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VALDEIR SOARES MONTEIRO

EMPIRICAL ESSAYS ON FISCAL POLICY IN BRAZIL

FORTALEZA - CE
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Tese apresentada ao Programa de Pós-Graduação em Economia (CAEN) da Universidade Federal do Ceará, como requisito parcial à obtenção do título de Doutor em Economia. Área de concentração: Métodos Quantitativos.

Orientador: Prof. Dr. Paulo Rogério Faustino Matos.

Coorientador: Prof. Dr. Cristiano da Costa da Silva

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A Deus.

Aos meus pais, Valdeci e Jaciara.

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“A riqueza de uma nação se mede pela riqueza do povo e não pela riqueza dos príncipes.” (Adam Smith, ano 1776, “A riqueza das nações”).

RESUMO

Esta tese tem três ensaios. Na primeira abordagem usamos wavelet incondicional para tratar da reação fiscal no Brasil no período de 2001 a 2022. Identificamos insolvência de médio prazo da dívida líquida apenas no período 2003-2004. Encontramos co-movimentos antifásicos (curto e médio prazo) liderados pelo saldo primário na crise do subprime, durante a crise fiscal de 2016-2017 e na recente pandemia. Embora o atual nível de dívida em relação ao PIB não seja insolvente, a previsão de 60% ao final de 2022 e o alto nível de inflação anual (12,13% em abril de 2022) requerem muita atenção.

No segundo ensaio propomos uma discussão sobre a situação recente dos investimentos dos governos estaduais no Brasil. Nossas principais conclusões são baseadas na estimativa de um painel dinâmico equilibrado ao longo do período de 2015 a 2021. Encontramos elasticidades significativas do investimento para a Receita Corrente Líquida (RCL) em resposta à sua própria defasagem, a dívida externa defasada para a RCL, a dívida interna defasada para a RCL e o caixa defasado para a RCL. Essa constatação sugere que os investimentos públicos têm reagido para garantir sua sustentabilidade, em resposta às mudanças observadas na dívida e no caixa.

No terceiro artigo exploramos o conjunto de dados do governo do estado do Ceará para buscar uma relação sistêmica entre capital público, dívida e caixa. Utilizando um modelo de vetor de integração, um impulso-resposta e uma wavelet, encontramos uma relação de longo prazo caracterizada por uma reação negativa (positiva) dos investimentos ao aumento da dívida (caixa). Choques de dívida e caixa impactam investimentos após sete bimestres nas direções esperadas, e tais respostas não se dissipam em quatro anos. Também encontramos comovimentos antifásicos com ciclos de investimentos liderados por ciclos de dívida e caixa nos anos de 2014 e 2015, período de intensa crise fiscal financeira nos estados brasileiros.

Palavras-chave: Solvência da Dívida; Wavelet coherency; Sustentabilidade no investimento público; Recurso de financiamento.

ABSTRACT

This thesis has three essays. The first address we use unconditional wavelet to address fiscal reaction in Brazil over the period from 2001 to 2022. We identify medium-term insolvency of net debt only in the period 2003-2004. We find antiphase (short and medium term) co-movements led by the primary balance in the subprime crisis, during the 2016-2017 fiscal crisis and in the recent pandemic. There is this same evidence with low frequency between 2013 and 2019. Mainly after the change of fiscal regime in 2017, government has issued public bonds as a solution to balance public accounts. Although the current level of debt to GDP is not insolvent, the forecast of 60% at the end of 2022 and the high annual inflation level (12.13% in April 2022) require a lot of attention.

The second essay we propose a discussion on the recent situation of investment by state governments in Brazil. Our main findings are based on the estimation of a dynamic balanced panel over the period from 2015 to 2021. We find significant elasticities of the investment to Net Current Revenue (NCR) in response to its own lag, the lagged external debt to NCR , the lagged domestic debt to NCR, and the lagged cash to NCR. This finding suggests that public investments have reacted to ensure its sustainability, in response to observed changes in debt and cash.

The third article we exploit dataset of the government state of Ceará to search for a systemic relationship between public capital, debt and cash. We using integration vector model, an impulse-response and wavelet, find a long-term relationship characterized by a negative (positive) reaction of investments to the increase in debt (cash). Debt and cash shocks impact after seven bimesters investments in expected directions, and such responses do not dissipate in four years. We also find anti-phasic co-movements with cycles of investments led by cycles of debt and cash in years 2014 and 2015, period of intense fiscal financial crisis in Brazilian states.

Keywords: Debt Solvency; Wavelet coherency; Public investment sustainability; Financing resource.

SUMÁRIO

1 INTRODUCTION

The discussion about the wealth of nations originates economic science. Extensive literature points to the importance of public investment in economic growth. A good health infrastructure can, for example, generate more productivity in economic activities because healthier workers are more participative at work. However, governments cannot spend indefinitely, in the economy resources are scarce. Around the 1950s, a more serious literature begins to be developed where the central concern is the conduct of government fiscal policy. Over time, this area of study becomes increasingly broad, technical and decisive for the economic conduct of countries.

The COVID-19 pandemic was responsible for a catastrophic health crisis where millions of people died. The need to regulate the movement of people has had an unprecedented impact on the current globalized economy. National governments have announced gigantic packages of financial aid both for health and for the maintenance of economic activities. The high level of government spending has revived concerns about the conduct of fiscal policy in countries, especially in emerging economies.

On April 11, 2022, the International Monetary Fund[1] (IMF) presented an article demonstrating great concern about the high level of public debt in countries, which reached the level of 40% of the total, the highest in 60 years. The institution commented on the need for global cooperation to reduce the impact of uncertainties. Furthermore, a high debt condition can strongly reduce growth and generate high inflation in developing countries as suggested by Reinhart and Rogoff (2010).

In Brazilian net debt to GDP used to range between 30% and 40% between 2001 and 2006. From 2007 it registered values between 20% and 30% until the end of 2016. During the fiscal crisis, and due to the change in the fiscal regime (control of real spending evolution instead of primary surplus target) this indebtedness started to grow again in the years of 2017 and 2019, reaching again the level of 40%. However, our main concern is in the unprecedented behavior due to the pandemic in the years 2020 and 2021. Net debt to GDP rose from 31.23% (Mar/20) to 48.34% (Apr/22).

Currently, the discussion in Brazil is about making the fiscal rule established in the 95th constitutional amendment (PEC 55/2016) more flexible, which limits federal spending to

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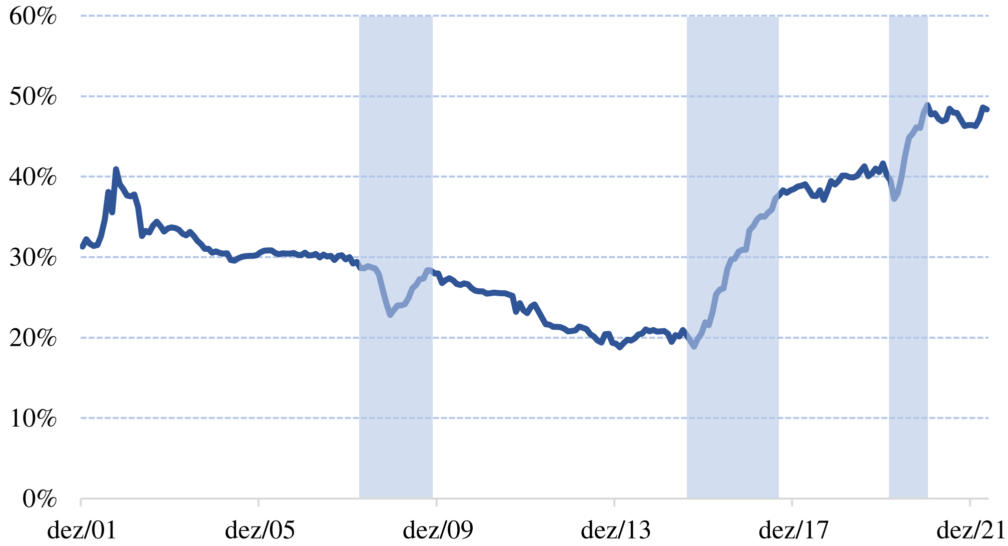
[1] https://www.imf.org/en/Blogs/Articles/2022/04/11/blog041122-dangerous-global-debt-burden-requires-decisive-cooperation

inflationary correction. The new fiscal framework[2] in March 2023 suggests fiscal targets for the coming years, moving from deficits of 0.5% in 2023 to surpluses in the following years, reaching 1% in 2026.

1.1 Modelling brazilian federal government fiscal reaction in the time-frequency domain

This context, we present this work that seeks to present a discussion on fiscal policy in Brazil, the work consists of three articles. All papers are produced with the cooperation of the Paulo Matos and Cristiano da Silva. The first article is titled "Modelling brazilian federal government fiscal reaction in the time-frequency domain", It was accepted for publication in the journal *Economics Bulletin*, in December 2022, and was presented for discussion at the 50º ANPEC Nacional 2022. We use wavelet to address fiscal reaction in Brazil over the period from 2001 to 2022. Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN. In Fig. 1, we report the related time series. We highlight three periods of historical importance. Between 2008 and 2009 the subprime crisis, in the period from 2015 to 2017 Brazilian fiscal crisis and in 2020 the COVID-19 pandemic.

Fig. 1. Debt to GDP and Primary Balance do GDP.



[2] https://www.gov.br/planejamento/pt-br/assuntos/noticias/2023/arcabouco-fiscal-e-crivel-transparente-e-foca-no-principal-que-sao-as-demandas-sociais-defende-tebet/apesentacao-arcabouco-fiscal.pdf

The analysis of indebtedness in nominal values suggests three periods with very clear characteristics. Between December 2001 and late 2015, there is an apparently linear growth trend, and from 2016 to March 2020 another trend with greater slope. In the last 2 years of the sample, there is a third phase of evolution, with more accentuated growth and with an additional element: high inflation. In this third moment, this net debt rose from R$ 2.8 trillion in March 2020 to R$ 4.4 trillion in April 2022.

Table 1. Descriptive statistic 1.

Descriptive statistics				
	Debt	Debt to GDP	Primary Balance	Primary Balance to GDP
n	245	245	245	245
average	1461,2404	30,9884	-31,6808	,3058
median	1010,4083	30,2600	40,4118	1,4200
standard error	1038,18300	7,99354	169,70707	2,77755
var.	1077823,944	63,897	28800,490	7,715
min.	411,77	18,79	-747,58	-9,98
max.	4367,68	48,90	119,65	2,99

We identify medium-term insolvency of net debt only in the period 2003-2004. We find antiphase (short and medium term) co-movements led by the primary balance in the subprime crisis, during the 2016-2017 fiscal crisis and in the recent pandemic. There is this same evidence with low frequency between 2013 and 2019. Mainly after the change of fiscal regime in 2017, government has issued public bonds as a solution to balance public accounts. Although the current level of debt to GDP is not insolvent, the forecast of 60% at the end of 2022 and the high annual inflation level (12.13% in April 2022) require a lot of attention.

1.2 A Note on the public investment-debt-cash linkages: a Brazilian cross-state analysis

Next, we present the paper entitled “A Note on the public investment-debt-cash linkages: a Brazilian cross-state analysis”, this paper was submitted to journal *Economics Bulletin* and it is revised according to suggestions from the editor and reviewer. We look forward to a possible acceptance. Summarizing the situation in terms of public investments in Brazil, the participation of state and municipal governments in the composition of public investment in Brazil is very significant and growing in recent years. According to Bonomo et al. (2021), over the period from 2015 to 2019, on average, municipal governments invested 0.54% of GDP, while state governments invested 0.48%, and the federal government invested 0.43%.

Regarding only real public investments by state governments, after this variable remained around R$ 50 billion over the period from 2015 to 2018, and subsequent reduction to the level of R$ 40 billion in 2019 and 2020, such investments reached almost R$ 76 billion in 2021, and R$ 114 billion in 2022 (Figure 1). It is important to identify in terms of financing, that the borrowing participation, which has already reached almost 58% in 2015, has shown successive reductions, with such participation reaching 12% in 2021. Among all 27 state governments, three have financed more than 50% of their respective investments through borrowing on average over this period: Piauí, Rio de Janeiro and Ceará. At the other extreme, Rondônia, Mato Grosso do Sul and Roraima showed the lowest levels of borrowing-to-investment ratio.

In this article, the central theme is the discussion on public investment and its sources of financing, in the linkage with debt and cash of subnational governments. The article was built nationally. Our main findings are based on the estimation of a dynamic balanced panel over the period from 2015 to 2021. We find significant elasticities of the investment to Net Current Revenue (NCR) in response to its own lag, the lagged external debt to NCR, the lagged domestic debt to NCR, and the lagged cash to NCR. This finding suggests that public investments have reacted to ensure its sustainability, in response to observed changes in debt and cash.

1.3 On the public investment-debt-cash linkages in the state government of Ceará

In the last article entitled “On the public investment-debt-cash linkages in the state government of Ceará”, we discussed sustainability of investments in Ceará. Of the three, the most technical and profound of all, submitted to *Revista Brasileria de Economia* (RBE), and accepted at the *World Finance Conference* (WFC) in Miami in December 2022. When we observe bimonthly series from 2011b6 to 2021b6 of real (R$ December 2021) public investments committed by the state government of Ceará – accumulated values over 6 bimesters –, we identify two atypical periods. The table 2 show the descriptive statistic.

Table 2. Descriptive statistic 2.

Descriptive statistic							
	INV	DC	CX	RCL	INV/RCL	DC/RCL	CX.RCL
n	61	61	61	61	61	61	61
average	4,22282	14,24266	4,95191	22,29727	19,39779	63,29044	21,97259
median	3,28409	13,47942	4,62963	21,9072	15,19762	63,86246	20,7439
standard error	2,248117	3,480678	1,59309	1,576623	11,18028	12,00717	5,791881
var.	5,054	12,115	2,538	2,486	124,999	144,172	33,546
min.	2,404	9,146	2,149	19,718	9,861	43,653	10,395
max	10,475	20,994	9,367	25,254	48,893	88,239	37,438

At the end of 2011 and 2014, the amounts of real investments committed were R$9.6 billion and R$10.5 billion, respectively. Over the remaining years, the real commitments oscillate around R$ 3 billion. More precisely, they ranged between R$ 2.4 billion (2019b5) and R$ 3.9 billion (2013b2). On consolidated debt, we see a level of approximately 50% of NCR over the first few years, and therefore we find higher levels of indebtedness, which culminated in more than 88% in 2020b3.

We add to the empirical literature on public investment sustainability by using an integration vector model, an impulse-response and a conditional multivariate wavelet framework. We find a long-term relationship characterized by a negative (positive) reaction of investments to the increase in debt (cash). Debt and cash shocks impact after seven bimesters investments in expected directions, and such responses do not dissipate in four years. We also find anti-phasic co-movements with cycles of investments led by cycles of debt and cash in the years 2014 and 2015, a period of intense fiscal financial crisis in Brazilian states.

The articles are not dependent on each other. References are provided at the end of each chapter.

2 MODELLING BRAZILIAN FEDERAL GOVERNMENT FISCAL REACTION IN THE TIME-FREQUENCY DOMAIN

2.1 Introduction

There is a strand of literature on public finance studying the drivers and consequences of indebtedness as collateral for issuing bonds or accessing borrowing to finance investments or even current expenditure. Some of the pioneers in this theoretical literature are Modigliani (1961), Mishan (1963) and Diamond (1965). In the 70's, the discussion promoted by Barro (1974) was important for the literature to recognize that public debt plays an important role in theoretical analysis of monetary and fiscal effects. We also highlight the theoretical approach proposed by Barro (1979), seen as the usual starting point of any discussion on sustainability of a public debt issue by a large national government.

Until then, one could expect a nonstationary debt-income ratio implied by some models of optimal government finance. However, a high and growing debt-GDP ratio was viewed as worrisome by macroeconomic models with limited taxation (Blanchard, 1984). Moreover, given the worrying debt levels in several countries from the 80's, the concept of solvency of public debt has become more explored in empirical studies, such as Hamilton and Flavin (1986), and Bohn (1998), for example. For instance, total public debt as a ratio of GDP in U.S. rose from 31.16% in December 1980 to 56.04% a decade later.

Following this literature, we define the debt solvency concept, by assuming that debt is sustainable so long as the probability of a debt explosion, and thus debt default, remains very low. Addressing this discussion is relevant given the longer-term macroeconomic implications of much higher public debt. In this context, Reinhart and Rogoff (2010) find that whereas the link between growth and debt seems weak at normal debt levels, median growth rates for countries with public debt over roughly 90% of GDP are about one percent lower than otherwise. Herndon et al. (2014) propose revisiting this issue and they argue – contrary to Reinhart and Rogoff's (2010) main conclusions – that the relationship between public debt and GDP growth varies significantly by period and country.

Despite empirical divergences on the macroeconomic role of high debt, the nonlinear effect of debt on growth can be related to a response of market interest rates as economies reach debt tolerance limits (Reinhart et al., 2003). This pass-through related to interest rates is also useful to understand the role played by inflation in GDP – a classic research agenda of monetary economics. [3] Specifically in the post-pandemic period, most of economies are registering high

[3] Matos et al. (2022) are studying the effects of high levels of inflation and debt on GDP in Brazil.

and unprecedented levels of indebtedness together with high inflation: a dangerous combination, and perhaps in some economies irreversible in the short term.

This scenario has motivated us to revisit the solvency of public debt in one of the world's largest emerging economies with a very interesting history of fiscal rules: Brazil.

Brazilian net debt to GDP used to range between 30% and 40% between 2001 and 2006. From 2007 it registered values between 20% and 30% until the end of 2016. During the fiscal crisis, and due to the change in the fiscal regime (control of real spending evolution instead of primary surplus target) this indebtedness started to grow again in the years of 2017 and 2019, reaching again the level of 40%. However, our main concern is in the unprecedented behavior due to the pandemic in the years 2020 and 2021. Net debt to GDP rose from 31.23% (Mar/20) to 48.34% (Apr/22). More specifically, to compensate in 2020 for the drop in revenue (R$ 112 billion with revenue not managed by the Federal Revenue) and the increase in extraordinary credit (R$ 426 billion with aid to the most vulnerable and in transfers to combat the pandemic), net debt grew by R$ 600 billion in 2020. The forecast of 60% at the end of 2022 by Focus report (Central Bank of Brazil) is also worrying.[4]

According to Berenguer-Rico and Carrion-i-Silvestre (2011), one can summarize the empirical contributions proposed until then, considering the role of non-stationarity time series analysis and observing that two different approximations can be found in the literature: a univariate approach that has focused on the stochastic properties of the stock of debt and, a multivariate one that has focused on the long-run properties of the flows of expenditures and revenues, i.e., in the stochastic properties of the deficit. Their main contribution is unifying these approaches considering the stock–flow system that fiscal variables configure.

Regarding this literature applied to Brazil, Rocha (1997) finds an intertemporal budget balance only until 1990 when financial assets are frozen. Issler and Lima (1998) and Cavalcanti and Silva (2010) find evidence of stationarity in the public deficit with adjustment always via taxes, with expenditures being exogenously determined, while Lima and Simonassi (2005) find that seigniorage affects deficit management and a fiscal reaction is taken when the deficit becomes greater than 2.2%. Simonassi (2007) finds a structural change in fiscal policy from 1995 onwards able to modify sustainable behavior in a low public sector response to deficit increases. Mendonça et al. (2009) find a post-2000 regime change and corroborate the previous

[4] The Focus Market Report provides weekly mean market expectations for inflation over following month, 12 months, and following year as well as expectations for SELIC target rate, real GDP growth, net public sector debt/GDP, industrial production growth, current account, and trade balance, collected from over 130 banks, brokers, and fund’s managers.

results of low fiscal reaction to deficits.

More recently, Campos and Cysne (2019) use Kalman filter, spline smoothing, and co-integration and they find a reduction in the variation of the primary deficit in response to the debt to GDP.

A first problem is that this broad discussion does not capture the most recent effects of the pandemic. Moreover, this literature has not measured yet the relation between primary balance and debt in the time frequency domain for Brazil.

Nonetheless, the most recent empirical literature on the relationship between fiscal, macroeconomic, and financial variables (Lo Cascio, 2015 and Matos et al., 2021, for instance) has employed continuous wavelet transformation tools to address empirical issues in U.S. This is a classical mathematical methodology, that has not yet been so explored in this literature for emerging markets, although the practical advantages. This mathematical framework enables us to infer on which primary balance cycle has been leading or lagging each debt cycle.

In other words, this technique offers us greater freedom in the analysis of co-movements, by allowing different responses, in intensity, and in the direction of the leadership of the cycles between the variables involved, over time and with different frequencies. This freedom in terms of time-varying results is relevant in a long-time horizon in a country with complex problems, as well as the freedom to respond with different frequencies is equally important when working with monthly series, even knowing that changes in the economy occur not in the short term, but after semesters or years.

We are conceptually and methodologically aligned to Lo Cascio (2015), which suggests the wavelet approach to verify fiscal sustainability in the U.S. from the annual time series of fiscal variables during the period between. She finds fiscal sustainability in the U.S. long term debt only until 1995, and she also shows that governments tend to respond more vigorously to budget deficits when the level of debt is high rather than low.

We add to this empirical literature on the sustainability in indebtedness, by analyzing for the first time the co-movements between primary balance and debt cycles in Brazil. Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN.

This paper is structured as follows. In Section 2 we have a theoretical discussion, and in Section 3 we show the wavelet framework. Section 4 reports our main findings, while Section 5 is devoted to the final discussion.

2.2 Debt Theory

The usual starting point of any discussion on sustainability of a public debt issue by a large national government is the model proposed by Barro (1979). The author assumes that the government can finance its expenditure through current taxation and public debt issue.[5] The government's budget in each period is

$$G_t + rb_{t-1} = \tau_t + (b_t - b_{t-1}) \tag{1}$$

In this model, real government expenditure in t, aside from interest rate payment on debt, is given by G_t, real tax revenue in t is denoted by τ_t, real stock of public debt outstanding ate the end of period t is denoted by b_t, and r means the rate of return on public and private debt. It is usual analyzing this same relationship powered by GDP. Solving the equation forward in time, this implies that the debt ratio in the future depends on the initial debt ratio, current and future interest and growth rates, and current and future primary balances. If the debt ratio is to remain constant, it is straightforward deriving a steady state relation between debt and the primary balance ratio. However, if the purpose is discussing on what debt level might be sustainable, we need to know more about the behavior of the primary balance. We follow Blanchard et al. (2021) by believing that a reassuring theoretical and empirical answer was given in an influential paper by Bohn (1998).

Bohn (1998) proposes an empirical approach trying to address two relevant questions: How do governments react to the accumulation of debt? Do they take corrective measures when the debt-GDP ratio starts rising, or do they let the debt grow? In a very didactic way, the author suggests that one can find direct evidence for corrective actions by examining the response of the primary (noninterest) budget surplus to changes in the debt-income ratio. A positive response shows that the government is taking actions to counteract the changes in debt, as reducing noninterest expenditures or raising tax. The idea is to search for a systematic relationship between debt to GDP (or debt-income ratio) and primary balance to GDP (or primary balance-income ratio), called fiscal reaction, by estimating the following regression:

$$s_t = \rho b_t + \alpha \boldsymbol{Z}_t + \varepsilon_t \tag{2}$$

where s_t means the primary balance to GDP, i.e., and $\boldsymbol{Z}_t$ is a set of other determinants and ε_t an error term. In this context, we propose to revisit this model of fiscal reaction, following methodologically Lo Cascio (2015), which applied unconditional wavelet to U.S. debt solvency. This technique makes it possible to infer, over time and with different frequencies, whether

[5] He does not deal with currency issue, although this type of finance could be included as one form of current taxation.

there are co-movements (in the same direction or not) and whether there is leadership of any of the two variables: primary surplus and net debt (as a ratio of GDP).

2.3 Methodology

The Fourier analysis can be considered one of the most important bases for the wavelet transform development. This analysis is a powerful tool to modelling time series on frequency domain. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands.

Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\, \psi_{\tau,s}^{*}(t) dt \quad (3)$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series in space and scale. To capture different frequencies of the signal, we use a mother wavelet that is stretched and shifted:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) \quad (4)$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequencies movements. So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) dt \quad (5)$$

The first wavelet measure that we will present it's the Wavelet Power Spectrum (WPS), which reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \quad (7)$$

To compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GWPS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \quad (8)$$

To study the dependencies between two time series $x(t)$ and $y(t)$ in time-scale/frequency plane, primary balance to GDP and debt to GDP in our exercise, we use wavelet coherency, a measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by:

$$W_{xy}(\tau, s) = W_x(\tau, s) W_y^*(\tau, s) \tag{9}$$

where $W_x(.)$ and $W_y(.)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and * denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $|W_{x,y}(\tau, s)|$. It computes the local covariance between two signals at each scale. The squared wavelet coherence is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherence is denoted as:

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1} W_{x,y}(\tau, s)\right)\right|^2}{S(s^{-1} W_x(\tau, s)^2) S(s^{-1} W_y(\tau, s)^2)} \tag{10}$$

where $S(.)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization fator ensuring the conversion to an energy density. Torrence and Webster (1999) note that in numerator of the squared wavelet coherence, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it's ensured that $0 \leq R^2 \leq 1$.

Hence, the main advantage of the wavelet coherence on XWT is the common measure unit to examine several combinations of signals. Torrence and Compo (1999) reveal that once the wavelet transforms conserves variance, the wavelet coherence is a good representation of the normalized covariance between two-time series, where the closer to zero (one) the coherence, the weaker (stronger) the local correlation between the time-series. The wavelet coherence has not theoretical distribution known; hence we follow the approach of Aguiar-Conraria and Soares (2011) deriving the confidence interval using Monte Carlo methods.

Although the wavelet coherence computes the degree of local linear correlation between two signals, it does not reveal patterns of lead-lag relationship neither if the movements are positives or negatives. To deal with these limitations, the phase-difference is commonly used to examine the delays in the fluctuations between the two time-series. Following Torrence and Webster (1999) we define the phase difference as:

$$\phi_{xy}(\tau, s) = \tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1} W_{x,y}(\tau, s)\right)\right\}}{\Re\left\{S\left(s^{-1} W_{x,y}(\tau, s)\right)\right\}}\right) \tag{11}$$

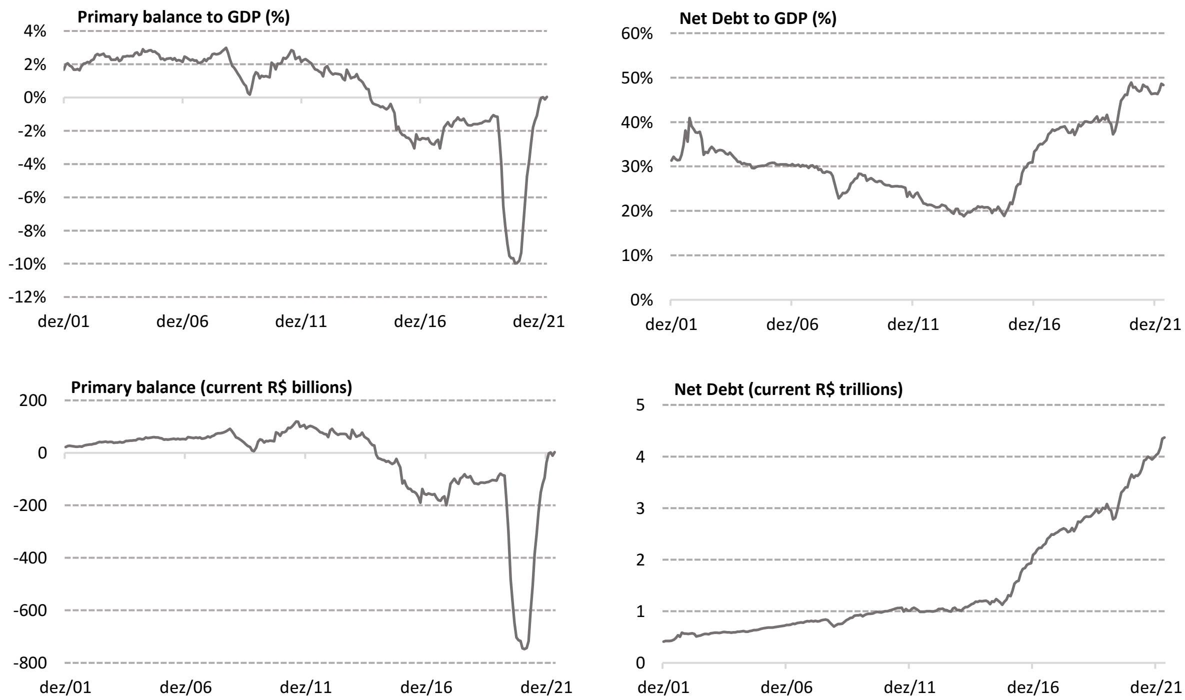
The smoothed real ($\Re$) and imaginary ($\Im$) parts should already be calculated in the wavelet coherence function. Both $R^2(\tau, s)$ and $\phi_{xy}(\tau, s)$ are functions of the position index (τ) and scale (s). Finally, we need the information on the signs of each part to determine the value of $\phi_{xy} \epsilon [-\pi, \pi]$. Next, we summarize the discussion of possible outcomes, depending on the region indicated in the phase difference:

- A phase-difference of zero indicates that the time-series move together at the specified frequency, while a phase-difference of $\phi_{xy} = \pm\pi$ indicates an anti-phase relation, and in both cases, there is no leadership,
- If $\phi_{xy} \epsilon \left(0, \frac{\pi}{2}\right)$ the series move in phase, but the time-series y leads x, while if $\phi_{xy} \epsilon \left(-\frac{\pi}{2}, 0\right)$ then it is x that is leading,
- If $\phi_{xy} \epsilon \left(\frac{\pi}{2}, \pi\right)$, then x is leading and time-series y is leading if $\phi_{xy} \epsilon \left(-\pi, -\frac{\pi}{2}\right)$, and in both cases, we find an out-of-phase relation.

2.4 Empirical Exercise

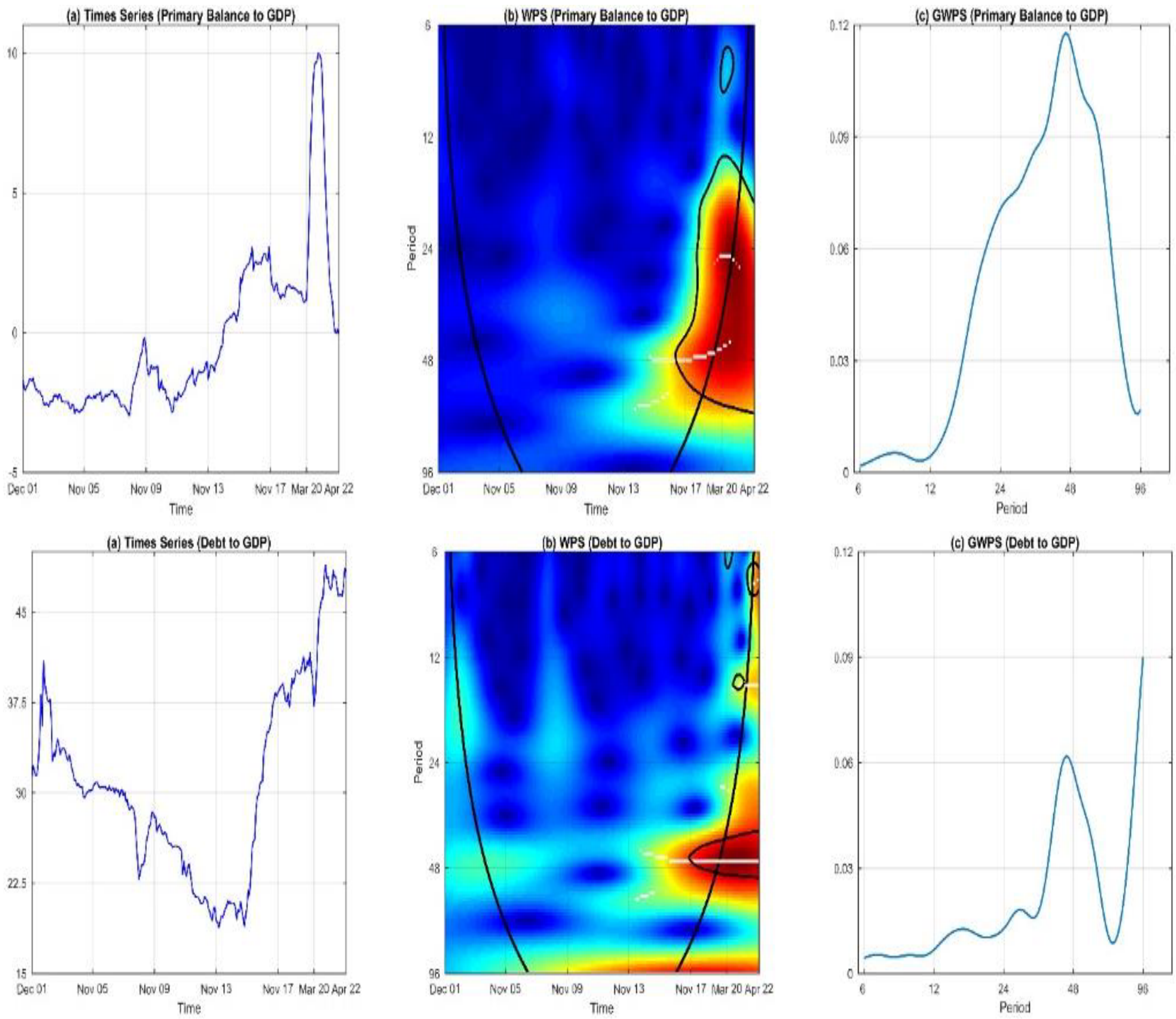
Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN. In Fig. 1, we report the related time series, while Fig. 2 shows WPS and GWPS for all series used here, and in Fig. 3 we report the results on wavelet coherency, phase-difference and gain.

Fig. 1. Primary Balance and Net Debt.



Source: Data can be found in the Brazilian Central Bank's Time Series Management System (SGS). Variable codes are: 4468 - Public Sector Net Debt (R$); 4503 - Public Sector Net Debt (% GDP); 5068 - NFSP Flow accumulated in 12 months without currency devaluation (R$); 5783 - NFSP without currency devaluation (% GDP).

Fig. 2. Debt to GDP and primary balance series (a), respective WPS (b) and Global WPS (c).



Notes: The black contours on the Wavelet Power Spectrum (WPS) plot refers to 5% significance and is theoretically obtained considering an AR (1) as the null hypotheses. In the heatmap, colder colors represent lower power while warmer colors represent higher power. The shaded area outside the Cone of Influence is subject to edge effects.

The analysis of indebtedness in nominal values suggests three periods with very clear characteristics. Between December 2001 and late 2015, there is an apparently linear growth trend, and from 2016 to March 2020 another trend with greater slope. In the last 2 years of the sample, there is a third phase of evolution, with more accentuated growth and with an additional element: high inflation. In this third moment, this net debt rose from R$ 2.8 trillion in March 2020 to R$ 4.4 trillion in April 2022. When weighted by GDP, the behavior presents different characteristics. After a strong increase of almost 10% at the beginning of the time window, the debt-GDP ratio reduces from 40.91% (September 2002) to around 19% at the end of 2013. In this period, there was just one unusual swing during the subprime crisis. We also find a stability around 20% in 2014 and 2015, and a subsequent growth trend, breaking the 40% level at the end of 2019. Once more, the concern is in the unprecedented behavior due to the pandemic: debt to GDP rose from 31.23% (March 2020) to 48.34% (April 2022).

The primary balance is the inverse of the public sector financing need (NFSP). Observing its accumulated value (12 months), the federal government used to record a surplus until October 2014. Even during the subprime crisis, the primary result was compromised, but there was no deficit. The first deficit was recorded in November 2014, followed by new deficits, with emphasis on the deficit value exceeding 200 billion in October 2017, during the fiscal crisis in the country. The recovery that began on that date was compromised by the pandemic.

We highlight the record deficit of almost R$ 750 billion in January 2021. The recovery was fast, and we see a deficit in December 2021 lower than the registered before the pandemic. The behavior of the GDP-weighted series is visually very similar. It is important to highlight the value of approximately 10% of GDP associated with the record deficit at the beginning of 2021.

Regarding the analysis based on Wavelet Power Spectrum (WPS) and Global Wavelet Power Spectrum (GWPS), we know that WPS reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane, while GPWS takes the average of wavelet power spectrum over all times. According to Fig. 2 (upside), WPS for primary balance shows a great power intensity between 2017 and 2020 with medium and low frequency, which is not recorded in the other periods. Regarding debt to GDP (Fig. 2. bottom), we observe that the WPS has high power at lower frequencies only between 2016 and 2017 (fiscal crisis), and in the other periods the variance is low. Still according to WPS analysis, we find that the spectrum varies so much along the time, indicating the non-stationarity of data and suitability of the method employed. The GWPS shows peaks only at long frequency, close to 48 months, between 6% and 9% for primary balance, and between 9% and 12% for debt.

Our main findings are based on the wavelet analysis, and they are reported in Fig. 3.

The first step in this analysis is to look at the heat map (left side of Fig. 3). The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain. Once we identify a red and significant region in each period, we need to identify the respective frequency (vertical axis of the heat map), and then proceeded to analyze the graph in the phase difference.

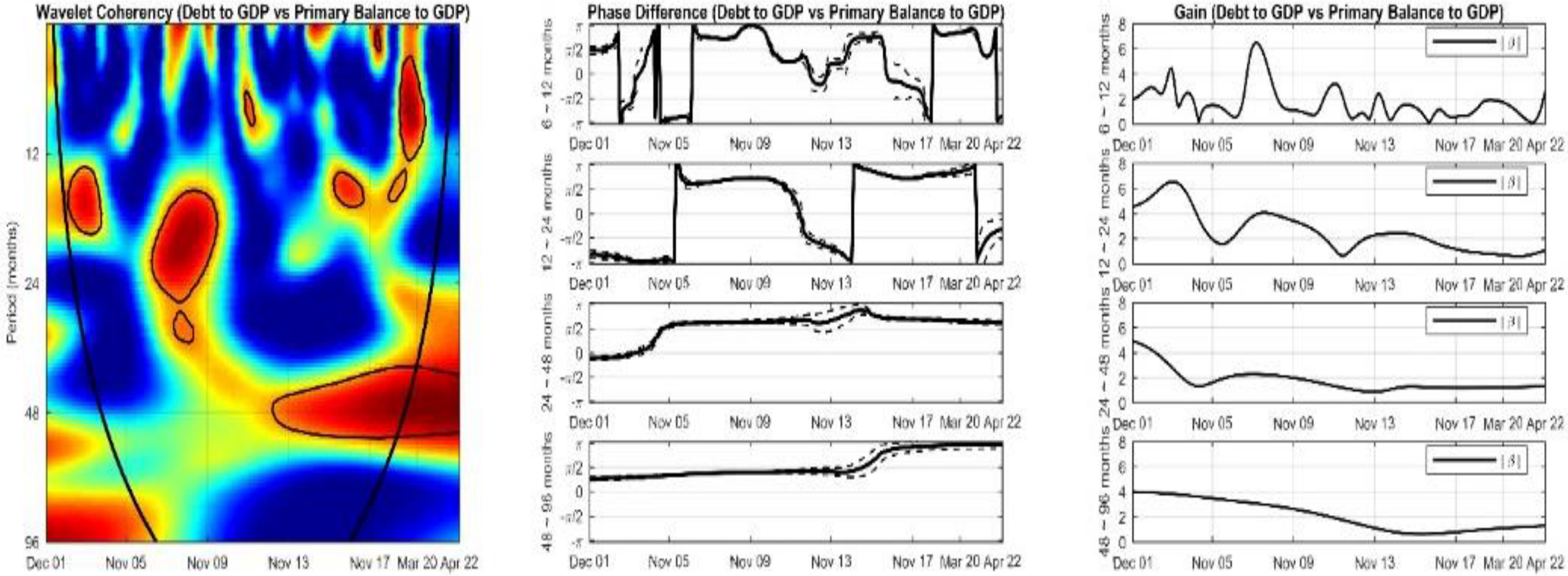
There are 4 phase difference graphs (center of Fig. 3). The first serves to identify the leadership of high frequency cycles (6 to 12 months), while the second is useful to analyze the cycles with frequency between 12 and 24 months, and so on, until the fourth charts show the cycles of low frequency, between 48 and 96 months. The analogy is the same for using the 4 gain graphs (right of Fig. 3).

Observing the heat map, we can highlight five periods in which the fiscal cycles show strong interaction. Analyzing these moments in chronological order, first we identify insolvency of Brazilian government net debt in the period 2003-2004 (period characterized by electoral turmoil followed by a new government administration) with frequency ranging from 1 year to 2 years period. This conclusion is associated with the presence of a red area with frequency between 12 and 24 months in the years 2003 and 2004, according to the heat map. Observing, the phase difference (graph with the same frequency), in this period the line is in the region between $(-\pi, -\pi/2)$, so we find an anti-phase relation$y$ with y (debt) leading x (primary balance). We can measure this insolvent behavior (increase in debt implying a reduction in the primary balance approximately 1 to 2 years ahead) with an elasticity ranging between 4 and 7, according to the respective gain graph. This is the reasoning for the next analysis.

Next, we find co-movements (frequency ranging from 1 year to 2 years period) with opposite direction led by the primary balance in the subprime crisis (2007 – 2008) and during the 2016-2017 fiscal crisis. There is this same evidence – antiphase leadership by primary balance – with low frequency (2 to 4 years) between 2013 and 2019. This finding on debt cycles anticipated by primary balance cycles can be measured or characterized by elasticities lower than 4.

More recently, once more we find that primary balance cycles leading debt cycles in the opposite direction with high frequency (less than 1-year period). All of our evidence are robust when using gross debt to GDP. We do not report these results, to save space, but we can disclose them if requested.

Fig. 3. Wavelet coherency: net debt to GDP vs primary balance to GDP



Notes: The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain of the explanatory variable, primary balance.

2.5 Conclusion

Since the Tax Liability Law (LRF) in 2000, Brazilian federal government has chosen for a primary surplus target. From 2017, we observe a new rule controlling real spending evolution. Mainly after this change of tax regime, government has not chosen for a fiscal adjustment through tax increases or expenditure reductions. Given that the interest payment on debt was not included in the previous rule (primary surplus target), nor in the new rule of spending ceiling, the consequence is that government has issued public bonds in local currency with the longest possible term as a solution to balance public accounts, instead of issuing currency. [6]

In this historical fiscal context, on the one hand, it is comforting finding here in our paper that the current debt is not insolvent in the current pandemic period, for instance, and that only in 2003 and 2004 there was a significant sign of insolvency. It is also important the preliminary evidence reported in Matos et al. (2022) that the level of indebtedness has not compromised the growth of real GDP per capita in recent decades, and that the villain of growth seems to be the inflation. Still according to them, the cycles of GDP per capita in the last two decades have determined the debt cycles in Brazil in an antiphase direction. Here, we find that the cycles of the primary result in periods of fiscal imbalances are also capable of driving in the opposite direction the cycles of indebtedness.

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[6] The inflation target regime was adopted in Brazil in 1999. Although high inflation is undesirable in a democracy that experienced hyperinflation in the 1990s, inflation was outside the tolerance range in 2001, 2002, 2003, 2015, 2016 and 2021.

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3 A NOTE ON THE PUBLIC INVESTMENT-DEBT-CASH LINKAGES: A BRAZILIAN CROSS-STATE ANALYSIS

3.1 Introduction

The relevance of strengthening public finance management by a federal or subnational government should not have a purpose in itself. In societies whose socio-macroeconomic conditions are unfavorable, the main role of an austere fiscal policy lies in its ability to raise per capita income and reduce its inequality. In this context, every contribution in monitoring fiscal variables is justifiable, with emphasis on the resilience and predictability of tax revenues and expenses with payroll and social charges, as well as the solvency of public debt. These are the minimum necessary conditions for the government – particularly in a low-income country and in times of crisis – to be a robust provider of essential services in areas such as health, education, security and pensions, and to be able to maintain the flow of public investments.

Regarding the public investments, policy makers should listen researchers and follow the literature on the linkages with the economic side, as well as the relationship between investments and other fiscal variables. The former issue has attracted increasing attention from the public finance literature and from international institutions, such as the International Monetary Fund (IMF) and the World Bank (WB), since the 1990s. We highlight the intuitive positive long-term consequences of public investment on economic growth. First, we mention a classic theoretical framework in Barro (1990), which builds a growth model including services and public investments as a productive input for private producers. Other seminal works also analyze the growth impact of public investment in the context of endogenous growth models, as Barro and Sala-i-Martin (1992) and Glomm and Ravikumar (1994), for instance.

A relevant contribution to this discussion is the incorporation of government debt in their endogenous growth framework suggested by Turnovsky (1999). More recently, Chatterjee and Turnovsky (2007) and Agenor (2010) have explored the importance of tied vs untied aid, and the role of infrastructure network effects and the efficiency of public investment in low-income countries, while Berg et al. (2010) look into the macroeconomic effects of aid-financed public investment, however disregarding the interaction of structural and policy conditions with debt dynamics. In this extensive theoretical literature, one of the most relevant contributions for us is Buffie et al. (2012). They propose a model to study the macroeconomic effects of public investment. A first differential is that their approach applies specifically to the reality of low-income countries, which seems adequate given our purpose to study Brazilian states. Second, they assume the role of the investment-growth linkages, public external and domestic debt accumulation, fiscal policy reactions necessary to ensure debt-solvency, and macroeconomic adjustment required to ensure internal and external balance.

This discussion which considers the relationship between fiscal variables is essential for us, given our purpose to model how public investments have reacted to ensure its sustainability, in response to observed changes in (external and domestic) debt and cash. More specifically, we propose an empirical exercise applied to state governments in Brazil based on a dynamic balanced panel over the longest available period, from 20015 to 2022, to better understand the public investment-debt-cash linkages.

We add to a recent discussion promoted by Simonassi et al. (2021) and Bonomo et al. (2021). The former work suggests that the increase in subnational government investments produces a virtuous cycle that contributes to subsequent increases in revenue that overlap with the respective increases in costing, over the period from 2008 to 2016. The second work suggests that public investment is not closely related to fiscal rules in Brazil but is mainly determined by fiscal conditions at state level. Our results allow us to add to this debate.

Moreover, we are aligned to a discussion promoted by Matos (2023). This author proposes an unprecedented theoretical accounting framework suitable for the budget execution of these subnational governments. In applied terms, there is an empirical exercise based on this modeling is proposed, with bimonthly data from 2015 to 2022 for all 27 governments, by estimating the vector model suitable for the stationarity of the time series.

The conduct of public policies by state governments is restricted by a set of old, rigid and universal rules. These subnational federative entities do not have revenue autonomy and cannot obtain contractual debt for current expenditure purposes. This context capable of compromising public investments motivates a specific analysis on the dynamics of the movements between: debt, cash, and investments. A study on the relationship between cash, debt and public investments by state governments is also relevant given the order of magnitude. Looking at the average real values (R$ dec/2022), during the period from 2015 to 2022, committed investments reached R$ 60.8 billion per year. The average cash value in the same period was R$ 157.3 billion. Domestic and external debts registered R$ 951.3 and R$ 137.9 billion, respectively.

Although we are not unconditional supporters of public investment, we recognize the role of the public sector as an investor, aiming to “complete markets”. In other words, investing in areas that seem to attract less attention and interest from the private sector, but which are still fundamental for a better business environment and for an increase in social infrastructure, mainly in low-income economies. Well-executed high-yielding public investment can imply in crowding-in, raise output and consumption and be self-financing in the long run.

Regarding the Brazilian states investment-growth linkage, Matos and dos Santos (2020) estimate an extended version of Barro-style growth panel regression, over the period from 2003 to 2017, controlling for household, enterprise, and government credit, exports, imports, years of schooling, government capital and current expenditures. They suggest that capital and current spending are relevant and different drivers of cross-state growth in Brazil. They find that government capital and current expenditure elasticities of GDP growth are: 1.01 and -1.75, respectively. Matos et al. (2022) suggest revisiting this issue by estimating a panel over the period from 2004 to 2019, however taking into account also for the same fiscal variables from municipal governments. They find a relevant role for both state and municipal governments capital spending.

Nonetheless, the positive macroeconomic impacts of public investments depend on the continuity, robustness, and sustainability of the flow of this type of capital expenditure, which is directly related to the sources of financing. It is precisely this last issue that our study empirically addresses. In this sense, theoretically, a subnational government in Brazil can finance public investment with official aid resources. Besides this exogenous source, we also need to assume that concessional borrowings may be available to any government state. However, these funding sources are not flowing as promised or desired.

Consequently, Brazilian state governments can try to use three sources of funds: accessing domestic borrowing, supplementing with external commercial borrowing, and covering the resulting gap with tax increases and/or spending cuts. The average (concessional and nonconcessional, external and internal) borrowing value over the period from 2015 to 2021 reached an annual level of R$ 16.0 billion, which is almost 32% of investments. According to Buffie et al. (2012), the financing of investments with own resources requires sharp macroeconomic adjustments, crowding out private investment and consumption and delaying the growth benefits of public investment, while nonconcessional external borrowing can smooth away difficult fiscal adjustments, reconciling scaling up of public investment with feasible constraints. They also argue that borrowing in the domestic debt market is ineffective in smoothing the path of fiscal adjustment and avoiding private sector crowding out. Moreover, commercial borrowing can make the economy more vulnerable to macroeconomic instability in the presence of persistent unexpected shocks. In this scenario, how are public investments by Brazilian government states reacting to the previous change in available cash and in debt? Are these public investments sustainable? Our work helps answer these two questions.

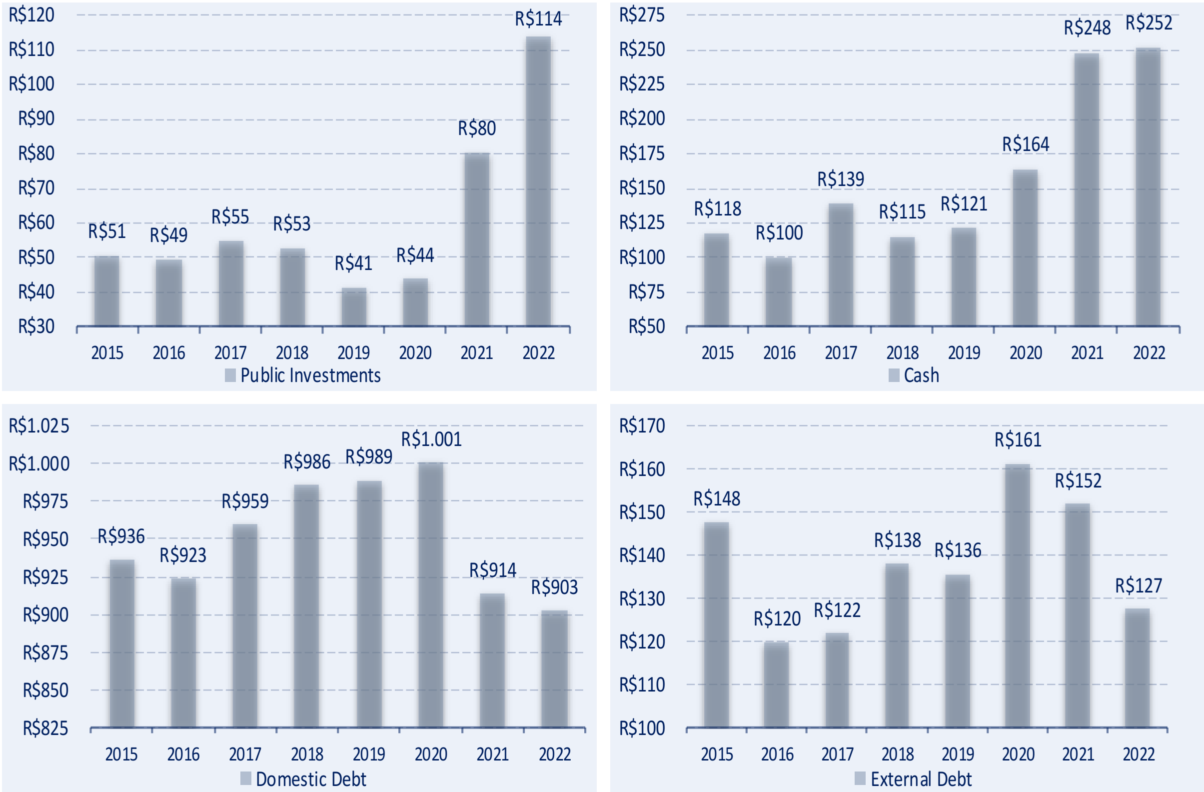
This paper is structured as follows. In Section 2 we discuss recent fiscal situation. Section 3 illustrates the setup of the empirical model, and reports main findings. Section 4 is devoted to the final discussion.

3.2 Brazilian cross-state fiscal situation

Summarizing the situation in terms of public investments in Brazil, the participation of state and municipal governments in the composition of public investment in Brazil is very significant and growing in recent years. According to Bonomo et al. (2021), over the period from 2015 to 2019, on average, municipal governments invested 0.54% of GDP, while state governments invested 0.48%, and the federal government invested 0.43%.

Regarding only real public investments by state governments, after this variable remained around R$ 50 billion over the period from 2015 to 2018, and subsequent reduction to the level of R$ 40 billion in 2019 and 2020, such investments reached almost R$ 76 billion in 2021, and R$ 114 billion in 2022 (Figure 1). It is important to identify in terms of financing, that the borrowing participation, which has already reached almost 58% in 2015, has shown successive reductions, with such participation reaching 12% in 2021. Among all 27 state governments, three have financed more than 50% of their respective investments through borrowing on average over this period: Piauí, Rio de Janeiro and Ceará. At the other extreme, Rondônia, Mato Grosso do Sul and Roraima showed the lowest levels of borrowing-to-investment ratio.

Fig. 1: Real Public investments, cash and debt (external and domestic) measured from the aggregation of the respective values of the 27 state governments (billion Dec/2022 R$) [a, b, c]



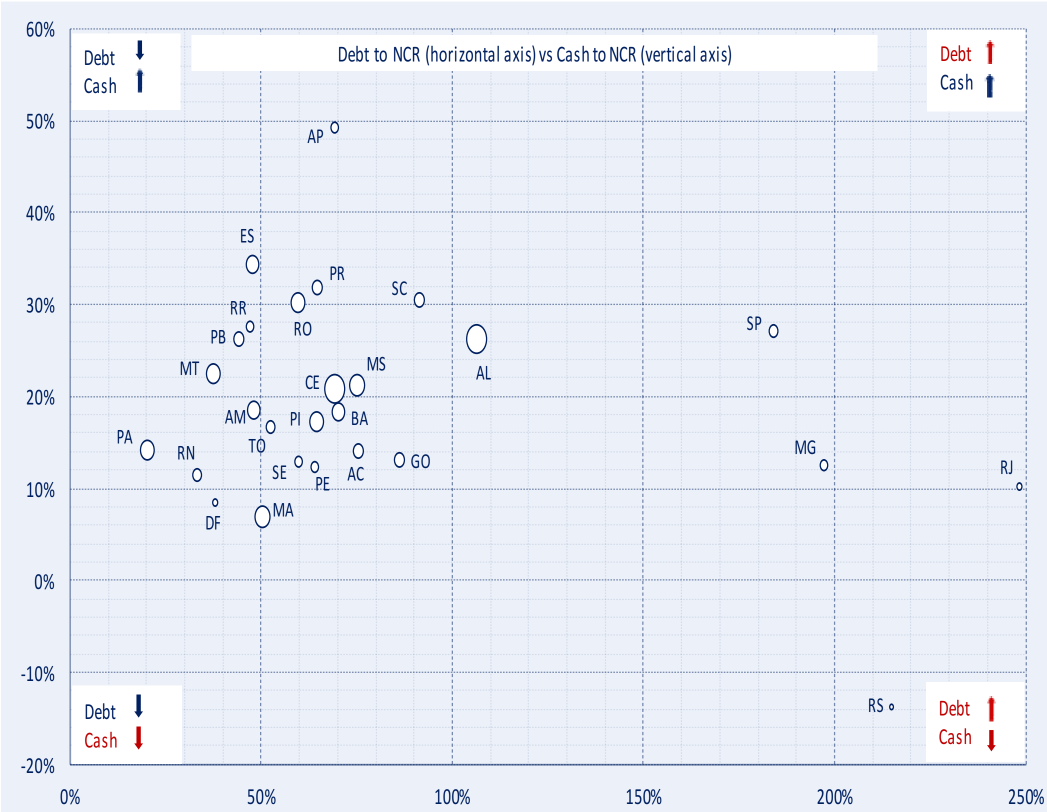
Source: Budget Execution Summary Report (RREO), Annex 1, and Tax Management Report (RGF), Annex 2, both available at SICONFI/STN. [a] We use investments committed each year. [b] Cash means the relative balance between financial assets (available cash and other financial assets) and processed remaining payables (except precatory). [c] We use the "gross" consolidated debt of state governments broken down into its external and internal components.

Observing the aggregate cash in Figure 1, there is a cyclical behavior in the first years of the sample, with continuous growth between 2018 and 2022. Over these 8 years, the real growth was almost 114%. In the same period, the aggregate consolidated debt of the states ranged only -5% from R$1.084 trillion (2015) to R$1.030 (2022). As a result, the cash-to-debt ratio has ranged from 10.9% in 2015 to 24.5% in 2022. Regarding the composition of the consolidated debt of the states, the external component has ranged between 11.3% in 2017 and 14.2% in 2021.

The scatter plot (Figure 2) allows you to summarize the investment-cash-debt linkage, based on the average values between 2015 and 2022 of each of these variables, as a ratio of the respective Net Current Revenue (NCR). The four richest states in the country show such behavior that they appear distant from the other states in a worrying area, because they have high indebtedness. It is true that these states do not appear among those that invest the most. Even more worrying is the situation in Rio Grande do Sul, as it still has a negative average cash position. In addition to these four rich and indebted states, only Alagoas has debt-to-NCR above 100%. Alagoas, however, leads the committed investment-to-NCR ranking between 2015 and 2022 with an average of 14.0%. In this ranking, we see next Ceará and Mato Grosso do Sul, both with cash-to-NCR close to 20%, and debt-to-NCR of 69.3% and 75.4%, respectively.

Figure 3 makes use of the same data used in Figure 2, but associating investments versus cash (left side) and versus debt (right side). There is considerable and visible dispersion in both cases, but it is intuitive to see a positive linear trend in the investments-to-NCR and cash-to-NCR ratio as well as a negative trend between investments-to-NCR and debt-to-NCR. Obviously, we know that a more robust answer to these relationships requires the use of a framework that allows statistical inference, like a dynamic balanced panel, for instance.

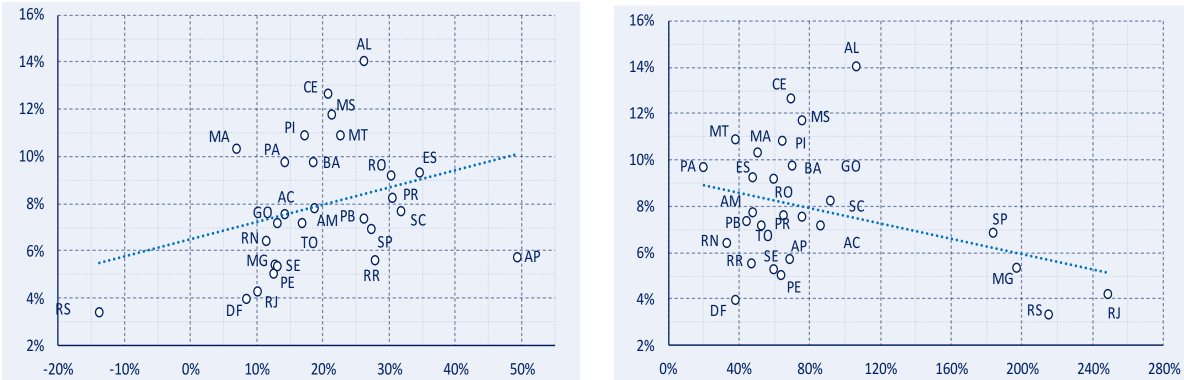
Fig. 2: Real debt, cash and public investments of government states (ratios calculated from the average of the respective values over the period from 2015 to 2022) [a, b, c, d, e]



	Inv to NCR
AL	14.0%
CE	12.7%
MS	11.7%
MT	10.9%
PI	10.8%
MA	10.3%
BA	9.7%
PA	9.7%
ES	9.3%
RO	9.2%
SC	8.2%
AM	7.8%
PR	7.6%
AC	7.5%
PB	7.4%
GO	7.2%
TO	7.2%
SP	6.9%
RN	6.4%
AP	5.7%
RR	5.6%
MG	5.4%
SE	5.3%
PE	5.0%
RJ	4.2%
DF	4.0%
RS	3.4%

Source: Budget Execution Summary Report (RREO), Annex 1, and Tax Management Report (RGF), Annex 2, both available at SICONFI/STN. [a] We use investments committed each year. [b] Cash means the relative balance between financial assets (available cash and other financial assets) and processed remaining payables (except precatory). [c] We use adjusted NCR, for the purpose of weighting the indebtedness. [d] The size of the circles on the scatterplot is proportional to the value of investments to NCR ratio reported in the table (right side). [e] We use the "gross" consolidated debt of state governments.

Fig. 3: Relation between public investments by state governments (% NCR) – vertical axis of each scatter plot – versus the respective cash (% NCR) – horizontal axis (left scatter plot) – and versus the respective debt (% NCR) – horizontal axis (right scatter plot) – (ratios calculated from the average of the respective values over the period from 2015 to 2022) [a, b, c, d, e]



Source: Budget Execution Summary Report (RREO), Annex 1, and Tax Management Report (RGF), Annex 2, both available at SICONFI/STN. [a] We use investments committed each year. [b] Cash means the relative balance between financial assets (available cash and other financial assets) and processed remaining payables (except precatory). [c] We use adjusted NCR, for the purpose of weighting the indebtedness. [d] We use the "gross" consolidated debt of state governments.

3.3 Empirical Exercise

We propose estimating dynamic panel model to assess the role of cash and debt as investment drivers, according to the following set of regressions:

$$\left(\frac{INV}{NCR}\right)_{i,t} = \alpha\left(\frac{INV}{NCR}\right)_{i,t-1} + \delta\left(\frac{DEBT}{NCR}\right)_{i,t-1} + \gamma\left(\frac{CASH}{NCR}\right)_{i,t-1} + \varepsilon_{i,t}$$
(1)

$$\left(\frac{INV}{NCR}\right)_{i,t} = \alpha\left(\frac{INV}{NCR}\right)_{i,t-1} + \vartheta\left(\frac{EXT\ DEBT}{NCR}\right)_{i,t-1} + \varphi\left(\frac{DOM\ DEBT}{NCR}\right)_{i,t-1} + \gamma\left(\frac{CASH}{NCR}\right)_{i,t-1} + \varepsilon_{i,t}$$
(2)

$$INV_{i,t} = \alpha INV_{i,t-1} + \delta DEBT_{i,t-1} + \gamma CASH_{i,t-1} + \varepsilon_{i,t}$$, and
(3)

$$INV_{i,t} = \alpha INV_{i,t-1} + \vartheta EXT\_DEBT_{i,t-1} + \varphi DOM\_DEBT_{i,t-1} + \gamma CASH_{i,t-1} + \varepsilon_{i,t}$$
(4)

In all regressions, the subscript i refers to each Brazilian state government among 27 states, and t to each year of our sample, from 2015 to 2022. Following this literature, in the regressions (1) and (2), we use all variables as a ratio of the respective Net Current Revenue (NCR), while in regressions (3) and (4), we real values. As usual, ε refers to the residual.

To control for endogeneity and omitted variable biases, our main conclusions are based on an instrumental variable difference-in-difference regression. We also use Arellano and Bond's (1991) efficient GMM estimate with fixed effects in the cross section and White's variance-covariance matrix in the temporal dimension. Concerning the dynamic panel instruments, we use Eviews default, by specifying a set of dynamic instruments associated with all series in each regression. Since the default set of instruments grows very quickly as the number of periods increases, and our sample is not so large ($T = 8$), we limit the number of lags to be used.

First, as a type of preliminary test we can see that not all variables are stationary at 5%, (Table 1). We address this issue by applying the transformations suggested in Arellano and Bond (1991). Thus, we estimate growth regressions in difference, i.e., we take the first difference of the equations (1) to (4).

Table 1. Panel unit root test [a, b]

	Variables as a ratio to NCR (%)	Real variables (R$ dec/2022)
Investments	0.4816 (0.6866)	10.6313 (1.0000)
Cash	-13.4758 ** (0.0000)	-5.3985 ** (0.0000)
Total Debt	4.7574 (1.0000)	-1.3686 (0.0856)
External Debt	-0.6422 (0.2604)	-3.5960 ** (0.0002)
Domestic Debt	3.7009 (0.9999)	-1.5890 (0.0560)

Notes: [a] Levin et al. (2002) panel unit root test with intercept over the period from 2015 to 2022 (H0: common unit root). [b] Respective p-values are reported in the parentheses. * p-value<0.05. ** p-value<0.01.

Our main findings are reported in Table 2.

Table 2. Results on investments reaction [a, b, c, d]

	Variables as a ratio to NCR (%)		Real variables (R$ dec/2022)	
	[1]	[2]	[3]	[4]
Main results - lagged explanatory variables				
Investments	0.0957 ** [0.0149]	0.0363 [0.0543]	0.4797 ** [0.0015]	0.5650 ** [0.0050]
Cash	0.1658 ** [0.0032]	0.1606 ** [0.0046]	0.2792 ** [0.0002]	0.1763 ** [0.0015]
Total Debt	-0.1044 ** [0.0089]		-0.0268 ** [0.0001]	
External Debt		-0.0516 [0.0296]		0.4915 ** [0.0035]
Domestic Debt		-0.1215 ** [0.0117]		-0.0496 ** [0.0007]
Complementary results				
Arellano-Bond test - AR(2)	-0.4878 (0.6257)	-0.9304 (0.3522)	0.7006 (0.4836)	0.3432 (0.7315)
Instrument rank	27	27	27	27
Sargan-Hansen test	25.4689 (0.3807)	24.9249 (0.3542)	23.8952 (0.4676)	21.7888 (0.5330)

Notes: [a] Dynamic balanced panel with the 26 states and Federal District, from 2015 to 2022. [b] Arellano and Bond's (1991) efficient GMM estimate with fixed effects in the cross section and White's variance-covariance matrix in the temporal dimension. [c] Instrument set: lagged dependent and exogenous variables (dynamic). [d] Respective standard errors are reported in the brackets, and p-values are reported in the parentheses. * p-value<0.05. ** p-value<0.01.

In all regressions, except for second model, we find a robust and significant values of the parameter associated with the investment's reaction in relation to its own lag. This positive reaction with this order of magnitude may suggest that there is an inertial behavior.

Regarding the role of the drivers able to explain the investments sustainability, the estimations of the cash to NCR parameter range from 0.161 to 0.166, while the values of real investments parameters range from 0.176 to 0.279. We find a robust finding on the positive reaction of investment to lagged available cash.

In models (1) and (3), which consider the effect of total debt, we again have a robust intuitive result, given by negative and significant (at 5%) parameters.

When we disaggregate the total debt into its internal and external components, according to the results of models (2) and (4), there is robustness on the negative impact of internal or domestic debt. The effect of the external debt (which represents less than 14% of the total debt in this period) was only significant in model (4), which considers the variables in real terms. This effect was positive, counterintuitively.

As complementary results, we also report in **Table 2** the results for Sargan–Hansen test for the overall validity of the instruments by analyzing the sample of the moment conditions used in the estimation process. We fail to reject the null hypothesis that such restrictions are valid for all specifications. Moreover, following Arellano and Bond's (1991) test we fail to reject the null hypotheses of no autocorrelation of the error term for autoregressive process in the models (1) to (4). [7]

3.4 Conclusion

Our first conclusion is that the empirical literature on public investments in Brazil is scarcer than it should be. Given the evolution of theoretical frameworks developed for public finance management, and the recent 2014 fiscal crisis, we observe that the indebtedness of state governments began to worry policy makers and researchers. In addition, we have also observed a literature on subnational tax revenue and payroll expenditure forecasting. The concepts of transparency, in the sense of mistaken predictability with intention or not, and resilience are even more recent and remains unexplored. Specifically on public investments, there is a research agenda relating investments or capital expenditures and growth. However, the agenda relating investments and other fiscal variables is very limited, and some recent contributions applied to Brazilian government states are Simonassi et al. (2021) and Bonomo et al. (2021). Our results allow us to add to this specific debate.

Second, we show a robust reduction of funding through borrowing of state government public investments over the period from 2015 to 2021: in 2015 this dependence was 57.9%, and in 2021, 12.1%. Since states' ability to invest autonomously — that is, with their own resources — is directly dependent on current savings, the significant improvement in the primary result in 2021 (after the pandemic) suggests that public investments by state governments may be sustainable in the short term.

Third, we show that Brazilian richest states appear distant from the other states in a worrying area, with high indebtedness. It is true that these states do not appear among those that invest the most. The states that invest the most appear to be in a comfortable situation in terms of debt versus cash. There are, however, states that could and should invest much more, considering their fiscal situation: Amapá, Roraima and the Federal District. The latter, despite its low cash value, has very low indebtedness, 38.2%.

Finally, our main findings based on a dynamic panel over the period from 2015 to 2022

[7] While subject to the usual caveats of cross-state instrumental variable regression – bias due to lagged dependent variable, potentially weak instruments, weak tests of overidentifying restrictions and lack of instruments for other explanatory variables – our findings suggest that the investment-debt-cash linkages are not driven by endogeneity, simultaneity or measurement biases.

suggest that state investment is not inertial or explosive, but cyclical, and it seems to be sustainable, given its positive reaction to lagged change in cash, and its negative reaction to lagged changes in domestic and external debt.

It is important to reconcile our evidence with the conduct of public policies. Brazil has a set of fiscal rules that constrains government spending at the federal and subnational levels, and according to Bonomo et al. (2021), those fiscal rules are not seem to blame for the collapse of public investment, but the sharp public investment contraction rather reflected the lack of fiscal discipline and uncontrolled fiscal expansion. Consequently, our research agenda can certainly be useful to redesign the fiscal framework with alternative fiscal rules and risk management framework, revisiting for instance, Constitutional Amendment 95/2016, entitled the "ceiling rule" of federal government spending, and he CAPAG's methodology. Moreover, we claim that we need to examine how fiscal policy can be retooled for the post-pandemic scenario to deliver sustainable investment. We are aligned to Blanchard et al. (2021) in the sense of proposing a replacement of old, fixed, constant and universal fiscal rules by dynamic state-specific assessments based on stochastic analysis to aiming to monitor public investment and debt sustainability.

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4 ON THE PUBLIC INVESTMENT-DEBT-CASH LINKAGES IN THE STATE OF CEARÁ

4.1 Introduction

The role of public capital, measured by its impact on socio-macroeconomic variables, has been theoretically addressed mainly in growth models, especially in the last five decades. Possibly, Arrow and Kurz (1970) are one of the first to discuss the role of public investments, by assuming that public capital, rather than the flow of public services, enters into the production function. We have observed in most theoretical (endogenous or exogenous) growth models that has been common to specify the source of funding for this public spending as a premise of the framework. In this context, we know that low-income countries can finance public investment preferentially with official aid resources, and concessional borrowings. When both sources are not flowing as desired, government can use three sources of funds: accessing domestic borrowing, supplementing with external commercial borrowing, and covering the resulting gap with tax increases and/or spending cuts.

Regarding the collateral of each financing source, there is a literature studying this issue, when the source comes from external aid. For instance, Chatterjee and Turnovsky (2007) and Berg et al. (2010) discuss the effect of external transfers, arguing that this aid is effective depending on the externalities associated with the public good it helps to finance. There is a wider and more specific literature studying the role of public investment, when it is financed with its own resources, through a fiscal effort to generate a flow of primary current surplus. Barro and Sala-i-Martin (1992) show that if the social return on public investment is greater than the private return, it is possible to change the long-term growth trajectory, and therefore it is important to find an optimal tax policy that connects the characteristics of the social service offered. They suggest that investment and growth incentives are high if the tax amount is fixed. Glomm and Ravikumar (1994) also contribute to the discussion about the optimal amount of tax for the sustainability of efficient public investment.

There is also a strand of this literature studying the effects of indebtedness as collateral for issuing bonds or accessing borrowing to finance investments or even current expenditure. One of the pioneers in this theoretical literature may have been Modigliani (1961), Mishan (1963) and Diamond (1965). Until then, one could expect a nonstationary debt-income ratio implied by some models of optimal government finance. However, a high and growing debt-GDP ratio was viewed as worrisome by macroeconomic models with limited taxation

(Blanchard, 1984). Moreover, given the worrying debt levels in several countries from the 80's, the concept of solvency of public debt has become more explored in empirical studies, such as Hamilton and Flavin (1986), and Bohn (1998), for example.

Regardless of the source, there seems to be a consensus on the relevance of the sustainability in the sense of continuity of public investments. In this context, Agenor (2010) addresses the growth dependence on public investment, by assuming that government is responsible for investment in infrastructure, which drives the production of commodities and health services, generating greater labor productivity. However, infrastructure efficiency has no linear relationship with investment, and it is necessary to maintain the governance of public investments to shift the trend from low productivity and growth to a steady state of high growth.

There is still another consensus: the importance of this topic to be addressed by international institutions capable of influencing and determining the conduct of public policies in economies. In Allen and Leipziger (2006), Buffie et al. (2012), and Melina, Yang and Zana (2014), International Monetary Fund (IMF) and World Bank (WB) researchers are studying and developing frameworks aiming to monitor debt sustainability, and its relation with public investments, while Blanchard et al. (2021) discuss the system of fiscal rules in the European Union and suggest that country-specific assessments using stochastic debt sustainability analysis is a good normative. Regarding specifically the cash, IMF (2021) is proposing that governments must address the challenge of meeting extended cash needs to finance COVID-19-related emergency spending. IMF researchers argue that a multi-pronged approach to cash management is required to ensure that liquidity is adequate to satisfy the government's payment obligations.

Specifically on subnational governments, we claim that we need a greater attention, given the scarcity of the related literature. In this specific context, we highlight a work by the World Bank Group (2016), in which researchers argue that subnational debt levels in developing countries are becoming increasingly significant as central governments continue to decentralize spending responsibilities, revenue-raising authority, and borrowing rights to subnational governments. Therefore, the World Bank, in collaboration with other partners, has developed a global knowledge program on subnational fiscal reform and debt management. The program aims to strengthen developing countries' institutional capacity to maintain subnational fiscal sustainability and prudent debt management alongside a stable macroeconomic framework.

Concerning this literature applied to Brazilian states, Matos and dos Santos (2020) and Matos et al. (2022) find a relevant role for state government capital spending in cross-state growth. Concerning the sustainability, Simonassi et al. (2021) find an inefficient policy in debt management, and an investment cycle that generates higher revenue returns than the cost increments. Bonomo et al. (2021) find that the main determinant of public investment is the fiscal condition of the country and states, and that controlling personnel expenses would be an effective instrument to increase fiscal space for investments. Even more in line with our paper, Matos and Monteiro (2022) find that public investments by state governments have reacted to ensure its sustainability, in response to changes in debt and cash, based on a dynamic balanced panel from 2015 to 2021.

In this paper, we add to this empirical literature on the sustainability of investment by state governments, with an emphasis on the state of Ceará. According to Brazilian Institute of Geography and Statistics (IBGE), this is one of the states with the highest proportion of poor people in the country (40.6% in 2020), and one with the lowest real GDP per capita: R$ 14,935.68 in 2019, (24th among 27 states). If this state were a country, and considering purchasing power parity, the state would rank 99th in the World Bank's GDP per capita ranking, between South Africa and Iraq. On the other hand, this state government has led the national ranking of Investment/Net Current Revenue (NCR) between 2017 and 2020, becoming 6th place in the in 2021.

Observing the scarce literature on public finance applied to a particular Brazilian local government such as Ceará, Santos and Matos (2021a), Matos (2022a), and Uchoa et al. (2022) used to infer on the analysis of the sustainability of investments, based on the solvency of public debt. This is a very useful indirect approach, considering that debt is a collateral of investment financed via credit operations. We add to this debate, by using a direct approach also used in Matos and Monteiro (2022). We follow theoretically the fiscal dynamics proposed by Matos (2022b) aiming to conclude on the sustainability (or not) of the flow of public investment by state government of Ceará, through its reaction in relation to debt and cash (in the short, mid and log run).

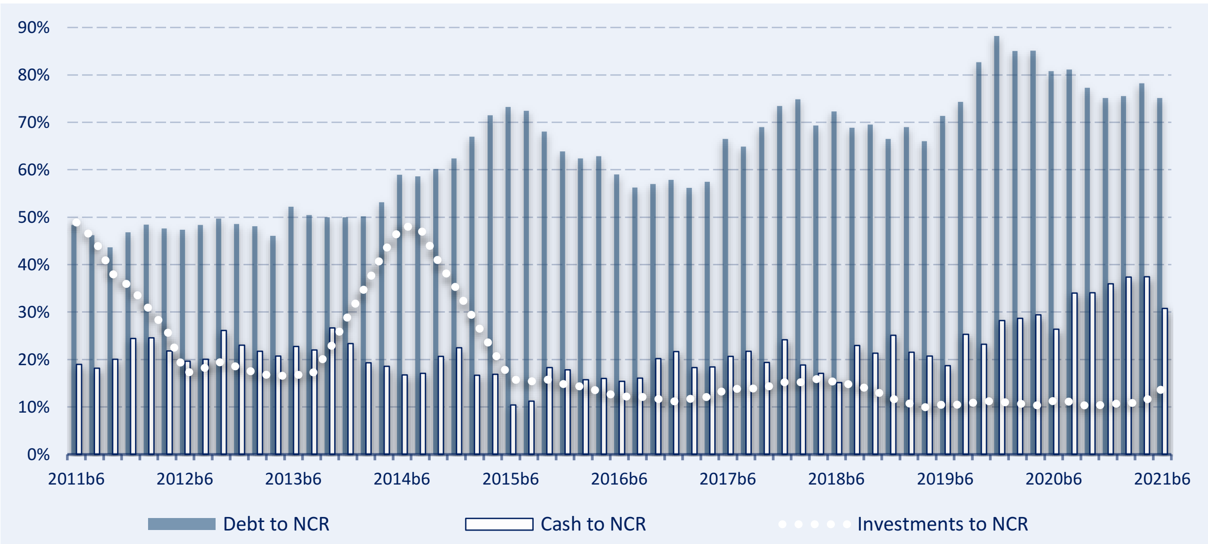
This paper is structured as follows. In Section 2 we discuss recent fiscal situation. Section 3 illustrates the setup of the empirical model, while section 4 reports main findings. Section 5 is devoted to the final discussion.

4.2 The fiscal situation of public investments of the state government of Ceará

When we observe bimonthly series from 2011b6 to 2021b6 of real (R$ December 2021) public investments committed by the state government of Ceará – accumulated values over 6 bimesters –, we identify two atypical periods. At the end of 2011 and 2014, the amounts of real investments committed were R$9.6 billion and R$10.5 billion, respectively. Over the remaining years, the real commitments oscillate around R$ 3 billion. More precisely, they ranged between R$ 2.4 billion (2019b5) and R$ 3.9 billion (2013b2). This visual pattern remains similar when we analyze the series of investments as a ratio of Net Current Revenue (NCR), also accumulated (Figure 1). It is worth noting that both the strong increase in the level of investments throughout 2014, as well as the strong reduction in the following year, both are accompanied by an increase in indebtedness and a reduction in cash.

On consolidated debt, we see a level of approximately 50% of NCR over the first few years, and therefore we find higher levels of indebtedness, which culminated in more than 88% in 2020b3. There is a recent downward trend, and this ratio reached 75% in 2021b6. The cash to NCR ratio, which ranged between 10% and 26% between 2011b6 and 2019b6, has showed a recent upward trend, reaching the highest value of 37% in 2021b5.

Fig. 1. Investments, cash and debt measured (as a ratio of Net Current Revenue): Ceará state government [a, b, c]



Source: Budget Execution Summary Report (RREO), Annexes 1 and 6, available at SICONFI/STN. [a] We use committed investments (nominal accumulated value in 12 months). [b] Cash means the relative balance between financial assets (available cash and other financial assets) and processed remaining payables (except precatory). [c] We use the "gross" consolidated debt of state government.

We know that the sustainability of the flow of this type of capital expenditure is directly related to the sources of financing. Considering the nominally accumulated value (6 bimonths) of domestic credit operations over the analyzed period, we see a heterogeneous behavior in terms of financing of committed investments. From mid-2012 to early 2014, there is a first higher peak of participation, with a representation ranging between 30% and 40% of investments. After that we see a more discreet participation, around 20%, and from 2016 onwards, we show a downward trend in the access to domestic credit by this state government. In 2017, 50% of the accumulated committed investments are financed with domestic credit. Once more, we find a cycle, with low representation in 2019, and a strong representation in 2020, with the peak of domestic financing: more than 70% of investments. At the end of 2021, around 30% of the R$ 3.48 billion of committed investments are being financed by domestic credit.

The external credit (accumulated 6 bimesters) shows stability at the beginning of the period, around 10% of total investment. It is important to note a crowding-out effect in mid-2015, with a reduction in domestic credit and an increase in the paradiplomatic resources. The peak is evident in 2016b5, with almost 50% of investments financed with external credit. Thereafter, there is a slightly negative downward trend. In December 2021, this share of external credit was 10% of the accumulated investments committed by the state government of Ceará.

4.3 Methodology

4.3.1 Methodological context

The positive macroeconomic impacts of public investments depend on the continuity, robustness, and sustainability of the flow of this type of capital expenditure. This issue is directly related to the sources of financing. In this sense, a subnational government in Brazil can finance public investment with official aid resources. Besides this exogenous source, we also need to assume that concessional borrowings may be available to any government state. However, these funding sources are not flowing as promised or desired. Consequently, Brazilian state governments can try to use three sources of funds: accessing domestic borrowing, supplementing with external commercial borrowing, and covering the resulting gap with tax increases and/or spending cuts. In other words, any state government needs to monitor the level of "stock variables" economically speaking: debt and cash. In this scenario, how are public investments by Ceará state government reacting to the previous change in its own available cash and in its debt? Are these public investments sustainable? Our paper helps answering such questions by using two different frameworks.

In the next subsection, we will briefly describe the most used technique in the case of time series modeling, when the endogenous and exogenous variables present non-stationary behavior.

We will therefore comment on the vector error correction model.

In subsection 3.3, we will address the Wavelet transform, a mathematical approach that enables us to investigate the behavior of a parameter over various time scales, besides enables a more flexible approach to deal with high and low frequency components. We also can work with non-stationary data.

4.3.2 Vector error correction

We follow this literature on time series, as Matos and Monteiro (2022), for instance, by estimating a system to assess the role of cash and debt as investment drivers, according to the following regressions:

$$INV_t = c + \alpha INV_{t-1} + \delta DEBT_{t-1} + \gamma CASH_{t-1} + \varepsilon_t \quad (1)$$

$$INV/NCR_t = c + \alpha INV/NCR_{t-1} + \delta DEBT/NCR_{t-1} + \gamma CASH/NCR_{t-1} + \varepsilon_t \quad (2)$$

In both regressions, the subscript t refers to each sample bimester, from 2011b6 to 2021b6. Following this literature, we use all variables in level (December 2021 R$) in equation (1), and as a ratio of the respective Net Current Revenue (NCR), equation (2). As usual, ε refers to the residual.

First, as a type of preliminary test we perform a unit root test to find whether the variables are stationary or not. The results will be reported in subsection 4.2, but we can here anticipate this discussion on the non-stationarity of all variables, aiming to explain how we intend to address this issue.

As usual, the first step in this case is to perform the conditional joint cointegration test, considering, as always, the same vector of instruments. In the scenario where trace test suggests that the null hypothesis that there is no cointegrating vector is rejected, the next step is to estimate the cointegration vector. This vector gives us values about the long-term relationship between the model variables. Next, we estimate the original model, but in difference, considering the cointegrating vector. The last step is to analyzing the response of investments to Cholesky one standard deviation innovations. All these results are reported and discussed in subsection 4.2.

4.3.3 Multivariate wavelet analysis

4.3.3.1 The continuous wavelet transform

The Fourier analysis can be considered one of the most important bases for the wavelet transform development. The central idea of Fourier analysis it's that any periodic function can be expressed by an infinite sum of trigonometric functions. By defining a basis of sines and cosines of different frequencies, the Fourier Transform capture the relative importance of each frequency on the original signals. This analysis is a powerful tool to modelling time series on frequency domain. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands.

Observing the evolution of this literature, the Wavelet transform has some additional features over original Fourier transform, as argued by In and Kim (2012): i) it can decompose the data into several time scales instead of the frequency domain (which allow us to investigate the behavior of a signal over various time scales); ii) it uses local base functions that adjust the window width to deal with different frequencies (this enables a more flexible approach to deal with high and low frequency components) and iii) it allows to work with non-stationary data. The latter feature is especially important to examine financial time series, once that heteroskedasticity, sudden regime shifts, structural breaks at unknown time points are common pattern trough the financial cycle paths. Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\, \psi^{*}_{\tau,s}(t) dt \tag{3}$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series in space and scale. To capture different frequencies of the signal, we use a mother wavelet that is stretched and shifted:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) \tag{4}$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequencies movements (Rua, 2012). So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) dt \tag{5}$$

The basis function $\psi_{\tau,s}$ must obey some criteria, such as: admissibility, similarity, invertibility, regularity, and vanishing moments. There are many options of wavelet mother functions to select. Aguiar-Conraria and Soares (2011) highlight the importance of that choice and suggest picked up an analytic wavelet to study the synchronism between oscillatory signals because its corresponding transform contains information on both amplitude and phase, providing an estimate of the instantaneous amplitude and instantaneous phase of the signal in the neighboring of each time/scale location (τ, s). On subset of analytic wavelet, the Morlet wavelet mother is the most popular alternative because some properties, which is given by:

$$\psi_{\omega_0}(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2} \tag{6}$$

where the non-dimensional frequency ω_0 is set $\omega_0 = 6$ to satisfy the admissibility condition. As the wavelet transform decomposes the original signal in a time-scale domain, which put us the necessity to convert scale into frequency. Lilly and Olhede (2009) point that this conversion can be made by associate the wavelet $\psi_{\tau,s}$ with one of three special frequencies (the peak frequency, the energy function, or the central instantaneous frequency), by using the formula $\omega(s) = \frac{\omega_\psi}{s}$, where ω_ψ denotes any of the three angular special frequency.

By the usual "Fourier" frequency f (cycles per unit time) we have that $f(s) = \frac{\omega_\psi}{2\pi s}$. In this sense, the Morlet wavelet is an ideal alternative since it provides us a unique relation between frequency and scale (the peak frequency, the energy frequency and the central instantaneous frequency are all equal) which makes it easier the conversion from scales to frequencies. The choice of $\omega_0 = 6$ give us a conversion ratio equal $f = \frac{6}{2\pi s} \approx \frac{1}{s}$, that direct correspondence between scale and frequency is ideal to simplify an effective interpretation of the results. Since CWT is applied on finite-length time series, border distortions will occur due the fact that values of the transform at the beginning and the end of the sample are imprecisely computed, which involves artificial padding on the extremes of the sample (the most common is set zero to extend the time series). As larger scales decrease the amplitude near the edges as more zeroes enter the analysis, the region that suffers from these edge effects is function of s. The Cone of Influence is the region of the wavelet spectrum in which edge effects become important by a factor of e^{-2}. In the case of the Morlet wavelet this is given by $\sqrt{2s}$.

3.3.2. Wavelet tools

The first wavelet measure that we will present it's the wavelet power spectrum (WPS), which reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \quad (7)$$

To compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GWPS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \quad (8)$$

To study the dependencies between two original time series $x(t)$ and $y(t)$ in time-scale/frequency plane, Torrence and Webster (1999) were the first to define the wavelet coherence. The measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by:

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s) \quad (9)$$

where $W_x(.)$ and $W_y(.)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and $*$ denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $\left|W_{x,y}(\tau, s)\right|$. It computes the local covariance between two signals at each scale. The squared wavelet coherence is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherence is denoted as:

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{x,y}(\tau, s)\right)\right|^2}{S(s^{-1}W_x(\tau, s)^2)S(\, s^{-1}W_y(\tau, s)^2)} \quad (10)$$

where $S(.)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization fator ensuring the conversion to an energy density. Torrence and Webster (1999) note that in numerator of the squared wavelet coherence, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it’s ensured that $0 \leq R^2 \leq 1$.

Hence, the main advantage of the wavelet coherence on XWT is the common measure unit to examine several combinations of signals. Torrence and Compo (1999) reveal that once the wavelet transforms conserves variance, the wavelet coherence is a good representation of the normalized covariance between two-time series, where the closer to zero (one) the coherence, the weaker (stronger) the local correlation between the time-series. The wavelet coherence has not theoretical distribution known, hence we follow the approach of Aguiar-Conraria (2011) deriving the confidence interval using Monte Carlo methods.

Although the wavelet coherence computes the degree of local linear correlation between two signals, it isn't reveals patterns of lead-lag relationship neither if the movements are positives or negatives. To deal with these limitations, the phase-difference is commonly used to examine the delays in the fluctuations between the two time-series. Following Torrence and Webster (1999) we define the phase difference as:

$$\phi_{xy}(\tau,s) = \tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}{\Re\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}\right) \tag{11}$$

The smoothed real ($\Re$) and imaginary ($\Im$) parts should already be calculated in the wavelet coherence function. Both $R^2(\tau,s)$ and $\phi_{xy}(\tau,s)$ are functions of the position index (τ) and scale (s). We also need the information on the signs of each part to completely determine the value of $\phi_{xy}\epsilon[-\pi,\pi]$. A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{xy}\epsilon\left(0,\frac{\pi}{2}\right)$ the series move in phase, but the time-series y leads x, while if $\phi_{xy}\epsilon\left(-\frac{\pi}{2},0\right)$ then it is x that is leading. A phase-difference of $\phi_{xy}=\pm\pi$ indicates an anti-phase relation. Finally, if $\phi_{xy}\epsilon\left(\frac{\pi}{2},\pi\right)$, then x is leading and time-series y is leading if $\phi_{xy}\epsilon\left(-\pi,-\frac{\pi}{2}\right)$.

Finally, aiming to capture the interdependence and/or causality between multiple time series, we follow Aguiar-Conraria and Soares (2014) and Aguiar-Conraria et al. (2018). We use high-order wavelet tools which allow us to investigate the dependency of one time series upon a set of other time series (multiple wavelet coherency) on the time-frequency plane as well to examine the co-movements between two time-series after controlling the oscillations of a subset of time series (partial wavelet coherency, partial phase-difference and partial wavelet gain) on the time-frequency plane. Given a set of p time series $(x_1,x_2,..,x_p)$ with $p>2$ and $x_i=\{x_{i_n}, n=0,..,T-1\}$, to compute multiple and partial wavelet measures is necessary to perform a smoothing operation on the cross-spectra. Let S_{ij} be the smoothed version of the cross-wavelet spectrum between the time series x_i and x_j ($S_{ij}=S(W_{ij})$ and $\mathcal{L}=\left(S_{ij}\right)_{i,j=1}^{p}$ denote the pxp matrix of all the smoothed cross-wavelet spectra. It is important to observe that this matrix is function of specific combination (τ,s) at which the spectra are being computed and it's a Hermitian matrix ($\mathcal{L}=\mathcal{L}^H$) where the symbol H denotes conjugate transpose. So, $S_{ij}=S_{ji}^*$, $\forall\, i\neq j$ and $S_{ii}=S(|W_i|^2)$ is a real (positive) number $\forall i$. The squared multiple wavelet coherency between the series $\boldsymbol{x_1}$ and all the other series $\boldsymbol{x_2}, .., \boldsymbol{x_p}$ is given by:

$$R^2_{1(2..p)} = 1 - \frac{\mathcal{L}^d}{S_{11}\mathcal{L}^d_{11}} \tag{12}$$

where d denotes the determinant of the matrix, and $\mathcal{L}^d_{ij}$ denotes the cofactor of the element in position (i, j), i.e. $\mathcal{L}^d_{ij} = (-1)^{i+j} det\mathcal{L}^j_i$, where $det\mathcal{L}^j_i$ denotes the determinant of the submatrix obtained by crossing out the row i and column j of $\mathcal{L}$ to convert it to $\mathcal{L}_{ij}$. The complex partial wavelet coherency of x_1 and x_j $(2 \leq j \leq p)$ allowing for all the other series will be computed by:

$$\varrho_{1j.q_j} = -\frac{\mathcal{L}^d_{j1}}{\sqrt{\mathcal{L}^d_{11}}\sqrt{\mathcal{L}^d_{jj}}} \tag{13}$$

The partial wavelet coherency of x_1 and x_j allowing for all the other series, is denoted by:

$$R_{1\,j.q_j} = \frac{|\mathcal{L}^d_{j1}|}{\sqrt{\mathcal{L}^d_{11}}\sqrt{\mathcal{L}^d_{jj}}} \tag{14}$$

The squared partial wavelet coherency of x_1 and x_j is simply the square of $R_{1\,j.q_j}$.

Since that the complex partial wavelet coherency measures the local correlation between the time series x_1 and x_j after controlling the influence of all the other time series $(x_i; i = 2, .., p \text{ and } i \neq j)$, Aguiar-Conraria and Soares (2018) define the partial phase-difference of x_1 over x_j as:

$$\phi_{1j.q_j} = \tan^{-1}\left(\frac{\Im\left(\varrho_{1j.q_j}\right)}{\Re\left(\varrho_{1j.q_j}\right)}\right) \tag{15}$$

Following them, we define the complex partial wavelet gain of the time-series x_1 over x_j as follow:

$$\mathcal{g}_{1j.q_j} = -\frac{\mathcal{L}^d_{j1}}{\mathcal{L}^d_{11}} \tag{16}$$

The partial wavelet gain is computed by the modulus of the complex partial wavelet *gain.* Then:

$$G_{1j.q_j} = \frac{|\mathcal{L}^d_{j1}|}{\mathcal{L}^d_{11}} \tag{17}$$

Note that the partial wavelet gain provides us a measure similar to the (modulus) coefficient of a multiple linear regression of x_1 against the explanatory variables $x_2,,,x_p$ (for $j = 2,..,p$) at each time and frequency.

These authors also highlight that these measures can be specified by wavelet complex coherencies. Consider a matrix $\mathfrak{L} = \left(\varrho_{ij}\right)_{i,j=1}^{P}$ of all the complex wavelet coherencies ϱ_{ij}. Hence, the multiple wavelet coherency can be computed as:

$$R^2_{1(2..p)} = 1 - \frac{\mathfrak{L}^d}{\mathfrak{L}^d_{11}} \tag{18}$$

The complex partial wavelet coherency $\varrho_{1j.q_j}$ and the partial wavelet coherency $R_{1j.q_j}$ are given by:

$$\varrho_{1j.q_j} = -\frac{\mathfrak{L}^d_{j1}}{\sqrt{\mathfrak{L}^d_{11}}\sqrt{\mathfrak{L}^d_{jj}}} \quad \text{and} \quad R_{1j.q_j} = \frac{\left|\mathfrak{L}^d_{j1}\right|}{\sqrt{\mathfrak{L}^d_{11}}\sqrt{\mathfrak{L}^d_{jj}}} \tag{19}$$

Finally, the complex partial wavelet gain $\mathcal{g}_{1j.q_j}$ and the partial wavelet gain $G_{1j.q_j}$, respectively, by:

$$\mathcal{g}_{1j.q_j} = -\frac{\mathfrak{L}^d_{j1}}{\mathfrak{L}^d_{11}}\frac{\sigma_1}{\sigma_j} \quad \text{and} \quad G_{1j.q_j} = \frac{|\mathfrak{L}^d_{j1}|}{\mathfrak{L}^d_{11}}\frac{\sigma_1}{\sigma_j} \tag{20}$$

The results on multivariate analysis will be reported in subsection 4.3.

4.4 Empirical Exercise

4.4.1 Database

We report in Table 1 some of the main statistics and the results on the stationarity of the main fiscal variables of state government of Ceará.

Table 1. Summary statistics and unit root test [a, b, c]

	Variables in level (R$ billion dec/2021)			Variables as a ratio of NCR (%)		
	Investments	Cash	Debt	Investments	Cash	Debt
Summary statistics						
Mean	R$4.22	R$4.95	R$14.24	19.40%	21.97%	63.29%
S.D.	R$2.25	R$1.59	R$3.48	11.18%	5.79%	12.01%
Minimum	R$2.40	R$2.15	R$9.15	9.86%	10.40%	43.65%
Maximum	R$10.47	R$9.37	R$20.99	48.89%	37.44%	88.24%
Δ (2011b6 - 2021b6)	-R$6.16	R$4.00	R$9.37	-35.08%	11.77%	26.73%
Unit root test						
ADF test	-1.1914	0.9728	1.6811	-1.3016	0.8840	0.8181
	[0.2110]	[0.9104]	[0.9764]	[0.1762]	[0.8967]	[0.8859]

Notes: [a] Source: Budget Execution Summary Report (RREO), Annexes 1 and 6, available at SICONFI/STN. [b] Augmented Dickey-Fuller unit root test without intercept nor trend over the period from 2011b6 to 2021b6 (H0: unit root). [c] Respective p-values are reported in the brackets. * p-value<0.05. ** p-value<0.01.

Public investments, both in level (R$ dec/2021) and in terms of NCR, show a more volatile behavior than cash and debt, if the average values of the time series are considered. Two aspects can help explain this pattern. Investments are flow-type variables, and are still influenced by the electoral period. According to Matos et al. (2022), the comparison between municipalities and states suggests a cyclical behavior, given that the main component of capital expenditures is investments, and in the years before municipal elections, municipalities commit more to investments than states. In 2019, for example, Brazilian municipal governments invested almost R$ 47 billion, while aggregate state governments invested R$ 34 billion. In the ratio of current revenue, investment since 2015 comes from a more constant trend between 10% and 16%, showing greater resistance to variations. From the peak of investment committed by the state government in 2014, this capital expenditure has remained constant at R$ 3 billion, while as a ratio of NCR, it also remains smooth ranging from 10% to 16%. On the other hand, cash showed a recent trend of real growth, reaching its maximum (R$ 9.37 billion) in 2021b4. The real debt reached its maximum value of almost R$ 21 billion in 2020b5, and since then it has shown a downward trend. It is important to note and record the reduction of more than R$ 2 billion in this debt if the period between the fifth quarter of 2020 and the end of 2021 is considered.

Finally, we perform the unit root test to all variables, in level, and as a ratio of NCR. We need to check whether all variables used are stationary before estimating an Autoregressive Vector model. According to the results on the augmented Dickey and Fuller (1979) test reported in Table 1, we cannot reject the hypothesis of nonstationary behavior of debt, cash, and investments, in level or as a ratio of NCR. We address this issue by applying the Vector Error Correction model (VEC). This nonstationary behavior is not a problem for using multivariate coherency method.

4.4.2 Vector error correction: results

According to the ADF stationarity tests, one cannot reject the null hypothesis that the cash and debt series are non-stationary, in level or as a ratio of the NCR. Therefore, we perform the conditional joint cointegration test, considering the same instruments. For both system specifications, the trace test does not reject by 10% the null hypothesis that there is a cointegrating vector. This finding suggests that investment flow, debt and cash of the Ceará state government may have a long-term structural equilibrium relationship with each other. Next we report the estimation of this cointegrating vector:

$$INV_t = 7.25X10^{-9} - 0.5624DEBT_t + 0.9831CASH_t + \varepsilon_t \qquad (21)$$
$$(-3.4279) \qquad (2.4204)$$

$$INV/NCR_t = 0.4261 - 0.7380DEBT/NCR_t + 1.0432CASH/NCR_t + \varepsilon_t \qquad (22)$$
$$(-3.2814) \qquad (1.9414)$$

In both cointegrating vectors, the long-term relationship between investment and debt or cash are significant at 5%, according to respective p-values reported in parenthesis. This finding characterizes the long-term sustainability of Ceará state government investments, given the positive signs of the relationship with cash, and mainly negative signs of the relationship with debt.

We report the results of the estimation of short-term relationships in Table 2. We find robust results for both system specification suggesting that the variation of investments is persistent but nor explosive, and depends on the cointegrating vector, and the lagged cash variation. The value of the explanatory power in both versions of the model (0.70 and 0.63) suggests a reasonable fit for the investments dynamics. We also find that changes in cash and debt seem to have an exogenous behavior, with the exception of the significant dependence of the cash change (as a ratio of NCR) on the second debt lag (as a ratio of NCR).

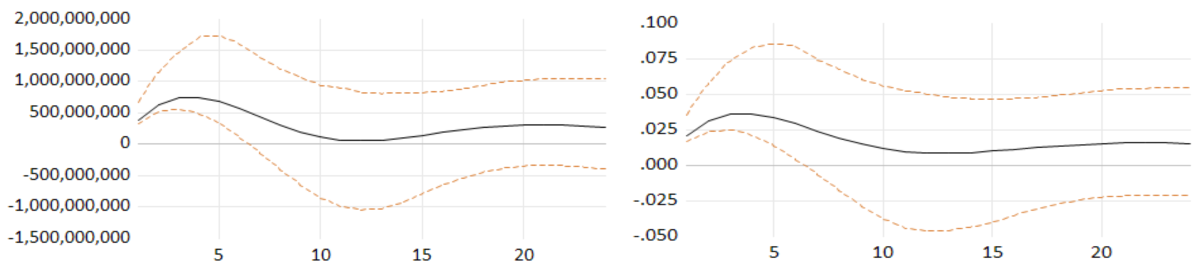
Table 2. Results on investments reaction [a, b]

	Variables in level (R$ billion dec/2021)			Variables as a ratio of NCR (%)		
Error correction	Δ (invest)	Δ (cash)	Δ (debt)	Δ (invest)	Δ (cash)	Δ (debt)
Constant	2.20×10^7	1.28×10^8	**1.63×10^8 ***	0.0010	0.0037	0.0053
	(0.3854)	(1.1459)	**(1.7182)**	(0.3352)	(0.7835)	(1.1387)
Cointeg.Equat.	**-0.1135 ****	0.0591	-0.0113	**-0.1027 ****	0.0550	-0.0168
	(-4.0103)	(1.0618)	(-0.2392)	**(-3.5599)**	(1.1426)	(-0.3565)
Δ (invest(-1))	**0.7270 ****	0.1507	-0.0304	**0.6257 ****	0.1279	-0.0747
	(5.8259)	(0.6145)	(-0.1468)	**(4.9707)**	(0.6094)	(-0.3624)
Δ (invest(-2))	0.0154	-0.1718	0.1079	0.0598	-0.1746	0.0775
	(0.1210)	(-0.6861)	(0.5093)	(0.4764)	(-0.8338)	(0.3769)
Δ (cash (-1))	**-0.1569 ***	-0.1758	-0.0224	**-0.1963 ***	-0.1916	-0.0840
	(-2.0160)	(-1.1497)	(-0.1728)	**(-2.2438)**	(-1.3138)	(-0.5862)
Δ (cash (-2))	-0.1017	0.0160	-0.1872	-0.1319	-0.0394	-0.2277
	(-1.3211)	(0.1054)	(-1.4624)	(-1.4958)	(-0.2680)	(-1.5774)
Δ (debt(-1))	0.0102	-0.0795	0.0042	-0.0286	-0.0463	0.0853
	(0.1216)	(-0.4837)	(0.0302)	(-0.3259)	(-0.3168)	(0.5940)
Δ (debt(-2))	0.0441	-0.2609	0.2129	0.0376	**-0.2673 ***	0.1506
	(0.5300)	(-1.5944)	(1.5377)	(0.4316)	**(-1.8410)**	(1.0564)
R^2	0.7026	0.1063	0.0994	0.6258	0.1313	0.0778

Notes: [a] Estimation of the model in autoregressive vector format with insertion of the error correction vector for the period from the sixth bimester of 2011 to the sixth bimester of 2021. [b] Respective t-statistics are reported in the parenthesis.
* p-value<0.05. ** p-value<0.01.

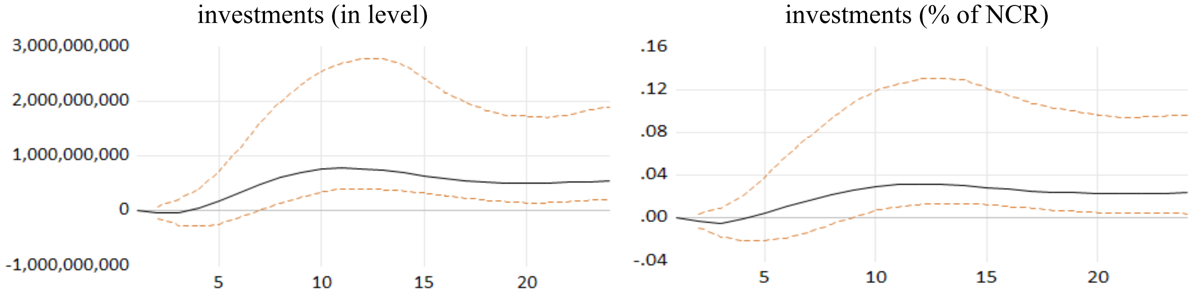
According to the impulse-response results reported in Figure 2, our robust finding show that investments committed by the state government tend to show persistence in the short term, due to a positive shock, which dissipates in one and a half year (six bimonths). Investments react (significantly) positively to increases in cash and negatively to increases in indebtedness, however only after the seventh or eighth bimester. In both cases, the reactions do not dissipate (even in a four-year horizon) and seem to stabilize after two years.

Fig. 2. Impulse-response [a, b]

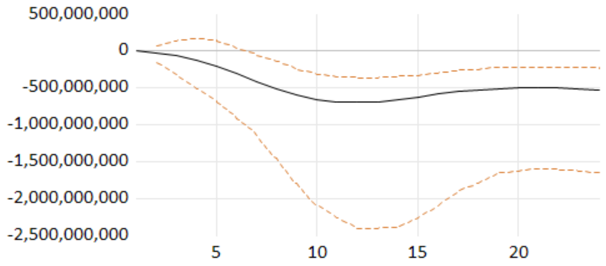


2.a. Response of real investments (in level) to real investments (in level)

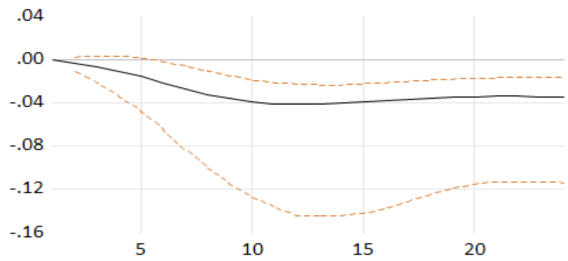
2.d. Response of investments (% of NCR) to investments (% of NCR)



2.b. Response of real investments (in level) to real cash (in level)



2.e. Response of investments (% of NCR) to cash (% of NCR)



2.c. Response of real investments (in level) to real debt (in level)

2.f. Response of investments (% of NCR) to debt (% of NCR)

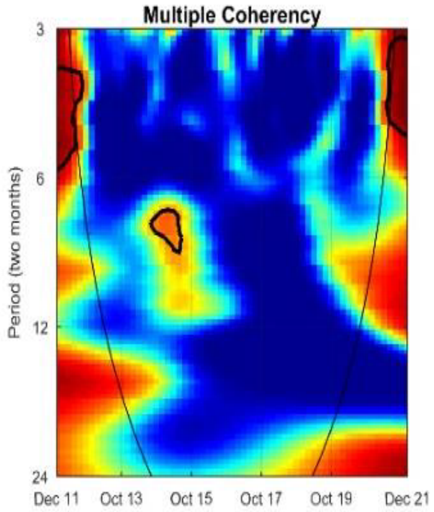
Notes: [a] Response to Cholesky one standard deviation (d. f. adjusted) innovations for the period from the sixth bimester of 2011 to the sixth bimester of 2021. [b] 95% Confidence interval using Hall´s studentized bootstrap with 999 bootstrap repetitions and 499 double bootstrap reps

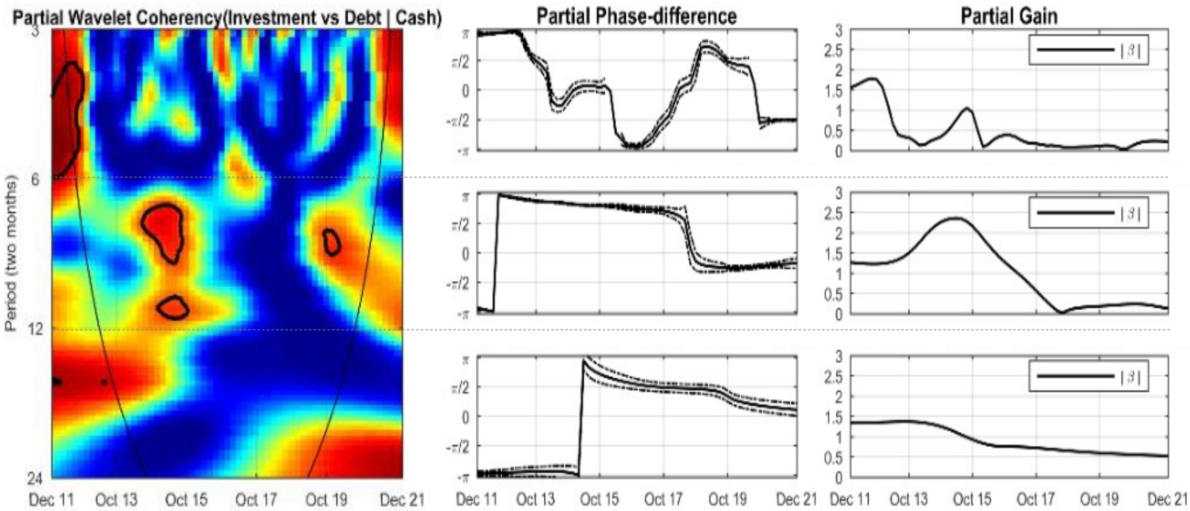
4.4.3 Multivariate coherency: results

Aiming to better understand investments-cash-debt linkages, we propose the use of the multiple partial coherence, phase-difference and gain. The first one measures the degree of adjustment of the explanatory variables on the dependent variable in the time-frequency domain. The other measures calculate the relationship between the fluctuations of investments versus debt, controlling the influence of cash on the oscillations in the time-frequency space, as well as the fluctuations of investments versus cash, controlling the influence of debt. According to Figure 3, we do not find a presence of successive areas with significance statistics in the multiple coherence, which denotes we do not have a good overall fit in the model. Next, we analyze whether cash and debt are useful to explain investments in the time-frequency location.

In both analyses, at the level or as a ratio of the NCR, the joint analysis suggests only the same significant area, during a short period between the second half of 2014 and the year 2015, associated with cycles with a frequency between 6 and 9 bimesters. This previous finding – for mid-run frequencies (6 ~9 bimesters), the multiple coherence is continuously significant from 2014 to 2015 – seem to be able to drive our findings on the controlled effects of both debt and cash. We observe that for both models (in level or as a % of NCR) the controlled effect of debt and cash are such that also at mid-run frequency (6~9 bimesters), the partial coherence is significant in a narrow area over the same period (2014 to 2015). For both explanatory variables (cash and debt) the partial phase-difference is set to $(\pi/2, \pi)$ indicating a negative significant co-movement.

Fig. 3. Investments-cash-debt linkages: joint wavelet coherency, partial phase-difference and partial gain. [a,b]





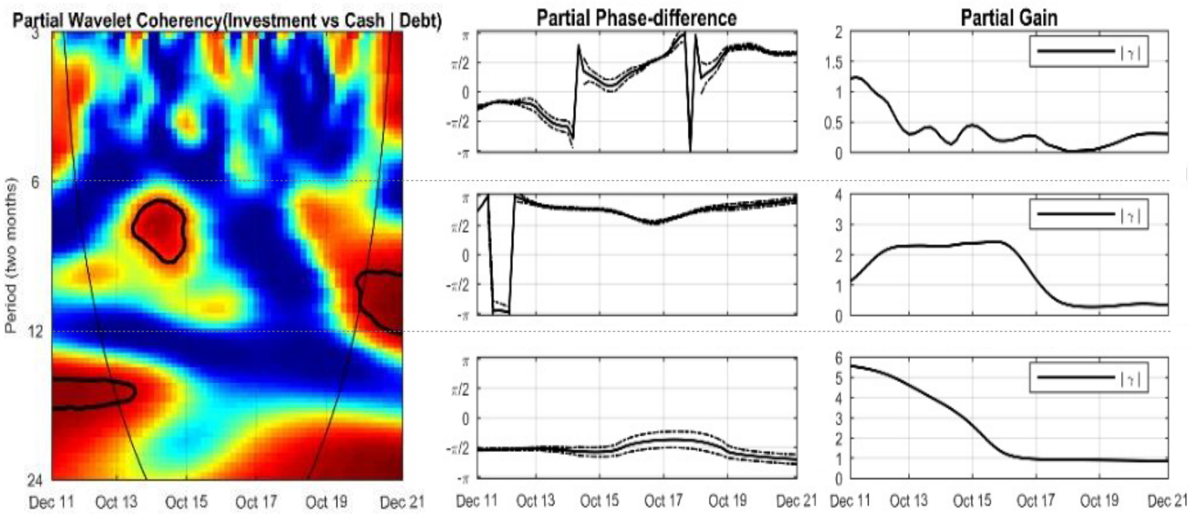


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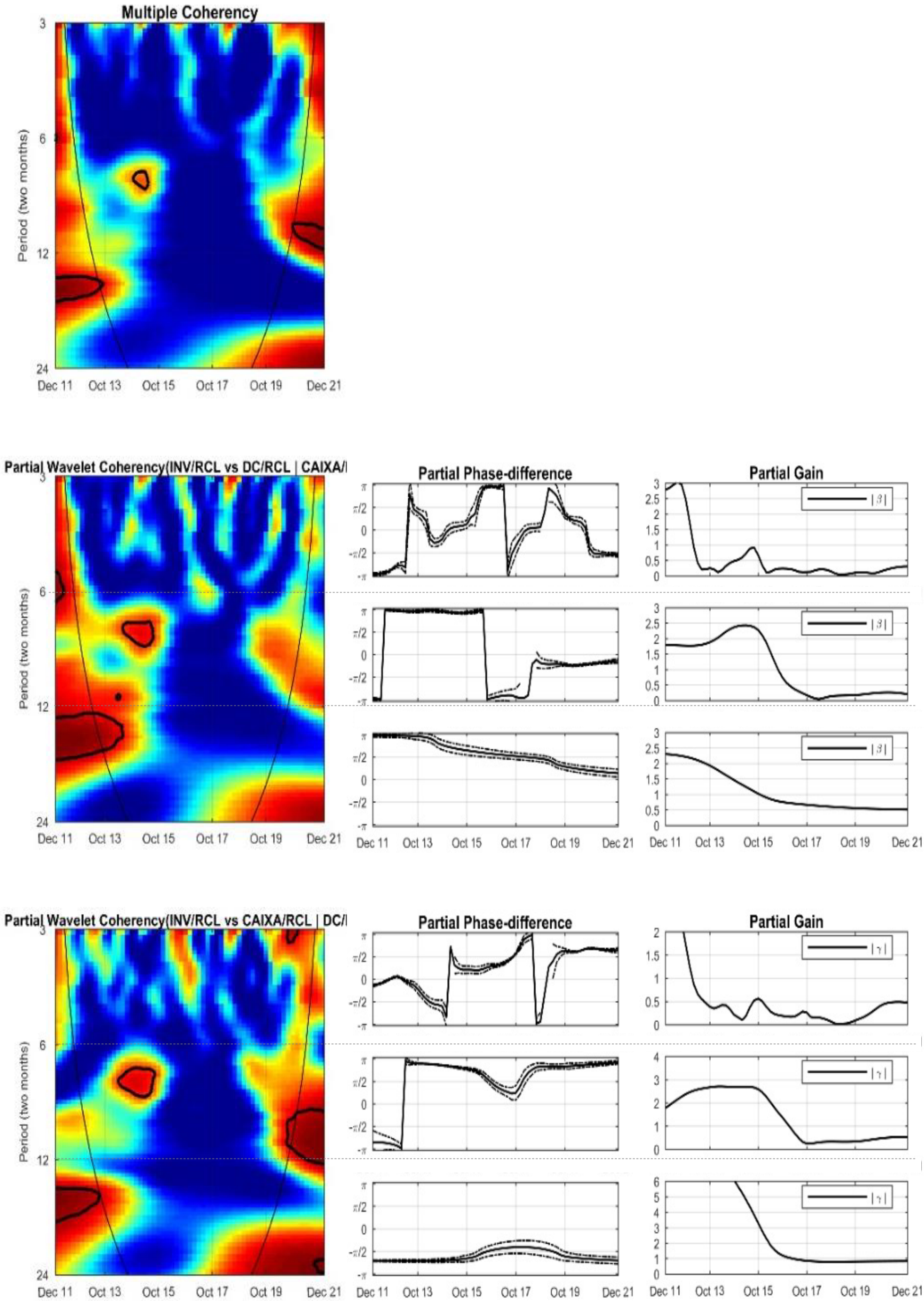
Fig. 3. Investments-cash-debt linkages: joint wavelet coherency, partial phase-difference and partial gain. [a,b]



Notes: [a] The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. [b] We also plot the phase-difference with plus and minus two standard deviations.

We also find that both cash and debt cycles are able to anticipate fluctuations of investment cycles during this period of fiscal instability for all state governments in Brazil. In both models, the partial gain associated with the leadership of debt cycles oscillates between 1 and 2.5, with an increasing trend throughout 2015. On the other hand, the partial gain associated with cash cycles is more stable over time, ranging between 2 and 3.

Even when we observe an area of significance of the co-movement of investment cycles and high-frequency cycles of debt to level, or low-frequency debt-to-NCR, in the first years of the sample, this co-movement is characterized by the leadership of debt cycles.

4.5 Conclusion

According to Uchoa et al. (2022), it is possible to infer (based on wavelet) that public investments by the state government of Ceará in equipment have influenced during the years 2006 to 2009 the revenue with the main tax: ICMS. These authors show that the commitment of resources in construction works is capable of influencing economic activity (IBCR-CE) between 2009 and 2012, industrial production between 2009 and 2010, and the volume of retail sales between 2012 and 2013. The authors also show that the investments in equipment between 2007 and 2011 and in construction works (during the period between 2007 and 2013, and during the years 2015 to 2018) were able to impact formal jobs.

This economic impact requires the sustainability of the flow of public investments, and this continuity depends on the sources of resources that finance the investment. This issue can be addressed by a theoretical approach: fiscal dynamics of public investments by Brazilian state governments, reported in Matos (2022b). This dynamic for state governments is specific and different from the dynamics of federal governments that may contract securities debt, as observed by the "Golden Rule" (Art. 167, III, of the Brazilian Federal Constitution, 1988) applied to states in Brazil. Our contribution empirically explores this theoretical dynamic for the state that led the national ranking of public investment in Brazil between 2017 and 2020. In other words, we analyze the relationship between the flow of committed investments and the non-flow variables: available cash and debt. Summarizing our results, we are able to find a long-term relationship suggesting sustainability, as well we find that cash and debt shocks on investment do not dissipate over a 4-year horizon, besides identifying the co-movements of the mid-run cycles of investments led or anticipated by debt and cash cycles, during the 2014 and

2015 fiscal crisis.

Finally, it is essential to make a caveat, because one of the main conceptual hypotheses of our paper is that the policy maker decision – responsive to past fluctuations observed in debt and cash – is the issuance of the purchase commitment note, i.e., the first stage of the execution of the contracting a construction works/installations, and acquisition of permanent material/equipment, for instance. It is important that the literature and the control institutions continue monitoring whether the stages of liquidation and payment of investments are following the amounts committed. In other words, it is reasonable to assume that the decision on whether or not to commit the investment is based on monitoring cash and debt. However, the social and macroeconomic benefits arising from the investment can only be observed and measured, if in fact such investment expenses have been finalized and paid. It is important in this context to question whether the indicators used to rank state governments should be based on committed or paid expenditures, for instance.

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