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Iran E. Lima Neto

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Technical note

Bubble plume modelling with new functional relationships

IRAN E. LIMA NETO, Assistant Professor, *Department of Hydraulic and Environmental Engineering, Federal University of Ceará, Campus do Pici, bl. 713, 60.451-970, Fortaleza, Brazil.*
Email: iran@deha.ufc.br

ABSTRACT
In this study, a new parameterization is proposed to be incorporated into the classical integral model for bubble plumes. The virtual origin of the flow, the entrainment coefficient and the momentum amplification factor were obtained by adjusting the model to a wide range of experimental data. This allowed to generate functional relationships based on the gas discharge and the water depth. Model simulations using these novel relationships resulted in a better agreement with experimental data than other approaches for bubble plumes, notably for tests under shallow to intermediate water depths. Therefore, the present model is proposed as a simpler approach to predict the flow induced by bubble plumes for a large range of operational conditions.

Keywords: Bubble, entrainment, integral equation, plume, two-phase flow

1 Introduction

Bubble plumes occur if gas is released into liquids (Fig. 1). These plumes are used for many environmental engineering applications (Socolofsky and Adams 2002, McGinnis *et al.* 2004, Lima Neto *et al.* 2008a, 2008b, 2008c).

Integral models based on the entrainment hypothesis have long been used to simulate the behaviour of bubble plumes (Cederwall and Ditmars 1970, Fannelop and Sjoen 1980, Milgram 1983, Wüest *et al.* 1992, Socolofsky *et al.* 2008). Overall, these provide a reasonable fit to the test data, especially for bubble plumes under deeper water conditions of ~50 m. However, a good match to a large set of test data including shallow, intermediate and deep-water depths only results with more complex models such as the three-dimensional Lagrangian approach for gas–liquid jets and plumes (Yapa and Zheng 1997). This study proposes simple functional relationships to be incorporated into the classical integral models to improve the prediction of the mean flow induced by bubble plumes from shallow to deep-water conditions.

2 Integral model and functional relationships

The integral model used herein is based on the classical bubble plume theory described by Socolofsky *et al.* (2002). Considering similar Gaussian distributions of liquid velocity and void fraction at various distances z from the gas release (Fig. 1), the equations for liquid volume and momentum conservation are, respectively,

$$\frac{\mathrm{d}(u_c b^2)}{\mathrm{d}z} = 2\alpha u_c b \tag{1}$$

$$\frac{\mathrm{d}(u_c^2 b^2)}{\mathrm{d}z} = \frac{2gQ_{g,a}H_a}{\gamma\pi(H_a + H_d - z)((u_c/(1+\lambda^2)) + u_s)} \tag{2}$$

in which u_c is the centreline liquid velocity, b the plume radius, where $u = 0.37u_c$, z the axial distance from the source, r the radial distance from the centreline, α the entrainment coefficient, γ the momentum amplification factor defined as the ratio of total momentum flux to the momentum flux carried by the mean flow, λ the spreading ratio of the bubble core radius relative to the entrained liquid radius, u_s the bubble slip velocity, $Q_{g,a}$ the volumetric gas discharge at atmospheric pressure, H_a the atmospheric pressure head, H_d the water head above the diffuser and g the gravity acceleration. Note that the bubble dissolution is neglected here. However, it is relevant in bubble plume applications related to deep lakes and reservoirs (e.g. Wüest *et al.* 1992, McGinnis *et al.* 2004).

Assuming that near the diffuser, the flow behaves similar to single-phase plumes only driven by buoyancy (Socolofsky *et al.* 2002), the starting conditions for the integration of Eqs. (1) and (2) are

$$u_{c,o} = \left[\frac{25gQ_{g,a}H_a(1+\lambda^2)}{24\alpha^2\pi(H_a+H_d)z_o}\right]^{1/3} \tag{3}$$

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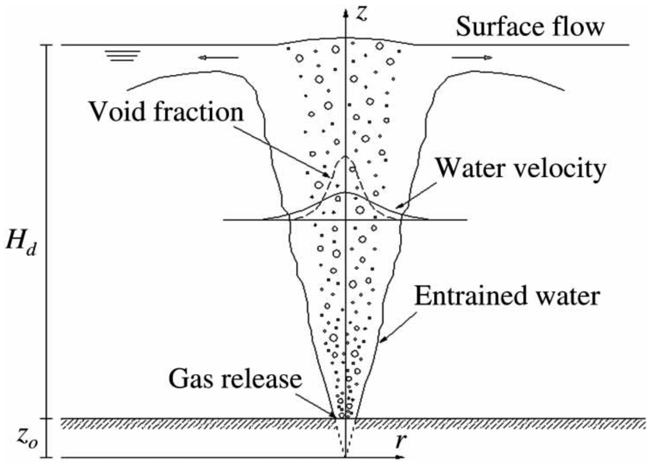

Figure 1 Definition sketch of a round bubble plume

$$b_o = \frac{6}{5}\alpha z_o \tag{4}$$

in which z_o is the virtual plume origin (Fig. 1). Herein, the virtual origin is considered proportional to a length scale of bubble plumes and bubbly jets as (Kobus 1968, Lima Neto *et al.* 2008a, 2008b, 2008c)

$$z_o = \delta \left(\frac{Q_g^2}{g}\right)^{1/5} \tag{5}$$

in which δ is a constant to be fitted by the test data. Equation (5) suggests that $z_o \sim Q_g^{0.4}$ with $Q_g = Q_{g,a}H_a/(H_a + H_d)$ as volumetric gas discharge at the diffuser.

Following Milgram (1983), constant values for the spreading ratio of the bubble core to the entrained liquid ($\lambda = 0.8$) and for the bubble slip velocity of $u_s = 0.35$ m/s are assumed (Lima Neto *et al.* 2008a, 2008b, 2008c). Hence, because the forces due to momentum and buoyancy dominate the flow, dimensional analysis describes the entrainment coefficient α and the momentum amplification factor γ by

$$[\alpha, \gamma] = f(\beta) \tag{6}$$

in which β is the dimensionless parameter:

$$\beta = \frac{g' Q_{g,a}}{H_d u_s^3} \tag{7}$$

where $g' = g(\rho_w - \rho_g)/\rho_w$, in which ρ_w and ρ_g are the water and gas densities, respectively. Since g' and u_s are taken constant, Eq. (7) implies that $\beta \sim Q_{g,a}/H_d$. Therefore, Eq. (6) is obtained by fitting the present model to the test data. Note that Eqs. (1) and (2) are solved using a Runge–Kutta fourth-order method.

3 Results and discussion

The virtual origin of the bubble plumes z_o was obtained by fitting lines to the data of plume radius b as a function of the distance from the source z, given by Kobus (1968), Topham (1975), Fannelop and Sjoen (1980), Milgram and Van Houten (1982), and Milgram (1983), resulting in standard deviations between the fit and the test data that were <10%. Figure 2 also shows the fit

$$z_o = 9.7 \left(\frac{Q_g^2}{g}\right)^{1/5} \tag{8}$$

resulting in an excellent agreement with a coefficient of determination $R^2 = 0.98$. Thus, Eq. (8) together with Eqs. (3) and (4) was used here to estimate the starting conditions for the plume. The initial velocity of the plume $u_{c,o}$ differed from those obtained from the condition of Wüest *et al.* (1992) within up to about ±100%. It is interesting to observe, however, that the initial conditions had little impact (<10% difference) on the overall model predictions, as already pointed out by Milgram (1983). Therefore, the use of Eqs. (3), (4) and (8) can be seen as a simpler way to estimate the starting conditions for bubble plumes.

Coefficients α and γ for each test condition were obtained by fitting the model to the experimental data, resulting in standard deviations for u_c and b that were <15%. This suggests that the combination of Eqs. (1)–(4) and (8) describes well the centreline velocity decay and flow spreading induced by bubble plumes. The fit values for α varied from 0.05 to 0.13, while these for γ varied from 1.0 to 2.0, that is, within the ranges reported by Milgram (1983). While α increases with β, the values of γ decrease. For $\beta > 2.0$, although the model provided slightly better fits for $\gamma < 1.0$, a minimum value of $\gamma = 1.0$ was adopted, implying that the momentum flux carried by turbulence is then negligible compared with that carried by the mean flow, as expected to occur in large-scale bubble plumes (Milgram 1983, Wüest *et al.* 1992). Figure 3 shows the fitted values of α ($R^2 = 0.96$) and γ ($R^2 = 0.91$) plotted versus β as

$$\alpha = 0.0148 \ln(\beta) + 0.101 \tag{9}$$

$$\gamma = 1.118(\beta)^{-0.181} \tag{10}$$

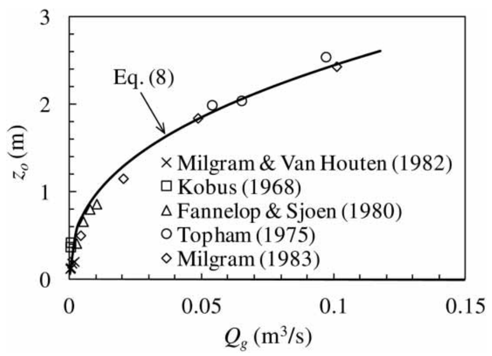

Figure 2 Relationship between virtual origin of bubble plumes z_o and diffuser gas discharge Q_g

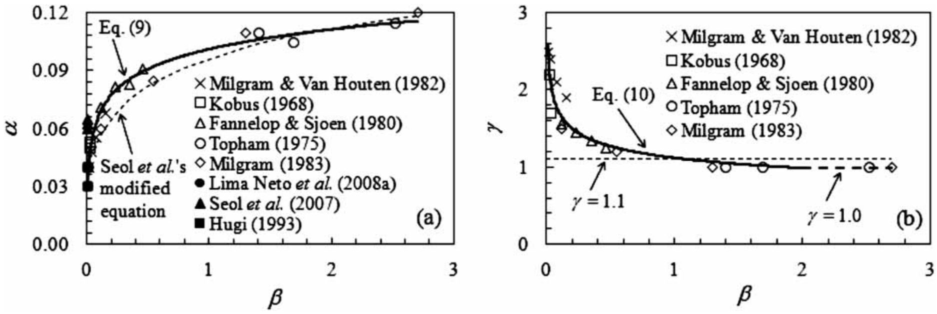

Figure 3 Functional relationships from curve fitting: (a) entrainment coefficient, $\alpha(\beta)$, and (b) momentum amplification factor, $\gamma(\beta)$. Note that for $\beta > 2$, a minimum of $\gamma = 1.0$ is adopted. Seol *et al.*'s modified equation for α and $\gamma = 1.1$ adopted from Socolofsky *et al.* (2008) are also included

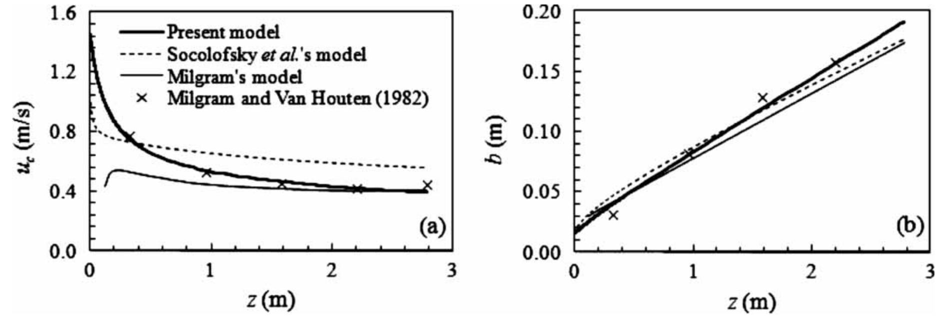

Figure 4 Comparison of various model predictions of (a) centreline velocity and (b) plume radius with small-scale experimental data for $H_d = 3.66$ m, $Q_{g,a} = 0.0005\,\mathrm{Nm^3/s}$ and $\beta = 0.03$

This suggests that parameter β is appropriate to describe the mean flow induced by bubble plumes. Note that Eq. (10) is valid for $0 < \beta < 2$, whereas for $\beta > 2$, $\gamma = 1.0$, as justified above. Also note that various dimensionless parameters such as $(H_a + H_d)/H_d$, $u_s/(g'H_\mathrm{d})^{1/2}$ and $Q_{g,a}/(u_s H_d^2)$ were tested (Bombardelli *et al.* 2007), but the resulting correlations indicated $R^2 < 0.90$. Furthermore, the equation of Seol *et al.* (2007) was modified to express the entrainment coefficient versus β, resulting in $\alpha = 0.18\exp(-1.16/\beta^{1/3}) + 0.04$. Do observe that here the axial distance from the source z was replaced by $H_d/2$. This equation also has a correction to account for $u_s = 0.30$ m/s (instead of $u_s = 0.35$ m/s, as adopted herein). Figure 3(a) shows that Eq. (9) agrees well with the fitted values of α. Additional data of Hugi (1993), Seol *et al.* (2007) and Lima Neto (2008a) are also included to confirm this trend. Figure 3(b) shows $\gamma = 1.1$, adopted from Socolofsky *et al.* (2008), to demonstrate that for small scales, a relationship for $\gamma(\beta)$ is indeed needed.

Figures 4 and 5 indicate different model predictions of centreline velocity and plume radius with the experimental data from small- and intermediate-scale tests (e.g. $\beta = 0.03$ and 0.23). For both scales, the present model results in a better agreement with the data than the classical model of Milgram (1983) and that of Socolofsky *et al.* (2008). However, the results were similar to those obtained with the complex Lagrangian model of Yapa and Zheng (1997). For larger scales ($\beta > 1$), similar results were obtained when comparing the present model with that of Socolofsky *et al.* (2008). This is consistent with Fig. 3, in which the values of α and γ obtained with the two approaches become close as $\beta > 1.0$.

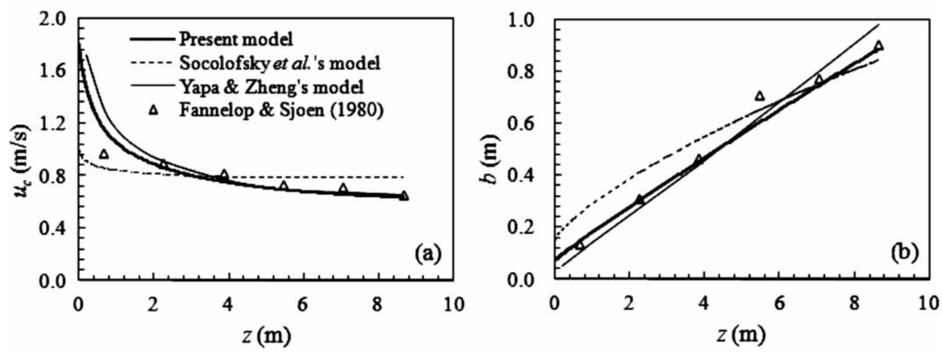

Figure 5 Comparison of various model predictions of (a) centreline velocity and (b) plume radius with intermediate-scale test data for $H_d = 10$ m, $Q_{g,a} = 0.01\,\mathrm{Nm^3/s}$ and $\beta = 0.23$

4 Conclusions

In this study, bubble plume modelling was performed using an integral approach with a new parameterization. Adjusting the model to a wide range of experimental data, the values for the virtual flow origin, the entrainment coefficient and the momentum amplification factor were fitted to obtain new functional relationships. Model predictions using these new relationships provide a better fit to the experimental data than others. Therefore, the present model is proposed to improve the prediction of the flow induced by bubble plumes for applications including surface aeration and circulation in tanks, lakes and reservoirs.

Notation

b = bubble plume radius (m)
b_o = bubble plume radius at source (m)
g = gravity acceleration (m/s^2)
H_a = atmospheric pressure head (m)
H_d = water head above diffuser (m)
$Q_{g,a}$ = volumetric gas discharge at atmospheric pressure (m^3/s)
Q_g = volumetric gas discharge at diffuser (m^3/s)
r = radial distance from centreline (m)
u = liquid velocity (m/s)
u_c = centreline liquid velocity (m/s)
$u_{c,o}$ = liquid velocity at source (m/s)
u_s = bubble slip velocity (m/s)
z = axial distance from source (m)
z_o = virtual origin of plume (m)
α = entrainment coefficient (–)
β = dimensionless parameter (–)
δ = constant in Eq. (5) (–)
γ = momentum amplification factor (–)
λ = spreading ratio of bubble core to entrained liquid (–)

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