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Outage and SER analysis of DF cooperative OFDM systems with nonlinear power amplifiers

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ABSTRACT

In this paper, a performance analysis of decode-and-forward (DF) cooperative OFDM systems is presented by considering the nonlinear distortions inserted by PAs at the source and relay. In particular, a theoretical formulation for the symbol error rate (SER) and for upper and lower bounds of the outage probability are developed considering fixed and selective DF relaying protocols. In the considered system model, the receiver exploits the signal received directly from the source as well as the signal transmitted through the relay using the maximum ratio combining (MRC) technique. The proposed analysis also studies the effects of the nonlinearities on the diversity order and it shows in what situations the PA nonlinearities affect significantly the system performance. The proposed theoretical expressions are validated by means of computer simulations.

1. Introduction

At the beginning of the last decade, cooperative communication has been proposed as a promising technology for wireless networks, due to its ability to emulate an antenna array through a distributed processing and transmission of the information [1,2]. A cooperative relaying network exploits spatial diversity, extends the coverage zone and improves the spectral capacity, without requiring significant improvements in hardware devices. Many relaying protocols have been proposed for cooperative systems [3], the best known being the amplify-and-forward (AF), fixed decode-and-forward (FDF) and selective decode-and-forward (SDF) [4]. In this paper, we are interested in the FDF and SDF protocols, which do not undergo noise amplification, contrarily to the AF protocol.

On the other hand, orthogonal frequency division multiplexing (OFDM) has been largely adopted by many wireless communications standards, such as IEEE 802.11a, IEEE 802.16 (WiMAX), 3GPP Long Term Evolution (LTE), Digital Video Broadcasting – Terrestrial (DVB-T) and Digital Audio Broadcasting – Terrestrial (DAB-T) [5], due to its high spectral efficiency, low complexity and robustness to inter-symbol (ISI) and inter-carrier interference (ICI). A major drawback of OFDM is the fact that the transmitted signals are characterized by a high peak-to-average power ratio (PAPR), which is a measure of how strong the power peaks are compared to the average power [5–7]. Due to the presence of nonlinear devices such as power amplifiers (PA), a high PAPR may introduce nonlinear distortions (NLD) in the received signals, significantly deteriorating the link quality, unless a high input back-off (IBO) is used [8–10]. The IBO is defined as the ratio between the PA input saturation power, i.e., the input power corresponding to the maximum output power, and the average PA input power. However, a high IBO provides a low-power efficiency and low signal-to-noise ratio (SNR) at the receiver.

Over the last years, OFDM has been widely associated with cooperative radio techniques. A study about the effects of nonlinear PAs in this kind of system is then of great relevance. In this paper, a performance analysis of a cooperative OFDM system is presented, using both FDF and SDF retransmission protocols and considering the effects of a nonlinear PA at the source and relay. It is assumed that the wireless channels undergo frequency-selective Rayleigh fading and that the receiver exploits the signal received directly from the source as well as the signal transmitted through the relay using the maximum ratio combing (MRC) technique. Specifically, the main original contributions of the present work can be summarized as follows:

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- A non-analytical exact formulation for the outage probability is derived for the considered OFDM system with the FDF and SDF protocols;
- Closed-form expressions for upper and lower bounds of the outage probability are developed;
- We perform an asymptotic outage analysis in order to observe the effects of the NLD on the diversity order;
- A closed-form expression for the symbol error rate (SER) of the considered cooperative system is proposed;
- Computer simulations corroborating the validity of the proposed analysis are presented.

In the literature, some works have studied the impact of nonlinear distortions in cooperative OFDM systems. However, no previous work has presented a performance analysis of nonlinear cooperative OFDM systems with DF relays. Ishibashi and Ono [11,12] studied the instantaneous power distribution of signals in a relaying communication system. However, no analytical SER or outage analysis was presented in [11,12]. In [13,14], analytical formulas for the outage probability and symbol error rate were derived for a nonlinear cooperative OFDM system considering one AF relay, showing that a nonlinear PA at the relay increases the outage probability only for high SNR thresholds. [15] extended the analysis of [13] to the case with multiple AF relays, where a relay selection is performed. However, in [15], it was considered that the wireless links between the source and the relays undergo frequency-flat fading. In [16] the outage probability of a AF cooperative OFDM system was studied, considering two different combining techniques, namely, MRC and selection combining (SC).

In [17], an error probability expression was developed for an AF cooperative OFDM system, assuming that the base station and the relay have nonlinear PAs, and that the source-relay channel is time-invariant. In [18], a bit error analysis of a cooperative OFDM system was presented considering AF relays and assuming an uplink scenario with both source and relay introducing nonlinear distortions and Doppler shifts, and the source-relay channel being time-invariant. Besides, [17,18] assumed that there is no direct path between the source and the destination, and none of them carried out an outage analysis. A theoretical outage analysis of a nonlinear cooperative OFDM system with relay selection was performed in [19]. However, in [19], only the source node has a nonlinear PA and there is no direct path between the source and the destination, contrarily to the present work. In [20], closed form outage probability expressions for two-way AF nonlinear OFDM relay networks are derived, with no direct path between the source and the destination.

As well as the analysis of the present paper, the performance studies of nonlinear OFDM systems of [13–20] are based on a single subcarrier. This approach has the advantage of providing an evaluation of the quality of each subcarrier. However, it does not give a global evaluation of the OFDM link due to the correlation among the subcarriers. An outage analysis of a cooperative nonlinear OFDM system based on the sum of the mutual information of all the subcarriers is presented in [21].

The rest of this work is divided as follows. In Section 2, the system model is described. Section 3 presents the outage analysis. In Section 4, the average SER formulas of the considered system are developed. Section 5 presents the simulation results and, finally, the conclusions and perspectives are given in Section 6.

2. System model

Let us consider a cooperative OFDM system composed of one source (S), one DF relay (R) and one destination (D), all of them equipped with a single antenna and operating in half-duplex mode. The communication is performed in two orthogonal time-slots, where the source transmits the OFDM signal using N subcarriers to the destination through a direct link and with the help of the relay. In which follows, two different relaying protocols are considered: FDF and SDF.

It is considered that both the source and relay nodes are equipped with a nonlinear memoryless PA. A MRC receiver is employed at the destination to combine the signals from the direct link (SD) and from the relaying link (RD). All the wireless links undergo independent and identically distributed (i.i.d.) Rayleigh block fading and additive white Gaussian noise (AWGN) with zero mean and variance σ_η^2.

The length L of the cyclic prefix included at the transmitted OFDM signal is considered to be long enough to guarantee the non-ISI and non-ICI conditions. It is also assumed that the transmitted data symbols $\mathbf{s} = \{s_1, s_2, \ldots, s_N\}$ in frequency domain are i.i.d., with an uniform distribution over a quadrature amplitude modulation (QAM) or phase-shift keying (PSK) alphabet with unit variance.

Let us denote by $f_s^{(NL)}(\cdot) : \mathbb{C}^N \to \mathbb{C}^N$ the function that models the nonlinear PA of the source applied to the time domain OFDM signal. We also consider that $\mathcal{F}_n(\cdot) : \mathbb{C}^N \to \mathbb{C}$ and $\mathcal{F}_n^{-1}(\cdot) : \mathbb{C}^N \to \mathbb{C}$ are respectively the functions that return the nth sample of the Discrete Fourier Transform (DFT) and the inverse DFT of the argument. Therefore, the signal received by the destination in frequency domain at the nth subcarrier through the SD link is given by

$$x_n^{(SD)} = h_n^{(SD)} \mathcal{F}_n\left(f_s^{(NL)}\left(\sqrt{P_s}\mathbf{s}'\right)\right) + \eta_n^{(SD)}, \tag{1}$$

for $1 \leqslant n \leqslant N$, where $h_n^{(SD)}$ and $\eta_n^{(SD)}$ are respectively the channel frequency response of the SD link and the corresponding noise in frequency domain, P_s is the transmitted signal power at the input of the source's PA and $\mathbf{s}' = \{s'_1, s'_2, \ldots, s'_N\} \in \mathbb{C}^N$ is the vector with the transmitted signals in time domain, with $s_n = \mathcal{F}_n(\mathbf{s}')$ and $s'_n = \mathcal{F}_n^{-1}(\mathbf{s})$.

For a high number N of subcarriers, $\mathbf{s}'$ can be modeled as a vector of stationary complex-valued Gaussian random variables [9]. Thus, from the extension of the Bussgang's Theorem [9,22], we can express the time domain output of the PA as

$$f_s^{(NL)}\left(\sqrt{P_s}\mathbf{s}'\right) = \alpha_s\sqrt{P_s}\mathbf{s}' + \mathbf{d}'_s, \tag{2}$$

where α_s is a complex-valued constant representing the attenuation and rotation imposed by the PA of the source on the symbols, $\mathbf{d}'_s$ is a random nonlinear distortion vector uncorrelated with $\mathbf{s}'$, with variance $\sigma_{d_s}^2$. Substituting (2) into (1), we get

$$x_n^{(SD)} = h_n^{(SD)}\alpha_s\sqrt{P_s}s_n + h_n^{(SD)}d_n^{(S)} + \eta_n^{(SD)}, \tag{3}$$

where $d_n^{(S)}$ represents the nth component of the DFT of the vector $\mathbf{d}'_s$. In a similar way, the signal received by the relay from the SR link in frequency domain at the nth subcarrier can be expressed as

$$x_n^{(SR)} = h_n^{(SR)}\alpha_s\sqrt{P_s}s_n + h_n^{(SR)}d_n^{(S)} + \eta_n^{(SR)}, \tag{4}$$

for $1 \leqslant n \leqslant N$, where $h_n^{(SR)}$ is the channel frequency response of the SR link and $\eta_n^{(SR)}$ corresponds to the noise in frequency domain.

In the FDF and SDF protocols, the signal $x_n^{(SR)}$ is decoded, re-encoded and then retransmitted to the destination. The SDF relay retransmits the signal only if the message is successfully decoded. Let $\hat{s}_n$ be the estimation of s_n performed by the relay and $\hat{\mathbf{s}}' = \{\hat{s}'_1, \hat{s}'_2, \ldots, \hat{s}'_N\} \in \mathbb{C}^N$ a vector containing the inverse DFT of $\{\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N\}$. Thus, assuming that $\hat{s}'_n$ is Gaussian, the signal received by the destination through the RD link in frequency domain can be written as

$$x_n^{(RD)} = h_n^{(RD)}\alpha_r\sqrt{P_r}\hat{s}_n + h_n^{(RD)}d_n^{(R)} + \eta_n^{(RD)}, \tag{5}$$

for $1 \leqslant n \leqslant N$, where $h_n^{(SR)}$ and $\eta_n^{(SR)}$ denote respectively the channel frequency response and the frequency domain noise at the nth subcarrier, α_r is the complex-valued constant representing the attenuation and rotation imposed by the PA of the relay on the symbols, $d_n^{(R)}$ is a random distortion vector uncorrelated with $\hat{s}_n$, with variance $\sigma_{d_r}^2$, and P_r is the transmitted signal power at the input of the relay's PA. It is also considered that the transmitted signals have a rectangular waveform, which means that $d_n^{(S)}$ and $d_n^{(R)}$ are statistically equal to AWGN, as shown in [9].

In this paper, we assume that the receiver detects the transmitted symbol using a NLD-aware MRC [23]. In addition, it is assumed that the receiver has perfect channel state information (CSI).

3. Outage analysis

In this section, an exact non-analytical formulation for the outage probability is developed, as well as analytical closed-form expressions for upper and lower bounds of the outage probability are developed for both FDF and SDF protocols. In addition, an asymptotic analysis is presented, allowing us to obtain the diversity order of the considered system.

3.1. Exact outage probability

From 3, 4, 5, the SNR of the n^{th} subcarrier associated with the SD, SR and RD links can be written respectively as follows

$$\gamma_{sd_n} = \frac{\left|h_n^{(SD)}\right|^2|\alpha_s|^2P_s}{\left|h_n^{(SD)}\right|^2\sigma_{d_s}^2 + \sigma_\eta^2}, \quad \gamma_{sr_n} = \frac{\left|h_n^{(SR)}\right|^2|\alpha_s|^2P_s}{\left|h_n^{(SR)}\right|^2\sigma_{d_s}^2 + \sigma_\eta^2}$$
$$\gamma_{rd_n} = \frac{\left|h_n^{(RD)}\right|^2|\alpha_r|^2P_r}{\left|h_n^{(RD)}\right|^2\sigma_{d_r}^2 + \sigma_\eta^2} \tag{6}$$

The cumulative density function (CDF) of the SD link can be obtained from (6) as follows:

$$P(\gamma_{sd_n} < \gamma_{th}) = \begin{cases} P\left(\left|h_n^{(SD)}\right|^2 < \frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right), & \text{if} \quad \gamma_{th} < |\alpha_s|^2P_s/\sigma_{d_s}^2, \\ 1, & \text{otherwise}, \end{cases} \tag{7}$$

where γ_{th} is a SNR threshold.

As Rayleigh fading is considered for all the wireless links, we may write

$$P(\gamma_{sd_n} < \gamma_{th}) = \left[1 - \exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right)u\left(\frac{|\alpha_s|^2P_s}{\sigma_{d_s}^2} - \gamma_{th}\right)\right]u(\gamma_{th}), \tag{8}$$

where $\sigma_{h_{sd}}^2$ denotes the mean power of $h_n^{(SD)}$ and $u(\cdot)$ represents the unit step function with $u(x) = 1$, for $x \geqslant 0$ and $u(x) = 0$ otherwise.

Similarly, the CDFs of the SNR of the SR and RD links are respectively given by

$$P(\gamma_{sr_n} < \gamma_{th}) = \left[1 - \exp\left(-\frac{1}{\sigma_{h_{sr}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right)u\left(\frac{|\alpha_s|^2P_s}{\sigma_{d_s}^2} - \gamma_{th}\right)\right]u(\gamma_{th}), \tag{9}$$

and

$$P(\gamma_{rd_n} < \gamma_{th}) = \left[1 - \exp\left(-\frac{1}{\sigma_{h_{rd}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_r|^2P_r - \gamma_{th}\sigma_{d_r}^2}\right)u\left(\frac{|\alpha_r|^2P_r}{\sigma_{d_r}^2} - \gamma_{th}\right)\right]u(\gamma_{th}), \tag{10}$$

where $\sigma_{h_{sr}}^2$ and $\sigma_{h_{rd}}^2$ are the mean powers of $h_n^{(SR)}$ and $h_n^{(SR)}$, respectively.

The instantaneous SNR at the output of the MRC receiver at the destination is given by $\gamma_{mrc_n} = \gamma_{sr_n} + \gamma_{rd_n}$, whose CDF is given by

$$P(\gamma_{mrc_n} < \gamma_{th}) = P(\gamma_{sd_n} + \gamma_{rd_n} < \gamma_{th}) = \int_{-\infty}^{\infty} f_{\gamma_{sd_n}}(x)P(\gamma_{rd_n} < \gamma_{th} - x)dx, \tag{11}$$

where $f_{\gamma_{sd_n}}(x)$ is the probability density function (PDF) of γ_{sd_n}, given by the derivative of (8) with respect to γ_{th}. Substituting (10) and the derivative of (8) in (11), and performing some algebraic manipulations, it follows that

$$\begin{aligned} P(\gamma_{mrc} < \gamma_{th}) = 1 - \Bigg[&\exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right) \\ &+ \int_{\max\left(\gamma_{th} - \frac{|\alpha_r|^2P_r}{\sigma_{d_r}^2}, 0\right)}^{\gamma_{th}} \frac{\sigma_\eta^2|\alpha_s|^2P_s}{\sigma_{h_{sd}}^2\left(|\alpha_s|^2P_s - \sigma_{d_s}^2x\right)^2} \\ &\times \exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{x\sigma_\eta^2}{|\alpha_s|^2P_s - x\sigma_{d_s}^2} - \frac{1}{\sigma_{h_{rd}}^2}\frac{(\gamma_{th} - x)\sigma_\eta^2}{|\alpha_r|^2P_r - (\gamma_{th} - x)\sigma_{d_r}^2}\right)dx\Bigg] \times u\left(\frac{|\alpha_s|^2P_s}{\sigma_{d_s}^2} - \gamma_{th}\right). \end{aligned} \tag{12}$$

In the FDF case, as the SR link is always used, the outage probability of the nth subcarrier is given by [4]

$$\begin{aligned} P_{out}^{(fdf)}(\gamma_{th}) &= P(\min(\gamma_{sr_n}, \gamma_{mrc}) < \gamma_{th}) \\ &= 1 - (1 - P(\gamma_{sr_n} < \gamma_{th}))(1 - P(\gamma_{mrc} < \gamma_{th})). \end{aligned} \tag{13}$$

Substituting (9) and (12) into (13), we obtain

$$\begin{aligned} P_{out}^{(fdf)}(\gamma_{th}) = 1 - &\exp\left(-\frac{1}{\sigma_{h_{sr}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right) \\ &\times \Bigg[\exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right) \\ &+ \int_{\max\left(\gamma_{th} - \frac{|\alpha_r|^2P_r}{\sigma_{d_r}^2}, 0\right)}^{\gamma_{th}} \frac{\sigma_\eta^2|\alpha_s|^2P_s}{\sigma_{h_{sd}}^2\left(|\alpha_s|^2P_s - \sigma_{d_s}^2x\right)^2} \\ &\times \exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{x\sigma_\eta^2}{|\alpha_s|^2P_s - x\sigma_{d_s}^2} - \frac{1}{\sigma_{h_{rd}}^2}\frac{(\gamma_{th} - x)\sigma_\eta^2}{|\alpha_r|^2P_r - (\gamma_{th} - x)\sigma_{d_r}^2}\right)dx\Bigg] \\ &\times u\left(\frac{|\alpha_s|^2P_s}{\sigma_{d_s}^2} - \gamma_{th}\right). \end{aligned} \tag{14}$$

On the other hand, in the SDF protocol, the relay link is used only if the SNR of the SR link is higher than γ_{th}, which leads to the following outage probability for the nth subcarrier [4]

$$P_{out}^{(sdf)}(\gamma_{th}) = P(\gamma_{sd_n} < \gamma_{th})P(\gamma_{sr_n} < \gamma_{th}) + P(\gamma_{mrc} < \gamma_{th})(1 - P(\gamma_{sr_n} < \gamma_{th})). \tag{15}$$

Substituting (6) and (12) into (15), we get the SDF outage probability as follows

$$P_{out}^{(sdf)}(\gamma_{th}) = 1 - \left[\exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right) + \exp\left(-\frac{1}{\sigma_{h_{sr}}^2}\frac{\gamma_{th}\sigma_\eta^2}{|\alpha_s|^2P_s - \gamma_{th}\sigma_{d_s}^2}\right) \times \int_{\max\left(\gamma_{th}-\frac{|\alpha_r|^2P_r}{\sigma_{d_r}^2},0\right)}^{\gamma_{th}} \frac{\sigma_\eta^2|\alpha_s|^2P_s}{\sigma_{h_{sd}}^2\left(|\alpha_s|^2P_s - \sigma_{d_s}^2x\right)^2}\right.$$
$$\left. \times \exp\left(-\frac{1}{\sigma_{h_{sd}}^2}\frac{x\sigma_\eta^2}{|\alpha_s|^2P_s - x\sigma_{d_s}^2} - \frac{1}{\sigma_{h_{rd}}^2}\frac{(\gamma_{th}-x)\sigma_\eta^2}{|\alpha_r|^2P_r - (\gamma_{th}-x)\sigma_{d_r}^2}\right)dx\right] \times u\left(\frac{|\alpha_s|^2P_s}{\sigma_{d_s}^2} - \gamma_{th}\right). \tag{16}$$

The outage probability is independent of the subcarrier due to the fact that the signal and channel statistical parameters are the same for all the subcarriers.

Both (14) and (16) represent non-analytical exact expressions for the outage probability of the considered system. However, due to the difficulty of finding primitive functions to the integrals in (14) and (16), in the next section we develop analytical closed-form expressions for upper and lower bounds of these integrals. It is worth noting that (14) and (16) can be solved numerically.

3.2. Upper and lower bounds

The SNR of the SD, SR and RD wireless links without accounting for the PA nonlinear distortions are respectively

$$\gamma_{sd_n^{(l)}} = \frac{\left|h_n^{(SD)}\right|^2|\alpha_s|^2P_s}{\sigma_\eta^2}, \quad \gamma_{sr_n^{(l)}} = \frac{\left|h_n^{(SR)}\right|^2|\alpha_s|^2P_s}{\sigma_\eta^2} \quad \text{and}$$
$$\gamma_{rd_n^{(l)}} = \frac{\left|h_n^{(RD)}\right|^2|\alpha_r|^2P_r}{\sigma_\eta^2}. \tag{17}$$

In addition, the SNR of the PA of source and relay are respectively given by $\gamma_{S_{PA}} = |\alpha_s|^2P_s/\sigma_{d_s}^2$ and $\gamma_{R_{PA}} = |\alpha_r|^2P_r/\sigma_{d_r}^2$. (6) can then be rewritten as

$$\gamma_{sd_n} = \frac{\gamma_{sd_n^{(l)}}\gamma_{S_{PA}}}{\gamma_{sd_n^{(l)}} + \gamma_{S_{PA}}}, \quad \gamma_{sr_n} = \frac{\gamma_{sr_n^{(l)}}\gamma_{S_{PA}}}{\gamma_{sr_n^{(l)}} + \gamma_{S_{PA}}} \quad \text{and} \quad \gamma_{rd_n} = \frac{\gamma_{rd_n^{(l)}}\gamma_{R_{PA}}}{\gamma_{rd_n^{(l)}} + \gamma_{R_{PA}}}. \tag{18}$$

The upper and lower bounds of the outage probability developed in the sequel are based on the following proposition, whose proof is straightforward:

Proposition 1. *If $x = yz/(y+z)$, then:* $\min(y,z)/2 \leqslant x \leqslant \min(y,z)$.

An upper bound for the SNR leads to a lower bound for the outage probability. Using Proposition 1, a lower bound for the cumulative density function (CDF) of γ_{sd_n} can then be derived as follows:

$$P_\downarrow(\gamma_{sd_n} < \gamma_{th}) = P(\min\left(\gamma_{sd_n^{(l)}}, \gamma_{S_{PA}}\right) < \gamma_{th})$$
$$= 1 - \left(1 - P\left(\gamma_{sd_n^{(l)}} < \gamma_{th}\right)\right)\left(1 - P\left(\gamma_{S_{PA}} < \gamma_{th}\right)\right), \tag{19}$$

where γ_{th} is a SNR threshold. As all the wireless channels are assumed to have Rayleigh fading and considering that the nonlinear PA model is time invariant, i.e., $P\left(\gamma_{S_{PA}} < \gamma_{th}\right) = u\left(\gamma_{th} - \gamma_{S_{PA}}\right)$, where $u(\cdot)$ represents the unit step function, (19) can be rewritten as

$$P_\downarrow(\gamma_{sd_n} < \gamma_{th}) = \left(1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}\right)u\left(\gamma_{S_{PA}} - \gamma_{th}\right)\right)u(\gamma_{th}), \tag{20}$$

where $\bar{\gamma}_{sd}$ is the mean SNR at the SD link. Similarly, the lower bounds for the CDF of the SR and RD paths are

$$P_\downarrow(\gamma_{sr_n} < \gamma_{th}) = \left(1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right)u\left(\gamma_{S_{PA}} - \gamma_{th}\right)\right)u(\gamma_{th}) \tag{21}$$

and

$$P_\downarrow(\gamma_{rd_n} < \gamma_{th}) = \left(1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{rd}}\right)u\left(\gamma_{R_{PA}} - \gamma_{th}\right)\right)u(\gamma_{th}), \tag{22}$$

where $\bar{\gamma}_{sr}$ and $\bar{\gamma}_{rd}$ are the mean SNRs of the SR and RD links, respectively.

Considering that the PDF of γ_{sd_n} is given by: $f_{\gamma_{sd_n}^{(UB)}}(x) = (1/\bar{\gamma}_{sd})e^{-\gamma_{th}/\bar{\gamma}_{sd}}u\left(\gamma_{S_{PA}} - \gamma_{th}\right)u(\gamma_{th})$. By replacing (22) and $f_{\gamma_{sd_n}^{(UB)}}(x)$ into (11), we obtain a lower bound for $P(\gamma_{mrc} < \gamma_{th})$:

$$P_\downarrow(\gamma_{mrc} < \gamma_{th}) = 1 - \exp\left(-\frac{\min(\gamma_{th}, \gamma_{S_{PA}})}{\bar{\gamma}_{sd}}\right)$$
$$- \int_{\max\left(\gamma_{th}-\gamma_{R_{PA}},0\right)}^{\min(\gamma_{th},\gamma_{S_{PA}})} \frac{1}{\bar{\gamma}_{sd}}\exp\left(-\frac{x}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}-x}{\bar{\gamma}_{rd}}\right)dx, \tag{23}$$

which must be solved by considering two cases separately: when $\bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}$ and $\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$.

If $\bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}$, (23) can be expressed as

$$P_\downarrow(\gamma_{mrc} < \gamma_{th}) = 1 - \exp\left(-\frac{\min(\gamma_{th}, \gamma_{S_{PA}})}{\bar{\gamma}_{sd_n}}\right)$$
$$- \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd} - \bar{\gamma}_{rd}}\left[\exp\left(-\frac{\min(\gamma_{th}, \gamma_{S_{PA}})}{\bar{\gamma}_{sd}} - \frac{\max(\gamma_{th} - \gamma_{S_{PA}}, 0)}{\bar{\gamma}_{rd}}\right)\right.$$
$$\left. - \exp\left(-\frac{\max(\gamma_{th} - \gamma_{R_{PA}}, 0)}{\bar{\gamma}_{sd}} - \frac{\min(\gamma_{th}, \gamma_{R_{PA}})}{\bar{\gamma}_{rd}}\right)\right], \tag{24}$$

and, if $\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$, (23) becomes

$$P_\downarrow(\gamma_{mrc} < \gamma_{th}) = 1 - \exp\left(-\frac{\min(\gamma_{th}, \gamma_{S_{PA}})}{\bar{\gamma}_{sd}}\right) - \frac{1}{\bar{\gamma}_{sd}}\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}\right)$$
$$\times \left(\min(\gamma_{th}, \gamma_{S_{PA}}) - \max(\gamma_{th} - \gamma_{R_{PA}}, 0)\right). \tag{25}$$

Substituting (21), (24) and (25) into (13) and considering $\gamma_{th} > 0$, the lower bound outage probability for the FDF protocol can be written as

$$P_{out_\downarrow}^{(fdf)}(\gamma_{th}) = \begin{cases} 1 - \left[1 + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}} - \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}}\exp\left(\frac{\min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{sd}} - \frac{\min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{rd}}\right)\right] & \\ \times \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right)u(\gamma_{S_{PA}} - \gamma_{th}), & \text{for } \bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}, \\ 1 - \left(1 + \frac{1}{\bar{\gamma}_{sd}}\min(\gamma_{th}, \gamma_{R_{PA}})\right)\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right)u(\gamma_{S_{PA}} - \gamma_{th}), & \text{for } \bar{\gamma}_{sd} = \bar{\gamma}_{rd}. \end{cases} \tag{26}$$

Similarly, replacing (21), (24) and (25) into (15), we obtain the lower bound outage probability for the SDF protocol, given by

$$P_{out_\downarrow}^{(sdf)}(\gamma_{th}) = \begin{cases} 1 - \Big[\exp\left(\frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right) + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}} \\ \quad - \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}} \exp\left(\frac{\min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{sd}} - \frac{\min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{rd}}\right)\Big] \\ \quad \times \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right) u(\gamma_{S_{PA}} - \gamma_{th}), & \text{for } \bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}, \\ 1 - \left(\exp\left(\frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right) + \frac{1}{\bar{\gamma}_{sd}} \min(\gamma_{th}, \gamma_{R_{PA}})\right) \\ \quad \times \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right) u(\gamma_{S_{PA}} - \gamma_{th}), & \text{for } \bar{\gamma}_{sd} = \bar{\gamma}_{rd}. \end{cases} \tag{27}$$

Moreover, from (26) and (27), we may write

$$P_{out_\downarrow}^{(sdf)}(\gamma_{th}) = P_{out_\downarrow}^{(fdf)}(\gamma_{th}) + \left[\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}}{\bar{\gamma}_{sr}}\right) - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}\right)\right] u(\gamma_{S_{PA}} - \gamma_{th}). \tag{28}$$

An important conclusion that can be drawn from (28) is that the outage probability difference between the SDF and FDF protocol is not affected by the nonlinearity of the relay PA. On the other hand, the source PA plays a prominent role on the system performance since, if $\gamma_{S_{PA}} < \gamma_{th}$, the outage probability equals to 1 for both protocols.

On the other hand, using Proposition 1, upper bounds for the outage probability can be obtained simply by replacing γ_{th} by $2\gamma_{th}$ at the lower bounds expressions (26) and (27), as follows

$$P_{out_\uparrow}^{(fdf)}(\gamma_{th}) = P_{out_\downarrow}^{(fdf)}(2\gamma_{th}) \quad \text{and} \quad P_{out_\uparrow}^{(sdf)}(\gamma_{th}) = P_{out_\downarrow}^{(sdf)}(2\gamma_{th}). \tag{29}$$

3.3. Asymptotic outage analysis

In this subsection, an asymptotic analysis of the outage probability is performed, allowing us to obtain the diversity order of the considered cooperative system using the FDF and SDF protocols. The next results are derived using the lower bound expressions (26) and (27). As it will be shown in the simulation results, the outage probabilities (26) and (27) are very close to the simulated outage probabilities in most of the tested cases. Similar expressions can be obtained for the upper bounds. It is noteworthy that, for both protocols, if $\gamma_{S_{PA}} < \gamma_{th}$, an outage event always occurs, which leads to a diversity order equal to 0. Thus, throughout the rest of this section, we assume $\gamma_{S_{PA}} > \gamma_{th}$.

We first consider the FDF OFDM cooperative system. By making $\bar{\gamma}_{sd} \to \infty$, with $\bar{\gamma}_{sr} = \omega\bar{\gamma}_{sd}$ and $\bar{\gamma}_{rd} = \tau\bar{\gamma}_{sd}$, where $\omega, \tau \in \mathbb{R}_+$ are fixed, and using a second order approximation of the exponential function, (26) can be approximated as

$$P_{out}^{(fdf)}(\gamma_{th}) \approx \begin{cases} \frac{\gamma_{th}(1+\omega^{-1}) - \min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{sd}} - \frac{\min(\gamma_{th}^2,\gamma_{R_{PA}}^2)(1-\tau^{-1})}{2\bar{\gamma}_{sd}^2} + \\ \frac{\gamma_{th}^2(1+\omega^{-1})^2}{2\bar{\gamma}_{sd}^2} - \frac{2\gamma_{th}\min(\gamma_{th},\gamma_{R_{PA}})(1+\omega^{-1})}{2\bar{\gamma}_{sd}^2} & \bar{\gamma}_{sd} \neq \bar{\gamma}_{rd} \\ \frac{\gamma_{th}(1+\omega^{-1}) - \min(\gamma_{th},\gamma_{R_{PA}})}{\bar{\gamma}_{sd}} - \frac{\gamma_{th}^2\min(\gamma_{th},\gamma_{R_{PA}})(1-\tau^{-1})}{2\bar{\gamma}_{sd}^2} + \\ \frac{\gamma_{th}^2(1+\omega^{-1})^2}{2\bar{\gamma}_{sd}^2} - \frac{2\gamma_{th}\min(\gamma_{th},\gamma_{R_{PA}})(1+\omega^{-1})}{2\bar{\gamma}_{sd}^2} & \bar{\gamma}_{sd} = \bar{\gamma}_{rd} \end{cases} \tag{30}$$

When $\bar{\gamma}_{sd} \to \infty$, (30) can be rewritten as

$$P_{out}^{(fdf)}(\gamma_{th}) \approx \begin{cases} \frac{\gamma_{th}}{\bar{\gamma}_{sd}}\omega^{-1}, & \text{for } \gamma_{th} \leqslant \gamma_{R_{PA}}, \\ \frac{\gamma_{th}(1+\omega^{-1}) - \gamma_{R_{PA}}}{\bar{\gamma}_{sd}}, & \text{for } \gamma_{th} > \gamma_{R_{PA}}, \end{cases} \tag{31}$$

which leads to a diversity order equal to 1, regardless of the value of $\gamma_{R_{PA}}$. This is due to the fact that the FDF protocol always uses the cooperative path, regardless of the quality of the SR link.

Let us now consider the SDF protocol. Applying the considerations of this subsection, (28) can be rewritten as

$$P_{out_\downarrow}^{(sdf)}(\gamma_{th}) \approx P_{out_\downarrow}^{(fdf)}(\gamma_{th}) - \frac{\gamma_{th}}{\bar{\gamma}_{sd}}\omega^{-1} + \frac{\gamma_{th}^2}{\bar{\gamma}_{sd}^2}\omega^{-1} + \frac{\gamma_{th}^2}{2\bar{\gamma}_{sd}^2}\omega^{-2}. \tag{32}$$

As $\bar{\gamma}_{sd} \to \infty$, (32) yields

$$P_{out}^{(sdf)}(\gamma_{th}) = \begin{cases} \frac{\gamma_{th}^2}{2\bar{\gamma}_{sd}^2}\left(\tau^{-1} + 2\omega^{-1}\right), & \text{for } \gamma_{th} \leqslant \gamma_{R_{PA}}, \\ \frac{\gamma_{th} - \gamma_{R_{PA}}}{\bar{\gamma}_{sd}}, & \text{for } \gamma_{th} > \gamma_{R_{PA}}. \end{cases} \tag{33}$$

Note that, when $\gamma_{th} > \gamma_{R_{PA}}$, the diversity order is equal to 1 and when $\gamma_{th} \leqslant \gamma_{R_{PA}}$, the diversity order is 2, due to the opportunistic use of the relay path by the SDF protocol.

By comparing the two protocols, we can conclude that the SDF protocol provides better diversity order than the FDF protocol when $\gamma_{th} \leqslant \gamma_{R_{PA}}$. On the other hand, when $\gamma_{th} > \gamma_{R_{PA}}$, the additional degree of diversity provided by the SDF is lost and the two protocols have the same diversity order. However, in this case, the asymptotic outage probability of the SDF protocol has a gain of $\gamma_{th}\omega^{-1}/\bar{\gamma}_{sd}$ with respect to the FDF case.

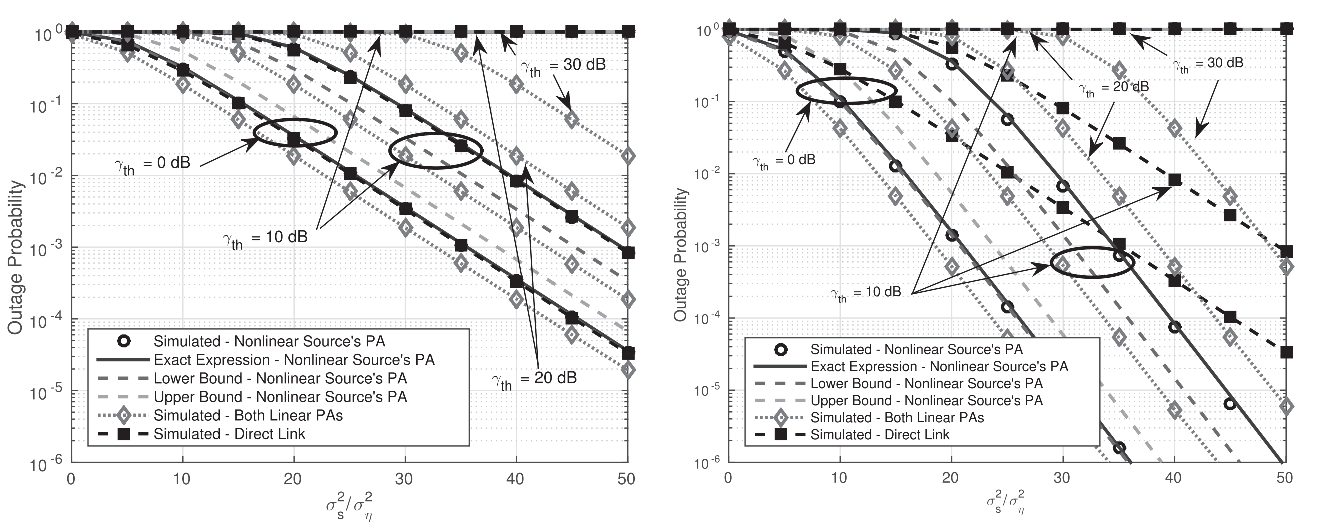


Fig. 1. Outage probability for the FDF (a) and SDF (b) protocols, with IBO = 0 dB, nonlinear PA at the source and linear PA at the relay.

Note that, from (26), (27), (31) and (33), for both protocols, the outage probability is not affected by the presence of the nonlinear PAs if $\gamma_{th} < \gamma_{R_{PA}}, \gamma_{S_{PA}}$. On the other hand, if $\gamma_{th} > \gamma_{R_{PA}}$, the outage probability is increased and the diversity order may be decreased.

A qualitatively similar conclusion can then be drawn for both protocols is: the nonlinear PAs increase the outage probability and decrease the system diversity only for high SNR thresholds. We can obtain an explanation for this fact by regarding the global SD channel as a series-cascade of two linear channels, one of the them being the fixed PA model. As the global SD link is limited by the worst of two links, when γ_{th} is too high, γ_{sd_n} is limited $\gamma_{S_{PA}}$, which means that the SD path is unable to provide diversity. A similar reasoning can be drawn for the SR and RD links.

4. Symbol error rate analysis

In this section, closed-form expressions for the average SER of the considered system is developed, considering both the FDF and SDF protocols. As the simulation results will show, the outage probability lower bound expressions (26) and (27) can used as approximations for the outage probability. The SER expressions are then derived using these lower bound outage probability expressions obtained in Section 3.2. Similar expressions can be obtained for the upper bounds.

A commonly used approximation for the error probability of any coherent modulation is given by [13,24–26]: $P_e = \left(a\sqrt{b}/\sqrt{\pi}\right)\int_0^\infty (1/\sqrt{\gamma_{th}})e^{-b\gamma_{th}}P_{out}(\gamma_{th})d\gamma_{th}$, where $a, b \in \mathbb{R}_+$ are the parameters that depend on the signal modulation. For instance, for M-QAM, $a = 2\left(1 - \frac{1}{\sqrt{M}}\right)$ and $b = \frac{3}{2(M-1)}$ [27,28]. By substituting (26) into the above P_e formula and performing some algebraic manipulations, we get the SER provided by the FDF protocol, given by

$$P_e^{(fdf)} = \begin{cases} a - \frac{\bar{\gamma}_{sd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}}\frac{a\sqrt{b}}{\sqrt{\beta}}\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}}\frac{a\sqrt{b}}{\sqrt{\kappa}}\operatorname{erf}\left(\sqrt{\kappa\min(\gamma_{S_{PA}},\gamma_{R_{PA}})}\right) \\ +\left\{\frac{\bar{\gamma}_{rd}\exp\left(\frac{\gamma_{R_{PA}}}{\bar{\gamma}_{sd}}-\frac{\gamma_{R_{PA}}}{\bar{\gamma}_{rd}}\right)}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}}\frac{a\sqrt{b}}{\sqrt{\beta}}\left[\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) - \operatorname{erf}\left(\sqrt{\beta\gamma_{R_{PA}}}\right)\right]\right\} \\ \times u(\gamma_{S_{PA}} - \gamma_{R_{PA}}), & \text{for } \bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}, \\ a - \frac{a\sqrt{b}}{\sqrt{\beta}}\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) - \frac{a\sqrt{b}}{2\bar{\gamma}_{sd}\sqrt{\beta^3}}\operatorname{erf}\left(\sqrt{\beta\min(\gamma_{S_{PA}},\gamma_{R_{PA}})}\right) \\ +\frac{a\sqrt{b\min(\gamma_{S_{PA}},\gamma_{R_{PA}})}}{\bar{\gamma}_{sd}\sqrt{\pi}\beta}\exp\left(-\beta\min(\gamma_{S_{PA}},\gamma_{R_{PA}})\right) \\ -\left\{\frac{\gamma_{R_{PA}}}{\bar{\gamma}_{sd}}\frac{a\sqrt{b}}{\sqrt{\beta}}\left[\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) - \operatorname{erf}\left(\sqrt{\beta\gamma_{R_{PA}}}\right)\right]\right\}u(\gamma_{S_{PA}} - \gamma_{R_{PA}}), & \text{for } \bar{\gamma}_{sd} = \bar{\gamma}_{rd}, \end{cases} \quad (34)$$

where $\beta = \frac{1}{\bar{\gamma}_{sd}} + \frac{1}{\bar{\gamma}_{sr}} + b$ and $\kappa = \frac{1}{\bar{\gamma}_{rd}} + \frac{1}{\bar{\gamma}_{sr}} + b$.

The SER provided by the SDF protocol can be derived from (34) and (28). Substituting (28) into the above P_e formula, we get

$$P_e^{(sdf)} = P_e^{(fdf)} + \frac{a\sqrt{b}}{\sqrt{\beta}}\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) - \frac{a\sqrt{b}}{\sqrt{\upsilon}}\operatorname{erf}\left(\sqrt{\upsilon\gamma_{S_{PA}}}\right), \quad (35)$$

where $\upsilon = \frac{1}{\bar{\gamma}_{sd}} + b$ and $P_e^{(fdf)}$ is given by (34). Note that, contrarily to the outage probability, the SER in (34) and (35) always depends on $\gamma_{S_{PA}}$ and $\gamma_{R_{PA}}$. From (35), we can conclude that, as $\beta \geqslant \upsilon$, then $\frac{a\sqrt{b}}{\sqrt{\beta}}\operatorname{erf}\left(\sqrt{\beta\gamma_{S_{PA}}}\right) \leqslant \frac{a\sqrt{b}}{\sqrt{\upsilon}}\operatorname{erf}\left(\sqrt{\upsilon\gamma_{S_{PA}}}\right)$, which implies that $P_e^{(sdf)} \leqslant P_e^{(fdf)}$ regardless of the channel state or the nonlinearities inserted by the source or relay PAs.

In order to evaluate the impact of the nonlinearities on the SER, let us eliminate the effect of the noise by considering $\bar{\gamma}_{sd}, \bar{\gamma}_{sr}, \bar{\gamma}_{rd} \to \infty$. Using this assumption, (34) can be rewritten as: $P_e^{(fdf)} = a\left(1 - \operatorname{erf}\left(\sqrt{b\gamma_{S_{PA}}}\right)\right)$. On the other hand, when $\bar{\gamma}_{sd}, \bar{\gamma}_{sr}, \bar{\gamma}_{rd} \to \infty$, we also have $P_e^{(sdf)} = P_e^{(fdf)}$. This means that for high values of SNR, the SERs of the two protocols converge to a same error floor, which depends exclusively on the SNR of the source PA and on the parameters a and b. This result confirms the fact that the nonlinear PA of the source has a higher impact on the system performance than the relay's PA.

5. Simulation results

In this section, simulation results are presented to validate the theoretical results of outage probability and SER derived in the previous sections. The results were obtained using 10^5 Monte Carlo simulations, each one generating independent channel, data and noise realizations. In all the tested cases, channels are assumed to be frequency-selective with a maximum multipath delay of 16 sampling periods and Rayleigh block fading. We also consider that $\sigma^2_{h_{sd}} = \sigma^2_{h_{sr}} = \sigma^2_{h_{rd}} = 1$, unless stated otherwise. The cooperative system operates using an OFDM signal with 256 subcarriers, a cyclic prefix of length equal to 16 and $P_s = P_r = 0.5$. We have used the soft-clipping model [9,22] to represent the PAs of the source and relay with the saturation amplitude being determined by the IBO value. We also assume that the signal is transmitted from the source with a cyclic redundancy check (CRC) code, which allows the relay to determine if the received symbol is correct or not. When SDF is considered, it is assumed that the relay can always detect a transmission error in the SR link.

5.1. Outage analysis

In order to observe the impact of each nonlinear PA on the outage probability, three cases were considered: (I) the source has a nonlinear PA and the relay has a linear PA; (II) the source has a linear PA and the relay has a nonlinear PA; (III) both source and relay have linear PAs. In all these cases, we simulated both the FDF and SDF protocols for several values of the SNR threshold (γ_{th}).

In Fig. 1, we present the results obtained in Case I, i.e., with a nonlinear source's PA and a linear relay's PA, for the FDF and SDF protocols. The figures show the outage probability obtained using computer simulations and using the developed exact outage formulations (obtained numerically), the lower and upper bound expressions, and the simulated outage probability obtained with only the direct link. At the source, we used IBO = 0 dB, which yields $\gamma_{S_{PA}} = 13.2$ dB and $\gamma_{R_{PA}} = +\infty$ dB. Notice that the simulated outage curves are very close to the exact outage probabilities numerically obtained and to the lower bound. It can also be viewed that, as expected, the simulated curves are always higher than the lower bound and smaller than the upper bound.

Moreover, Fig. 1 confirms the orders of diversity predicted in Section 3.3. Indeed, as $\gamma_{th} < \gamma_{R_{PA}}$, when $\gamma_{th} < \gamma_{S_{PA}}$, FDF provides an order of diversity of 1 and SDF an order of diversity of 2. Besides, when $\gamma_{th} > \gamma_{S_{PA}}$, both protocols provide an order of diversity equal to zero.

Fig. 1 also shows the simulated outage probabilities obtained in Case III, i.e. when the source and relay have linear PAs. It can be viewed that nonlinearity due to the PA of the source increases the outage probability for both protocols, even if $\gamma_{th} < \gamma_{S_{PA}}$. For instance, when $\gamma_{th} = 10$ dB, the FDF system with both amplifiers being linear outperforms the case where the source PA is nonlinear by 7 dB. This gain is around 6dB for the SDF protocol. However, when $\gamma_{th} > \gamma_{S_{PA}}$ dB, the difference between Cases I and III is more significant. Indeed, when a linear PA is considered for the source and relay, the diversity order is equal 1 for the FDF and 2 for the SDF. This means that the source PA changes the diversity order only when $\gamma_{th} > \gamma_{S_{PA}}$, as expected.

In Fig. 2, we present the results obtained for Case II, i.e., with a nonlinear relay's PA and a linear source's PA, for IBO = 0 dB at the relay, which yields $\gamma_{R_{PA}} = 13.2$ dB and $\gamma_{S_{PA}} = +\infty$ dB. These figures show the simulated outage probability and the exact outage probability obtained numerically, as well as the corresponding asymptotic curves, obtained from (31) and (33), and the simulated outage probability obtained with only the direct link. The simulated outage probabilities obtained in Case III, i.e. when the source and relay have linear PAs, are also shown. Note that, at high SNR values, the asymptotic curves are quite close to the results obtained via simulation, for both protocols.

Moreover, as predicted by the theoretical developments, when $\gamma_{th} < \gamma_{R_{PA}}$ the nonlinearities due to the relay's PA do not impact the outage probability. However, when $\gamma_{th} > \gamma_{R_{PA}}$, Case II provides outage probabilities higher than the ones obtained with Case III. Note also that, in this case, the impact of the relay's nonlinear PA are more noticeable for the SDF protocol than for the FDF protocol. Indeed, when $\gamma_{th} > \gamma_{R_{PA}}$, the difference between the curves obtained with the FDF system in Cases II and III is approximatively of 3dB. On the other hand, when $\gamma_{th} > \gamma_{R_{PA}}$, it can be viewed in Fig. 2 that the outage probabilities obtained with the SDF system in Cases II and III correspond to different orders of diversity. From these results, we conclude that, in terms of outage probability, the impact of a nonlinear PA at the source is more significant than the impact of a nonlinear PA at the relay.

Note that, in both Figs. 1 and 2, when high nonlinear distortions are considered the FDF system does not achieve considerable performance improvements over the non-cooperative case. On the other hand, the SDF system takes advantage of the cooperative link, achieving a higher diversity gain, as expected.

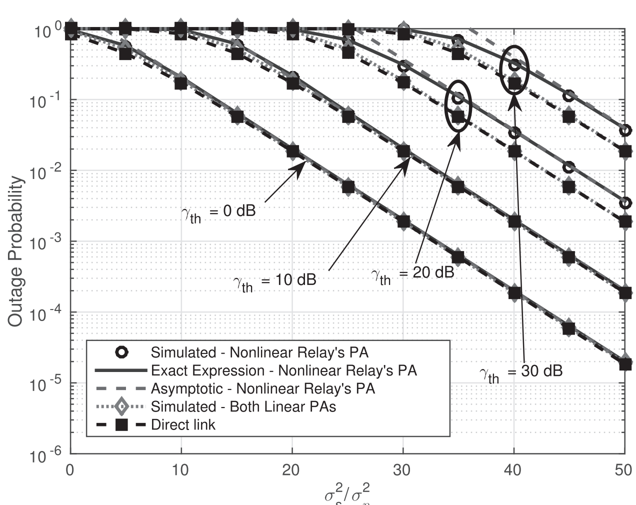
5.2. SER analysis

Simulation results that illustrate the SER analysis developed in Section 4 are presented in this subsection. When using the SDF protocol, we assume that the relay retransmits the received signal to the destination if the signal received by the relay is error-free. Otherwise, the relay does not retransmit the received symbol.

Fig. 3 shows the SER provided by several modulations (4-PSK, 16-64-256-QAM) using a nonlinear PA for both source and relay, with IBO = 5dB ($\gamma_{S_{PA}} = \gamma_{R_{PA}} = 23.6$dB) and $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd}$. In these figures, for each considered modulation, four curves are presented: simulated SER, theoretical SER, simulated SER that has been obtained considering that both source and relay PAs are linear, and the simulated SER obtained with only the direct link. The theoretical SER curves were obtained using (34) for the FDF protocol and (35) for the SDF. Notice that simulated results are very close to the theoretical curves in most of the cases.

Moreover, for 64 and 256-QAM transmitted signals, it can be viewed that there are SER floors for high SNRs. In fact, all the tested modulations have shown SER floors, however, the saturation of the SER only begins at very SNRs for 4-PSK and 16-QAM. These results were omitted for improving the quality of the presentation of the curves. This behavior was predicted in Section 4. Indeed, increasing the links SNR yields $\beta, \kappa, \nu \to b$. Assuming balanced links ($\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$) and considering that $\bar{\gamma}_{sd} \to \infty$, as shown in Section 4, the SER has a saturation bound that depends on the modulation order and on the source PA, given by $P_e^{(fdf)} = a\left(1 - \mathrm{erf}\left(\sqrt{b\gamma_{S_{PA}}}\right)\right)$ for both protocols.

Comparing the SER curves obtained with linear and nonlinear PAs, it can be viewed that the nonlinear PAs have a higher effect on the SER for high-order modulations and medium and high val-



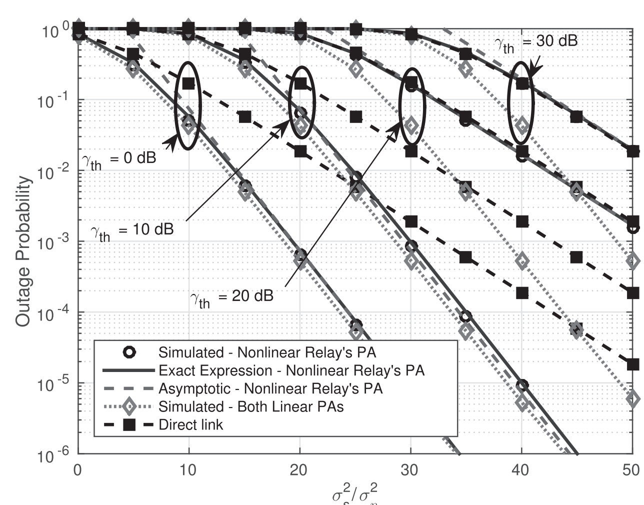


Fig. 2. Outage probability for the FDF (a) and SDF (b) protocols, with IBO = 0 dB, nonlinear PA at the relay and linear PA at the source.

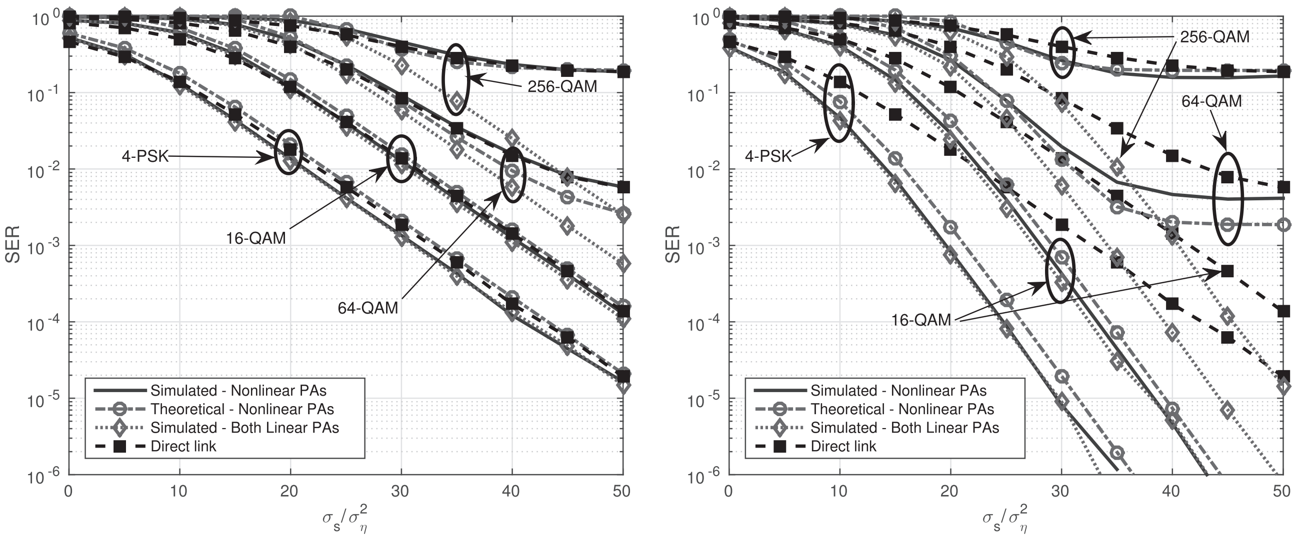


Fig. 3. SER for the FDF (a) and SDF (b) protocols, with IBO = 5 dB, nonlinear PAs at the source and relay, and balanced links ($\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$).

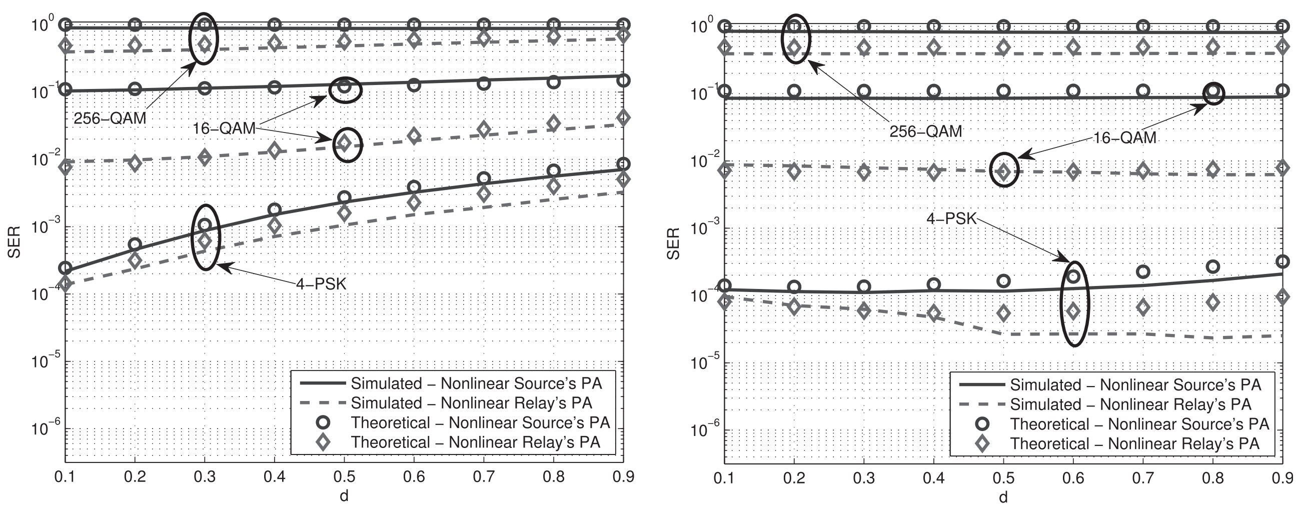


Fig. 4. SER vs. relay position for the FDF (a) and SDF (b) protocols, with a nonlinear PA at the source, a linear PA at the relay, IBO = 0dB and $\sigma_s^2/\sigma_\eta^2 = 25$ dB.

ues of σ_s^2/σ_η^2, where σ_s^2 is the variance of s_n. This can be explained as follows: from (34), using the fact that $\lim_{x\to+\infty}\mathrm{erf}(x) = 1$, it can seen that when $\bar{\gamma}_{sd} \neq \bar{\gamma}_{rd}$ and $\gamma_{S_{PA}} = \gamma_{R_{PA}} \to +\infty$, we have $P_e^{(fdf)} \to \frac{\bar{\gamma}_{sd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}} \frac{a\sqrt{b}}{\sqrt{\beta}} + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd}-\bar{\gamma}_{rd}} \frac{a\sqrt{b}}{\sqrt{\kappa}}$. On the other hand, when b is large (small constellation cardinality), and σ_η^2 is large, $P_e^{(fdf)}$ converges to this same value. This means that for small-order modulations and small values of the mean SNR, the nonlinearities have a small impact on $P_e^{(fdf)}$. A similar reasoning can be applied to the FDF protocol when $\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$, as well as for the SDF.

Besides, it is also possible to view in Fig. 3 that the SDF protocol presents better SERs than the FDF protocol. The lower the modulation order, the higher the gain of the selective protocol compared to the fixed one. This occurs due to the fact that, as above explained, the nonlinear distortions have a more significant effect on the SER for high-order modulations, eliminating the diversity provided by the SRD link and, hence, canceling the gains that the SDF protocol have over the FDF one.

It is important to highlight that when high nonlinear distortions are considered, the FDF system does not present considerable gains over the non-cooperative case, as already observed in the outage analysis. However, the SDF system makes a better use of the cooperative link, providing a lower SER than the direct link.

Finally, we have also been interested in investigating the effect of the distance between the source and the relay on the SER system performance for different modulations (4-PSK, 16 and 256-QAM). We consider a scenario in which source, relay and destination are placed at the same straight line with the relay somewhere between the source and the destination. The SD distance is set to one, the SR distance is denoted by d and the RD distance is $1-d$, with a path loss exponent equal to 2.

Fig. 4 shows the theoretical and simulated SER obtained in the case where the source's PA is nonlinear and the relay's PA is linear, and in the case with a linear source's PA and a nonlinear relay's PA, with IBO = 0dB and $\sigma_s^2/\sigma_\eta^2 = 25$dB. From these figures, notice that theoretical and the simulated curves are very close, which validates the obtained SER expressions for unbalanced links. Another important conclusion that can be drawn from these figures is that the FDF protocol performs better when the SR distance is small. This is due to the fact that the performance of this protocol depends strongly on the quality of the SR link. On the other hand, as it can be verified in this figure, in the SDF protocol, this distance has little influence on the system SER performance.

6. Conclusion

In this paper, a performance analysis of a cooperative OFDM system using a MRC receiver and the FDF and SDF retransmission protocols was developed by considering the effects of a nonlinear PA at the source and relay. In particular, an exact non-analytical expression for the outage probability was presented, as well as closed-form expressions for upper and lower of the outage probability. Moreover, an asymptotic outage analysis was derive, allowing us to obtain the system diversity order. A closed-form expression for the SER of the considered system was also developed.

The simulation results are very close to the exact outage probabilities numerically obtained and to the theoretical lower bound in most of the tested cases. The results corroborate that the impact of a nonlinear PA at the source is more significant than the impact of a nonlinear PA at the relay. Besides, although the SDF protocol performs generally better SERs than the FDF protocol, the impact of the nonlinearities are more noticeable for the SDF protocol than for the FDF protocol.

The results also confirm that a nonlinear PA increases the outage probability only if the target SNR is higher than some parameters that depend on the PA models and on the transmission powers. In this case, the diversity gains are lost and diversity order of the system is decreased. Moreover, the results also confirmed that the considered system has a SER saturation floor that depends on the modulation order and on the PA's SNR. We also concluded that the nonlinear PAs have a higher effect on the SER for high-order modulation cardinalities and small values of the noise power.

In future works, this performance analysis should be extended by taking into account more general fading and system models. Moreover, other diversity combining methods should be considered.

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