LYAPUNOV EXPONENTS OF QUANTUM DOT LASER SYSTEMS UNDER OPTICAL FEEDBACK: INFLUENCE OF CAVITY LENGTH

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Abstract— Performance of quantum dot lasers subject to optical feedback is numerically studied in this work from Lang-Kobayashi and multi-population rate-equation models. The approach adopted to study the characteristics of the laser response is based on the calculation of the Lyapunov exponents of the system and revealed the existence of chaos in shorter devices. Influence of the time-delay of the reflected back electrical field has also been addressed.

Keywords— semiconductor laser system, Lyapunov exponents, quantum dots

Resumo— O desempenho de lasers de pontos quânticos quando submetidos a realimentação óptica é estudado numericamente neste trabalho, a partir dos modelos de equações de taxa de Lang-Kobayashi e de multi-populações. A abordagem adotada para estudar as características da resposta laser é baseada no cálculo dos expoentes de Lyapunov do sistema, e revelou a existência de caos em dispositivos curtos. A influência da constante de atraso do campo elétrico refletido para a cavidade também é incluída na análise.

Palavras-chave— sistema laser semicondutor, expoentes de Lyapunov, pontos quânticos

1 Introduction

Semiconductor quantum dot (QD) lasers have been intensively studied in the last years because of their potential compared to quantum well and bulk lasers, as well as due to the particular properties associated with the 3D confinement in the quantum dots, such as the high differential gain which should lead to reduced linewidth enhancement factor and low chirp. Many papers have indeed addressed this issue and shown there is significant dependence of the alpha-factor on both internal and external factors, like carrier scattering dynamics (Melnik, 2006), wetting layer carrier population (Carrol, 2006), cavity length, temperature (Carrol, 2005), etc. As a consequence the device becomes sensitive to out-of-phase optical field, and even the coupling to optical fiber could be a problem due to the delayed optical field reflected back to the cavity (Gioannini, 2008a).

Among those factors, the influence of the cavity length is particularly interesting because it is a design issue, and theoretical information may guide manufacturers when choosing between short or long devices. Following previous works which revealed the existence of different operating regimes of quantum dot lasers, ranging from stable to chaotic-like solutions as the injection current increased (Gioannini, 2008b), it is natural to ask if the alpha-factor dependence on device length is enough to lead to chaotic-like behaviour.

To give that an answer, in this work it is studied the response of quantum dot lasers of different length from the calculated Lyapunov exponents of the dynamic system (Monteiro, 2002). This paper is organized as follows: in the next section the rate-equations based model to take into account the delayed optical field and the direct capture path is presented, and a discussion of the Lyapunov exponents is added. In section III results are presented and discussed. Finally, we draw the conclusions.

2 Modeling

2.1 Rate-equations

The model here used is based on that of (The, 2012), which considers separate dynamics for electron and hole populations of InAs quantum dots inserted in InGaAs quantum well, with GaAs separate confinement heterostructure (SCH). This assumption comes from the expected band diagram for this material, as obtained from NextNano3D simulation tool and shown in Figure 1. The diagram reveals the existence of one fundamental (GS) and two excited (ES1, ES2) confined energy states for electrons in the dots; higher energy states have been assumed to form an upper quasi-continuum (grey shaded part in Fig. 1a, called wetting layer) since they are weakly confined in the dots and can be quantum mechanically coupled together.

In order to correctly include the inhomogeneous dot size distribution which takes place in the Stranski-Krastanow growth technique, a multi-population approach has been adopted (Rossetti, 2007); this means that quantum dots of an ensemble are separated into *k* small groups of dots similar in size, thus leading to k equations for carrier number of each confined state. Moreover, there are also equations for carriers of the conduction band in the SCH and WL states. Electrons are first injected in the SCH and may undergo different physical processes, like diffusion, radiative and non-radiative recombination, capture and relaxation scattering, etc.

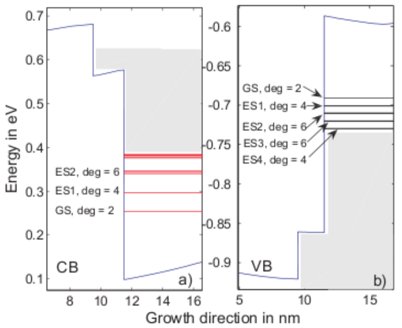

Figure 1. Band diagram of quantum dot material: a) Conduction Band b) Valence Band.

These phenomena are traditionally described from a set of first-order time-derivative equations as in the following short notation:

$$\begin{aligned} \dot{n}_{sch} &= f_1(I, n_{sch}, n_{wl}) \\ \dot{n}_{wl} &= f_2(n_{sch}, n_{wl}, n_{es2}, n_{es1}, n_{gs}) \\ \dot{n}_{es2k} &= f_{3k}(n_{wl}, n_{es2}, n_{es1}, E_0) \\ \dot{n}_{es1k} &= f_{4k}(n_{wl}, n_{es2}, n_{es1}, n_{gs}, E_0) \\ \dot{n}_{gk} &= f_{5k}(n_{wl}, n_{es1}, n_{gs}, E_0) \end{aligned} \tag{1}$$

Each velocity field equation of the above autonomous system also depends on time constants which characterize the various physical processes just mentioned.

For what concerns carriers of the conduction band, assumption of quasithermal equilibrium between quantum-dot and wetting layer states (The, 2010) makes things simpler, and the system looks like:

$$\begin{aligned} \dot{n}_{sch} &= f_6(I, n_{sch}, n_{wlDot}) \\ \dot{n}_{wlDot} &= f_7(n_{sch}, n_{wlDot}) \end{aligned} \tag{2}$$

The main difference between the model here used and that of (The, 2012) is that the photon equations for the cavity resonant modes have been replaced by one equation for the internal electrical field intensity, E_0 and one for its phase, Φ. This follows the classical Lang-Kobayashi model (Lang, 1980) used widely to study the effects of optical feedback in semiconductor single-mode lasers (Otto, 2010; O'Brien, 2004). Here are the remaining two equations:

$$\begin{aligned} \dot{E}_0 = & \frac{-E_0}{2\mathrm{t}_{ph}} + \frac{c}{2\mathrm{n}_r} + B_{sp} \\ & - \sum \left(g_{nES2} + g_{nES1} + g_{nGS} \right) \cdot E_0 \\ & + k \cdot E_0 \left(t - t_d \right) \cdot \cos\left(w_0 t_d + \Delta(t) \right) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\Phi} = & \frac{-k \cdot E_0 \left(t - t_d \right)}{E_0(t)} \cdot \sin\left(w_0 t_d + \Delta(t) \right) \\ & + 2\pi\delta f \end{aligned} \tag{4}$$

In the above equations t_{ph} is the photon lifetime in the laser cavity, c is the free-space light velocity, n_r is the active material refractive index, k is the intensity of feedback light, w_0 is the angular frequency of the solitary laser, t_d is the external cavity roundtrip time and δf is the frequency chirp calculated according to (Gioannini, 2007). Finally, the terms inside the summation operator is the material gain coupling cavity photons and carriers of the ground-, first- and second-excited states.

2.2 Lyapunov exponents

According to the theory of dynamical systems, only nonlinear dissipative systems may experience chaotic behavior, being chaos related to sensitivity to initial conditions and characterized by a time evolution towards a strange attractor in the phase space (Monteiro, 2002). A widely used approach to test sensitivity to initial conditions of nonlinear systems and, therefore, conclude about the existence of chaos requires the calculation of Lyapunov exponents.

Given a dynamic system with p velocity fields associated to state variables, there are two requirements to be satisfied before concluding if the process is chaotic:

a) at least one of the Lyapunov exponents associated to the velocity equations is positive: this is to guarantee divergence of adjacent trajectories (those starting at slightly different initial conditions);

b) the sum of all Lyapunov exponents associated to the whole set of velocity field equations must be negative: this is to ensure the system is dissipative (and therefore phase space evolution towards a strange attractor takes place).

In the present work the calculation of the Lyapunov exponents is based on the following formula:

$$\Lambda_i = \frac{1}{N} \cdot \sum_n^{N-1} \log_e\left(\frac{f_i(x_0 + \delta_0) - f_i(x_0)}{\delta_0} \right) \tag{5}$$

In this expression, N is the size of the discrete time vector corresponding to the last time instants of every simulation (transient is discarded), and δ_0 is the small deviation between two different initial conditions.

3 Results and discussion

In the following three devices are considered, with 0.6 mm, 0.7 mm and 1.0 mm length; all of them are 4 microns-wide edge-emitting single-mode lasers emitting at 1285 nm from the fundamental state. This device has been chosen to allow for better comprehension of already reported results, especially the chaotic-like solutions which had not been explained in terms of Lyapunov exponents (Gioannini, 2008a e 2008b).

First result shown in Figure 2 is the time-domain evolution of the sum of the whole set of Lyapunov exponents calculated for the carrier and photon population equations of the quantum dot laser. This figure has been obtained for a 300 mA switch-on driving, and the exponents have been calculated considering the last instants of simulation (to discard the transient). Although for l mm-long laser it seems stationary conditions have not been achieved, globally the requirement b) of section 2.2 has been satisfied.

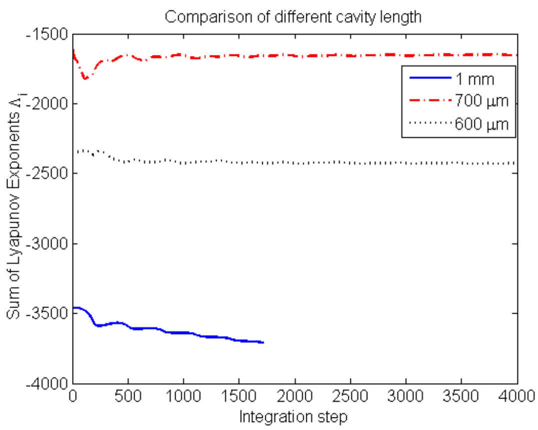

Figure 2. Time-domain evolution of the sum of calculated Lyapunov exponents for different laser length.

Individual contribution of every velocity field equation for the stationary value of the Lyapunov exponents is reported in Figure 3. Main purpose of this plot is to report the existence of one positive Lyapunov exponent for 700 microns- and 600 microns-long lasers, whereas the longer device has no positive exponent.

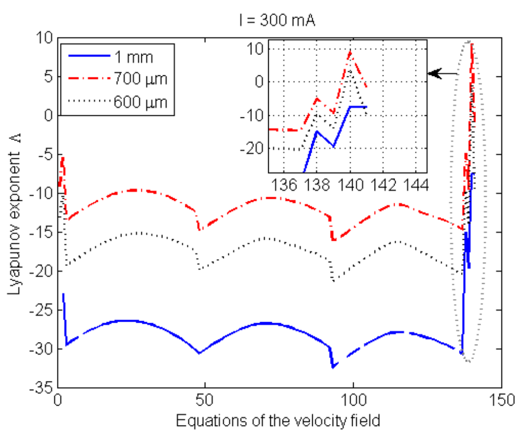

Figure 3. Calculated Lyapunov exponents for each velocity field equations of the laser system.

Indeed a phase-space portrait for this device reveals evolution towards a fixed-point solution for the 1 mm-long device, as shown in Figure 4 top-left and top-right parts. Left column refers to variation of output photon number as function of wetting layer and SCH populations, whereas right column refers to variation of photons as function of the carriers confined in the quantum dot. Bottom parts reveal instead the phase-space for the 700 microns-long device.

To make the study more complete, the influence of the external cavity has also been investigated. The existence of an external cavity is, actually, the basis for the Lang-Kobayashi model since it allows for the mathematical modeling of the optical feedback as a time-delayed electrical field. Thus, changing the external cavity length means changing the delay between the forward and backward components of the electrical field inside the device.

Compared to the results just discussed, plot of Figure 5 refers to a longer external cavity device, with 625 ps round-trip time, instead of 500 ps used so far. It is a 15 ns-long time-domain response for 300 mA switch-on driving, for the 700 microns-long device. There are, however, two time-series in the picture, which deviate from instant t = 9.5 ns, showing completely different responses at longer times. Solid and dotted lines refer to the same variable (output photon number), as the system responds to two slightly different initial conditions in some of the state variables (from $1.0e^{-3}$ to $1.1e^{-3}$). One can see the sensitivity to initial conditions which is very characteristic of chaotic behaviour.

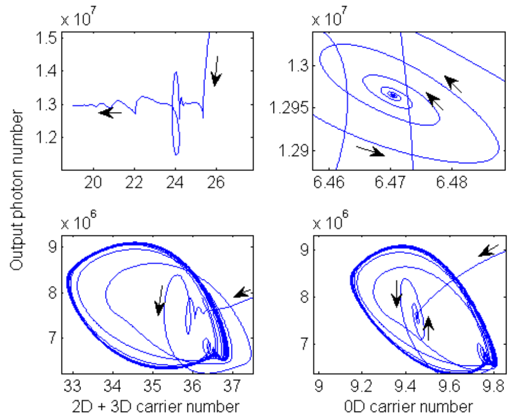

Figure 4. Phase-space portrait for 1 mm-long laser (top part) and 700 microns-long device (bottom part).

For completeness, in Figure 6 the deviation between the output photon number obtained for two very close initial conditions is compared in time-domain for the two external cavity configurations analyzed, i.e., 500 ps (top part) and 625 ps round-trip time (bottom part).

For what concerns the short external cavity (500 ps, top figure), we got a very regular profile, which gives no suggestion of tendency to exponential divergence of adjacent trajectories. On the other hand, when the longer external cavity is used, we got two remarks: first, we see no regular profile on the deviation between close trajectories (look at the inset, on the bottom part); second, to emphasize the result of Figure 5, at about instant t = 9.5 ns adjacent trajectories experience strong deviation.

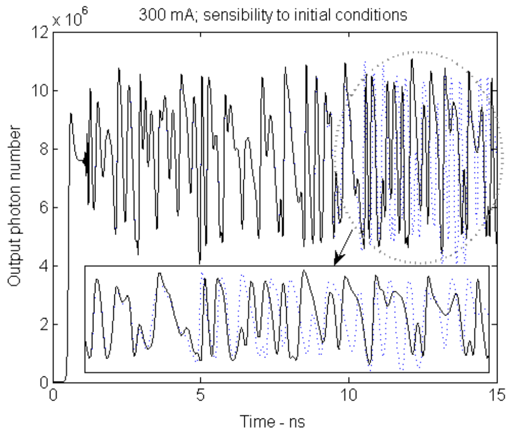

Figure 5. Time-domain laser response after 300 mA switch-on driving, for two slightly different initial conditions.

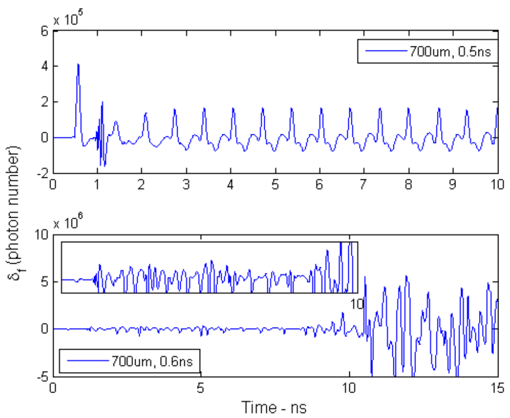

Figure 6. Plot of the separation between adjacent trajectories of the output photon number, obtained for 500 ps (top) and 625 ps (bottom) external cavity round-trip time. Inset: detailed view of the first 10 ns.

5 Conclusions

Sensitivity of quantum dot lasers to optical feedback has been investigated in this work on the basis of a Lang-Kobayashi-like description of the optical feedback effect and a multi-population numerical model for quantum dot devices.

The adopted approach was based on the calculation of the Lyapunov exponents of the laser dynamical system, and the analysis confirmed the dependence of the laser behaviour at different cavity length, as reported in (Carroll, 2005), according to which longer devices are more stable.

In addition, simulation results indicate that longer external cavity round-trip time delays lead to more pronounced sensitivity to initial conditions in quantum dot laser devices, pointing to chaotic-like behaviour.

Knowledgments

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