Estimation of Frequency-Selective Block-Fading MIMO Channels Using PARAFAC Modeling and Alternating Least Squares

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***Abstract*— The parametric estimation of Multiple-Input Multiple-Output (MIMO) channels is an important step towards the space-time characterization of the radio channel as well as for the design of efficient space-time signaling techniques. In this paper, we address the problem of multipath parameter estimation for frequency-selective block-fading MIMO channels by means of Parallel Factor (PARAFAC) analysis [1]–[2]. First, we present a fourth-order PARAFAC model for a specular MIMO radio channel, which jointly captures the space-time signature (angles of arrival, angles of departure and delays) and the time-varying fading amplitudes. Based on this model, an Alternating Least Squares (ALS) algorithm is used for estimating the multipath signals from a collection of received data blocks. After the multipath resolution stage, final estimates of the angles, delays and fading amplitudes are obtained by exploiting the knowledge of a multi-block training sequence. Numerical results are provided to illustrate the performance of this estimation approach.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) antenna systems have been extensively studied due to their ability of providing high data-rates and performance gains compared to traditional single-antenna systems [3], [4]. Perfect channel estimation is generally assumed in the analysis of MIMO antenna systems. However, in practice, the MIMO channel has to be estimated, typically using training/pilot symbols. An accurate channel estimation is important in coherent MIMO communication systems as well as it allows the design of efficient space-time signaling techniques that better exploit the MIMO channel. Parametric channel estimation techniques relying on a physical description of the MIMO channel (i.e. multipath angles, delays and fading amplitudes) are of great interest in wireless position-location systems and future wireless intelligent networks.

It is known that in time-varying wireless propagation environments, the multipath parameters are submitted to different varying rates. In practical mobile-radio channels, the multipath angles and delays experience a much slower rate of variation than the fast-fading amplitudes, and can be considered stationary over several data transmission blocks. In other words, while the fading amplitudes can vary completely when either the transmitter or the receiver moves as little as fractions of the wavelength, the angles of departure/arrival and propagation delays remain constant over changes in position of several (ten to thousands of) wavelengths [5]. For block transmission (i.e. time-slotted) MIMO systems, a “block-fading" MIMO channel model is generally considered for characterizing the time-varying nature of the radio channel. In the block-fading model, the path fading amplitudes vary between two successive blocks but they are considered constant over an entire data block.

Literature review: The exploitation of the algebraic structure of the wireless channel for the purpose of multipath parameter estimation was addressed in several works in the context of Single-Input Multiple-Output (SIMO) channels (see e.g. [5]–[6] and the references therein). Most of these approaches are based on high-resolution subspace methods, which exploit shift-invariance properties and/or the knowledge of the pulse shape function. In [7], a blind method for explicit angles and delay estimation is proposed, which uses a collection of previous unstructured estimates of the space-time channel impulse response obtained from multiple transmission blocks.

Training sequence based channel estimation methods exploiting the multi-block invariance of angles and delays have been proposed in [5], [8], [6] for an unstructured estimation of the space-time channel. These methods aim at showing that an increased estimation accuracy can be obtained by using *multi-slot processing*, which consists of appropriately combining the information from multiple transmission blocks (or time-slots) at the receiver so as to extend the effective training sequence.

In the context of MIMO channels, [9] proposes a modal analysis/filtering concept which exploits the different varying rates of the multipath parameters for estimating time-varying (block-fading) frequency-selective MIMO channels. The authors show that more accurate channel estimates with respect to the standard Least Squares (LS) estimation method can be obtained. The approach proposed in [10] is based on spectral factorizations of the specular channel into stationary (space) and non-stationary (fading amplitudes) signature subspaces, and uses linear prediction for estimating/tracking the time-varying channel. In [11], a subspace-based approach is proposed for a joint estimation of the Angles-Of-Arrival (AOAs), Angles-Of-Departure (AODs) and propagation delays of physical MIMO channels. This approach works on a previous unstructured estimate of the MIMO channel, and performs a subspace decomposition of the channel covariance matrix to determine AOAs, AODs and delays.

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In this work, a new parametric approach for estimating frequency-selective block-fading MIMO channels is proposed. It is based on the observation that the considered MIMO channel model has a tensor structure and follows a fourth-order PARAFAC model [1]–[2]. By extending a training sequence over multiple data transmission blocks and collecting these blocks at the receiver, we show that the received signal can be interpreted as a third-order PARAFAC model. This model is exploited for estimating the complete set of MIMO multipath parameters (AOAs, AODs, delays and fading amplitudes). The estimation method consists in using the Alternating Least Squares (ALS) algorithm, followed by a final estimation stage that relies on the knowledge of the training sequence. In [12], we proposed a multipath parameter estimation technique for SIMO channels, which is also based on a PARAFAC decomposition for modeling the time-varying multipath channel. This work can be viewed as an extension of [12] to MIMO channels.

The organization of this paper is as follows. Section II describes the system model and assumptions. Section III formulates the MIMO channel and the received signal using PARAFAC modeling. A sufficient condition for identifiability of the proposed PARAFAC model is established in Section IV. Section V describes the ALS-based method for estimating the MIMO channel parameters while in Section VI, some simulation results are presented for performance evaluation. The paper is concluded in Section VII.

II. System model and assumptions

Let us consider a MIMO antenna system with M_T transmit and M_R receive antennae. The spacing between any two antennae at both the transmit and receive arrays is assumed to be half-wavelength, so that we can apply the far-field approximation by assuming a locally plane wave. In this case, the MIMO channel can be characterized by specular multipath propagation, i.e., the channel between each transmit and receive antenna can be parameterized as the superposition of L paths. Figure 1 illustrates the considered MIMO propagation scenario. Each path is associated with a different scatterer located between the transmitter and the receiver. The location of the l-th scatterer determines an Angle Of Departure (AOD) ϕ_l and an Angle Of Arrival (AOA) θ_l (with respect to the transmit/receive array broadside) and a relative propagation delay τ_l for the l-th path It is also assumed that the maximum path delay exceeds the inverse of the coherence bandwidth so that the channel is frequency-selective. The finite support of the channel impulse response is equal to K symbol periods and the oversampling factor at the receiver is equal to P times the symbol rate. Let us define the following matrices collecting the transmitter and the receiver array responses and as well as the combined transmitter/receiver pulse shape responses:

$$\mathbf{A}^{(T)}(\boldsymbol{\phi}) = [\mathbf{a}^{(T)}(\phi_1)\cdots\mathbf{a}^{(T)}(\phi_L)] \in \mathbb{C}^{M_T\times L}$$
$$\mathbf{A}^{(R)}(\boldsymbol{\theta}) = [\mathbf{a}^{(R)}(\theta_1)\cdots\mathbf{a}^{(R)}(\theta_L)] \in \mathbb{C}^{M_R\times L}$$
$$\mathbf{G}(\boldsymbol{\tau}) = [\mathbf{g}(\tau_1)\cdots\mathbf{g}(\tau_L)] \in \mathbb{C}^{KP\times L}.$$

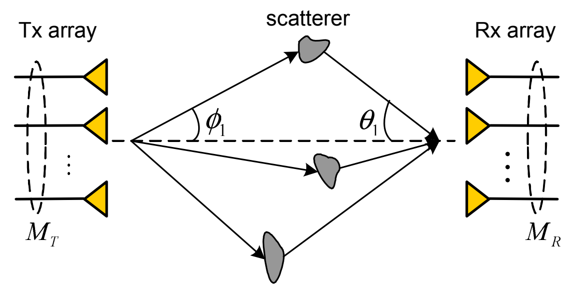

Fig. 1. MIMO multipath propagation scenario

A. Block-fading channel model

We adopt a block-fading model for characterizing the time-varying propagation channel The block-fading model is based on the fact that, in a time-varying environment, angles and delays (*long-term* parameters) experience a much slower rate of variation than the fast-fading amplitudes (*short-term* parameters). In our model, the path fading amplitudes are considered constant over an entire data transmission block, but vary between two blocks. Such an inter-block variation of the fading amplitudes characterizes the time-varying nature of the MIMO channel[1]. On the other hand, ϕ_l, θ_l and τ_l (which depend only on the propagation geometry) are assumed to be constant over an interval of stationarity spanning I blocks. This block-fading channel model is reasonable in most of mobile communication systems with time-slotted transmission, and has been exploited in, e.g. [9], [10], for purposes of MIMO channel estimation. Assuming a transmission of I blocks, the fading amplitude of the l-th path during the i-th block is represented by $b_{i,l}$. A matrix collecting the path fading gains during the I blocks is defined as:

$$\mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,L} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ b_{I,1} & b_{I,2} & \cdots & b_{I,L} \end{bmatrix} \in \mathbb{C}^{I\times L},$$

where the i-th row of $\mathbf{B}$ collects the L fading amplitudes for the i-th block. We assume that the envelope of each fading amplitude $b_{i,l}$ follows a Rayleigh distribution while the associated phase is uniformly distributed. Moreover, the amplitudes corresponding to different paths are assumed to be statistically independent.

B. Multi-block training sequence

At the transmitter, each transmission block is organized in M_T data streams that are transmitted by the M_T transmit antennae. The structure of these data streams depend on the considered particular scheme (e.g., spatial multiplexing,

[1]The block-fading assumption holds if the transmission block length is smaller than the coherence time of the multipath fading channel. This is the case considered here.

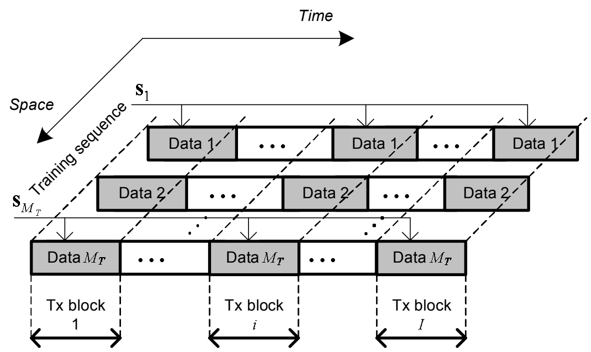

Fig. 2. Multiblock transmission structure with training sequence reuse

space-time coding, etc). Each one of the M_T data streams has a training sequence of N symbols known at the receiver.

The length-N training sequence at the m_T-th transmit antenna for the i-th transmission block is represented by:

$$\mathbf{s}_{m_T}(i) = [s_{m_T}(i,1) \cdots s_{m_T}(i,N)]^T \in \mathbb{C}^N.$$

In this work, the following assumptions hold concerning the design of the training sequences:

A.1 The M_T training sequence vectors $\mathbf{s}_1, \ldots, \mathbf{s}_{M_T}$ are linearly independent[2].

A.2 The length N of each training sequence $\mathbf{s}_{m_T}$, $m_T = 1, \ldots, M_T$, satisfy $N \geq M_T K$.

A.3 The training sequence $\mathbf{s}_{m_T}$, $m_T = 1, \ldots, M_T$, is reused across I successive transmission blocks, and we have $\mathbf{s}_{m_T}(i) = \mathbf{s}_{m_T}, \quad \forall i \in [1, I]$.

Figure 2 outlines the multiblock transmission structure with training sequence reuse across transmission blocks. It is to be noted that one transmission block comprises M_T parallel data blocks, each one of which having its own training sequence. Note also that the figure indicates that the same set of training sequences is inserted into I transmission blocks. Each data block has $N_{\text{block}} = N + N_{\text{data}}$ symbols, N_{data} denotes the number of "useful" data symbols of each data block. Throughout the paper, for signal modeling and channel estimation purposes, we focus only on the training sequence portion of each data block. After having estimated the channel, the useful data portion can be processed/recovered in a subsequent step by means of space-time processing.

III. PARAFAC MODELING

The block-fading MIMO channel can be viewed as a fourth-order tensor $\mathcal{H} \in \mathbb{C}^{I \times M_R \times M_T \times KP}$, i.e., an array having four dimensions. Let us define $h_{i,m_R,m_T,k'}$ as a scalar component of the MIMO channel tensor $\mathcal{H}$, which represents the impulse response of the k'-th tap of the channel between the m_T-th transmit and m_R-th receive antenna for the i-th fading block, $k' = (k-1)P + p$. We propose to use the PARAFAC (*Parallel Factor*) decomposition [1], [2] to model the block-fading MIMO channel. In PARAFAC notation, the scalar component $h_{i,m_R,m_T,k'}$ of the L-path block-fading MIMO channel can be written as:

$$h_{i,m_R,m_T,k'} = \sum_{l=1}^{L} b_{i,l} a^{(R)}_{m_R,l} a^{(T)}_{m_T,l} g_{k',l}, \tag{1}$$

where $a^{(R)}_{m_R,l} = [\mathbf{A}^{(R)}(\boldsymbol{\theta})]_{m_R,l}$, $a^{(T)}_{m_T,l} = [\mathbf{A}^{(T)}(\boldsymbol{\phi})]_{m_T,l}$, $g_{k',l} = [\mathbf{G}(\boldsymbol{\tau})]_{k',l}$, $b_{i,l} = [\mathbf{B}]_{i,l}$.

After baseband conversion and oversampling at each receive antenna, we collect NP received samples at each receive antenna. Let us define $x_{i,m_R,n'}$ as a scalar component of the received signal tensor $\mathcal{X} \in \mathbb{C}^{I \times M_R \times NP}$, representing the n'-th received signal sample at the m_R-th antenna for the i-th transmission block, and $n' = (n-1)P + p$. Ignoring the additive noise for notation simplicity, $x_{i,m_R,n'}$ can be written as:

$$x_{i,m_R,n'} = \sum_{m_T=1}^{M_T} \sum_{k=1}^{K} h_{i,m_R,m_T,k'}\, s_{n',m_T,k'}, \tag{2}$$

where

$$s_{n',m_T,k'} = [\mathbf{S}]_{n',(k'-1)M_T+m_T},$$

is an arbitrary element of $\mathbf{S} = \mathbf{S}_o \otimes \mathbf{I}_P \in \mathbb{C}^{NP \times M_T KP}$, and

$$\mathbf{S}_o = BlockToeplitz(\mathbf{s}_1, \cdots, \mathbf{s}_{M_T}) \in \mathbb{C}^{N \times M_T K} \tag{3}$$

is a block-Toeplitz training sequence matrix. Let us define $\mathbf{H}_{i\cdot\cdot} \in \mathbb{C}^{M_R \times M_T KP}$ as the i-th matrix-slice obtained by slicing the MIMO fourth-order tensor $\mathcal{H}$ along its first dimension. It can be shown that $\mathbf{H}_{i\cdot\cdot}$ can be expressed as a function of the multipath parameters as:

$$\mathbf{H}_{i\cdot\cdot} = \mathbf{A}^{(R)}(\boldsymbol{\theta}) D_i(\mathbf{B}) \mathbf{W}^T(\boldsymbol{\tau}, \boldsymbol{\phi}), \quad i = 1, \ldots, I, \tag{4}$$

where $D_i(\mathbf{B})$ is a diagonal matrix holding the i-th row of $\mathbf{B}$ on its diagonal and $\mathbf{W}(\boldsymbol{\tau}, \boldsymbol{\phi}) \in \mathbb{C}^{M_T KP \times L}$ is defined as

$$\mathbf{W}(\boldsymbol{\tau}, \boldsymbol{\phi}) = \mathbf{G}(\boldsymbol{\tau}) \diamond \mathbf{A}^{(T)}(\boldsymbol{\phi}). \tag{5}$$

The i-th matrix-slice of the received signal, denoted by $\mathbf{X}_{i\cdot\cdot} \in \mathbb{C}^{M_R \times NP}$, can be written as:

$$\mathbf{X}_{i\cdot\cdot} = \mathbf{H}_{i\cdot\cdot}\mathbf{S} = \mathbf{A}^{(R)}(\boldsymbol{\theta}) D_i(\mathbf{B}) \mathbf{C}^T(\boldsymbol{\tau}, \boldsymbol{\phi}), \quad i = 1, \ldots, I,$$

where

$$\mathbf{C}(\boldsymbol{\tau}, \boldsymbol{\phi}) = \mathbf{S}\left[\mathbf{G}(\boldsymbol{\tau}) \diamond \mathbf{A}^{(T)}(\boldsymbol{\phi})\right] \in \mathbb{C}^{NP \times L} \tag{6}$$

is a combined space-time channel response at the receiver side, i.e., a convolution between the receiver space-time signatures and the training symbols, and $\diamond$ stands for the Khatri-Rao (column-wise Kronecker) product. Figure 3 illustrates the interpretation of the received signal tensor as a collection of I matrix-slices.

[2]We remark that the "independence" assumption does not lead to an optimal training sequence set for estimating the MIMO channel. An optimal design should ensure that the training sequences have perfect periodic autocorrelations and cross-correlations within $K-1$ temporal shifts [13], where K is the temporal span of the channel impulse response. In this work, we are not concerned with optimal training sequence design, and we simply assume independent training sequences. As will be shown later in our simulation results, the independency assumption is enough to guarantee accurate estimates of the MIMO channel using the proposed approach.

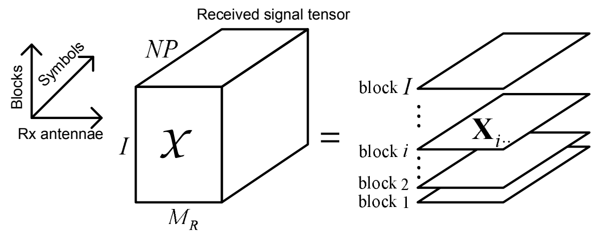

Fig. 3. Received signal third-order tensor as a collection of I matrix-slices.

Let us stack I slices $\mathbf{X}_{1\cdot\cdot}, \ldots, \mathbf{X}_{I\cdot\cdot}$ in a matrix $\mathbf{X}_1 \in \mathbb{C}^{M_R I \times NP}$, and I slices $\mathbf{H}_{1\cdot\cdot}, \ldots, \mathbf{H}_{I\cdot\cdot}$ in a matrix $\mathbf{H}_1 \in \mathbb{C}^{M_R I \times M_T KP}$:

$$\mathbf{X}_1 = \begin{bmatrix} \mathbf{X}_{1\cdot\cdot} \\ \vdots \\ \mathbf{X}_{I\cdot\cdot} \end{bmatrix}, \quad \mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{1\cdot\cdot} \\ \vdots \\ \mathbf{H}_{I\cdot\cdot} \end{bmatrix}.$$

$\mathbf{X}_1$ and $\mathbf{H}_1$ are the "unfolded representations" for the tensors $\mathcal{X}$ and $\mathcal{H}$, respectively. Using (4)-(5), we obtain the following input-output relation:

$$\mathbf{X}_1 = \mathbf{H}_1\mathbf{S} = \left[\mathbf{B} \diamond \mathbf{A}^{(R)}(\boldsymbol{\theta})\right] \mathbf{C}^T(\boldsymbol{\tau}, \boldsymbol{\phi}), \qquad (7)$$

where $\mathbf{H}_1 = \left[\mathbf{B} \diamond \mathbf{A}^{(R)}(\boldsymbol{\theta})\right] \mathbf{W}^T(\boldsymbol{\tau}, \boldsymbol{\phi})$. Thanks to the symmetry of the third-order PARAFAC model, two other unfolded matrix representations can be obtained: $\mathbf{X}_2 = \left[\mathbf{A}^{(R)}(\boldsymbol{\theta}) \diamond \mathbf{C}(\boldsymbol{\tau}, \boldsymbol{\phi})\right] \mathbf{B}^T \in \mathbb{C}^{NPM_R \times I}$ and $\mathbf{X}_3 = \left[\mathbf{C}(\boldsymbol{\tau}, \boldsymbol{\phi}) \diamond \mathbf{B}\right] \mathbf{A}^{(R)T}(\boldsymbol{\theta}) \in \mathbb{C}^{INP \times M_R}$.

IV. Identifiability

Identifiability of (7) allows one to uniquely determine (up to trivial ambiguities) the parameters of the L multipaths from the observed received signal tensor $\mathcal{X} \in \mathbb{C}^{I \times M_R \times NP}$. According to the identifiability results of the PARAFAC model, the identifiability of $\mathbf{A}^{(R)}$, $\mathbf{B}$, and $\mathbf{C}$ is linked to the concept of k-rank of these matrices [2]. Shortly, the k-rank $k_{\mathbf{A}}$ of a matrix $\mathbf{A}$ is equal to r if any set of r columns of $\mathbf{A}$ is linear independent, but any set of $r+1$ columns of $\mathbf{A}$ is linear dependent. The Kruskal condition says that, if $k_{\mathbf{A}^{(R)}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2(L+1)$, then $\mathbf{A}^{(R)}$, $\mathbf{B}$ and $\mathbf{C}$ can be identified from $\mathbf{X}_1$ up to trivial permutation and column scaling ambiguities [2]. In our context, a sufficient condition for identifying the MIMO multipath parameters can be obtained by recalling useful results on the k-rank of a matrix having Khatri-Rao product structure as well as on the k-rank of a Vandermonde matrix. These results are derived in [14] (c.f. Lemmas 1 and 2, respectively). A sufficient identifiability condition for our model can be obtained by applying the identifiability theorem of [14] to our context:

Theorem: Given $\mathbf{X}_{i\cdot\cdot} = \mathbf{A}^{(R)}(\boldsymbol{\theta}) D_i(\mathbf{B}) \mathbf{C}^T(\boldsymbol{\tau}, \boldsymbol{\phi})$, suppose that the L multipaths have statistically independent propagation (i.e. distinct AODs, AOAs and delays). Provided that the fading amplitudes are temporally uncorrelated across successive transmission blocks, a sufficient condition for almost-sure identifiability is:

$$\min(I, L) + \min(M_R, L) + \min(M_T + KP - 1, L) \geq 2(L+1)$$

Proof: Use assumptions **A.1**-**A.2** to conclude that $\mathbf{S}$ is full rank (and full k-rank) to deduce $\text{rank}(\mathbf{C}(\boldsymbol{\tau}, \boldsymbol{\phi})) = \text{rank}(\mathbf{W}(\boldsymbol{\tau}, \boldsymbol{\phi}))$. Apply Lemma 1 in [14] by making the following correspondences: $\mathbf{A} \rightarrow \mathbf{G}(\boldsymbol{\tau})$, $\mathbf{B} \rightarrow \mathbf{A}^{(T)}(\boldsymbol{\phi})$, to verify that $\text{rank}(\mathbf{W}(\boldsymbol{\tau}, \boldsymbol{\phi})) \geq \min(k_{\mathbf{G}(\boldsymbol{\tau})} + k_{\mathbf{A}^{(T)}(\boldsymbol{\phi})} - 1, L)$. Finally, use the fact that the k-rank of a matrix is equal to its rank with probability one whenever its columns are drawn independently from an absolutely continuous distribution. Thus, we have $k_{\mathbf{A}^{(R)}} = \min(M_R, L)$, $k_{\mathbf{B}} = \min(I, L)$, $k_{\mathbf{C}} = \min(M_T + KP - 1, L)$. ■

Remarks:

1) *The identifiability condition established in the previous theorem is sufficient but not necessary. Assuming $M > 1$ and $N > 1$ (irrespective of the oversampling factor P), a necessary condition is $k_{\mathbf{B}} \geq 2$ [15]. In practice, this means that at least $I \geq 2$ transmission blocks must be collected at the receiver to ensure a uniqueness of model (7).*

2) *Column permutation is unremovable although not relevant in our context, since the ordering of the multipath responses is unimportant for channel estimation purposes. Scaling ambiguity can be eliminated by exploiting prior knowledge of the space-time manifold structure i.e., the array geometry and the pulse shape function.*

V. Estimation of the MIMO channel parameters

The estimation of the MIMO multipath parameters is done in two-stages. The first one is blind, and consists in using the trilinear Alternating Least Squares (ALS) algorithm [2] for fitting a third-order PARAFAC model to the received signal tensor $\mathcal{X} \in \mathbb{C}^{I \times M_R \times NP}$. Each iteration of the ALS algorithm is composed of three estimation steps. In each step one component matrix is updated by fixing the two others to their values obtained in previous steps. Given the unfolded representations $\mathbf{X}_{i=1,2,3}$ of the received signal tensor, the conditional LS updates at the r-th iteration are given by:

$$\widehat{\mathbf{C}}^T_{(r)} = \left[\widehat{\mathbf{B}}_{(r-1)} \diamond \widehat{\mathbf{A}}^{(R)}_{(r-1)}\right]^{\dagger} \mathbf{X}_1,$$

$$\widehat{\mathbf{B}}^T_{(r)} = \left[\widehat{\mathbf{A}}^{(R)}_{(r-1)} \diamond \widehat{\mathbf{C}}_{(r)}\right]^{\dagger} \mathbf{X}_2,$$

$$\widehat{\mathbf{A}}^{(R)T}_{(r)} = \left[\widehat{\mathbf{C}}_{(r)} \diamond \widehat{\mathbf{B}}_{(r)}\right]^{\dagger} \mathbf{X}_3,$$

where $\widehat{\mathbf{A}}^{(R)}_{(r)} = \widehat{\mathbf{A}}^{(R)}_{(r)}(\boldsymbol{\theta})$ and $\widehat{\mathbf{C}}_{(r)} = \widehat{\mathbf{C}}_{(r)}(\boldsymbol{\tau}, \boldsymbol{\phi})$. At the first iteration ($r = 1$), $\widehat{\mathbf{A}}^{(R)}_{(0)}$ $\widehat{\mathbf{B}}_{(0)}$ can be randomly initialized or using previous knowledge of the multipath parameters (e.g. AOAs). Let $e(r) = \left\|\mathbf{X}_1 - \left[\widehat{\mathbf{B}}(r) \diamond \widehat{\mathbf{A}}^{(R)}(r)\right] \widehat{\mathbf{C}}^T(r)\right\|_F$ denote the estimation error at the end of the r-th iteration. The convergence is declared when $|e(r) - e(r-1)| \leq 10^{-5}$. The second stage consists in using the training sequence matrix $\mathbf{S}$ to find an LS estimate of $\mathbf{W}(\boldsymbol{\tau}, \boldsymbol{\phi}) = \mathbf{G}(\boldsymbol{\tau}) \diamond \mathbf{A}^{(T)}(\boldsymbol{\phi})$ as $\widehat{\mathbf{W}}(\boldsymbol{\tau}, \boldsymbol{\phi}) = \mathbf{S}^{\dagger}\widehat{\mathbf{C}}^{(conv)}(\boldsymbol{\tau}, \boldsymbol{\phi})$, where $\widehat{\mathbf{C}}^{(conv)}(\boldsymbol{\tau}, \boldsymbol{\phi})$ is the estimated value of $\mathbf{C}(\boldsymbol{\tau}, \boldsymbol{\phi})$ at the convergence. Separated estimations of $\widehat{\mathbf{A}}^{(T)}(\boldsymbol{\phi})$ and $\widehat{\mathbf{G}}(\boldsymbol{\tau})$ as well as the elimination of the scaling factors can be carried out by exploiting the Vandermonde structures of $\mathbf{A}^{(R)}(\boldsymbol{\theta})$ and $\mathbf{A}^{(T)}(\boldsymbol{\phi})$ and using proper pulse shape design.

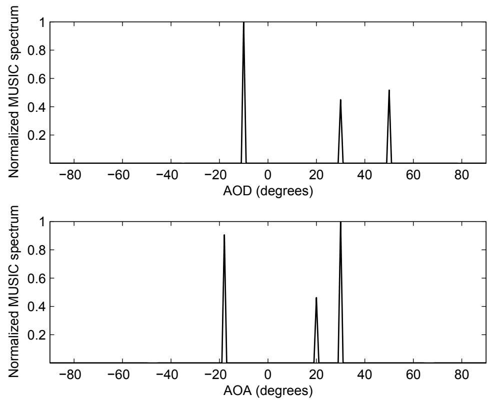

Fig. 4. The normalized MUSIC spectrum for AOD and AOA estimation.

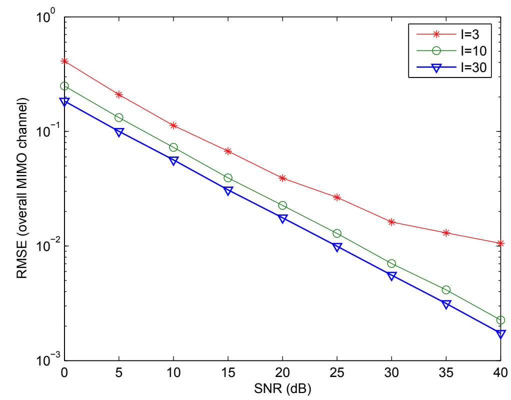

Fig. 5. RMSE performance of the proposed estimator.

VI. Simulation results

In this section, some simulation results are shown to illustrate the performance of the proposed parametric MIMO channel estimator. We assume $N = 10$ training symbols per transmit antenna. The training symbols are modulated using Binary Phase Shift Keying (BPSK). The oversampling factor is assumed to be $P = 2$. The pulse shape function is a raised cosine with roll-off 0.35. We consider $L = 3$ specular multipaths with equal average power. The vector containing the AODs, AOAs and delays of the multipaths are respectively $\boldsymbol{\phi} = [-10^o, 30^o, 50^o]$, $\boldsymbol{\theta} = [-18^o, 20^o, 35^o]$ and $\boldsymbol{\tau} = [0, T, 2T]$, where T denotes the symbol period (K=3 is assumed). Uncorrelated fading across the successive blocks is modeled by assuming that $b_{i,l} \sim N(0,1)$.

In order to evaluate the accuracy of the proposed method in estimating the spatial signatures, Fig. 4 depicts the normalized MUSIC spectrum for the AODs and AOAs. We have assumed $M_T = M_R = 4$, $I = 10$ and a Signal to Noise Ratio (SNR) of 20dB. We can see that accurate estimates of the transmitter and receiver spatial signatures are obtained.

Figure 5 shows the Root Mean Square Error (RMSE) between estimated $\widehat{\mathbf{H}}$ and true $\mathbf{H}$ MIMO channel matrices as a function of the SNR at each receive antenna. These results are an average over 1000 independent realizations assuming $M_T = M_R = 2$ and $I = 3$, 10 or 30. In this simulation, 95% of the runs were retained for plotting the results. The 5% worst (ill-convergent) runs were discarded. Convergent runs have converged within 30 iterations in average. Note that the estimation performance improves as the number of transmission blocks is increased. In fact, fading amplitudes variation across the blocks is converted into temporal diversity for resolving the multipath signals.

VII. Conclusion

In this paper, a PARAFAC-based tensor model for frequency-selective block-fading MIMO channels has been proposed. Using a multi-block training sequence (i.e. a training sequence that is reused across successive transmission blocks), a third-order PARAFAC model has been considered for the received signal. Based on this model, an ALS-based estimation method has been used for a complete determination of the multipath parameters (AOAs, AODs, delays and fading amplitudes). In future work, we shall provide performance comparisons of the proposed method with competing ones under realistic channel models.

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