficient, that is, the amount of j^{th} nutrient in i^{th} feed; b_j is the requirement of j^{th} nutrient.

Linear programming is an effective method to find the least possible cost that satisfies the set of all nutrient requirements and to select proportionaly the feeds to be mixed in a ration. However, it is also true that the linear programming technique implicity assumes constant marginal products and constant returns to scale for the production response which can hardly be true as demonstrated earlier. That is, linear programming does not allow for trade offs, for instance, between protein and energy, the main nutrients in a broiler ration.

The broiler industry, in general, has available microcomputers with linear programming packages that are used to formulate rations. Feed formulations are processed weekly depending on availability of feeds and their price variation.

One linear programming least cost feed formulation for a finishing ration used by the broiler industry is as follows. Appendix C presents prices of the feed ingredients for two different periods of time, each of which make up the objective function to be minimized. The feed ingredients available are also shown in Appendix C. The right hand side (b_j) of each constraint corresponding to nutrients (1-12) and feed ingredient (13-19) are shown in Table 3-1.

The feed mix formulation in Table 3-1, made on April 12, 1982, provided a cost/ton of $179.63 ($.0898/lb.). A formulation on May 31, 1982 had exactly the same minimum constraints and available feed ingredients. However, feed prices changed slightly resulting in

Table 3-1. The Linear Programming for Finishing Ration Used by the Broiler Firm on April 12, 1982

Nutrients and Feed Ingredients	Unit	Right Hand Side (b_j) Minimum($\geq$)	Maximum($\leq$)	Actual
1. Protein	%	21.7		21.343
2. Met. Energy	Kcal./lb	1480		1480
3. Crude Fat	%	4.1		7.904
4. Crude Fiber	%		3.9	2.505
5. Calcium	%	.8	.9	.8
6. Available Phosphorus	%	.4		.4
7. Sodium	%	.18	.23	.18
8. Lysine	%	1.04		1.04
9. Methionine	%	.42		.479
10. Methionine & Cystine	%	.83		.83
11. Choline	Mg/lb	708.0		708
12. Xanthophyl	Mg/lb	6.0	14.6	6.236
13. Animal Fat	%	2.0	6.0	2.469
14. Meat & Bone 50	%		6.0	6
15. Po Tank 50 Soy	%	14.75	15.0	14.75
16. TM Mix 430	%	.05	.05	.05
17. Broiler Vit. Mix	%	.05	.05	.05
18. LSS Solubles	%	.75	.75	.75
19. Fixed	%	.375	.375	.375

different actual values on the right hand side of some constraints, such as: protein (21.7), crude fat (7.697) and xanthophyl (6.321). The feed cost also changed slightly to \$178.75/ton (\$.0899/lb.).

Feed Formulation by Quadratic Programming

As shown earlier, an optimum solution to the problem of least cost production must lie on the expansion path of the production function (say quadratic) which is defined as the locus of all points with:

$$-\frac{dP}{dE} = \frac{r_e}{r_p}$$

where,

-dP/dE is the marginal rate of technical substitution between protein and metabolizable energy;

r_e is the constant price per unit of metabolizable energy;

r_p is the constant price per unit of protein.

This condition and two optimum solutions are characterized by the points A and B in Figure 3-1. This figure, depicited in the protein (P) and metabolizable energy (E) space, shows two levels of pre-fixed total feed cost, that for prices r_p and r_e are all combinations of P and E on the lines F_1F_1' and F_2F_2' respectively. Two levels of broiler liveweight on the production surface are represented by the isoquant $\overline{W}_1$, and $\overline{W}_2$. The JJ' line is the expansion path. As indicated in Chapter II, an optimum solution, say point A, can be met either by minimizing the

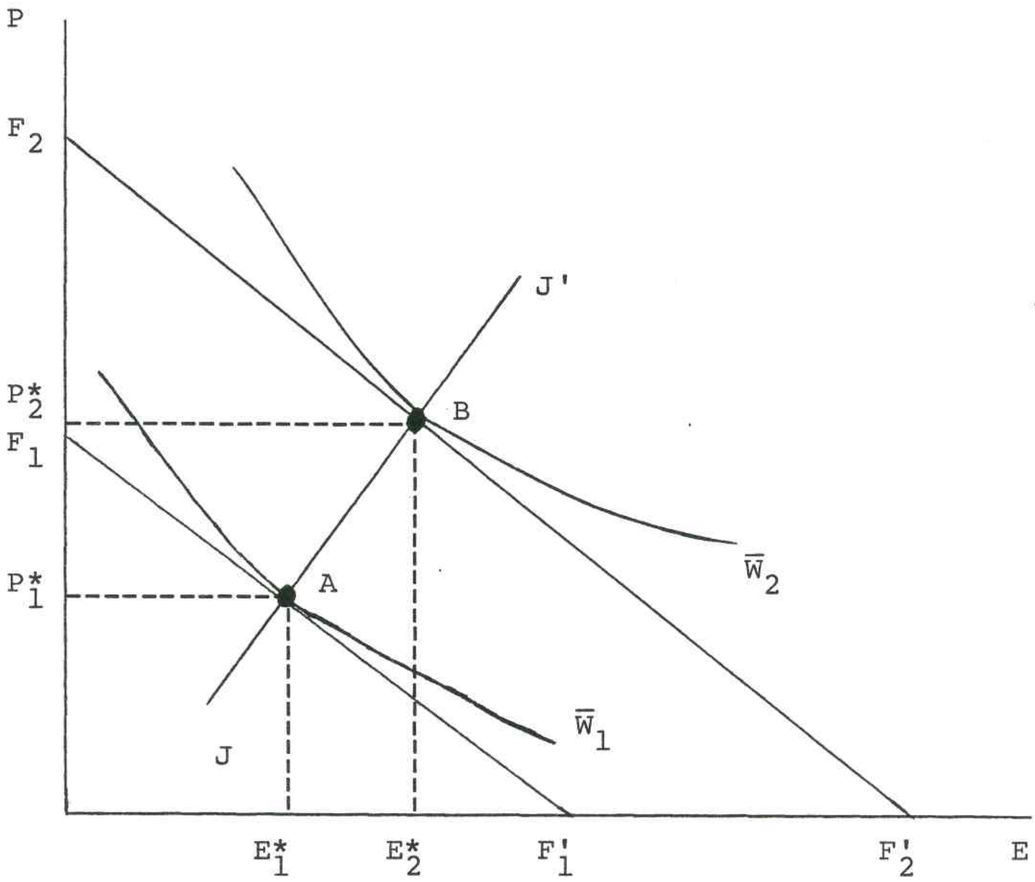

Figure 3-1. Representation of Two Least Cost Solutions on the Expansion Path for Broiler Production Using Protein (P) and Energy (E).

total feed cost,

$$Z_1 = r_p P + r_e E$$

Subject to a fixed level of bird weight ($\overline{W}_1$),

$$f(P, E) = \overline{W}_1$$

or by maximizing bird weight,

$$W_1 = f(P, E)$$

Subject to a fixed level of total feed cost ($\overline{Z}_i$),

$$r_p P + r_e E = \overline{Z}_1$$

Note that in either economic model, application of a simple lagrange technique would lead to an optimum solution at point A or point B in Figure 3-1. However, two problems arise. First, inequality restrictions have to be added to the formulation in order to satisfy both nutrients and feed ingredients requirements that may be necessary for nutritional balance. This means that an extreme optimum solution, say point A, may not be attained as inequality restrictions are inserted into the world. Hence, a mathematical programming technique needs to be employed to search for a solution at or as close as possible to point A. In a mathematical programming framework, the first model described above would be difficult for application as the production response is not linear resulting in non-linear constraints. Second, both outlined economic models are intractable for direct application in the sense that prices of the nutrients, protein and metabolizable energy, are not available. Therefore, a suitable transformation from the nutrient space to the feed ingredient space is required. Consider the set of n available feed ingredients defined by the vector.

$$x' = (x_1, x_2, \ldots, x_n).$$

Define two n-dimensional vectors of technical coefficients as,

$$M_p' = (M_{p1}, M_{p2}, \ldots, M_{pn})$$

$$M_e' = (M_{e1}, M_{e2}, \ldots, M_{en})$$

where,

M_{pj}, j = 1, 2,..., n, is the provision of protein per unit of the j^{th} feed ingredient.

M_{ej}, j = 1, 2,..., n, is the provision of metabolizable energy per unit of the j^{th} feed ingredient.

Then, the basic nutrients, protein (P) and metabolizable energy (E), can be expressed in terms of the n feed ingredients by the relations,

$$P = M_p' x$$

$$E = M_e' x$$

or, in matrix notation,

$$\begin{bmatrix} P \\ E \end{bmatrix} = \begin{bmatrix} M_p' \\ M_e' \end{bmatrix} x$$

Taking the transpose of both sides,

$$\begin{bmatrix} P & E \end{bmatrix} = x' \begin{bmatrix} M_p & M_e \end{bmatrix}$$

So, any mathematical relation defined in the nutrient space can now be transposed to the feed ingredient space.

The two models that lead to the same optimum levels of protein and metabolizable energy on the expansion path of the production function (P_1^* and E_1^* at point A in Figure 3-1) take into account a quadratic response specified as,

$$W = b_0 + b_1P + b_2E + b_3P^2 + b_4E^2 + b_5PE$$

or,

$$W^* = W - b_0 = b_1P + b_2E + b_3P^2 + b_4E^2 + b_5PE$$

Using definition of a quadratic form, the production function can be rewritten in matrix notation as,

$$W^* = \begin{bmatrix} b_1 b_2 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} + \begin{bmatrix} P & E \end{bmatrix} \begin{bmatrix} b_3 & 1/2b_5 \\ 1/2b_4 & b_4 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix}$$

Substituting the vectors $\begin{bmatrix} P \\ E \end{bmatrix}$ and $\begin{bmatrix} P & E \end{bmatrix}$ by the relations found above, it follows,

$$W^* = \begin{bmatrix} b_1 b_2 \end{bmatrix} \begin{bmatrix} M_p' \\ M_e' \end{bmatrix} x + x' \begin{bmatrix} M_p M_e \end{bmatrix} \begin{bmatrix} b_3 & 1/2b_5 \\ 1/2b_5 & b_4 \end{bmatrix} \begin{bmatrix} M_p' \\ M_e' \end{bmatrix} x$$

or simply,

$$W^* = f(x) = d'x + x'Dx.$$

A quadratic programming model to maximize broiler weight for a given feed cost can now be defined as:

Maximize: $W^* = d'x + x'Dx$

Subject to: $C'x = \overline{Z}$

$Ax \leq b$

$x \geq 0$

where,

x: an n x 1 vector (n is number of feed ingredients available);

d': an 1 x n vector;

D: an n x n matrix

C: an 1 x n vector of the prices of the n feed ingredients;

$\overline{Z}$: a given fixed level of total feed cost;

A: an n x n matrix of the nutrients coefficients. An element of A, say a_{ij}, specifies the amount of nutrient $i(i = 1, 2, \ldots, n)$ per unit of feed ingredient $j(j = 1, 2, \ldots, n)$;

b: an n x 1 vector of minimum or maximum amount of each n nutrients to achieve nutritional balance.

Minimum and maximum values for feed ingredients are also included here as needed for nutrition.

To meet one of the goals of this study, a quadratic programming model (Appendix D) was applied to two different right hand sides (b vector). It is significant to note at this point that the b vector, c vector and A matrix may be the same values specified for linear programming models currently used throughout the feed industry. The model can be explained to industry as constraining the LP problem to alternative levels of feed cost per bird, $\overline{Z}$, in order to produce the largest possible bird, W*, for that cost. Thus, industry constraints used by a major firm in the broiler industry were applied as the vector described in Table 3-1. Also, a set of biological constraints was specified by Dr. Gene Pesti of the Poultry Department at the University of Georgia. The biological constraints vector is presented in Table 3-2. Table 3-2 contains only constraints that differ from those in Table 3-1. So, the constraints left out in Table 3-2 are exactly the same as those in Table 3-1. The levels of protein and metabolizable energy in Table 3-2 define the range of the experimental data generated by the feeding experiment constructed for this study. The data for lysine, methionine and methionine and lysine are referred to as a percentage relative to the protein level in the ration, as opposed to those similar constraints in Table 3-1 where the percentages for the same nutrients refer to percentage in the ration.

Finally an important contrast between linear and quadratic programming formulation can be made. A characteristic of the linear programming technique is that it tends to provide optimum solutions around the lower bound of the nutrient and feed ingredients constraints (See Table 3-1). Under this technique, broilers withdrawn at different time periods, consequently at different liveweights, have been grown on the same ration. For instance, the data in Table 1-5 show a range of observed average broiler liveweights from 3.65lbs. to 4.05lbs. for different periods of time where broilers were fed on the same ration. On the other hand, the quadratic programming technique can predict not only the final broiler liveweight but also

Table 3-2. Critical Biological Constraints for a Broiler Finishing Ration that May Differ From Industry Constraints Used in Linear Programming

Nutrients and Feed Ingredients	Unit	Right Hand Side (b_j) Minimum (≥)	Maximum (≤)
1. Protein	%	17.5	22.0
2. Met. Energy	Kcal/lb.	1315	1542
8. Lysine	%	4.35	4.35
9. Methionine	%	2.17	2.17
10. Methionine and Cystine	%	4.04	4.04
13. Animal Fat	%	1.0	6.0
15. Po Tank 50 Soy	%		15.0

it can adjust the nutritional requirements as heavier broilers are produced along, or as close as possible, to points on the expansion path. Along the expansion path for increased weight, the changing marginal productivity of nutrients the level of feed prices are the important criteria for determining the least cost per pound of broiler produced. In linear programming, only the feed prices and the right hand side are important. Marginal productivity of feed is never considered by linear programming as a factor that cause the nutritional requirements in the right hand sides to vary according to the weight of bird that is desired or according to changing prices of feed ingredients. A significant benefit of the quadratic programming model is that the model determines the optimum levels of P and E that are consistent with feed prices and the least cost bird (Chapter II).

CHAPTER IV

DATA AND ESTIMATIONS

Data for the research were generated through two experiments, called experiment two and three, conducted by Dr. Gene Pesti in the department of Poultry Science at the University of Georgia.

Experiment Two

The objective of the experiment was to characterize the response of male broiler chickens to diets of various protein and energy levels. Two thousand and sixteen central Soya (Peterson x Hubbard) feather sexed day-old chicks were used in the experiment (from a commercial hatchery). These chicks were randomly assigned to 48 pens with 42 chicks per pen. The birds were fed *ad libitum* with nine different diets made up of five protein levels (17.5, 18.6, 19.8, 20.9 and 22.0%) and five metabolizable energy (kcal/lb) levels (1315, 1374, 1429, 1488 and 1542) (Figure 4-1). These levels of nutrients were derived from the composition of representative basal diets which is in Table 4-1. The experiment was designed so that there were five replicates on each ration. Observations on average pen weights, average pen feed consumption and average

Table 4-1. Composition of Representative Basal Diets

Ingredients	Test Diets I	II	III	IV
	Percentage of Diet			
Ground yellow corn	61.63	61.92	71.53	54.85
Poultry by-product meal	5.00	5.00	5.00	5.00
Soybean meal, dehulled	28.02	23.36	16.05	19.31
Wheat middlings	-	-	2.70	16.20
Poultry oil	2.68	7.01	1.50	1.50
Dicalcium phosphate	-	-	1.57	1.60
Defluorinated phosphate	1.49	1.65	-	-
Limestone	.49	.39	.95	.88
DL-Methionine	.09	.08	.05	.07
L-lysine (98%)	-	-	.04	-
Mineral mix[a]	.05	.05	.05	.05
Vitamin mix[b]	.25	.25	.25	.25
Salt	.30	.30	.30	.30

Calculated Contents (NRC, 1977)

	I	II	III	IV	NRC, 1977 Recommendations[c]
Metabolizable energy (Kcal/g)	3.15	3.40	3.15	2.90	3.20
Protein (%)	22.0	19.8	17.5	19.8	20.0
Lysine (%)	1.18	1.03	.88	.99	1.00
Methionine (%)	.46	.42	.37	.40	.38
TSAA[d] (%)	.78	.71	.64	.70	.72
Lysine (% of protein)	5.4	5.2	5.0	5.0	5.0
Methionine (% of protein)	2.1	2.1	2.1	2.0	1.9
TSAA (% of protein)	3.6	3.6	3.7	3.6	3.6

Analyzed Contents

	I	II	III	IV
Crude protein, crumbles	21.4	20.7	17.3	19.0
Crude protein, pellets	22.9	20.0	18.2	18.6
Metabolizable energy				

[a]Vitamin premix provides (per kg/diet): vitamin A, 11,000 IU; vit. D_3, 1100 ICU; vit. E, 11 IU; riboflavin, 4.4 mg; Ca pantothenate, 12 mg; nicotinic acid, 44 mg; choline Cl, 220 mg; vit. B_{12}, 6.6 mcg; vit. B_6, 2.2 mg; menadione, 1.1 mg (as MSBC); folic acid, 0.55 mg; d-biotin, 0.11 mg; thiamine, 2.2 mg (as thiamine mononitrate); ethoxyquin, 125 mg.

[b]Trace mineral mix provides (ppm of diet): Mn, 60; Zn, 50; Fe, 30; Cu, 5; I, 1.05, Ca, 75 (min.) and 90 (max.).

[c]For 3-6 week old broilers

[d]Total sulfur-containing amino acids

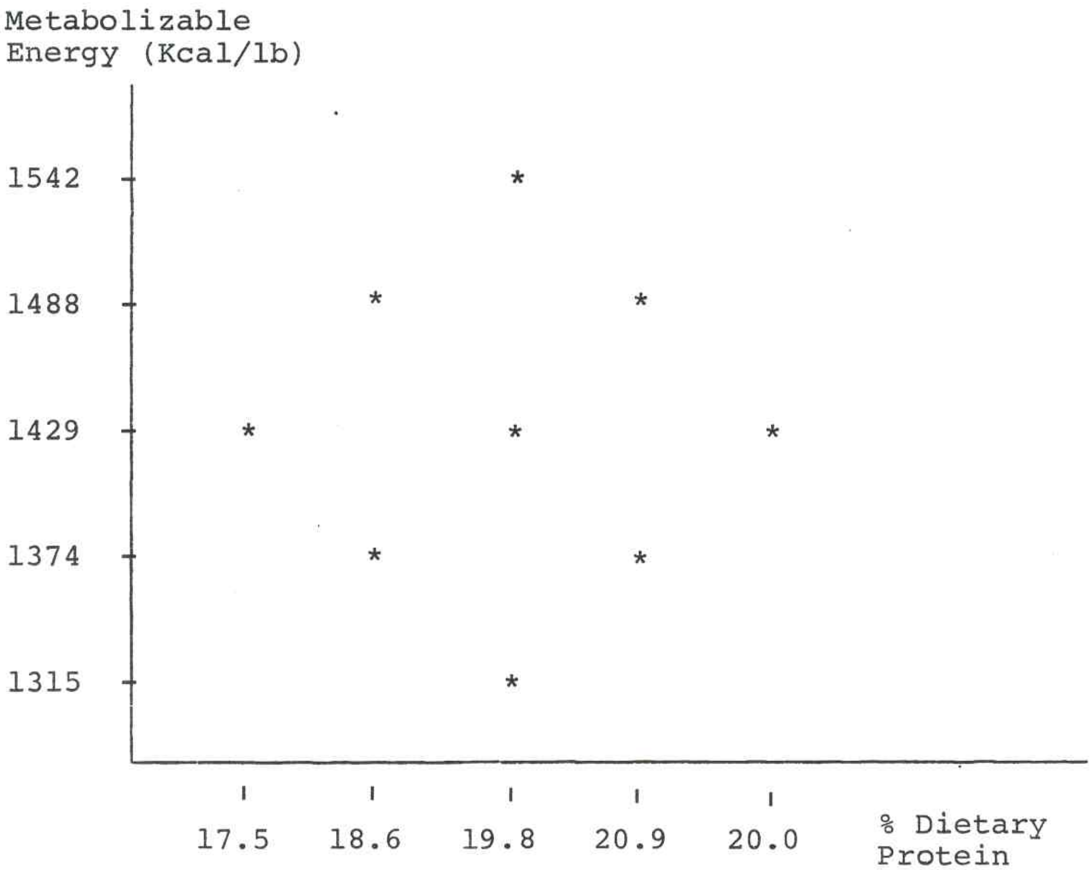

Figure 4-1. Nine Diet Combinations of Protein and Metabolizable Energy Used in Experiment Two.

percentage of fat in the carcass were recorded at the end of 5, 6, 7 and 8 weeks.

Experiment Three

This experiment had the same objectives, feeding characteristics and allocation of chickens per pen of experiment two. However, as specified by the author, there was a slight difference from experiment two in the protein and metabolizable energy levels. Two extreme diets were added to the design (Figures 4-2) in an attempt to generate substitution effects of protein and energy over a wider range than the usual experiment. The composition of the basal diets were similar to experiment two.

Also, to define consumption data, observations on average pen weights and average pen feed consumption were recorded half-weekly from 3 to 8 weeks. Percentage of carcass fat was observed at the end of 6, 7 and 8 weeks as a means of adding a quality dimension to the data base.

Data regarding available feed ingredients in broiler operations and their prices for selected periods were collected from a major broiler producing firm. Also, data on average broiler liveweights withdrawn from feeding at different periods of time were collected from broiler producers that utilized the ration formulation of the firm referenced above.

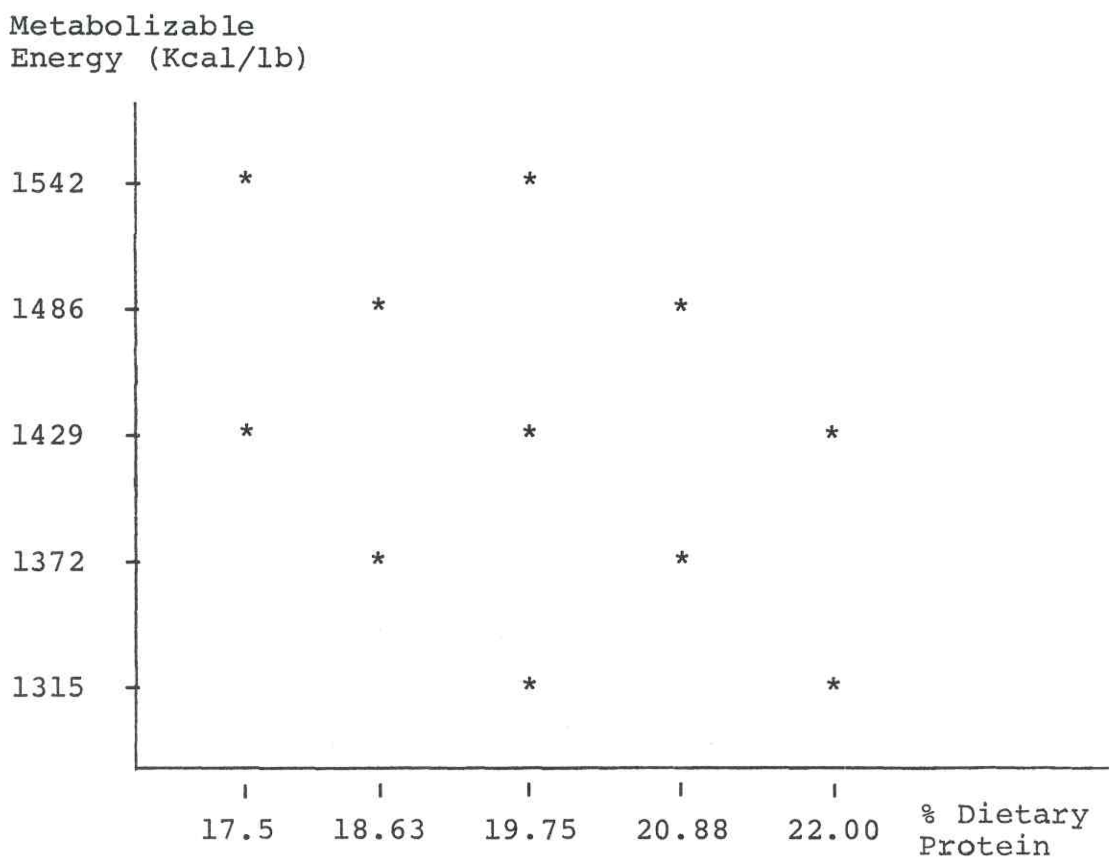

Figure 4-2. Eleven Diet Combinations of Protein and Metabolizable Energy Used in Experiment Three.

An important difference between these two experiments is that experiment two was conducted in the summer while experiment three was conducted in spring.

The design of all experiments were of the central composite type (13). Choice of this type of design seemed to be appropriated since extreme combinations of protein and metabolizable energy may not fit the nutritional need of chickens (37). Furthermore, the design provides a balance between the number of replications per treatment and number of treatments and factors (13).

Empirical Estimates of Production Response, Fat Response and Time of Consumption Response

Production Response

In order to accomplish the objectives through application of quadratic programming, empirical estimates of quadratic response were obtained from both experiment two and experiment three. Based on the available data from these experiments, the general form of the production response was:

$$W_{rst} = f(P_{rst}, E_{rst})$$

where,

W_{rst}: Cumulative average of liveweight per broiler (kilogram) of the r^{th} pen fed on the s^{th} ration at the end of the t^{th} week.

P_{rst}: Cumulative average of crude protein intake (kilogram) per broiler of the r^{th} pen fed on the s^{th} ration at the end of the t^{th} week.

E_{rst}: Cumulative average of metabolizable energy intake (kilogram) per broiler of the r^{th} pen fed on the s^{th} ration at the end of the t^{th} week.

The explicit specification of the production response was, $W_{rst} = a_0 + a_1P_{rst} + a_2E_{rst} + a_3P^2_{rst} + a_4E^2_{rst} + a_5P_{rst} \cdot E_{rst} + e_{rst}$. According to previous applications of this type of response to livestock (6, 9, 10, 22), specially to broilers, it is expected to have a_0, a_1 and a_2 positive, a_3 and a_4 negative and a_5 positive or negative. It is also expected, based on other studies that the estimate of a_i (i=0, 1,..., 5) should be significant at most at the 5% significant level and the coefficient of determination (R^2) should be around .98.

From the way the experiments were conducted and data were collected, time was the only fixed variable. All other variables (feed inputs and output) are, therefore, stochastic. This happens because each average pen was the experimental unit and each pean was observed over time at pre-fixed time periods. Hence, two estimation problems might arise. The first problem refers to autocorrelated errors. As Dillon (13, pp. 161-2) points out,

> "Response experiments often involve repeated observations on the same experimental unit; for example, weekly readings of the liveweight of animals in a group, multiple cuttings of hay from a plot, crop rotation sequences on the same field, yields of perennial crop. With such data sets combining cross-section and time-series observations, the error assumptions of ordinary least-squares regression are likely to be upset by autocorrelation due to sequential observation on the same unit not being satistically independent."

The second problem concerns the stochastic explanatory variables (inputs) which are assumed to be fixed in many statistical models. As a result of these problems, biased and inconsistent estimators could be obtained. The only way to avoid the stochastic variable problem is to have an experimental design in such a way that the amount of feed intake per pen is pre-fixed at different levels, and have data on liveweight and time elapsed to eat the feed as recorded at those levels of total feed consumption. Although this procedure would provide unbiased and consistent estimates for the response equation, there would be loss in terms of prediction since more dispersion on the liveweight variable would be observed for each level of feed consumption. On the other hand, in an experiment where time is fixed (experiments two and three) the observations on liveweight levels follow a smooth pattern providing good fit for prediction. In any event, working with biased estimators is not an uncommon problem in applied economic research. In this regard, Gujarati (21, p. 324) has a point:

> "In reality, what is usually done is to "assume away" the error of measurement problem by supposing that they are not present; if they are present, we suppose that they are of sufficiently small magnitude so that we can proceed with the usual estimation procedure."

The assumptions of the model in study are then:

1. $E(e_{rst})=0$
2. $E(e_{rs,t};e_{rs,t-1}) \neq 0$ for all r and s
3. $E(e_{r_1st};e_{r_2st})=0$ for all s and t
4. $E(e_{rs_1t};e_{rs_2t})=0$ for all r and t

Therefore, given Dillon's argument and the above assumptions, autocorrelation has been assumed to be present in the model and the first order autocorrelation process[1/] has been used to correct the model. Using Durbin's method (31, p. 289), the estimates of the quadratic responses corrected for autocorrelation are:

<u>Experiment Two</u>:

$$W = .053277 + \underset{(.159)^*}{1.148502P} + \underset{(.011)^*}{.102896E} - \underset{(.939)^*}{2.295389P^2} - \underset{(.0031)^*}{.099087E^2} + \underset{(.119)^{**}}{.25587PE}$$

[1/]First order autocorrelation process assumes that, $e_{rs,t} = \rho e_{rs,t-1} + v_{rst}$.

$R^2 = .99$

Standard errors are in parentheses,

(*) Significant at 1%,

(**) Significant at 5%.

<u>Experiment Three:</u>

$$W = \underset{(.1892)^*}{.041988} + \underset{(.0118)^*}{1.457695P} + \underset{(.6727)^*}{.109539E} - \underset{(.0027)^*}{1.758822P^2} -$$

$$-\underset{(.0027)^*}{.007404E^2} + \underset{(.084)^{**}}{.163387PE}$$

$R^2 = .99$

Standard errors are in parentheses,

(*) Significant at 1%,

(**) Significant at 5%.

One of the investigations of this work is to study whether the results derived from applying experiment three (spring) production response are different from the one from applying experiment two (summer) production respose.

Both quadratic response are concave functions (See Appendix A and Figures 4-1 and 4-2). The sign of the coefficients, significance of the coefficients and magnitude of R^2 are according to the expected results found in other studies and from the viewpoint of economic theory (Chapter II).

The difference in magnitude of the estimated coefficients between the two response functions might be attributed to a seasonal factor (environmental temperature) It is a biological fact that chickens tend to weighed

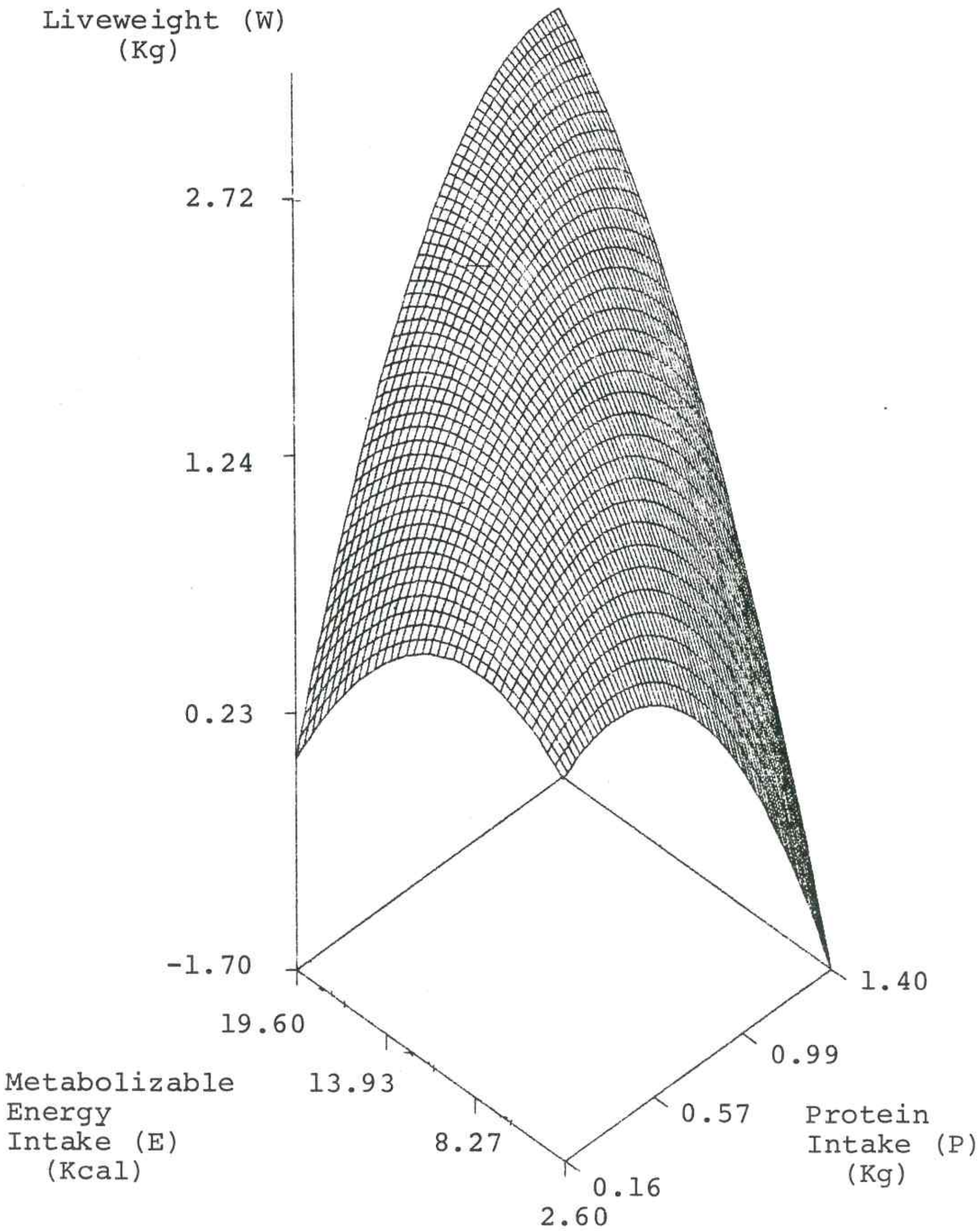

Figure 4-1. Graphical Representation of the Estimated Production Response of Experiment Two.

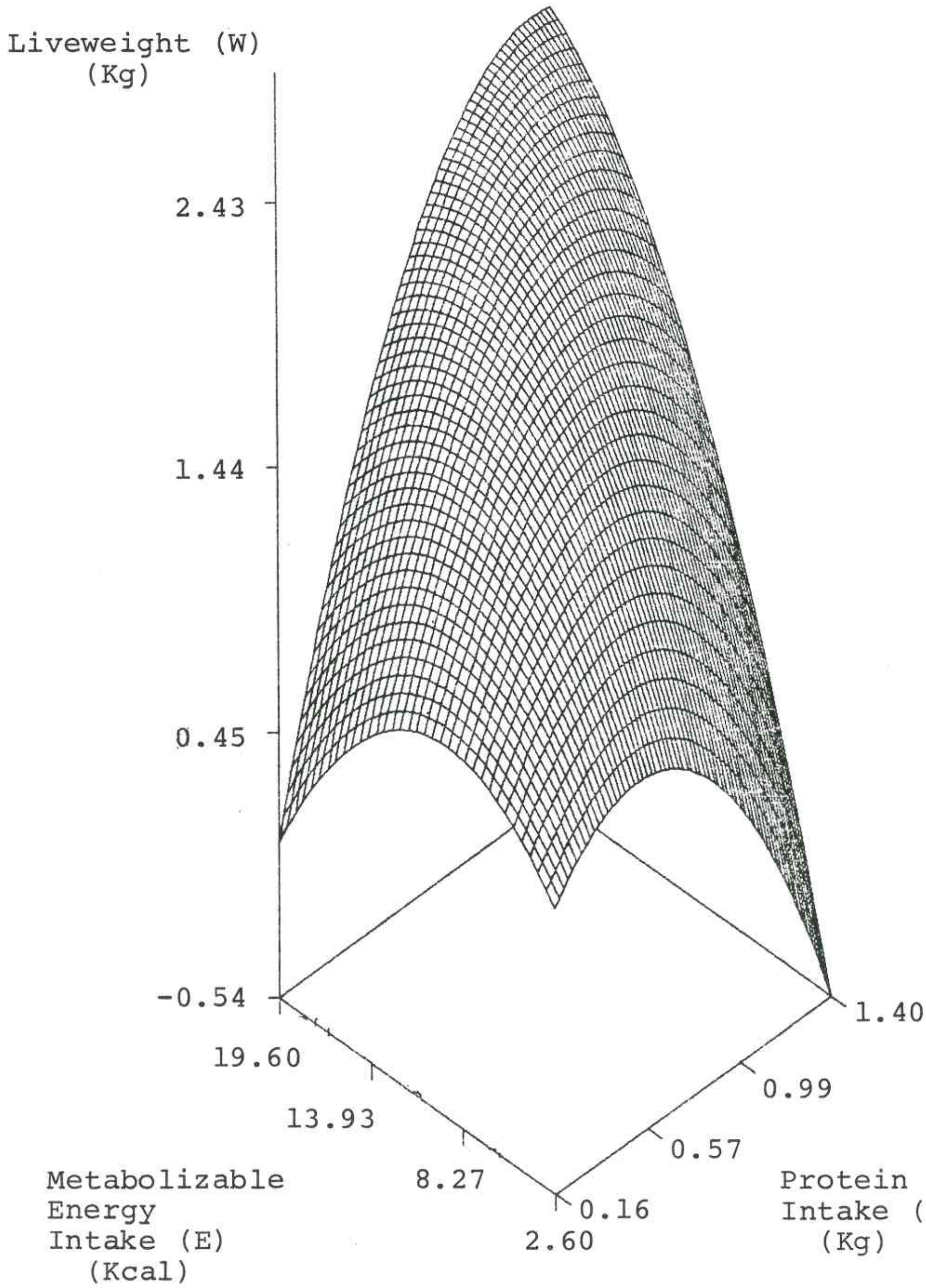

Figure 4-2. Graphical Representation of the Estimated Production Response of Experiment Three.

less during the warmer seasons (10). This is confirmed by the estimates of the quadratic responses. The estimated spring response (experiment three) reaches the absolute maximum for liveweight at 10.5 lbs. (4.7kgs), while the estimated summer response (experiment 2) reaches that absolute maximum at 8.3 lbs. (3.7kgs). Consequently, results of the quadratic programming derived from the spring experiment response will provide heavier birds than the one derived from the summer experiment response, based on the same feed ingredients prices and feed intake.

Fat Response

Quality of the broiler will be taken into account by inserting broiler's body fat content into the quadratic programming models. Certainly, the fat content in the bird's carcass depends on the ration which the bird is consuming. Birds on a high energy ration are fatter than birds on low energy ration.

The fat equation is defined as a function showing the relationship between bird's body fat and protein and metabolizable energy intake. That is:

$$F_{rst} = g(P_{rst}, E_{rst})$$

where,

F_{rst}: Cumulative average of a broiler's body fat (kilogram) of the r^{th} pen fed on the s^{th} ration of the end of the t^{th} week; P_{rst} and E_{rst} are defined as in the production response.

A fat response equation can be incorporated into the quadratic programming model as a constraint. The fat equation was specified as,

$$F = b_0 + b_1 P + b_2 E.$$

It should be expected that $b_1 < 0$ and $b_2 > 0$, meaning that for a given level of energy, increase in the protein level provides a leaner bird and, for a given level of protein, increase in the energy level gives a fatter bird. Research by Pesti has resulted in similar findings in prior study (36).

Fat equations were fitted on data form experiments two and three. All the estimation problems were investigated following the same procedures used to evaluate the production response. The equation fitted to experiment three data was statistically poor and values of the coefficients did not match with expectations. On the other hand, the fit on data from experiment two was statistically good and values of the coefficients were as expected, i.e.,

$$F = .013651 - \underset{(.0233)*}{.088859P} + \underset{(.0015)*}{.016912E} \qquad R^2 = .74$$

Standard errors are in parentheses,

(*) Significant at 1%.

Therefore, this was the fat equation chosen for prediction in the quadratic programming model.

Feed Consumption and Time Equation

The time elasped for a chicken to consume a certain amount of feed plays an important role in broiler feed formulation. As an example, consider a broiler ration formulated using two different sets of constraints (say minimum levels of certain nutrients). If the rations result in two different broiler liveweights, consider that the set of constraints that gave a heavier bird might have taken much longer than the time required to produce the lighter bird. Furthermore, for given constraints and prices of feed, cost per bird could have been the same. Based on the example, an appropriate economic problem is: given C cents feed cost that yields W lbs. broiler in T days, is it possible to produce a W lbs. broiler with C cents feed cost in fewer days than T? If so, resources other than feed will be more productive. Also, quality of the bird as measured by fat content in the bird, might be a factor to take into consideration. If time-consumption response and fat response are included in the ration formulation (say quadratic programming) the economic problem could be solved.

As in the previous equations, the time of consumption equation is also specified to be a function of protein and metabolizable energy intake:

$$T_{rsc} = h(P_{rsc}, E_{rsc})$$

where,

T_{rsc}: Time elasped (days) for a broiler of the r^{th} pen fed on the s^{th} ration to eaz the c^{th} kilogram of feed;

P_{rsc}: Cumulative average of crude protein intake (kilogram) per broiler of the r^{th} pen fed on the s^{th} ration at the end of the c^{th} kilogram of feed consumed;

E_{rsc}: Cumulative average of metabolizable energy intake (kilogram) per broiler of the r^{th} pen fed on the s^{th} ration at the end of the c^{th} kilogram of feed consumed.

The specification of the function h is based on the following argument. For each additional small increase in feed consumption, the time elasped for a chicken to eat more food, obviously, also increases, possibly, at a decreasing rate. Mathematically, this argument is expressed as; let C be a certain amount of feed intake and T be the time elasped to eat that amount of feed. Then, the following two conditions must hold:

$$\frac{dT}{dC} > 0 \text{ and } \frac{d^2T}{dC^2} < 0$$

Besides the two estimation problems mentioned in estimating production response the time of consumption equation presents another one. The experiments were designed as time being fixed, i.e., time is a fixed variable. As such it cannot be used as a dependent variable in an econometric equation. The suggestion then is to make the time variable stochastic by examining feed consumption at different levels and then "calculate" the time elasped for a chicken to eat a fixed amount of feed[1/]. Appendix B describes the procedure of the calculations. Estimation of the time equation derived from experiment three proved reliable estimates for prediction. Reliability was due to the accuracy of the calculations for the time of consumption variable when observations on consumption were taken at half week intervals. The time of consumption variable derived from experiment two when only weekly observations were taken, was not as accurate. Hence, only the time equation from experiment three was used in further analysis.

Previous works on the specification of the time of consumption equation cite two possible functions that satisfy the above referenced two conditions: square root or quadratic, which are reprectively,

$$T = c_0 + c_1P + c_2E + c_3P^{.5} + c_4E^{.5} + c_5(PE)^{.5}$$

$$T = c_0' + c_1'P + c_2'E + c_3'P^2 + c_4'E^2 + c_5'PE$$

Heady et. al., (25) using corn (high energy content) and soybean (high protein content) as independent variables in the time equation showed that the square root function performed better than the quadratic function. Both specifications were fitted in the nutrient space (P and E), and the square root estimates did perform better than the

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[1/]Feed consumption was examined at 4.4092, 6.6139, 8.8185, 11.023 and 13.2277 pounds which correspond to 2, 3, 4, 5 and 6 kilograms, unit by which the experiments were conducted.

quadratic estimates. Therefore, the time of consumption equation that was considered most accurate was:

$$T = \underset{}{4.587262} + \underset{(102.926)^{*}}{243.612384P} + \underset{(6.487)^{*}}{16.578949E} + \underset{(15.763)^{*}}{50.935784P^{.5}} +$$

$$\underset{(2.674)^{**}}{5.34376E^{.5}} - \underset{(51.485)^{*}}{131.133132(PE)^{.5}}$$

$R^2 = .99$

Standard errors are in parantheses,

(*) Significant at 1%,

(**) Significant at 5%.

The value of the coefficients match those found by Heady et. al., (25).

In order to solve the economic problem raised earlier in the section, it is necessary to have a time equation as one of the constraints of the quadratic programming formulation. The non-linearity of the above time equation makes it impossible to use in a set of linear constraints. To avoid non-linear constraint a linear equation was also specified and it estimates were,

$$T = 24.403404 + \underset{(2.352)^{*}}{10.511097P} + \underset{(.144)^{*}}{1.010951E} \qquad R^2 = .95$$

Standard errors are in parantheses,

(*) Significant at 1%.

The square root time equation was more accurate than the linear time equation in terms of prediction. That is, the linear equation tended to underestimate in relation to the predicted time from the square root equation. However this differential bias was estimated and used to adjust the linear equation that will enter as a constraint in the feed formulation problem.

CHAPTER V

EMPIRICAL RESULTS

The Linear Programming Results

The main economic indicators showing broiler production responses as derived from feed formulated by linear programming (LP) were obtained from feed mill records. Observations were obtained during two periods on broiler average liveweight and time elasped for a bird to reach each level of liveweight. The observations suggested that broiler producers withdraw birds from the feeding process at different ages. This happens for several reasons. Producers might market birds early because the added value might be less than the added cost of feeding. If birds are at least a minimum weight demanded, they might be withdrawn earlier or later to match the schedule of the processing plant. However, the right hand side values (see Table 3-1) of a linear programming feed formulation cannot reflect the least cost method of producing alternative liveweight of broilers. The LP ration is formulated for least cost of a unit of feed with no objective means of predicting the cost of alternative liveweights produced by the ration.

It might be worthwhile pointing out at this point that the data on LP (production response) were observed for all broiler producers affiliated to the broiler firm that formulates the ration. Given these data, the average liveweight of 46 days old broilers 3.62 to 3.65 pounds was computed among all producers that delivered broilers at that age (Table 5-1).

The liveweight growth rate, in both observation periods, was much higher from day 48 to day 49 than the growth rate in earlier days. For example, on April 12, the growth rate from 48 to 49 days was 7%, while the growth rates from 46 to 47 days and from 47 to 48 days approximately 2% (Table 5-1). On May 31, the later day's growth rate was also 7%, but in the earlier days that rate was 1% or lower (Table 5-1). Unless there was a minor aggregation error, the liveweight growth rate might be an indication that growing broilers longer (at least up until 49 days) might be more profitable, depending on the levels of added costs.

Birds grown on feed formulated April 12 weighed slightly more than the birds grown on feed formulated May 31. At 46 and 47 days, the weight difference was about 1 and 2% respectively, while at 48 and 49 days that difference increased to about 4% (Table 5-1). It is supposed here that some seasonal factor, such as temperature, might have influenced the growth in the two periods.

Table 5-1. Observed Average Liveweights from Industry Records in Four Periods on April 12, 1982 and May 31, 1982

	Average Liveweight	
Time	April 12, 1982	May 31, 1982
(days)	(lbs)	(lbs)
46	3.65	3.62
47	3.72	3.65
48	3.81	3.67
49	4.08	3.93

A principal result of using linear programming feed formulation was that 72.2 cents of feed cost were required on average to produce a 3.78 pounds broiler in an average time between 47 and 48 days for feed formulated on April 12 (Table 5-2). For the formulation made on May 31, 69.7 cents of feed cost were required on average to produce a bird weighing an average of 3.72 pounds in just more than 48 days (Table 5-2). A main concern of broiler producers is how these results can be improved. There are two ways this can be done. First, biological research can improve technical and, thus, efficiency in broiler production. Second, selecting the optimum nutrient levels that produce least cost of broiler will point to improved economic efficiency in broiler production.

Industry observations on linear programming restriction levels in Table 3-1 and resulting outputs in Table 5-1 and 5-2 will serve as the reference point for comparison with the results that came from estimating price efficient input levels that lead to maximum growth and least cost of broiler production, i.e., via quadratic programming in this instance.

The Quadratic Programming Results

In order to accomplish objective 1 of the thesis the theoretical framework of Chapter III indicates an investigation on whether a right hand side of basic nutrient requirements can be specified best by industry or

Table 5-2. Response Data from Linear Programming Formulation on April 12, 1982 and May 31, 1982

Results	April 12, 1982	May 31, 1982
Time Weighted Avg. Liveweight[a] (lbs)	3.78	3.72
Avg. Technican Feed Efficiency[b] (Conversion)	2.10	2.08
Ave. Feed Consumption[c] (lbs)	7.97	7.74
Feed Cost (cent/lb): Starter	9.34	9.26
Finisher	8.98	8.94
Av . Feed Cost[d] (cents broiler)	72.2	69.7
Ave. Feed Cost per Pound of Broiler (cents/lb)	19.1	18.7

[a] Time weighted ave. liveweight $= \sum_{i=1}^{n} t_i w_i / \sum_{i=1}^{n} t_i$. Calculations were based on Table 5-1.

[b] Monthly average pounds of feed/lb. of broiler liveweight.

[c] Avg. feed consumption=(av . technical feed efficiency). (time eight ave. liveweight).

[d] It is assumed that the starting ration takes, on average based on experimental data, 21% of the total feed consumption. Then, ave. feed cost is computed, for instance on April 12, 1982, as:

$$(.21)\ (7.93)\ (9.34) + (.79)\ (7.93)\ (8.98) = 72.2$$

by biological constraints for economic modeling by quadratic programming. Results of testing both constraint sets are shown in Table 5-3 (industry constraints) and in Table 5-4 (biological constraints). Data in these two Tables were derived by using production response from experiment three and feed ingredient prices on April 12, 1982. The decision on which constraint set provides the least cost result is to be based on the following: given a certain level of feed cost per broiler which set of constraints produces maximum weight at least cost per pound in the shortest length of time and produces the highest quality bird (least carcass fat).

At the 69 cents feed cost level the difference between feed cost per pound of broiler in the formulations using industry constraints (Table 5-3) and the formulation using biological constraints (Table 5-4) is 17.16-16.87=.29 cents/pound in favor of industry constraints. On the other hand, at the same feed cost level the difference in rate of consumption between the two formulation is 46.4-46.2=.2 days in favor of biological constraints. These approximate levels of difference were observed for all levels of feed cost. Since least time is undobutly valuable but unknown in this study, these results are conflicting and inconclusive for making a choice.

An important point on the argument about quality of the bird produced concerns the diet composition of the two formulations. In both cases, the methematical

Table 5-3. Results of Varying Feed Cost Using Production Response from Experiment Three, Selected Feed Ingredient Prices[a/] and Industry Constraints

	Quadratic Programming Results						Rate of Consumption	
Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Time[c/] (Sq. Root) (days)	Time[d/] (Linear) (days)
69	22.00	1487	7.77	4.09	1.90	16.87	46.4	44.2
70	22.00	1487	7.88	4.13	1.91	16.95	46.7	44.5
71	22.00	1487	7.99	4.17	1.92	17.03	46.9	44.8
72	22.00	1487	8.11	4.21	1.93	17.10	47.2	45.1
73	22.00	1487	8.29	4.24	1.95	17.22	47.5	45.4
74	22.00	1487	8.33	4.28	1.95	17.29	47.7	45.7
75	22.00	1487	8.44	4.32	1.95	17.36	48.0	46.0
76	22.00	1487	8.56	4.35	1.97	17.47	48.3	46.2
77	22.00	1487	8.67	4.39	1.97	17.53	48.5	46.5

a/ April 12, 1983.

b/ Technical feed efficiency = feed intake/broiler liveweight.

c/ Estimated values from the square root time equation (p. 67).

d/ Estimated values from the linear time equation (p. 68).

Table 5-4. Results of Varying Feed Cost Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a] and Biological Constraints

	Quadratic Programming Results						Rate of Consumption	
Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Time[c] (Sq. Root) (days)	Time[d] (Linear) (days)
69	22.00	1432	7.75	4.02	1.93	17.16	46.2	43.8
70	22.00	1435	7.85	4.06	1.93	17.24	46.5	44.0
71	22.00	1438	7.94	4.09	1.94	17.36	46.7	44.3
72	22.00	1441	8.04	4.13	1.95	17.43	46.9	44.5
73	22.00	1444	8.13	4.17	1.95	17.51	47.2	45.8
74	22.00	1447	8.23	4.20	1.96	17.62	47.4	45.1
75	22.00	1450	8.32	4.23	1.97	17.73	47.6	45.3
76	22.00	1452	8.42	4.27	1.97	17.80	47.9	45.6
77	22.00	1455	8.51	4.30	1.98	17.91	48.1	45.9

[a] April 12, 1982.

[b] Technical efficiency=feed intake/broiler liveweight.

[c] Estimated values from the square root time equation (p. 67).

[d] Estimated values from the linear time equation (p. 68).

programming solution provides a stable percentage of protein in the ration, at 22%, at all levels of feed cost. However, higher levels of metabolizable energy are found in the industry constraints solution, which implies that a fatter bird would be produced, given that protein is held constant. Although fat response could be estimated from experiment three data, estimates from other experiments support the possibility that the difference in body fat would be relevant. To predict the possible difference, the fat response of experiment two was used. At 69 cents feed cost, the industry constraints were predicted to produce .2 pounds of body fat more than the biological constraints.

Choosing between the two formulations was difficult because of conflicting results and no data on the value of time. However, due to the inference made on fat, biological constraints (See Table 3-2) were chosen to develop further quadratic programming results. Birds that were too fat, oily bird syndrome, was a significant industry problem at the time of the study and further reinforces the choice of biological constraints.

Input Prices and Least Cost of Production--Experiment Three

The selection of best constraint set was made for one set of feed ingredient prices only. Certainly, the setting of selected constraints in a programming problem is one of many conditions that affects the results.

In fact, one of the important results of this analysis is to find out how changes in feed ingredient prices determine protein and energy levels in the right hand side so that feed cost per pound of broiler is minimized.

According to Table 5-4 and 5-5 which were calculated using feed prices on April 12, 1982 and May 31, 1982, protein in the ration was the only factor that did not respond to changes in feed ingredient prices occurring during the period. Protein was steady at 22%, which was the upper limit of biological constraint in the experiments. On the other hand, metabolizable energy did respond to the price change. Metabolizable energy levels on May 31 compared to the levels on April 12 decreased at all values of feed cost. At 69 cents feed cost, in the April 12 formulation, the value for crude metabolizable energy was 1432 kcal/lb. For each 1 cent increase in feed cost per bird metabolizable energy increased by about 3 kcal/lb. On the May 31 formulation, the metabolizable energy level started at 1428 kcal/lb. and increased by 2 to 3 kcal/lb. for each additional 1 cent increment in feed cost. For least cost of production the amount of crude metabolizable energy in the ration should increase if heavier birds are produced. In spite of the fact that protein remains constant, the results are consistent from the standpoint of biological requirement for chicken growth.

Table 5-5. Results of Varying Feed Cost Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a] and Biological Constraints

Feed Cost/ Broiler (Cents)	Quadratic Programming Results: Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/ Pound Broiler (Cent/lb)	Rate of Consumption: Time[c] (Sq. Root) (days)	Time[d] (Linear) (days)
69	22.00	1428	7.53	3.94	1.91	17.51	45.7	43.2
70	22.00	1429	7.63	3.98	1.92	17.59	45.9	43.4
71	22.00	1432	7.73	4.01	1.93	17.71	46.2	43.7
72	22.00	1435	7.82	4.05	1.93	17.78	46.4	43.9
73	22.00	1437	7.91	4.08	1.94	17.89	46.7	44.2
74	22.00	1440	8.00	4.12	1.94	17.96	47.0	44.5
75	22.00	1443	8.09	4.15	1.95	18.07	47.2	44.7
76	22.00	1446	8.19	4.18	1.96	18.18	47.4	45.0
77	22.00	1448	8.28	4.22	1.96	18.25	47.7	45.2

[a] May 31, 1982.

[b] Technical feed efficiency=feed intake/broiler liveweight.

[c] Estimated values from the square root time equation (p. 67).

[d] Estimated values from the linear time equation (p. 68).

The level of the protein and metabolizable energy nutrients found in a ration are crucial to determine other variables in broiler production. In the results presented in Table 5-4 or Table 5-5, increased spending on feed cost/broiler is obviously related to the greater amount of feed intake required to produce a heavier bird. These results indicate increasing feed efficiency[1/], as typically measured by industry, which is consistent with a quadratic production response. This measure of technical efficiency is directly related to cost per pound of broiler which is the more important goal. Feed cost/pound of broiler increases with feed per pound of broiler as birds become heavier. This result means that for each additional 1 cent of feed cost per bird, the marginal gain of liveweight is less than a unit of weight. In other words, average feed cost per pound was always increasing with increasing bird weight.

The relationship of changing costs over time was examined through use of function showing rate of feed consumption. The function was estimated by both square root and linear time equations. According to experiment results, at 69 cents feed cost, 46.2 days (square root equation was required for a broiler to eat 7.75 pounds of feed and reach 4.02 pounds of liveweight. These levels of feed intake and liveweight were estimated to be obtained

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[1/] The increases are clearly observed in the third decimal places, which are not shown in the figures.

in 43.8 days according to the linear time equation Table 5-4. There was an increase of .2 to .3 days, using either the square root or linear equazion for each cent increase in feed cost and average cost per pound per day was increasing with time. Thus, there was a consistent bias of 2.3 to 2.5 days between the square root and linear equation prediction, for any given level of feed cost.

Input Prices and Least Cost of Production--Experiment Two

Certainly, the main effects of feed ingredient price changes on the economic variables of broiler production are similar regardless of the production function that is analyzed. So, this section emphasizes the effect of a production response change on the least cost of broiler production. This effect will be investigated by comparing the results from experiment two wizh the results from experiment three. Table 5-4 of experiment three and Table 5-6 of experiment two are based on comparable prices (April 12)[1/].

As earlier, production response in experiment three was more technically efficient than the production response in experiment two. That is, to reach a certain level of liveweight, birds grown in spring (experiment three) ate less feed than birds grown in summer (experiment two). This is seen by comparing 4.13 pounds broilers in Table 5-4

[1/] A comparison based on May 31 prices would lead to the same conslusions.

Table 5-6. Results of Varying Feed Cost Using Production Response from Experiment Two, Selected Feed Ingredients Prices[a] and Biological Constraints

				Quadratic Programming Results			
Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Technical Broiler Liveweight (lbs)	Feed Efficiency[b] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Broiler's Body Fat[c] (lbs)
69	22.00	1464	7.60	3.91	1.94	17.65	.2962
70	22.00	1466	7.70	3.95	1.95	17.72	.302
71	22.00	1468	7.79	4.00	1.95	17.75	.3043
72	22.00	1470	7.89	4.04	1.95	17.82	.3083
73	22.00	1471	7.99	4.09	1.95	17.85	.3123
74	22.00	1473	8.09	4.13	1.96	17.92	.3164
75	22.00	1475	8.19	4.17	1.96	17.98	.3204
76	22.00	1477	8.29	4.22	1.96	18.01	.3244
77	22.00	1478	8.39	4.26	1.97	18.07	.3285

a/ Technical feed efficiency=feed intake/broiler liveweight.

b/ April 12, 1982.

c/ Estimated values from the fat response (p. 64).

and 5-6. To reach that weight under experiment three, the least cost feed intake is 8.04 pounds at a cost of 72 cents per bird, while to reach the same weight under experiment two, 8.09 pounds of feed were required at a cost of 74 cents. For the same reason, the technical feed efficiencies to produce a 4.13 pounds broiler using results of experiment two and three were 1.96 and 1.95 respectively.

The least cost protein level did not change between different production responses. It remained constant at the upper limit of 22% at all levels of feed cost per bird in both production response results. On the other hand, with feed prices constant, the metabolizable energy level varied with change in the production response. At any given value of total feed cost, the metabolizable energy level from experiment two response is higher than the metabolizable energy level from experiment three response. For instance, at 69 cents feed cost least cost energy levels were 1464 and 1432 kcal/lb. of metabolizable energy using experiment two (summer) or three (spring) response function, respectively. From a biological point of view these results may seem to be inconsistent because during warmer seasons birds have been found to intake less energy than during cooler seasons (10). However, from the economic standpoint the explanation for such a result is based on the marginal productivity of energy. Consider the production response of experiment two and three.

The marginal productivity of energy from experiments two and three production responses are respectively:

Summer: $(MP)_{12} = .102896 - .18174E + .25587P$

Spring: $(MP)_{13} = .109539 - .014808E + .163387P$

For given levels of protein intake and metabolizable energy intake the two marginal productivities can be calculated. Calculations made for a given feed cost level and a given amount of intake are:

69 cents feed cost: Summer $(MP)_{12}$ = .21 pounds

Spring $(MP)_{13}$ = .16 pounds

As implied by economic theory, the higher marginal productivity of energy will dictate more of its use given fixed input and output price levels. Therefore higher levels of energy produced least cost birds at the same weight from the production response of experiment two (summer) as compared to spring production.

The superiority in technical efficiency of experiment three over experiment two reflects into economic efficiency as measured by feed cost/pound of broiler. At any given level of feed cost, production response in experiment three can provide a heavier bird than the production response of experiment two. Consequently, feed cost/pound of broiler is lower for results derived from experiment three production response. This result suggests that during cooler season broiler producers should make more profit than during warmer seasons.

Least Cost and Broiler's Quality

Generally the heavier the bird, the more fat is found in the carcass. The fat content in the bird's carcass is the variable used to measure quality of the bird. To fulfill objective two of the thesis, an economic analysis on the possible trade off between least cost broiler and quality of broiler was conducted. Estimated carcass fat content of least cost birds varied from .2872 to .3186 pounds within the feed cost range of 69 to 77 cents and bird weight of 3.81 to 4.15 pounds, Table 5-7. That range of carcass fat indicates a basis for setting the right hand side of a fat constraint thaz will yield an analysis of cost effects within a quadratic programming formulation.

To remain consistent with experiment two data, five levels of carcass fat were chosen for the right hand side constraint, i.e., .2645, .2866, .3086, .3307 and .3527 pounds. These fat values are in a range wide enough to produce leaner and fatter birds than birds derived from the least cost application where fat was unconstrained (Table 5-7). The results from imposing the fat constraint in the optimization problem are shown in Table 5-8.

The two lowest levels of fat, .2645 and .2866 pounds, in Table 5-8 are lower than the minimum fat level estimated from least cost formulation in Table 5-7. At those two values for fat, at any given feed cost, the fatter the bird, the higher the energy level and the heavier the

Table 5-7. Results of Varying Feed Cost Using Production Response from Experiment Two, Selected Feed Ingredients Prices[a/] and Biological Constraints

		Quadratic Programming Results					
Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Technical Broiler Liveweight (lbs)	Feed Efficiency[b/] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Broiler's Body Fat[c/] (lbs)
69	22.00	1459	7.38	3.81	1.94	18.11	.2872
70	22.00	1461	7.48	3.85	1.94	18.18	.2912
71	22.00	1463	7.57	3.90	1.94	18.21	.2952
72	22.00	1465	7.67	3.94	1.95	18.27	.2992
73	22.00	1465	7.67	3.94	1.95	18.27	.3029
74	22.00	1469	7.86	4.03	1.95	18.37	.3069
75	22.00	1471	7.95	4.07	1.95	18.42	.3108
76	22.00	1473	8.05	4.11	1.96	18.49	.3147
77	22.00	1474	8.14	4.15	1.96	18.55	.3186

[a/] May 31, 1982.

[b/] Technical feed efficiency=feed intake/broiler liveweight.

[c/] Estimated values from the fat response (p., 64).

Table 5-8. Results of Varying Feed Cost Using Production Response from Experiment Two, Selected Feed Ingredient Prices[a/], Biological Constraints and Fat Constraints

	Broiler Body Fat (lbs)											
	.2645						.2866					
Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
69	22.00	1386	7.29	3.66	1.99	18.85	22.00	1454	7.40	3.81	1.94	18.11
70	22.00	1381	7.34	3.67	2.00	19.07	22.00	1431	7.58	3.85	1.97	18.18
71	22.00	1373	7.41	3.68	2.01	19.29	22.00	1425	7.64	3.86	1.98	18.39
72	22.00	1365	7.48	3.70	2.02	19.46	22.00	1413	7.74	3.88	1.99	18.56
73	22.00	1356	7.56	3.71	2.04	19.62	22.00	1408	7.79	3.89	2.00	18.77
74	22.00	1349	7.63	3.72	2.05	19.89	22.00	1402	7.83	3.90	2.01	18.97
75	22.00	1341	7.70	3.74	2.06	20.05	22.00	1397	7.88	3.91	2.02	19.18
77	22.00	1326	7.85	3.76	2.09	20.48	22.00	1380	8.04	3.94	2.04	19.54

(Continued)

Table 5-8 (Continued)

Feed Cost/ Broiler (Cents)	Broiler Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
69	20.56	1506	7.35	3.80	1.93	18.16	18.67	1521	7.49	3.73	2.01	18.49
70	20.91	1504	7.43	3.84	1.93	18.22	19.03	1520	7.57	3.79	2.00	18.47
71	21.23	1500	7.51	3.89	1.93	18.25	19.34	1518	7.64	3.84	1.99	18.49
72	21.49	1493	7.62	3.94	1.93	18.27	19.74	1516	7.71	3.90	1.98	18.46
73	21.74	1485	7.72	3.98	1.94	18.34	20.08	1514	7.78	3.95	1.97	18.48
74	21.98	1478	7.83	4.03	1.94	18.36	20.42	1513	7.86	4.01	1.96	18.45
75	22.00	1457	8.01	4.07	1.96	18.43	20.75	1511	7.93	4.06	1.95	18.47
76	22.00	1435	8.20	4.11	1.99	18.49	21.04	1507	8.02	4.10	1.95	18.54
77	22.00	1429	8.26	4.15	1.99	18.55	21.28	1500	8.12	4.15	1.95	18.55

(Continued)

Table 5-8 (Continued)

Feed Cost/ Broiler (Cents)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
	Broiler Body Fat (lbs) .3527					
69	--	--	--	--	--	--
70	--	--	--	--	--	--
71	17.58	1531	7.78	3.75	2.07	18.93
72	17.95	1530	7.85	3.81	2.06	18.40
73	18.30	1528	7.92	3.88	2.04	18.81
74	18.65	1527	7.99	3.94	2.03	18.78
75	18.99	1525	8.07	3.99	2.02	18.80
76	19.33	1524	8.14	4.05	2.01	18.77
77	19.65	1522	8.21	4.11	2.00	18.73

a/ May 31, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

bird, given that protein was stable at 22%. To produce a lean bird with fat constant, the protein content in the ration must be high and the energy content must decrease as size and age of bird increase Table 5-8.

Concerning technical feed efficiency and feed cost/ pound broiler in Table 5-8, results for .2866 pounds of fat are superior to the results for .2645 pounds of fat, at all levels of feed cost. Specifically, the differences in technical feed efficiency and feed cost/pound broiler for the two fat levels are .03 to .05 and .74 to .9 cents/ lbs., respectively, in favor of the .2866 pounds of fat constraint.

Attention is now turned to higher levels of fat, .3086, .3307 and .3527 pounds, which are also presented in Table 5-8. The lower level, .3086 pounds of fat, is included in the range of the estimated fat values from the least cost solutions in Table 5-7, while the two highest levels of fat, .3307 and .3527 pounds, are not. At any of these higher fat levels, there is a trade-off between protein and energy, as feed cost varies from 69 to 77 cents. That trade-off means that for lower feed cost, protein is low and energy is high; and for higher feed cost, protein is high and energy is low. This trade-off is needed to stabilize fat at a fixed level. Observe what happens to protein and energy when feed cost is at any given amount, and fat varies from .3086 to .3527: protein decreases and energy increases. This last protein-energy trade off occurs in such a way that, at any level of feed cost, the addition of fat to the birds carcass causes: a) reduction of liveweight; b) increase in technical feed efficiency; c) increase in feed cost/pound broiler. These results suggest important implications in broiler producing decision. In sum, high fat birds might not be of economic advantage to produce.

Excluding the .3307 and .3527 pounds of fat levels of the analysis, emphasis must be made on the three lowest levels of fat, that is, .2645, .2866 and .3086 pounds. If a decision must be made on which of these fat content is best, several factors have to be considered. Birds with .2645 pounds of fat are leaner and lighter and provide higher feed efficiency and feed cost/pound broiler, than birds with higher levels of fat, compared at any given feed cost. Consider now a comparison between the .2866 and .3086 pounds of fat birds. At 69 and 70 cents feed cost birds with .2866 pounds of fat provide lower feed cost/ pound broiler than the birds with .3086 pounds of fat. However, at any feed cost level greater than 70 cents, the fatter birds are more economically efficient (lower feed cost/pound broiler) than the lighter birds.

As far as feed cost/pound broiler is concerned, .3086 pounds can be said to be the best level of fat for feed cost at 71 cents or greater. At 69 and 70 cents feed cost, .2866 pounds of fat would be the best level. It is worthwhile noting that .2866 and .3086 pounds of fat levels are within the range of least cost formulation (Table 5-7).

The least fat (.2645 pounds) results cannot be discarded. Despite the higher feed cost/pound and less liveweight, the .2645 pounds of fat broiler are of better quality than the higher fat content birds. It leaner birds are worth more in the market, then the least fat broiler in the present analysis could be viable for production. Beside that, producing a lighter and less fat bird will probably take less time than producing a heavier and fatter bird. This fact might improve the viability of producing the least fat bird.

Least Cost and Least Time--Experiment Three

Incorporation of the time constraint into the quadratic programming formulation will give information whether broiler grown in less time have any cost advantage or disadvantage relative to broilers grown over a larger period. The idea is then to investigate the possibility of reducing time to produce a specified level of broiler liveweight.

The right hand side in the time constraint was normalized around the estimated time from the least cost solution without respect to time. Feed ingredients prices on April 12 were used in the analysis. So, results in

Table 5-4, from least cost formulation, will serve as reference to the time constrained results presented in Table 5-9.

As noted earlier, there was a consistent bias between the time estimated values from the time square root equation and the time linear equation. Since the square root equation is more reliable for prediction, that bias was taken into account for setting the right hand side for time. For instance, 41 days in the linear time "constraint' would correspond to approximately 43 days in the square root time equation. The best estimate of time is reported in the Tables.

The 43 and 44 days time constraint would represent very least time results, since the values are outside the range of the predicted time in the least cost results of Table 5-4. The upper values of the time constraint, 45, 46 and 47 days, fall in the time range of Table 5-4.

Two special results are quite noticeable in Table 5-9:

a) For any assigned length of time, broiler liveweight remains constant at all levels of feed cost;

b) Broiler liveweight increases with time, at any given level of feed cost.

Condition b implies that there exists a trade off between protein and energy that makes it possible to produce heavier birds in a longer period of time, at the same

Table 5-9. Results of Varying Feed Cost Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a/], Biological Constraints and Time Constraint

Feed Cost/ Broiler (Cents)	Rate of Consumption-Time (days)											
	43						44					
	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
69	19.55	1343	7.25	3.61	2.01	19.11	20.73	1393	7.34	3.77	1.95	18.30
70	19.26	1331	7.33	3.61	2.03	19.39	20.43	1381	7.42	3.77	1.97	18.57
71	18.97	1319	7.42	3.61	2.05	19.67	20.14	1369	7.51	3.77	1.99	18.83
72	18.60	1315	7.49	3.61	2.07	19.94	19.85	1357	7.59	3.77	2.01	19.01
73	18.16	1315	7.56	3.61	2.09	20.22	19.57	1345	7.67	3.77	2.03	19.36
74	17.94	1315	7.59	3.61	2.10	20.49	19.30	1334	7.76	3.77	2.06	19.63
75	17.5	1315	7.67	3.60	2.13	20.83	19.03	1323	7.84	3.77	2.08	19.89
76	17.5	1315	7.67	3.60	2.13	21.11	18.72	1315	7.92	3.77	2.10	20.16
77	17.5	1315	7.67	3.60	2.13	21.39	18.32	1315	8/00	3.76	2.12	20.48

(Continued)

Table 5-9 (Continued)

Feed Cost/ Broiler (Cents)	Rate of Consumption-Time (days)											
	45						46					
	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/ Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
69	21.98	1435	7.44	3.91	1.90	17.65	22.00	1420	7.89	4.05	1.95	17.04
70	21.55	1429	7.52	3.92	1.92	17.86	22.00	1438	7.83	4.05	1.93	17.28
71	21.27	1418	7.60	3.92	1.94	18.11	22.00	1472	7.72	4.05	1.91	17.53
72	20.98	1405	7.68	3.92	1.96	18.37	22.00	1450	7.79	4.05	1.92	17.78
73	20.69	1393	7.76	3.92	1.98	18.62	21.84	1434	7.86	4.05	1.94	18.02
74	20.40	1381	7.85	3.92	2.00	18.88	21.48	1427	7.94	4.06	1.95	18.23
75	20.13	1370	7.93	3.92	2.02	19.13	21.20	1416	8.02	4.06	1.97	18.47
76	19.86	1359	8.01	3.92	2.04	19.39	20.92	1404	8.10	4.06	1.99	18.72
77	19.59	1347	8.10	3.92	2.07	19.64	20.65	1393	8.19	4.06	2.02	18.96

(Continued)

Table 5-9 (Continued)

Feed Cost/ Broiler (Cents)	Rate of Consumption-Time (days)					
	47					
	Protein (%)	Metabolizable Energy (Kcal/ lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/ Pound Broiler (Cents/lb)
69	--	--	--	--	--	--
70	--	--	--	--	--	--
71	--	--	--	--	--	--
72	--	--	--	--	--	--
73	22.00	1425	8.27	4.19	1.97	17.42
74	22.00	1456	8.17	4.19	1.95	17.66
75	22.00	1465	8.14	4.19	1.94	17.90
76	22.00	1443	8.21	4.19	1.96	18.14
77	21.72	1434	8.29	4.19	1.98	18.38

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

level of feed cost[1/]. This result has important implication for broiler producers by current feeding practices. Actually, some broiler producers feed broilers for longer periods of time (see Table 5-1) to obtain heavier birds, however, at the expenses of higher feed cost. This is so because the linear programming technique does not incorporate performance of the bird in the formulation. When a ration is formulated, it is determined by feed prices regardless of the desired weight of bird or time desired for feeding.

According to the conclusion in a) under least time formulation, the least cost of feed per pound of bird is not attained. The explanation is that, to achieve lower cost of feed in broiler production, total broiler liveweight must increase with feed cost. But what has been observed in Table 5-9 is that for increases in feed cost broiler liveweight does not change at all, given a fixed length of time. Increases in feed cost cause a trade off between protein and energy in such a way that different points on a single isoquant are selected. As a consequence of that, feed cost/pound broiler increases with feed cost. Then, if broiler producers decided to produce in the very least time, 43 days, the rational decision would be to spend only 69 cents on feed to obtain a 3.61 pound broiler.

[1/] It should be recalled at this point that linear programming does not give this trade off.

The same rational is applied to produce broilers at the second and third least time, 44 and 45 days, where 3.77 and 3.92 pounds broilers could be produced at 69 cents feed cost.

For longer length of time, 46 and 47 days, the results are comparable with the ones in Table 5-4 for least cost of feed per pound of broiler. An appropriate comparison is made between least cost and least time results at day 46.

Suppose now that 46 days is still applied, but feed cost is reduced to 69 cents. It is shown in Table 5-9 that broiler liveweight remains at 4.05 pounds but feed cost/pound broiler is reduced substantially to 17.04 cents/lb. The reasoning is that, the iso-time line being fixed, as isocost line shifts down it reaches a higher isoquant curve from changing the level of the nutrient inputs. In conclusion, least time formulation can be used to improve efficiency in broiler production by selecting better combination of the nutrient inputs, that is, protein and energy.

Least Time and Bird's Quality--Experiment Three

So far, time and fat constraints have been applied independently. Combination of time and fat constraints in the same quadratic programming formulation was possible by using the linear time equation from experiment three and the fat equation from experiment two. Results from that combination are presented in Table 5-10 through 5-18, using

Table 5-10. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 69

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .2645 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.2866 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.7	1338	7.24	3.61	2.00	19.11	18.53	1391	7.25	3.60	2.01	19.17
44	21.53	1378	7.27	3.76	1.93	18.35	20.20	1419	7.34	3.77	1.95	18.30
45	22.00	1335	7.75	3.89	1.99	17.74	21.80	1444	7.44	3.92	1.90	17.60
46	--	--	--	--	--	--	--	--	--	--	--	--
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-10 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs): .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	17.5	1455	7.20	3.56	2.02	19.38	--	--	--	--	--	--
44	19.08	1474	7.33	3.74	1.96	18.45	17.97	1529	7.32	3.69	1.98	18.70
45	20.70	1499	7.43	3.91	1.90	17.65	19.45	1541	7.48	3.88	1.93	17.78
46	--	--	--	--	--	--	--	--	--	--	--	--
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-10 (Continued)

Rate of Consumption Time (days)	.3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.68	1536	7.76	3.82	2.03	18.06
46	--	--	--	--	--	--
47	--	--	--	--	--	--

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-11. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 70

Rate of Consumption Time (days)	Broiler's Body Fat (lbs)											
	.2645						.2866					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.52	1325	7.31	3.61	2.02	19.37	18.32	1375	7.33	3.60	2.04	19.44
44	21.33	1365	7.34	3.76	1.95	18.62	19.97	1402	7.42	3.77	1.97	18.57
45	22.00	1336	7.75	3.89	1.99	17.99	21.57	1429	7.52	3.92	1.92	17.86
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	17.33
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-11 (Continued)

Rate of Consumption Zime (days)	Broiler's Body Fat (lbs)											
	.3086						.3307					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	17.5	1455	7.20	3.56	2.02	19.66	--	--	--	--	--	--
44	18.87	1457	7.42	3.74	1.98	18.72	17.77	1512	7.41	3.69	2.01	18.97
45	20.48	1482	7.51	3.91	1.92	17.90	19.39	1536	7.50	3.88	1.93	18.04
46	21.62	1478	7.76	4.06	1.91	17.24	--	--	--	--	--	--
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-11 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.75	1542	8.73	3.82	2.02	18.32
46	--	--	--	--	--	--
47	--	--	--	--	--	--

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-12. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 71

Rate of Consumption Time (days)	Broiler's Body Fat (lbs)											
	.2645						.2866					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.37	1315	7.36	3.61	2.04	19.67	18.11	1359	7.42	3.60	2.06	19.72
44	21.12	1352	7.41	3.76	1.97	18.88	19.74	1387	7.51	3.77	1.99	18.83
45	22.00-	1336	7.75	3.89	1.99	18.25	21.35	1415	7.60	3.92	1.94	18.11
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	17.57
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-12 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	18.66	1441	7.50	3.74	2.00	18.98	17.57	1495	7.49	3.69	2.03	19.24
45	20.26	1467	7.59	3.91	1.94	18.16	19.18	1519	7.59	3.88	1.95	18.30
46	21.67	1481	7.74	4.06	1.91	17.49	20.38	1516	7.83	4.05	1.93	17.53
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-12 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.75	1542	7.73	3.82	2.02	18.59
46	17.70	1435	8.54	4.01	2.13	17.71
47	--	--	--	--	--	--

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-13. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a/], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 72

Rate of Consumption Time (days)	Broiler's Body Fat (lbs)											
	.2645						.2866					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.37	1315	7.36	3.61	2.04	19.94	17.91	1344	7.50	3.60	2.08	20.00
44	20.93	1339	7.48	3.76	1.99	19.15	19.52	1371	7.59	3.77	2.01	19.10
45	22.00	1335	7.75	3.89	1.99	18.51	21.13	1400	7.68	3.92	1.96	18.36
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	17.82
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-13 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3527 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	18.45	1424	7.58	3.74	2.03	19.25	17.5	1489	7.52	3.69	2.04	19.51
45	20.04	1451	7.68	3.91	1.96	18.41	18.98	1504	7.67	3.88	1.98	18.55
46	21.57	1474	7.78	4.06	1.92	17.73	20.43	1520	7.81	4.05	1.93	17.78
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-13 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.75	1542	7.73	3.82	2.02	18.85
46	18.89	1532	8.00	4.01	1.99	17.95
47	--	--	--	--	--	--

[a] April 12, 1982.

[b] Technical feed efficiency = feed intake/broiler liveweight.

Table 5-14. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 73

	Broiler's Body Fat (lbs)											
	.2645						.2866					
Rate of Consumption Time (days)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.37	1315	7.36	3.61	2.04	20.22	17.70	1329	7.59	3.60	2.11	20.68
44	20.72	1326	7.56	3.76	2.01	19.41	19.31	1356	7.68	3.77	2.04	19.36
45	22.00	1336	7.75	3.89	1.99	18.77	20.92	1386	7.75	3.92	1.98	18.62
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	18.07
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-14 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086						.3307					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	18.24	1409	7.67	3.74	2.05	19.52	17.5	1489	7.52	3.69	2.04	19.78
45	19.82	1435	7.76	3.91	1.98	18.67	18.77	1487	7.75	3.88	1.99	18.81
46	21.34	1459	7.89	4.06	1.93	17.98	20.31	1511	7.85	4.05	1.94	18.02
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-14 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.68	1536	7.76	3.82	2.03	19.11
46	19.02	1542	7.95	4.01	1.98	18.20
47	--	--	--	--	--	--

[a/] April 12, 1982.

[b/] Technical feed efficiency = feed intake/broiler liveweight.

Table 5-15. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 74

Rate of Consump-tion Time (days)	Broiler's Body Fat (lbs) .2645 Protein (%)	Metabo-lizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Effici-ency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.2866 Protein (%)	Metabo-lizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Live-weight (lbs)	Technical Feed Effici-ency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	19.37	1315	7.36	3.61	2.04	20.50	17.53	1315	7.66	3.60	2.13	20.55
44	20.56	1315	7.62	3.76	2.03	19.68	19.10	1341	7.76	3.77	2.06	19.63
45	22.00	1335	7.75	3.89	1.99	19.02	20.73	1373	7.83	3.92	2.00	18.88
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	18.32
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-15 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	18.05	1394	7.75	3.74	2.07	19.79	17.5	1489	7.52	3.69	2.04	20.05
45	19.61	1419	7.85	3.91	2.01	18.92	18.57	1471	7.84	3.88	2.02	19.07
46	21.14	1445	7.94	4.06	1.95	18.23	20.10	1495	7.93	4.05	1.96	18.27
47	22.00	1429	8.26	4.19	1.97	17.66	21.20	1491	8.18	4.19	1.95	17.66

(Continued)

Table 5-15 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.52	1522	7.83	3.82	2.05	19.37
46	19.02	1542	7.95	4.01	1.98	18.45
47	18.92	1443	8.73	4.17	2.09	17.74

a/ April 12, 1982

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-16. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 75

	Broiler's Body Fat (lbs)											
	.2645						.2866					
Rate of Consumption Time (days)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	17.53	1315	7.66	3.60	2.13	20.83
44	20.56	1315	7.62	3.76	2.03	19.95	18.90	1327	7.84	3.77	2.08	19.89
45	22.00	1336	7.75	3.89	1.99	19.28	20.54	1361	7.90	3.92	2.01	19.13
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	18.56
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-16 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	17.85	1379	7.84	3.74	2.09	20.05	--	--	--	--	--	--
45	19.40	1404	7.93	3.91	2.03	19.18	18.37	1456	7.92	3.88	2.04	19.33
46	20.92	1430	8.03	4.06	1.98	18.47	19.90	1480	8.01	4.05	1.98	18.52
47	22.00	1429	8.26	4.19	1.97	17.90	21.25	1494	8.16	4.19	1.95	17.90

(Continued)

Table 5-16 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.5	1521	7.84	3.82	2.05	19.63
46	18.87	1530	8.01	4.01	2.00	18.70
47	20.03	1527	8.24	4.17	1.98	17.98

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-17. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 76

	Broiler's Body Fat (lbs)											
	.2645						.2866					
Rate of Consumption Time (days)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	20.56	1315	7.62	3.76	2.03	20.21	18.73	1315	7.92	3.77	2.10	20.16
45	21.84	1326	7.81	3.89	2.01	19.53	20.35	1348	7.97	3.92	2.03	19.39
46	--	--	--	--	--	--	22.00	1384	8.01	4.04	1.98	18.81
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-17 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency b/ (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	17.66	1364	7.92	3.74	1.92	20.32	--	--	--	--	--	--
45	19.20	1390	8.01	3.91	2.05	19.44	18.18	1440	8.01	3.88	2.06	19.59
46	20.70	1415	8.11	4.06	2.00	18.72	19.69	1465	8.10	4.05	2.00	18.76
47	22.00	1429	8.26	4.19	1.97	18.14	21.15	1487	8.20	4.19	1.96	18.14

(Continued)

Table 5-17 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	17.50	1521	7.84	3.82	2.05	19.89
46	18.68	1515	8.09	4.01	2.02	18.95
47	20.07	1531	8.22	4.17	1.97	18.22

a/ April 12, 1982.

b/ Technical feed efficiency = feed intake/broiler liveweight.

Table 5-18. Results of Using Production Response from Experiment Three, Selected Feed Ingredients Prices[a/], Biological Constraints, Time Constraint and Fat Constraint Feed Cost/Broiler = 77

	Broiler's Body Fat (lbs)											
	.2645						.2866					
Rate of Consumption Time (days)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b/] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	--	--	--	--	--	--	18.73	1315	7.92	3.77	2.10	20.42
45	21.68	1316	7.86	3.89	2.02	19.79	20.17	1336	8.04	3.92	2.05	19.64
46	--	--	--	--	--	--	21.81	1372	8.08	4.04	2.00	19.06
47	--	--	--	--	--	--	--	--	--	--	--	--

(Continued)

Table 5-18 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3086 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)	.3307 Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--	--	--	--	--	--	--
44	17.50	1351	8.00	3.74	2.14	20.59	--	--	--	--	--	--
45	19.00	1375	8.10	3.91	2.07	19.69	17.99	1425	8.09	3.88	2.08	19.84
46	20.48	1400	8.19	4.06	2.02	18.96	19.49	1450	8.18	4.05	2.02	19.01
47	21.96	1426	8.28	4.19	1.98	18.38	20.94	1472	8.28	4.19	1.98	18.38

(Continued)

Table 5-18 (Continued)

Rate of Consumption Time (days)	Broiler's Body Fat (lbs) .3527					
	Protein (%)	Metabolizable Energy (Kcal/lb)	Feed Intake (lbs)	Broiler Liveweight (lbs)	Technical Feed Efficiency[b] (ratio)	Feed Cost/Pound Broiler (Cents/lb)
43	--	--	--	--	--	--
44	--	--	--	--	--	--
45	--	--	--	--	--	--
46	18.49	1499	8.17	4.01	2.04	19.20
47	19.96	1522	8.27	4.17	1.98	18.46

a/ April

b/ Technical feed efficiency = feed intake/broiler liveweight.

feed ingredient prices on April 12, 1982. Each of these was made for a given level of feed cost. As in the previous results, time was fixed at 43, 44, 45, 46 and 47 days and fat was fixed at .2645, .2866, .3086, .3307 and .3527 pounds.

Important results in Table 5-10 through 5-18 show both the effect on broiler growth from reducing time to grow a broiler at a fixed amount of fat and from reducing amount of fat in the bird's carcass given a fixed length of time.

Assume feed cost at 69 cents (Table 5-10). At any given level of fat, the more time allowed for growing a broiler, the heavier the birds. As a consequence, a lower feed cost/pound broiler is obtained. That is, to keep fat constant but to increase time and feed intake, there is a trade off between protein and energy that brings the solution to a higher isoquant and improve efficiency in cost of feed. Then, if a level of fat is selected, better economic results are obtained from longer growing periods, as far as only feed cost is concerned. Suppose now that time is fixed at some length and fat is allowed to vary. In this situation, as fat increases, broiler liveweight also increases and feed cost/pound broiler decreases up to a certain level of fat and then start to decrease and increase, respectively. For example, consider feed cost and time at 69 cents and 45 days respectively. As fat increases from .2645 to .3527, broiler liveweight increases and feed cost/pound broiler decreases only up to .2866 pounds of fat level, after then, feed cost/pound broiler increases. Again, the reason for that is the occurence of trade off between protein and energy. At low levels of fat, protein is high and energy is low. As more fat is added to the bird, protein and energy levels are progressively decreased and increased respectively, which is biologically consistent. Consider now the extreme case of 77 cents feed cost and hold time fixed at 46 days. Then, the most efficient level of fat that yields the lowest feed cost/pound broiler occurs at .3086 pounds. In conclusion, at given fixed levels of feed cost and time, there exist the most efficient level of fat which is some level between the lowest and the highest level that were analyzed.

It is also observed in the results of Table 5-10 through 5-18 that certain combinations of fat level and length of time were impossible to obtain. At any level of feed cost, it was either not possible to produce an extremely lean bird in more than 45 days or to produce an extremely fat bird in less than 45 days. In both situations, the fat and time constraints could not be satisfied simultaneously.

Unrestricted Protein and Energy Levels

The main goal of this work was to formulate economically efficient rations for broilers. Attaining economic efficiency in production should be the primary objective of any firm. In an attempt to show the possibility of improvement in broiler production from the economic standpoint, an application of the model without restrictions on any feed ingredient or on protein and energy was performed. However, restrictions on other nutrients were kept. The response function was from experiment three. April 12 feed ingredients prices were used and production was 72 cents feed cost per bird, as these seemed to be representative of recent industry condition. The least cost results were:

Protein = 24.43%

Metabolizable Energy = 1437 Kcal/lb.

Feed Intake = 8.80 lbs.

Broiler Liveweight = 4.42 lbs.

Technical Feed Efficiency = 1.99

Feed Cost/Pound Broiler = 16.29

Rate of Consumption-Time = 49.7 days

Compared to the results in Table 5-4, where protein and energy are constrained, at 72 cents feed cost the partially unconstrained model certainly provided a heavier bird at a cost per pound advantage of 1.14 cents per pound. However, it took two days longer to obtain the larger liveweight. Since the exact value of time in broiler operation was not available, the results of the constrained and unconstrained models are not strictly comparable.

Using the data, another unconstrained model, having only a feed cost constraint, was analyzed. In this model, prices of corn and soybean were increased and decreased parametrically from the original price by .45 cents/lb. up to a 1.35 cents total increase and .90 cent total decrease. Results of this model provided estimates of economic efficient levels of protein and energy, since the model causes an isoquant (is forced) to be tangent to the isocost line. Also, changing the prices of corn (most frequently used source of energy) and soybean (most frequently used source of protein) served as a basis for investigating trade-off effect between protein and energy as their prices change.

Table 5-19 displays the results of the trade-off analysis. Compared to the results in Table 5-4 and the results of the partially unconstrained model at the original prices of corn and soybean higher levels of protein and energy and a heavier bird were obtained. These results reflect the economically efficient point of production. However, 53 days were needed to obtain the 4.84 pounds birds which is 3 days longer than the time required in the partially unrestricted model to produce a 4.42 pounds bird.

Results of the Table 5-19 are consistent. If prices of corn or soybean or both increase, the isocost line shifts down as long as the feed ingredient is in the solution. If either corn or soybean is not in the

Table 5-19. I put-Output Results of the Unrestricted Model for Protein (P) Metabolizable Energy (M.E.) and Broiler Liveweight (LWT) from Varying Corn and Soybean Meal Prices, Using Experiment Three, and April 12, 1982 Prices

Price of Corn (cents/lb)	Input Output[a]	Price of Soybean Meal (cents/lb.) 10.35	10.80	11.25	11.70	12.15	12.60
4.53	P	22.31	22.21	22.10	21.99	21.87	21.87
	M.E.	1531	1531	1530	1528	1525	1525
	LWT	4.98	4.98	4.98	4.98	4.98	4.98
4.98	P	22.64	22.53	22.42	22.29	22.19	22.19
	M.E.	1527	1527	1525	1523	1521	1521
	LWT	4.90	4.90	4.90	4.90	4.90	4.90
5.43[b]	P	22.94	22.83	22.71	22.58	22.48	22.48
	M.E.	1519	1519	1517	1515	1513	1513
	LWT	4.84	4.84	4.84	4.84	4.84	4.84
5.88	P	23.21	23.10	22.98	22.85	22.75	22.75
	M.E.	1508	1507	1506	1504	1502	1502
	LWT	4.79	4.79	4.79	4.79	4.79	4.79
6.33	P	23.43	22.10	22.98	22.85	22.75	22.75
	M.E.	1494	1507	1506	1504	1502	1502
	LWT	4.77	4.77	4.77	4.77	4.77	4.77
6.78	P	23.61	23.33	23.20	23.07	22.97	22.97
	M.E.	1478	1493	1492	1490	1488	1488
	LWT	4.75	4.75	4.75	4.75	4.75	4.75

[a] Units are percent protein (P), Kilocalories of energy (M.E.) and Pounds of liveweight (LWT).

[b] Average price levels of Spring, 1982.

solution, its price change can only affect the level of output (broiler liveweight) to the point where it is excluded from the ration. Consider soybean price at 11.25 cents/lb. As corn price increases, the isocost line rotates and touchs a lower isoquant. Another cent increase in the price of corn (above 6.79) excludes it from the solution; therefore, the isocost is not affected from high price of corn.

A final analysis was made to evaluate current industry specifications of P and E. Protein and energy levels from the linear programming solution, i.e., 21.7% and 1480 Kcal/lb. (Table 3-1) were analyzed in two separate models. In the first of these models protein and energy restrictions were fixed at industry levels and feed cost was set at 72 cents/brolier. These were the only constraints in the model.

The results were:

Protein = 21.7%

Metabolizable Energy = 1480 Kcal/lb.

Feed Intake = 7.88 lbs.

Broiler Liveweight = 4.82 lbs.

Technical feed efficiency = 2.14

Feed Cost/Pound Broiler = 14.94 cents/lb.

Rate of Consumption-Time = 52.8 days

Compared to the underlined results of Table 5-19 (row 3-Column 3) birds were lighter as a consequence of the imposition of the protein and energy restrictions, but, the results were very close to the economic efficient point production.

In the second model, those levels of protein and energy were applied to the restricted model of Table 5-4. The results were:

Protein = 21.7%

Metabolizable Energy = 1480 Kcal/lb.

Feed Intake = 8.07 lbs.

Broiler Liveweight = 4.12 lbs.

Technical Feed Efficiency = 1.96

Feed Cost/Pound Broiler = 17.47

Rate of Consumption-Time = 46.8 days

With respect to broiler liveweight and time, three is very slight difference between the above results and the ones in Table 5-4. Concerning protein and energy levels difference, the results in Table 5-4 would produce better quality birds, since the levels of protein and energy are higher and lower respectively.

CHAPTER VI

CONCLUSIONS

Objectives of the thesis were pursued based on estimations of production responses, feed consumption rate and fat response. The response data were incorporated into a quadratic programming model of restricted economic and biological efficiency. The good statistical estimations of responses from a feeding experiment designed for the study projects good reliability of inferences made on broiler liveweight, feed consumption over time and broiler carcass fat. The best production response for broiler was broiler liveweight characterized as a quadratic function of protein and energy. Therefore, quadratic programming was the basic technique used to find optimum operational points in broiler production. Optimum production points were found from maximizing production (broiler liveweight) given a fixed level of cost (feed cost/broiler) and a set of inequality constraints on nutrients and feed ingredients. Economic theory was used to show that such a model will project cost per pound of broiler production within specified time intervals and for given levels of broiler quality as measured by broiler carcass fat. However, because of inequality constraints in the model, strict economic efficient levels of protein and metabolizable energy used in production cannot be attained. Several constraints such as calcium, fiber, phosphorus, etc. are necessary for adequate chicken growth. Application of the technique should provide results closer to economically efficient solutions that will be found in the current techniques of linear programming. Although, levels of protein and energy currently used in least cost feed mix problems prove to be well within economically efficient ranges.

From the objectives that were proposed and studied and given that broilers are currently grown for varying lengths of time and marketed at average liveweights varying from 3.65 to 4.08 pounds, the major conclusions of this study are:

a) Finishing rations should vary in nutrient composition with size of the bird desired. This is consistent with selecting a point on the expansion path of production response. Using data from either of two experimental designs (called experiment two or experiment three), the ration change was characterized by an increase in the metabolizable energy of 2 to 3 kcal/lb for each additional cent in the feed cost/broiler and consequent increase in bird weight. The protein level remained constant at the upper limit allowed (22%) at all values of feed cost/broiler.

b) Changes in feed ingredients prices in the period between April 12 and May 31, 1982 did not change the level of protein (remaining at 22% at all levels of feed cost/ broiler), but did reduce the efficient level of metabolizable energy by 4 to 7 kcal/lb of feed. The ingredients prices in the period did not change enough to significantly affect the ration composition. From April 12 to May 31 the price of corn increased 2.6% and price of soybean meal decreased by only .72%. Although these two feed ingredients correspond to about 72% of the ration, the small price change affected only the reduced level of metabolizable energy and would have reduced the least cost weight of broiler by only .08 pounds at a common level of industry feed inputs (72 cents feed cost/broiler). Large effects of changes in corn and soybean meal prices were documented in a trade off analysis of protein and energy use (Table 5-19). Based on the price set of April 12, 1982 where corn and soybean meal were 5.43 and 11.25 cents/lb. (underlined values in Table 5-19), price of corn and soybean meal were parameterically increased and decreased by a rate of .45 cents/lb. The increase and decrease in corn price were from 5.43 to 6.78 cents/lb and from 5.43 to <u>4.53</u> cents/lb respectively. The price of soybean was increased from 11.25 to <u>12.60</u> cents/lb and decreased from 11.25 to <u>10.35</u> cents/lb. Changing the price of soybean meal by an increment of .45 cents/lb in the range of 10.35 to 12.15 cents/lb, with the price of corn fixed at 5.93 cents/lb, caused decreased of .10% of protein and 2 kcal/lb of metabolizable energy. Liveweight remained constant at 4.84 pounds. On the other hand, when price of soybean meal is fixed at 11.25 cents/lb, and price of corn is increased from 4.53 to 6.33 cents/lb by increments of .45 cents/lb, the protein level is increased gradually by .10% and the level of metabolizable energy is progressively decreased by 5, 8, 11, and 14 kcal/lb. Liveweight also progressively is decreased by .08, .06, .05 and .02 pounds. Therefore, prices of corn and soybean meal may incicate levels of protein and metabolizable energy and broiler liveweight if prices of other feed ingredients do not change.

c) Rations formulated in summer were different from rations formulated in spring. Results of the experiment three (spring) and experiment two (summer) with either April 12 or May 31 prices indicated that experiment two provided 30 to 35 kcal/lb of metabolizable energy more than experiment three. However, feed intake was .15 lbs lower in summer than in spring. The results on feed intake were consistent with empirical finding on temperature stress on birds where feed intake is lower in warmer seasons. Biologically, the difference in metabolizable energy appears inconsistent. However, economic analysis showed that the marginal productivity of metabolizable energy intake was higher in summer than in spring, dictating a higher use with feed ingredient prices fixed. Moreover, birds from the spring experiment average, .11 lbs more than the brids from the summer experiment. In conclusion, at any given feed cost/broiler, feed cost/pound broiler was lower in spring than in summer, i.e., birds grown in spring had least cost per pound relative to birds grown in summer.

d) It is possible to specify a ration that will grow a bird in a given length of time. The ration for a specific bird size will produce a bird that has least cost per pound under that restriction. At any constrained length of time, broiler liveweight was held constant at all levels of feed cost/broiler. In 43, 44, 45, 46 and 47 days, broiler liveweight was fixed at 3.61, 3.77, 3.91, 4.05 and 4.19 pounds respectively. Therefore, if a bird's carcass fat is not a factor, for a given length of time and the respective broiler liveweight, feed cost/broiler can be reduced as a direct consequence of a trade off between protein and metabolizable energy. This trade off was such that for each one cent reduction in feed cost/broiler, protein and metabolizable can be increased by .30% and 12 kcal/lb respectively and feed intake decreased .08 to .09 pounds, so that liveweight remained unchanged. This is so because in the restricted model, production occurs in an economically inefficient point. Then, as feed cost line shifts down different points on the same isoquant are selected. Certainly, there was a limit in the trade off. In the longest length of time of 47 days a 4.19 pounds broiler could not be produced with less than 73 cents of feed cost/broiler.

e) Control of a birds carcass fat is possible by ration. Response functions estimated by the study show that lean or fat birds are produced according to the levels of protein and metabolizable energy which change with the production of a desired fat level. The two lowest levels of broiler body fat analyzed, .2645 and .2866 pounds, were certainly associated with high protein (22%) and low metabolizable energy (between 1386 and 1454 kcal/lb). At the lowest level of fat, metabolizable energy varied from 1326 to 1386 kcal/lb. which correspond to range of feed cost/broiler of 69-77 cents. However, the reduction of feed cost/broiler from 77 to 69 cents was accompanied by a reduction in broiler liveweight and feed cost/pound broiler from 3.76 to 3.66 pounds and from 20.48 to 18.85 cents/lb. respectively. As fat production was increased to the upper levels, .3086, .3307 and .3527 pounds per bird, less protein and more energy were in the ration. At 72 cents feed cost broiler of fat is increased from .3086 to .3527 pounds, protein was reduced from 21.49 to 17.95% and metabolizable energy was increased from 1493 to 1530 kcal/lb. For any given level of feed cost, the feed cost/pound broiler can be lower for a leaner bird, as a result of a trade off between protein and metabolizable energy. At 72 cents feed cost/broiler, as fat level is increased progressively from .2645 to .3527, feed cost/ pound broiler is respectively: 19.46, 18.56, 18.27, 18.46 and 18.90 cents/lb. For all levels at feed cost/broiler, fat level at .3086 pounds per bird provided the lowest feed cost/pound broiler. Therefore, producing leaner birds might meet the goals at profit maximization of the producer and satisfaction of the consumers.

f) simultaneously combining time and fat constraints in the model, any combination of fat or lean bird per length of time can be obtained within the feed cost/broiler levels that were studied. The leanest bird (.2645 pounds of fat) grown in the shortest length of time (43 days) could be produced with 69 cents feed cost/broiler weighting 3.61 pounds the bird would have a feed cost per pound of 19.11 cents. The characteristics of the ration were 19.7% protein and as low as 1338 kcal/lb of metabolizable energy. It would be possible to keep feed cost/broiler and broiler body fat at low levels of 69 to 72 cents and .2645 and .2866 respectively, while increasing length of time up to 45 days to produce heavier birds weighting 3.89 to 3.92 pounds. In the range of 69 to 72 cents feed cost/broiler, .2866 of broiler body fat provided, in general, the lowest levels of feed cost/pound broiler; for time varying from 43 to 45 days. For higher levels of feed cost/broiler, the lowest levels of feed cost/pound broiler occurred for higher level of fat and longer length of time. Feed cost/broiler of 73 cents, .3086 pounds of fat and 46 days provided the lowest level of feed cost/pound broiler which was 17.98 cents/lb.

g) There was indication that the right hand side for protein (21.7%) and metabolizable energy (1480 kcal/lb.) used in linear programming feed formulations by the broiler firm studied is not far from the economic efficient point of production found without nutrient restrictions. By entering those levels of protein and metabolizable energy in the right hand side of a protein-energy restricted model and using 72 cents feed cost/broiler, the result was 4.82 pounds broiler while the economic efficient point of production, using same data, occurred at 4.84 pounds broiler. Now, entering those values of protein and metabolizable energy in the general restricted (model) the result was a 4.12 pounds broiler. Without these restrictions, it was demonstrated (Table 5-4) that a 4.13 pounds broiler and 22% protein and 1441 kcal/lb. metabolizable energy could be obtained. The lower level of energy in the model would provide a better quality bird.

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APPENDIXES

Appendix A. Condition for the Existence of Concave Production Function and Convexity of the Isoquants

Let $Y=f(x_1, x_2 \ldots, x_n)$ be a general production function for n variable inputs to be maximized subject to a constraint that $g(x_1, x_2 \ldots, x_n)=0$. Then, for this constrained optimization problem, the function of f is said to be a strict concave function if the determinants of the principal minors of the following bordered hessian matrix alternate in sign starting with plus. That is,

$$\begin{bmatrix} f_{11} & f_{12} \cdots f_{1n} & g_1 \\ f_{21} & f_{22} \cdots f_{2n} & g_2 \\ \cdots & \cdots & \cdots \\ f_{n1} & f_{n2} \cdots f_{nn} & g_n \\ g_1 & g_2 \cdots g_n & 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{11} & f_{12} & g_1 \\ f_{21} & f_{22} & g_2 \\ g_1 & g_2 & 0 \end{bmatrix} >0; \begin{bmatrix} f_{11} & f_{12} & f_{13} & g_1 \\ f_{21} & f_{22} & f_{23} & g_2 \\ f_{31} & f_{32} & f_{33} & g_3 \\ g_1 & g_2 & g_3 & 0 \end{bmatrix} <0; \ldots$$

For a two inputs production function to be maximized subject to a given total input cost, $\overline{C}$, the lagrange function, first order conditions and the bordered hessian matrix are:

$$L=f(x_1, x_2) + \lambda(\overline{C} - \sum_{i=1}^{2} r_i x_i)$$

$$\frac{\partial L}{\partial x_i} = f_i - \lambda r_i = 0 \qquad (i=1, 2)$$

$$\frac{\partial L}{\partial \lambda} = \overline{C} - \sum_{i=1}^{2} r_i x_i = 0$$

$$H = \begin{bmatrix} f_{11} & f_{12} & -r_1 \\ f_{12} & f_{22} & -r_2 \\ -r_1 & -r_2 & 0 \end{bmatrix} \tag{1}$$

The condition for a maximum (concavity of f) is then det H > 0. Subtituting $r_1=f_1/\lambda$ and $r_2=f_2/\lambda$ into (1) and expanding det H it comes,

$$\det H = \frac{-f_1}{\lambda}\left(\frac{-f_2}{\lambda} f_{12} + \frac{f_1}{\lambda} f_{22}\right) + \frac{f_2}{\lambda}\left(\frac{-f_2}{\lambda} f_{11} + \frac{f_1}{\lambda} f_{12}\right) > 0$$

$$\det H = \frac{1}{\lambda^2}\left(f_1f_2f_{12} - f_1^2f_{22} - f_{11}f_2 + f_1f_2f_{12}\right) > 0$$

Since $\lambda^2 > 0$, the condition for det H > 0 is

$$-f_1^2f_{22} + 2f_1f_2f_{12} - f_{11}f_2^2 > 0$$

or equivalent, by multiplying through by minus 1:

$$f_1f_{22} - 2f_1^2f_2f_{12} + f_{11}f_2^2 > 0. \tag{2}$$

Let $Y=f(x_1, x_2)$ define a production function for two variable inputs. By taking the total differential of this function and equating it to zero in order to explicit the slope of an isoquant, it is then obtained:

$$dy = f_1dx_1 + f_2dx_2 = 0 \tag{3}$$

Rearrange terms in (3) and get:

$$\frac{dx_1}{dx_2} = \frac{-f_2}{f_1} \tag{4}$$

[1/]

Equation (4) represents the slope of an isoquant. Since f_1 and f_2 are positive for a strict concave function, a typical isoquant derived from a concave function up to the point of its absolute maximum is downward sloping. However, the concavity of the isoquant is given by the derivative of (4) which is:

$$\frac{d^2x_1}{dx_2^2} = \frac{-d(f_2/f_1)}{dx_2} = \frac{-\left[\left(f_{22} + f_{12}\frac{dx_1}{dx_2}\right) f_1 - \left(f_{12} + f_{11}\frac{dx_1}{dx_2}\right) f_2\right]}{f_1^2} \tag{5}$$

Substituting (4) into (5) it becomes:

$$\frac{d^2x_1}{dx_2^2} = -\frac{\left[\left(f_{22} - \frac{f_{12}f_2}{f_1}\right) f_1 - \left(f_{12} \quad \frac{f_{11}f_2}{f_1}\right) f_2\right]}{f_1^2}$$

$$= -\frac{1}{f_1^3}\left[(f_{22}f_1^2 - f_{12}f_1f_2) - (f_{12}f_1f_2 - f_{11}f_2^2\right]$$

$$= -\frac{1}{f_1^3}\left[f_{22}f_1^2 - 2f_{12}f_1f_2 + f_{11}f_2^2\right] \tag{6}$$

[1/] This ratio defines the marginal rate of technical substitution of input x_1 for input X_2.

It has been shown in equation (2) that the expression in brackets in (6) is positive. Therefore,

$$\frac{d^2x_1}{dx_2^2} < 0$$

which implies that an isoquant of a concave production function is convex to the origin.

Appendix B. Calculation of the Time Variable

The basic assumption for the calculation of time elapsed for a chicken to eat a certain amount of feed within a short period of time is that the daily consumption rate is fixed within that period. This assumption is more accurate the shorter the period of time.

The equation used for calculation of the daily feed consumption is:

$$C_{t_n} = C_{t_o} (1 + r)^{t_n - t_o}$$

where,

C_{t_n} and C_{t_o} are total feed consumption at the end of periods t_n and t_o;

r is the daily feed consumption rate (pounds per day);

t_n and t_o are total elasped time ($t_n > t_o$ and $t_n - t_o$ = three to four days at most).

The application of the formula is as follows: For a given diet and a prior fixed feed consumption levle, say $\bar{C}$, find, in the experimental data, two consecutive levels of feed consumption (C_{t_n} and C_{t_o}) that include $\bar{C}$. Then the daily feed consumption rate r* may be calculated as,

$$C_{t_n} = C_{t_o} (1 + r^*)^{t_n - t_o}$$

Applying log to both sides,

$$\log C_{t_n} = \log C_{t_o} (1 + r^*)^{t_n - t_o}$$

Using properties of log,

$$\log C_{t_n} = \log C_{t_o} + (t_n - t_o) \log (1 + r^*)$$

Rearranging terms, determine the rate as;

$$1 + r^* = (C_{t_n}/C_{t_o})^{1/t_n - t_o}$$

To find the time elasped for a chicken to eat $\overline{C} - C_{t_o}$ of feed, say t^*, use the known rate r^* back in the equation, i.e., $\overline{C} = C_{t_o} (1 + r^*)^{t^*}$

$$\log \overline{C} = \log C_{t_o} + t^* \log (1 + r^*)$$

$$t^* = \frac{\log (\overline{C}/C_{t_o}}{\log (1 + r^*)}$$

Therefore, the time elasped to consume $\overline{C}$ of feed in $t_o + t^*$. An example: Consider observed data for broilers on a particular diet (protein = 18.63% and metabolizable energy = 1486 Kcal/lb. Let $\overline{C} = 6.6139$ lbs. Two consecutive levels of feed consumption that include $\overline{C}$ are

$$C_{t_n} = C_{42} = 7.33037$$

$$C_{t_o} = C_{38} = 6.186722$$

$$t_n - t_o = 42 - 38 = 4.$$

Then, $1 + r^* = (7.3304/6.1867)^{1/4}$

$1 + r^* = 1.043319$

and the rate is .043319 and t* is,

$$t^* = \frac{.06677}{.042407} = 1.5745$$

So, the time elapsed to eat 6.6139 lbs. of feed is,

$$t_o + t^* = 38 + 1.6 + 39.6 \text{ days}$$

Finally, a check on the accuracy of r* is carried out by predicting C_{42}, say C^*_{42}, from C_{38}; that is,

$$C^*_{42} = 6.1867\ (1.043319)^4 = 7.330397$$

Hence, $C_{42} = C^*_{42} = -.000027$, which shows that the technique is very accurate. This level of accuracy was observed for all diets and levels of $\overline{C}$.

Appendix C. Prices of Available Feed Ingredients On April 12 and May 31, 1982

Feed Ingredients	Price (cents/kg) April 12	May 31
Corn (x_1)	11.9711	12.2797
Animal fat (x_3)	33.6205	34.7228
Pro pak (x_4)	37.4345	37.4347
Po tank 50 soy (x_5)	27.3594	27.4696
Meat of bone 50 (x_6)	24.802	24.6258
Soybean meal (x_8)	9.3696	9.3696
Grind limestone (x_{11})	1.9842	1.9842
Defluor phos (x_{12})	29.0128	29.0128
Salt, plain (x_{13})	6.2832	6.2832
Choline cl 35 (x_{14})	41.8878	41.8878
Methionine mha (x_{15})	264.5547	230.824
Wheat midds (x_{16})	12,4782	10.2735
Feather meal (x_{17})	27.5578	27.668
Gluten feed (x_{18})	14.1537	14.1537
Poultry by prod (x_{19})	28.5499	28.5499
Rice mil feed (x_{20})	6.3934	6.3934
Dried whey (x_{21})	28.8806	28.8806
Soy hulls (x_{22})	9.9208	9.9208
Wafer meal (x_{23})	12.1254	12.1254
Poultry tm (x_{24})	28.6601	29.5419
Broiler vit. mix (x_{25})	317.4657	303.1356
Fixed ingredient (x_{26})	264.5547	264.5503
Tm mix 430 (x_{27})	2204.58	3295.85

Appendix D. Description of a Quadratic Programming Feed Formulation Model

The purpose of the appendix is to describe and show how to set up quadratic programming for a broiler feed formulation that will yield least cost/pound of broiler under given constraints. The problem maximizes production response, w=f(P, E) subject to a set of nutrient and feed ingredients restrictions, and a given level of feed cost/broiler. For simplicity, consider only two feed ingredients, corn and soybean meal, production response of experiment three and April 12 prices. The composition of corn and soybean meal are:

	Protein (%) (P)	Metabolizable Energy (Kcal/g)[1/] (E)
Corn (x_1)	8.6	3.4392
Soybean meal (x_8)	48.5	2.425

The production response to estimate liveweight of a broiler (w*) from experiment three can be expressed in matrix notation as (See Chapter III):

$$W^* = \begin{bmatrix} 1.457695 & .109539 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} + \begin{bmatrix} P & E \end{bmatrix} \begin{bmatrix} -1.758822 & .081693 \\ .081693 & -.007404 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix}$$

[1/] To match with the results of a complete quadratic programming, metric system unit is used.

To define this function in the feed ingredient space, use the nutrient coefficients, i.e.,

$$\begin{bmatrix} P \\ E \end{bmatrix} = \begin{bmatrix} .086 & .485 \\ 3.4392 & .425 \end{bmatrix} \begin{bmatrix} x_1 \\ x_8 \end{bmatrix} \qquad (2)$$

Substituting the vector $\begin{bmatrix} P \\ E \end{bmatrix}$ or its transpose from equation two into the production response, equation one, and writing out the results, broiler liveweight is expressed as a function of corn (x_1) and soybean meal (x_8) as:

$$w^* = .502088\ x_1 + .972614\ x_8 - .052257\ x_1^2 - .265095\ x_8^2 + .01893\ x_1 x_8 \qquad (3)$$

This is the function to be maximized[1/]. Fat and time equations are also transformed from the nutrient space into feed ingredient space.

Fat equation:

$$\text{Fat} = .013651 - .088859P + .016912E$$
$$= .013651 + \begin{bmatrix} -.088859 & .016912 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} \qquad (4)$$

By substituting the vector $\begin{bmatrix} P \\ E \end{bmatrix}$ from (2) into (4) and writing the result out:

$$\text{Fat} = .013651 + .050522\ x_1 - .002085\ x_8$$

Time equation:

$$\text{Time} = 24.403404 + 10.511097P + 1.010951E$$
$$= 24.403404 + \begin{bmatrix} 10.511097 & 1.01095 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix} \qquad (5)$$

1/ The quadratic programming package (Rand QP 30) used can only be applied for minimization problems. Hence the function is multiplied by minus one and minimized without altering final results.

By substituting the vector $\begin{bmatrix} P \\ E \end{bmatrix}$ from (2) into (5) and writing the result out:

$$\text{Time} = 24.403404 + 4.380817\ x_1 + 7.549438\ x_8$$

Protein and energy constraints must likewise be transformed into feed ingredient space. Consider an example for protein less than or equal to 22% and metabolizable energy greater than or equal to 2.9 kcal/g.

For protein:

$$\frac{.08\ x_1 + .485\ x_8}{x_1 + x_8} \leq .22 \qquad (6)$$

or,

$$-.134\ x_1 + .265\ x_8 \leq 0 \qquad (7)$$

For metabolizable energy:

$$\frac{3.4392\ x_1 + 2.425\ x_8}{x_1 + x_8} \geq 2.9$$

or,

$$-.5392\ x_1 + .475\ x_8 \leq 0 \qquad (9)$$

where equation 7 and 9 are appropriate constraints for equation three.

The cost constraint at 72 cent feed cost/broiler is readily written in feed ingredient space by multiplying cents per kilograms of each ingredient times the amount of ingredient to be used.

$$11.971\ x_1 + 24.802\ x_8 = 72$$

The same general procedure would be followed if a larger number of feed ingredients is used.

Next, a print out of a complete quadratic programming run on the Rand QP 360 package is described. It was made using experiment three production response, April 12, 1982 prices and 72 cents feed cost/broiler.

There are 26 restrictions in the model named R_1, $R_2, \ldots, R_{29}$[1/] which are stated in lines 1392-1417 of the print out. A plus sign and a blank preceding the restriction name refers to less than or equal to and equality restriction type respectively. $LINEAR in Line 1391 corresponds to the linear part of the objective function. It is always, preceded by a dollar sign.

The restrictions are:

R_1: protein $\geq$ 17.5%

R_2: protein $\leq$ 22.0%

R_3: metabolizable energy $\geq$ 2.9 kcal/g

R_4: metabolizable energy $\leq$ 3.4 kcal/g

R_5: crude fat $\geq$ 4.1%

R_6: curde fiber $\geq$ 3.9%

R_7: calcium $\geq$ .8%

R_8: calcium $\leq$ .9%

R_9: available phosphous $\geq$ .4%

R_{10}: sodium $\geq$ .18%

R_{11}: sodium $\leq$ .23%

R_{12}: lysine = 4.35%

[1/] Note that R_{19}, R_{23} and R_{25} were unused.

R_{13}: methionine = 2.17%

R_{14}: methionine and eystine = 4.04%

R_{15}: choline $\geq$ 1560.87 mg/kg

R_{16}: xanthophyl $\geq$ 13.23 mg/kg

R_{17}: xanthophyl $\leq$ 32.19 mg/kg

R_{18}: feed cost = 72 cents

Restrictions R_{20} to R_{29} refers to restriction of the percentage of a feed ingredient in the ration

R_{20}: $x_3 \geq 1.0\%$

R_{21}: $x_3 \leq 6.0\%$

R_{22}: $x_5 \leq 15\%$

R_{24}: $x_6 \leq 6.0\%$

R_{26}: $x_{10} = .75\%$

R_{27}: $x_{25} = .05\%$

R_{28}: $x_{26} = .375\%$

R_{29}: $x_{27} = .05\%$

Lines 1420 to 2197 describes the coefficients of each feed ingredient in the objectives function and in the constraints. Take x_1 as an example, .502088 in line 1420 is the coefficient of x_1 in the linear part of the objective function; .089 is the coefficient of x_1 in the restriction R_1, and so on; .0522571 and .142266, in lines 1447 and 1448 are the coefficients of the terms x_1^2 and x_1x_3 in the nonlinear part of the objective function, and so on.

Lines 2200-2225 refer to the right hand side of the constraint. As shown earlier in the example for corn and soybean meal the right hand side of all constraints in zero except the feed cost constraint which in this case is 72 cents.

Lines 2232-2261 describe a SAS program. It was possible to write a program in the QP 360 package to separate the primal variable solution (feed ingredients) and store it on a disc in vector form. Hence a SAS program in matrix form was written using that solution vector to make further transformations. The SAS program is as follows:

Lines	Description
2239	A 27 x 1 vector of the feed ingredients in the solution including the ones with zero value;
2240	An 1 x 27 vector of d's;
2241-2244	A 27 x 2 matrix of coefficients of protein and metabolizable energy of the 27 feed ingredients;
2245	An 1 x 2 vector of coefficients of the linear part of the production response;
2246	A 2 x 2 matrix that defines the non-near part of the production response;
2247	A 2 x 1 vector of protein and metabolizable energy intake;

1/The author is grateful to Mr. John Mackert for writing this program.

Lines	Description
2248	Total feed intake in kilograms;
2249	A 27 x 1 vector of percentages of each feed ingredient in the ration;
2250	Calculation of broiler liveweight from the linear part of the production response;
2251	Calculation of broiler liveweight from the non-linear part of the production response;
2252	Broiler liveweight from the linear and non-linear parts of the production response;
2253	An 1 x 2 vector of protein (%) and metabolizable energy (kcal/kg);
2254	An 1 x 2 vector..4535925 is the conversion factor of kilogram to pound;
2255	Broiler liveweight in kilogram (.041988 is the intercept of the production response)[1/];
2256	Broiler liveweight in pounds;
2257	Metabolizable energy in kcal/lb;
2258	Total feed intake in pounds;
2259	An 1 x 2 vector of coefficients of the linear time equation;
2260	Estimate of time without intercept term;
2261	Estimate of time[1/];

[1/]Estimate of time can be done in a single statement by combining expressions in lines 2259-2261.

```
//NRCPX72  JOB ,'GAHLINHA',USER=AECINST,
// PASSWORD=      ,TIME=(,30),NOTIFY=AECINST
// EXEC FORTHG,REGION.GO=750K,PARM='SIZE=725000',DSET='WEBSMA.BIGQP'
//LOADERIN DD DSN=AECINST.QUADPROG,DISP=SHR
//SYSLOUT DD SYSOUT=*
//FT15F001 DD DSN=AECINST.RON(NRCPX72),DISP=OLD
//GO.SYSIN DD *
BEGIN
ROWS
           $LINEAR
           +R1
           +R2
           +R3
           +R4
           +R5
           +R6
           +R7
           +R8
           +R9
           +R10
           +R11
            R12
            R13
            R14
           +R15
           +R16
           +R17
            R18
           +R20
           +R21
           +R22
           +R24
            R26
            R27
            R28
            R29
END
MATRIX
      X1    LINEAR -.502088
      X1    R1     .089
      X1    R2     -.134
      X1    R3     -.5392
      X1    R4     .0392
      X1    R5     .6
      X1    R6     -1.4
      X1    R7     .765
      X1    R8     -.865
      X1    R9     .3
      X1    R10    .174
      X1    R11    -.224
      X1    R12    .0941
      X1    R13    -.02338
      X1    R14    -.02256
      X1    R15    965.62
      X1    R16    -8.82
      X1    R17    -10.14
      X1    R18    11.9711
      X1    R20    -.06
```

X1	R21	.01	00144000
X1	R22	-.15	00144100
X1	R24	-.06	00144200
X1	R26	-.0075	00144300
X1	R27	.0005	00144400
X1	R28	.00375	00144500
X1	R29	.0005	00144600
X1	X1	.0522571	00144700
X1	X3	.142266	00144800
X1	X4	-.0273825	00144900
X1	X5	-.00347303	00145000
X1	X6	-.0242027	00145100
X1	X8	-.0181935	00145200
X1	X10	.0104318	00145300
X1	X15	-.103761	00145400
X1	X16	.0145733	00145500
X1	X17	-.0483244	00145600
X1	X18	.00235827	00145700
X1	X19	-.0223359	00145800
X1	X20	.00306241	00145900
X1	X21	.0192648	00146000
X1	X22	-.00133754	00146100
X1	X23	.0581631	00146200
X3	LINEAR	-.845203	00146300
X3	R1	.175	00146400
X3	R2	-.22	00146500
X3	R3	-4.816	00146600
X3	R4	4.316	00146700
X3	R5	-95.9	00146800
X3	R6	-3.9	00146900
X3	R7	.8	00147000
X3	R8	-.9	00147100
X3	R9	.4	00147200
X3	R10	.18	00147300
X3	R11	-.23	00147400
X3	R12	0.	00147500
X3	R13	0.	00147600
X3	R14	0.	00147700
X3	R15	1560.87	00147800
X3	R16	13.23	00147900
X3	R17	-32.19	00148000
X3	R18	33.6205	00148100
X3	R20	.94	00148200
X3	R21	-.99	00148300
X3	R22	-.15	00148400
X3	R24	-.06	00148500
X3	R26	-.0075	00148600
X3	R27	.0005	00148700
X3	R28	.00375	00148800
X3	R29	.0005	00148900
X3	X3	.440803	00149000
X3	X4	-.223299	00149100
X3	X5	-.124998	00149200
X3	X6	-.189228	00149300
X3	X8	-.167182	00149400
X3	X10	.0163291	00149500
X3	X15	-.504278	00149600
X3	X16	.00745614	00149700

X3	X17	-.32794	00149800
X3	X18	-.0429576	00149900
X3	X19	-.206291	00150000
X3	X20	-.00444829	00150100
X3	X21	.0309028	00150200
X3	X22	-.0315613	00150300
X3	X23	.157367	00150400
X4	LINEAR	-1.1876	00150500
X4	R1	-.431	00150600
X4	R2	.386	00150700
X4	R3	.1222	00150800
X4	R4	-.6222	00150900
X4	R5	-3.9	00151000
X4	R6	-1.9	00151100
X4	R7	-5.2	00151200
X4	R8	5.1	00151300
X4	R9	-2.6	00151400
X4	R10	.18	00151500
X4	R11	-.23	00151600
X4	R12	-1.3639	00151700
X4	R13	-.35498	00151800
X4	R14	-.05176	00151900
X4	R15	1560.87	00152000
X4	R16	13.23	00152100
X4	R17	-32.19	00152200
X4	R18	37.4345	00152300
X4	R20	-.06	00152400
X4	R21	.01	00152500
X4	R22	-.15	00152600
X4	R24	-.06	00152700
X4	R26	-.0075	00152800
X4	R27	.0005	00152900
X4	R28	.00375	00153000
X4	R29	.0005	00153100
X4	X4	.427996	00153200
X4	X5	.323122	00153300
X4	X6	.355658	00153400
X4	X8	.336696	00153500
X4	X10	.0302997	00153600
X4	X15	.613368	00153700
X4	X16	.0834178	00153800
X4	X17	.534739	00153900
X4	X18	.131673	00154000
X4	X19	.393567	00154100
X4	X20	.0336245	00154200
X4	X21	.0539556	00154300
X4	X22	.0749647	00154400
X4	X23	-.00853172	00154500
X5	LINEAR	-1.0935	00154600
X5	R1	-.325	00154700
X5	R2	.28	00154800
X5	R3	-.4289	00154900
X5	R4	-.0711	00155000
X5	R5	-13.7	00155100
X5	R6	-1.4	00155200
X5	R7	-.1	00155300
X5	R8	.0	00155400
X5	R9	-.2	00155500

X5	R10	-.18	00155600
X5	R11	.13	00155700
X5	R12	-.085	00155800
X5	R13	.395	00155900
X5	R14	.48	00156000
X5	R15	-1106.68	00156100
X5	R16	13.23	00156200
X5	R17	-32.19	00156300
X5	R18	27.3594	00156400
X5	R20	-.06	00156500
X5	R21	.01	00156600
X5	R22	.85	00156700
X5	R24	-.06	00156800
X5	R26	-.0075	00156900
X5	R27	.0005	00157000
X5	R28	.00375	00157100
X5	R29	.0005	00157200
X5	X5	.249803	00157300
X5	X6	.268016	00157400
X5	X8	.255334	00157500
X5	X10	.0253796	00157600
X5	X15	.485969	00157700
X5	X16	.0686235	00157800
X5	X17	.427391	00157900
X5	X18	.106499	00158000
X5	X19	.315727	00158100
X5	X20	.027413	00158200
X5	X21	.0452549	00158300
X5	X22	.0603955	00158400
X5	X23	-.00175309	00158500
X6	LINEAR	-.970337	00158600
X6	R1	-.325	00158700
X6	R2	.28	00158800
X6	R3	.6954	00158900
X6	R4	-1.1954	00159000
X6	R5	-5.9	00159100
X6	R6	-1.4	00159200
X6	R7	-7.2	00159300
X6	R8	7.1	00159400
X6	R9	-3.1	00159500
X6	R10	-.55	00159600
X6	R11	.5	00159700
X6	R12	-.225	00159800
X6	R13	.54	00159900
X6	R14	.92	00160000
X6	R15	-621.67	00160100
X6	R16	13.23	00160200
X6	R17	-32.19	00160300
X6	R18	23.6997	00160400
X6	R20	-.06	00160500
X6	R21	.01	00160600
X6	R22	-.15	00160700
X6	R24	.94	00160800
X6	R26	-.0075	00160900
X6	R27	.0005	00161000
X6	R28	.00375	00161100
X6	R29	.0005	00161200
X6	X6	.295589	00161300

X6	X8	.279694	00161400
X6	X10	.0230003	00161500
X6	X15	.559448	00161600
X6	X16	.0675371	00161700
X6	X17	.475175	00161800
X6	X18	.112758	00161900
X6	X19	.345785	00162000
X6	X20	.0280611	00162100
X6	X21	.040752	00162200
X6	X22	.0649943	00162300
X6	X23	-.0246831	00162400
X8	LINEAR	-.972614	00162500
X8	R1	-.31	00162600
X8	R2	.265	00162700
X8	R3	.475	00162800
X8	R4	-.975	00162900
X8	R5	3.2	00163000
X8	R6	-.5	00163100
X8	R7	.6	00163200
X8	R8	-.7	00163300
X8	R9	.1	00163400
X8	R10	.172	00163500
X8	R11	-.222	00163600
X8	R12	-1.05025	00163700
X8	R13	.32945	00163800
X8	R14	.4894	00163900
X8	R15	-1205.88	00164000
X8	R16	13.23	00164100
X8	R17	-32.19	00164200
X8	R18	24.802	00164300
X8	R20	-.06	00164400
X8	R21	.01	00164500
X8	R22	-.15	00164600
X8	R24	-.06	00164700
X8	R26	-.0075	00164800
X8	R27	.0005	00164900
X8	R28	.00375	00165000
X8	R29	.0005	00165100
X8	X8	.265095	00165200
X8	X10	.0229166	00165300
X8	X15	.523938	00165400
X8	X16	.0657878	00165500
X8	X17	.448742	00165600
X8	X18	.10778	00165700
X8	X19	.327751	00165800
X8	X20	.0270541	00165900
X8	X21	.0406771	00166000
X8	X22	.0618725	00166100
X8	X23	-.0180987	00166200
X10	LINEAR	-.217953	00166300
X10	R1	.105	00166400
X10	R2	-.15	00166500
X10	R3	1.8418	00166600
X10	R4	-2.3418	00166700
X10	R5	-.8	00166800
X10	R6	-2.4	00166900
X10	R7	.66	00167000
X10	R8	-.76	00167100

X10	R9	.31	00167200
X10	R10	.07	00167300
X10	R11	-.12	00167400
X10	R12	.0645	00167500
X10	R13	-.0381	00167600
X10	R14	-.0072	00167700
X10	R15	877.45	00167800
X10	R16	13.23	00167900
X10	R17	-32.19	00168000
X10	R18	9.3697	00168100
X10	R20	-.06	00168200
X10	R21	.01	00168300
X10	R22	-.15	00168400
X10	R24	-.06	00168500
X10	R26	.9925	00168600
X10	R27	.0005	00168700
X10	R28	.00375	00168800
X10	R29	.0005	00168900
X10	X10	.00480629	00169000
X10	X15	.0293356	00169100
X10	X16	.0101795	00169200
X10	X17	.0346674	00169300
X10	X18	.0116131	00169400
X10	X19	.0283702	00169500
X10	X20	.00349644	00169600
X10	X21	.00870728	00169700
X10	X22	.00603325	00169800
X10	X23	.0118315	00169900
X11	R1	.175	00170000
X11	R2	-.22	00170100
X11	R3	2.9	00170200
X11	R4	-3.4	00170300
X11	R5	4.1	00170400
X11	R6	-3.9	00170500
X11	R7	-37.2	00170600
X11	R8	37.1	00170700
X11	R9	.4	00170800
X11	R10	.18	00170900
X11	R11	-.23	00171000
X11	R12	.0	00171100
X11	R13	.0	00171200
X11	R14	.0	00171300
X11	R15	1560.87	00171400
X11	R16	13.23	00171500
X11	R17	-32.19	00171600
X11	R18	1.984	00171700
X11	R20	-.06	00171800
X11	R21	.01	00171900
X11	R22	-.15	00172000
X11	R24	-.06	00172100
X11	R26	-.0075	00172200
X11	R27	.0005	00172300
X11	R28	.00375	00172400
X11	R29	.0005	00172500
X12	R1	.175	00172600
X12	R2	-.22	00172700
X12	R3	2.9	00172800
X12	R4	-3.4	00172900

X12	R5	4.1	00173000
X12	R6	-3.9	00173100
X12	R7	-31.2	00173200
X12	R8	31.1	00173300
X12	R9	-17.6	00173400
X12	R10	-5.32	00173500
X12	R11	5.27	00173600
X12	R12	.0	00173700
X12	R13	.0	00173800
X12	R14	.0	00173900
X12	R15	1560.87	00174000
X12	R16	13.23	00174100
X12	R17	-32.19	00174200
X12	R18	29.013	00174300
X12	R20	-.06	00174400
X12	R21	.01	00174500
X12	R22	-.15	00174600
X12	R24	-.06	00174700
X12	R26	-.0075	00174800
X12	R27	.0005	00174900
X12	R28	.00375	00175000
X12	R29	.0005	00175100
X13	R1	.175	00175200
X13	R2	-.22	00175300
X13	R3	2.9	00175400
X13	R4	-3.4	00175500
X13	R5	4.1	00175600
X13	R6	-3.9	00175700
X13	R7	.8	00175800
X13	R8	-.9	00175900
X13	R9	.4	00176000
X13	R10	-38.42	00176100
X13	R11	38.37	00176200
X13	R12	.0	00176300
X13	R13	.0	00176400
X13	R14	.0	00176500
X13	R15	1560.87	00176600
X13	R16	13.23	00176700
X13	R17	-32.19	00176800
X13	R18	6.2832	00176900
X13	R20	-.06	00177000
X13	R21	.01	00177100
X13	R22	-.15	00177200
X13	R24	-.06	00177300
X13	R26	-.0075	00177400
X13	R27	.0005	00177500
X13	R28	.00375	00177600
X13	R29	.0005	00177700
X14	R1	.175	00177800
X14	R2	-.22	00177900
X14	R3	2.9	00178000
X14	R4	-3.4	00178100
X14	R5	4.1	00178200
X14	R6	-3.9	00178300
X14	R7	.8	00178400
X14	R8	-.9	00178500
X14	R9	.4	00178600
X14	R10	.18	00178700

X14	R11	-.23	00178800
X14	R12	.0	00178900
X14	R13	.0	00179000
X14	R14	.0	00179100
X14	R15	-260784.81	00179200
X14	R16	13.23	00179300
X14	R17	-32.19	00179400
X14	R18	41.8878	00179500
X14	R20	-.06	00179600
X14	R21	.01	00179700
X14	R22	-.15	00179800
X14	R24	-.06	00179900
X14	R26	-.0075	00180000
X14	R27	.0005	00180100
X14	R28	.00375	00180200
X14	R29	.0005	00180300
X15	LINEAR	-1.16616	00180400
X15	R1	-.625	00180500
X15	R2	.58	00180600
X15	R3	2.9	00180700
X15	R4	-3.4	00180800
X15	R5	4.1	00180900
X15	R6	-3.9	00181000
X15	R7	.8	00181100
X15	R8	-.9	00181200
X15	R9	.4	00181300
X15	R10	.18	00181400
X15	R11	-.23	00181500
X15	R12	3.48	00181600
X15	R13	-78.264	00181700
X15	R14	-76.768	00181800
X15	R15	1560.87	00181900
X15	R16	13.23	00182000
X15	R17	-32.19	00182100
X15	R18	264.5547	00182200
X15	R20	-.06	00182300
X15	R21	.01	00182400
X15	R22	-.15	00182500
X15	R24	-.06	00182600
X15	R26	-.0075	00182700
X15	R27	.0005	00182800
X15	R28	.00375	00182900
X15	R29	.0005	00183000
X15	X15	1.12565	00183100
X15	X16	.104651	00183200
X15	X17	.910198	00183300
X15	X18	.200051	00183400
X15	X19	.647562	00183500
X15	X20	.0469313	00183600
X15	X21	.0510761	00183700
X15	X22	.118419	00183800
X15	X23	-.111433	00183900
X16	LINEAR	-.454241	00184000
X16	R1	.01	00184100
X16	R2	-.055	00184200
X16	R3	.9489	00184300
X16	R4	-1.4489	00184400
X16	R5	-.1	00184500

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X16   R6      4.1                00184600
X16   R7      .66                00184700
X16   R8      -.77               00184800
X16   R9      .17                00184900
X16   R10     .175               00185000
X16   R11     -.225              00185100
X16   R12     .14775             00185200
X16   R13     .21705             00185300
X16   R14     .3686              00185400
X16   R15     485.03             00185500
X16   R16     13.23              00185600
X16   R17     -32.19             00185700
X16   R18     12.4782            00185800
X16   R20     -.06               00185900
X16   R21     .01                00186000
X16   R22     -.15               00186100
X16   R24     -.06               00186200
X16   R26     -.0075             00186300
X16   R27     .0005              00186400
X16   R28     .00375             00186500
X16   R29     .0005              00186600
X16   X16     .0234696           00186700
X16   X17     .104804            00186800
X16   X18     .030398            00186900
X16   X19     .081396            00187000
X16   X20     .00855477          00187100
X16   X21     .0183487           00187200
X16   X22     .0164432           00187300
X16   X23     .0168094           00187400
X17   LINEAR  -1.45094           00187500
X17   R1      -.605              00187600
X17   R2      .56                00187700
X17   R3      .034               00187800
X17   R4      -.534              00187900
X17   R5      -5.9               00188000
X17   R6      -2.4               00188100
X17   R7      .6                 00188200
X17   R8      -.7                00188300
X17   R9      -.35               00188400
X17   R10     -.52               00188500
X17   R11     .47                00188600
X17   R12     2.393              00188700
X17   R13     1.3596             00188800
X17   R14     .8012              00188900
X17   R15     679.04             00189000
X17   R16     13.23              00189100
X17   R17     -32.19             00189200
X17   R18     27.5578            00189300
X17   R20     -.06               00189400
X17   R21     .01                00189500
X17   R22     -.15               00189600
X17   R24     -.06               00189700
X17   R26     -.0075             00189800
X17   R27     .0005              00189900
X17   R28     .00375             00190000
X17   R29     .0005              00190100
X17   X17     .765634            00190200
X17   X18     .179093            00190300
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X17    X19      .554749                                                 00190400
X17    X20      .0441058                                                00190500
X17    X21      .0612776                                                00190600
X17    X22      .103736                                                 00190700
X17    X23      -.0501955                                               00190800
X18    LINEAR   -.504225                                                00190900
X18    R1       -.045                                                   00191000
X18    R2       .0                                                      00191100
X18    R3       1.2245                                                  00191200
X18    R4       -1.7245                                                 00191300
X18    R5       2.4                                                     00191400
X18    R6       5.6                                                     00191500
X18    R7       .5                                                      00191600
X18    R8       -.6                                                     00191700
X18    R9       .19                                                     00191800
X18    R10      .173                                                    00191900
X18    R11      -.223                                                   00192000
X18    R12      .687                                                    00192100
X18    R13      .2664                                                   00192200
X18    R14      .4228                                                   00192300
X18    R15      41.91                                                   00192400
X18    R16      -13.225                                                 00192500
X18    R17      -5.735                                                  00192600
X18    R18      14.1537                                                 00192700
X18    R20      -.06                                                    00192800
X18    R21      .01                                                     00192900
X18    R22      -.15                                                    00193000
X18    R24      -.06                                                    00193100
X18    R26      -.0075                                                  00193200
X18    R27      .0005                                                   00193300
X18    R28      .00375                                                  00193400
X18    R29      .0005                                                   00193500
X18    X18      .0456859                                                00193600
X18    X19      .133284                                                 00193700
X18    X20      .0119402                                                00193800
X18    X21      .0207563                                                00193900
X18    X22      .0257118                                                00194000
X18    X23      .00352756                                               00194100
X19    LINEAR   -1.20425                                                00194200
X19    R1       .425                                                    00194300
X19    R2       .38                                                     00194400
X19    R3       -.1093                                                  00194500
X19    R4       -.3907                                                  00194600
X19    R5       -7.9                                                    00194700
X19    R6       -1.4                                                    00194800
X19    R7       -2.7                                                    00194900
X19    R8       2.6                                                     00195000
X19    R9       -1.6                                                    00195100
X19    R10      -1.07                                                   00195200
X19    R11      1.02                                                    00195300
X19    R12      .04                                                     00195400
X19    R13      .182                                                    00195500
X19    R14      .324                                                    00195600
X19    R15      -4391.51                                                00195700
X19    R16      13.23                                                   00195800
X19    R17      -32.19                                                  00195900
X19    R18      28.5499                                                 00196000
X19    R20      -.06                                                    00196100
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X19	R21	.01	00196200
X19	R22	-.15	00196300
X19	R24	-.06	00196400
X19	R26	-.0075	00196500
X19	R27	.0005	00196600
X19	R28	.00375	00196700
X19	R29	.0005	00196800
X19	X19	.405216	00196900
X19	X20	.0334636	00197000
X19	X21	.0503594	00197100
X19	X22	.0765052	00197200
X19	X23	-.0222004	00197300
X20	LINEAR	-.154117	00197400
X20	R1	.114	00197500
X20	R2	-.159	00197600
X20	R3	2.3048	00197700
X20	R4	-2.8048	00197800
X20	R5	-1.3	00197900
X20	R6	23.5	00198000
X20	R7	.72	00198100
X20	R8	-.82	00198200
X20	R9	.34	00198300
X20	R10	.14	00198400
X20	R11	-.19	00198500
X20	R12	.12535	00198600
X20	R13	.06237	00198700
X20	R14	.11644	00198800
X20	R15	1119.95	00198900
X20	R16	13.23	00199000
X20	R17	-32.19	00199100
X20	R18	6.3934	00199200
X20	R20	-.06	00199300
X20	R21	.01	00199400
X20	R22	-.15	00199500
X20	R24	-.06	00199600
X20	R26	-.0075	00199700
X20	R27	.0005	00199800
X20	R28	.00375	00199900
X20	R29	.0005	00200000
X20	X20	.00323538	00200100
X20	X21	.00627834	00200200
X20	X22	.00659487	00200300
X20	X23	.00364227	00200400
X21	LINEAR	-.395211	00200500
X21	R1	.049	00200600
X21	R2	-.094	00200700
X21	R3	.9688	00200800
X21	R4	-1.4688	00200900
X21	R5	3.8	00201000
X21	R6	-3.1	00201100
X21	R7	-.06	00201200
X21	R8	-.04	00201300
X21	R9	.19	00201400
X21	R10	-.25	00201500
X21	R11	.2	00201600
X21	R12	-.4019	00201700
X21	R13	.10942	00201800
X21	R14	.03004	00201900

X21	R15	205.05	00202000
X21	R16	13.23	00202100
X21	R17	-32.19	00202200
X21	R18	28.8806	00202300
X21	R20	-.06	00202400
X21	R21	.01	00202500
X21	R22	-.15	00202600
X21	R24	-.06	00202700
X21	R26	-.0075	00202800
X21	R27	.0005	00202900
X21	R28	.00375	00203000
X21	R29	.0005	00203100
X21	X21	.015779	00203200
X21	X22	.0107517	00203300
X21	X23	.0218359	00203400
X22	LINEAR	-.259444	00203500
X22	R1	.055	00203600
X22	R2	-.1	00203700
X22	R3	2.1284	00203800
X22	R4	-2.6284	00203900
X22	R5	2.	00204000
X22	R6	29.1	00204100
X22	R7	.75	00204200
X22	R8	-.85	00204300
X22	R9	.35	00204400
X22	R10	-.07	00204500
X22	R11	.02	00204600
X22	R12	.522	00204700
X22	R13	.2604	00204800
X22	R14	.4848	00204900
X22	R15	1560.87	00205000
X22	R16	13:23	00205100
X22	R17	-32.19	00205200
X22	R18	9.9208	00205300
X22	R20	-.06	00205400
X22	R21	.01	00205500
X22	R22	-.15	00205600
X22	R24	-.06	00205700
X22	R26	-.0075	00205800
X22	R27	.0005	00205900
X22	R28	.00375	00206000
X22	R29	.0005	00206100
X22	X22	.0146067	00206200
X22	X23	-.000978256	00206300
X23	LINEAR	-.568371	00206400
X23	R1	.075	00206500
X23	R2	-.12	00206600
X23	R3	-.958	00206700
X23	R4	.458	00206800
X23	R5	-5.9	00206900
X23	R6	-2.6	00207000
X23	R7	.7	00207100
X23	R8	-.8	00207200
X23	R9	.21	00207300
X23	R10	-1.17	00207400
X23	R11	1.12	00207500
X23	R12	.115	00207600
X23	R13	.057	00207700

X23	R14	.064	00207800
X23	R15	-26.43	00207900
X23	R16	-.44	00208000
X23	R17	-18.52	00208100
X23	R18	12.1254	00208200
X23	R20	-.06	00208300
X23	R21	.01	00208400
X23	R22	-.15	00208500
X23	R24	-.06	00208600
X23	R26	-.0075	00208700
X23	R27	.0005	00208800
X23	R28	.00375	00208900
X23	R29	.0005	00209000
X23	X23	.0647544	00209100
X24	LINEAR	.0	00209200
X24	R1	.175	00209300
X24	R2	-.22	00209400
X24	R3	2.9	00209500
X24	R4	-3.4	00209600
X24	R5	4.1	00209700
X24	R6	-3.9	00209800
X24	R7	-19.2	00209900
X24	R8	19.1	00210000
X24	R9	.4	00210100
X24	R10	.18	00210200
X24	R11	-.23	00210300
X24	R12	.0	00210400
X24	R13	.0	00210500
X24	R14	.0	00210600
X24	R15	1560.87	00210700
X24	R16	13.23	00210800
X24	R17	-32.19	00210900
X24	R18	28.6601	00211000
X24	R20	-.06	00211100
X24	R21	.01	00211200
X24	R22	-.15	00211300
X24	R24	-.06	00211400
X24	R26	-.0075	00211500
X24	R27	.0005	00211600
X24	R28	.00375	00211700
X24	R29	.0005	00211800
X24	X24	.0	00211900
X25	R1	.175	00212000
X25	R2	-.22	00212100
X25	R3	2.9	00212200
X25	R4	-3.4	00212300
X25	R5	4.1	00212400
X25	R6	-3.9	00212500
X25	R7	.8	00212600
X25	R8	-.9	00212700
X25	R9	.4	00212800
X25	R10	.18	00212900
X25	R11	-.23	00213000
X25	R12	.0	00213100
X25	R13	.0	00213200
X25	R14	.0	00213300
X25	R15	1560.87	00213400
X25	R16	13.23	00213500

X25	R17	-32.19	00213600
X25	R18	317.4657	00213700
X25	R20	-.06	00213800
X25	R21	.01	00213900
X25	R22	-.15	00214000
X25	R24	-.06	00214100
X25	R26	-.0075	00214200
X25	R27	-.9995	00214300
X25	R28	.00375	00214400
X25	R29	.0005	00214500
X26	R1	.175	00214600
X26	R2	-.22	00214700
X26	R3	2.9	00214800
X26	R4	-3.4	00214900
X26	R5	4.1	00215000
X26	R6	-3.9	00215100
X26	R7	.8	00215200
X26	R8	-.9	00215300
X26	R9	.4	00215400
X26	R10	.18	00215500
X26	R11	-.23	00215600
X26	R12	.0	00215700
X26	R13	.0	00215800
X26	R14	.0	00215900
X26	R15	1560.87	00216000
X26	R16	13.23	00216100
X26	R17	-32.19	00216200
X26	R18	264.5547	00216300
X26	R20	-.06	00216400
X26	R21	.01	00216500
X26	R22	-.15	00216600
X26	R24	-.06	00216700
X26	R26	-.0075	00216800
X26	R27	.0005	00216900
X26	R28	-.99625	00217000
X26	R29	.0005	00217100
X27	R1	.175	00217200
X27	R2	-.22	00217300
X27	R3	2.9	00217400
X27	R4	-3.4	00217500
X27	R5	4.1	00217600
X27	R6	-3.9	00217700
X27	R7	.8	00217800
X27	R8	-.9	00217900
X27	R9	.4	00218000
X27	R10	.18	00218100
X27	R11	-.23	00218200
X27	R12	.0	00218300
X27	R13	.0	00218400
X27	R14	.0	00218500
X27	R15	1560.87	00218600
X27	R16	13.23	00218700
X27	R17	-32.19	00218800
X27	R18	2204.58	00218900
X27	R20	-.06	00219000
X27	R21	.01	00219100
X27	R22	-.15	00219200
X27	R24	-.06	00219300

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     X27   R26    -.0075
     X27   R27    .0005
     X27   R28    .00375
     X27   R29    -.9995
END
RHS
           R1     0.
           R2     0.
           R3     0.
           R4     0.
           R5     0.
           R6     0.
           R7     0.
           R8     0.
           R9     0.
           R10    0.
           R11    0.
           R12    0.
           R13    0.
           R14    0.
           R15    0.
           R16    0.
           R17    0.
           R18    72.
           R20     .0
           R21     .0
           R22     .0
           R24     .0
           R26     .0
           R27     .0
           R28     .0
           R29     .0
END
PRMODE     USE 0042222234 FOR PRINT CONTROL
0042222234
ERRORS
SOLVE
EXIT
//SASSTEP EXEC SAS
//DISK DD DSN=AECINST.RON(NRCPX72),DISP=OLD
//SYSIN DD *
DATA VECTOR;
INFILE DISK;
INPUT X 1-20;
PROC MATRIX PRINT;
FETCH CON72;
ONE=1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1;
PE=.086 3.4392 / .12 3.0864 / 0 7.716 / .606 2.7778 / .5 3.3289 / .5 2.2
   .502 4.2438 / .485 2.425 / .36 1.5983 / .07 1.0582 / 0 0 / 0 0 / 0 0
   0 0 / .8 0 / .165 1.9511 / .78 2.866 / .22 1.6755 / .6 3.0093 / .061
   .126 1.9312 / .12 .7716 / .1 3.858 / 0 0 / 0 0 / 0 0 / 0 0;
L=1.457695 .109539;
Q=-1.758822 .0816935 / .0816935 -.0074039;
N=(PE'*CON72);
TCON=(ONE*CON72);
FEED=(TCON**-1)*CON72;
W1=(L*N);
W2=(N'*Q*N);
```

```
WT=(W1 + W2);
PCT=(TCON**-1)*N;
KGLB=0 .4535924;
WTT=(WT + .041988);
WTLB=(2.2046226*WTT);
PCTLB=(KGLB*PCT);
TCONLB=(2.2046226*TCON);
TCOE=10.511097 1.010951;
TT=(TCOE*N);
TTT=(24.403404 + TT);
//
```