Rotation-Invariant Image Description from Independent Component Analysis for Classification Purposes

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Abstract: Independent component analysis (ICA) is a recent technique used in signal processing for feature description in classification systems, as well as in signal separation, with applications ranging from computer vision to economics. In this paper we propose a preprocessing step in order to make ICA algorithm efficient for rotation invariant feature description of images. Tests were carried out on five datasets and the extracted descriptors were used as inputs to the k-nearest neighbor (k-NN) classifier. Results showed an increasing trend on the recognition rate, which approached 100%. Additionally, when low-resolution images acquired from an industrial time-of-flight sensor are used, the recognition rate increased up to 93.33%.

1 INTRODUCTION

The human ability to recognize objects regardless of eventual rotation, translation or scalling transformation is one of the most basic and important features for human-environment interaction (Cichy, 2013). This recognition ability also provides human beings with the unique ability of sensing and actuating in a wide range of situations. In addition, it enables object labeling wherever it is located and whatever it is oriented on a scene.

In computer vision applications, a fundamental issue is to recognize objects regardless of viewpoint transformations. Particularly, in industrial applications such as object counting and selection in conveyor belts, pattern recognition is worldwide used. In these applications, object recognition implies label assignment according to its feature description. By object description it is meant one with as few data as possible, thus allowing for fast and, eventually, all-embedded implementations.

Classical methods for 2-D object recognition include B- Spline moment method (Huang and Cohen, 1996), moment methods (Hu, 1962), (Zhao and Chen, 1997), (Mukundan, 2001), Fourier and Wavelet transform methods based on object contour (Oirrak et al., 2002), (Khalil and Bayoumi, 2002) (Huang et al., 2005).

In the last decade, ICA has been claimed to offer powerful feature description from a reduced set of descriptors. Essentially, it is a blind source separation technique, which estimates components that are as independent as possible (Hyvärinen et al., 2001). Pioneering this field was the work on the separation of two physiological signals (Jutten and Herault, 1991), and it has been established as an interesting tool for research. In fact, significant advances have been achieved in terms of efficiency of algorithms and range of applications where ICA can be used, as well. Therefore, interest concerning this technique has increased in electrical power field (Lima et al., 2012), computer vision (Pan et al., 2013), face recognition (Sanchetta et al., 2013), neuroimaging (Khorshidi et al. 2014) (Tong et al., 2013), neurocomputing (Park et al. 2014) (Rojas et al. 2013), biomedical signal processing (Sindhumol et al. 2013), computational statistics (Chattopadhyay et al., 2013), economic modeling (Lin and Chiu, 2013), chemistry (Masoum et al. 2013), etc.

ICA solves the problem of suitably representing multivariate data by linearly decomposing a random vector $\boldsymbol{x}$, into components, $\boldsymbol{s}$, that are statistically independent, according to Eq. (1) below. Main goal is to estimate the independent components (ICs), or the mixing matrix $\boldsymbol{A}$ only from the observed data $\boldsymbol{x}$.

$$\boldsymbol{x} = A\ \boldsymbol{s} \tag{1}$$

In order to make ICA estimation possible, the ICs must be non-gaussian; this non-gaussianity assumption in ICA mixture modeling is probably the main reason of the conducted researches on the field (Hyvärinen and Oja, 2000). Another known restriction in ICA is that it is not rotation invariant; this means that rotation of the observers affects the estimation of the mixing matrix and the ICs, as well. Therefore, the ability of representing rotating objects would be compromised, in principle.

In the literature, works discussing the issued of rotation invariance generally refer to (Huang et al., 2005) and (Ali et al., 2006), whose methods also consider the translation and scaling transformations. The former introduces a new scheme for affine invariant description and affine motion estimation contour-based depiction extracted by ICA. The latter is an invariant description method based on a normalized affine-distorted and noise-corrupted object boundary.

As an alternative to face the second restriction, in this paper we propose to use ordering preprocessing step as a way to make ICA robust to rotation transformation of the observers, so that getting rotation-invariant image descriptors. This ordering step is accomplished by making the input vector to undergo a nonlinear transformation, here expressed in terms of a matrix $\boldsymbol{\lambda}$.

Then, to evaluate the proposal, we performed a simple k-Nearest Neighbor classifier (k-NN) on various image database to show that more efficient image descriptors can be obtained if this preprocessing takes place.

This paper is organized as follows: Section 2 briefly describes fundamentals of ICA and the proposed preprocessing step for ICA-based rotation invariant image feature extraction. In section 3 the datasets are described and discussed along with the experimental results of a classification system. Finally, in section 4 conclusions are drawn.

2 BASICS OF ICA

ICA is a mathematical technique that reveals hidden factors that underlie a set of random variables, which are assumed non-gaussian and mutually statistically independent. It is also described as a statistical signal processing technique whose goal is to linearly decompose a random vector into components that are not only uncorrelated, but also as independent as possible (Fan et al., 2002). Thus, ICA can be considered as a generalization of the principal component analysis (PCA). PCA generates a representation of data inputs based on uncorrelated variables, whereas ICA provides a representation based on statistically independent variables (Déniz et al., 2003).

The basic definition of ICA is given in the following. Given a set of observations of random variables $x_1(t), x_2(t). ...x_n(t),$ where t is the time or sample index, assume that they are generated as a linear mixture of independent components $s_1(t), s_2(t). ...s_n(t)$ (Huang et al., 2005):

$$x = A\,(s_1(t), s_2(t), \ldots, s_n(t))^T = As \qquad (2)$$

where $\boldsymbol{A}$ is an unknown mixture matrix, $\boldsymbol{A} \in R^{n \times n}$ (Huang et al., 2005). The ICA model, Eq. (1), describes how the observed data are generated by a process of mixing the independent components $\boldsymbol{s}$. ICs are latent variables, what means that they cannot be directly observed. Thus, the classic ICA problem consists in estimating $\boldsymbol{A}$ and $\boldsymbol{s}$, when only $\boldsymbol{x}$ is observed, provided that the observers, collecting the mixtures and representing the rows of $\boldsymbol{A}$, be independent, so that $\boldsymbol{A}$ is invertible (Bizon et al., 2013) (Huang et al., 2005).

After estimating the matrix $\boldsymbol{A}$ properly, the problem stated by Eq. (1) can be rewritten as:

$$s = A^{-1} x = Wx \,, \qquad (3)$$

in such a way that a linear combination $s = Wx$ is the optimal estimation of the independent source signals $\boldsymbol{s}$ (Bizon et al., 2013).

Under the assumption of the statistical independence of the components, and that they are characterized by a non-gaussian distribution, the basic ICA problem stated in Eqs. (1) and (2) can be solved by maximizing the statistical independence of the estimates $\boldsymbol{s}$ (Bizon et al., 2013).

On the process of finding the matrix $\boldsymbol{W}$, some useful preprocessing techniques are used in order to facilitate the calculation (Fan et al., 2002). There are two quite standard preprocessing steps in ICA. The first one moves the data center to the origin by subtracting the data mean as follows

$$\tilde{x} = x - E\{x\} \qquad (4)$$

The second step consists in whitening data, i.e., by applying a data transform and providing uncorrelated components of unit variance,

$$z = V\tilde{x} \,, \qquad (5)$$

where $\boldsymbol{V}$ is the whitening matrix and $\boldsymbol{z}$ is the whitened data.

ICA applications on pattern recognition of rotated images require as training step the random variables to be the training images. Letting $\boldsymbol{x_i}$ to be a vectorized image, we can construct a training image set $\boldsymbol{x_1}, \boldsymbol{x_2},...,\boldsymbol{x_n}$, with n random variables which are assumed to be the linear combination of m unknown independent components $\boldsymbol{s}$, denoted by $\boldsymbol{s_1}, \boldsymbol{s_2},...,\boldsymbol{s_m}$ converted into vectors and denoted as $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2},...,\boldsymbol{x_n})^T$ and $\boldsymbol{s} = (\boldsymbol{s_1}, \boldsymbol{s_2},...,\boldsymbol{s_n})^T$. From this relationship, each image $\boldsymbol{x_i}$ is represented as a linear combination of $\boldsymbol{s_1}, \boldsymbol{s_2},...,\boldsymbol{s_m}$ with weighting coefficients $a_{i1}, a_{i2},...,a_{im}$, related to the matrix $\boldsymbol{A}$. When ICA is applied to extract image features, the columns of $\boldsymbol{A_{train}}$ are features, and the coefficients $\boldsymbol{s}$ signal the presence and the amplitude of the i-th feature in the observed data $\boldsymbol{x_{train}}$(Fan et al., 2002). Futhermore, the mixing matrix $\boldsymbol{A_{train}}$ can be considered as features of all training images (Yuen and Lai, 2002). Accordingly, $\boldsymbol{x_{test}}$ must be multiplied by the vector $\boldsymbol{s}$ for the characteristics $\boldsymbol{A_{test}}$ as:

$$A_{test} = x_{test}\, s^{-1} \tag{6}$$

Finally, this matrix contains the main feature vectors of the image under test, which is the input to the classifier, as Figure 1 illustrates.

There are several algorithms that perform ICA and they are named FastICA (Hyvärinen et al., 2001), Jade (Cardoso, 1989), ProDenICA (Hastie and Tibshirani, 2003), orInfomax (Bell and Sejnowski, 1995), KernelICA (Bach and Jordan, 2002). Here, we perform ICA by applying FastICA because it is simple and allows program code modification and maintenance.

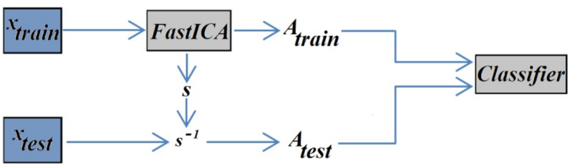

Figure 1: Steps of the classification process.

2.2 Proposed Technique

The proposed technique consists in arranging the vectorized images such that pixel intensities are ordered (this does not modify the intensity distribution and Probability Density Function of image pixels under study). As our results reveal, it improves the ICA estimation, thus providing better representation of images that have undergone rotations.

The ordering procedure is accomplished by multiplying input vector $\boldsymbol{x}$ by a matrix, hereafter referred as $\boldsymbol{\lambda}$, which is unique for each sample image and is responsible for ordering the vector. Combining this procedure with Eq. (1), it can be written as

$$\lambda x = \lambda A s = x_{order} = B\, s_{order} \tag{7}$$

Eq. (6) shows that the ICA model for the ordered input vector remains valid.

The matrix here proposed is not a permutation matrix used in basic linear algebra to permute rows or columns of a matrix, but it is actually $n \times n$ matrix able to reorder the elements of n-size vector, and assumes the following form:

$$\lambda = \begin{pmatrix} \cdots & 0 & r_{1k} & \cdots \\ \cdots & 0 & r_{2k} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & 0 & r_{nk} & \cdots \end{pmatrix}_{nxn} \tag{8}$$

There is only one non-zero column, whose elements can be obtained from the following pseudo-code:

```
x = INPUT;//waits for the input image vector
maxValue=MAX(x);//finds the maximum of input vector, x
maxIndex=FIND(maxValue,x);// returns index
λ = ZEROS(SIZE(x),SIZE(x));//initialization as zero-matrix
count = 1;
maxCounter = SIZE(x);
REPEAT
        minValue=MIN(x);//finds the minimum of input vector, x
        minIndex=FIND(minValue,minValue);//returns index
        λ(count,maxIndex) = minValue/maxValue;//fills matrix
        CLEAR x(minIndex);//eliminates minIndex-th element

UNTIL count = maxCounter
```

As results will reveal, the adoption of the ordering preprocessing leads to improvement in classification accuracy, which will be associated later to the non-gaussianity of the data in the ICA model.

2.3 Classification

Classification is the final stage of any image processing system where each unknown pattern is assigned to a category. The degree of difficulty in a classification problem depends on the variability of feature values that characterize objects belonging to a same category with regard to differences between feature values of objects belonging to different categories (Mercimek et al., 2005). In this paper, we use the k-Nearest Neighbor classifier (k-NN) for supervised pattern recognition, a classical technique proposed by (Cover, 1968) as a reference method to evaluate the performance of more sophisticated techniques (Coomans and Massart, 1981).

Our main purpose is to investigate the effect of the ordering preprocessing on the ICA estimation. Thus, the comparison among several classifiers is out of the scope of this paper.

3 DATASETS

The performance evaluation of the ordering procedure of a ICA-based classification system has been done in a very straightforward manner. It simply compares the classifier accuracy obtained when the input vectors are ordered (to some extent) to the case when they are not. This is done for different image sets, described in the following.

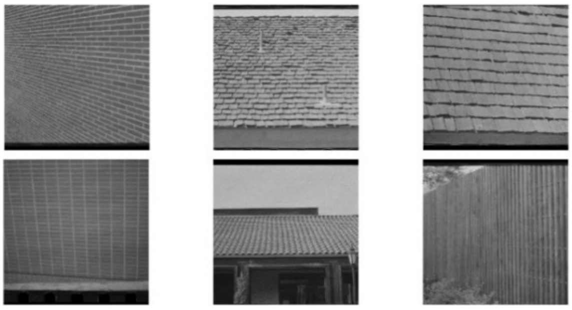

Figure 2: Examples of 1024 x 1024 images from Dataset A.

3.1 Datasets A and B (Small Database)

The first dataset used in this experiment includes 12 images of 1024 × 1024 pixels and 7 images of 512 × 512 pixels, acquired from the database of the Ming Hsieh Department of Electrical Engineering of the University of Southern California.

Each image was rotated 1° step from 0° to 360°, thus forming 361 samples for every image. Those corresponding to 0° were used for training and the others used for testing. Figure 2 shows some image samples of dataset A.

3.2 Dataset C (Low-resolution Images)

In order to evaluate the proposed method and extend conclusions for industrial-like applications, thus broadening the range of interested readers, we performed tests on an additional dataset. In this experiment, tests were carried out on images extracted from the 3D sensor effector pmd E3D200, from ifm electronic ®, which is a low-resolution time-of-flight 50 × 64 pixels sensor.

Another purpose of this experiment is to prospect real-time implementation of all-industrial image classification systems using ICA-based description. Dataset C contains pictures of three small packages, just different in size, which were acquired after randomly rotating the packages on a conveyor belt. This was done in a bad illuminated scenario, as it can be seen in the poor quality of images in Figure 3 below.

It is worth emphasizing that this experiment was performed on three image classes. The number of prototypes per class is 6, each one referred to a side of every box, in such a way that the database available for training contains 18 images.

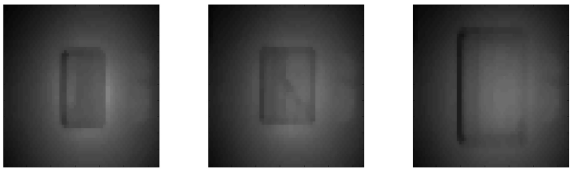

Figure 3: 50 x 64 pixels pictures of three packages with dimensions 15×10.5×7.2 cm, 15×14×6 cm and 21.5×16.2×9.6 cm, respectively.

3.3 Dataset D (Large-Size Database)

To further evaluate the performance of the proposed method for large datasets, another experiment was necessary, this time having 77 images. To create this database, other 58 textures images acquired from the database of the Ming Hsieh Department of Electrical Engineering of the University of Southern California were resized and added to datasets A and B.

Each image was rotated with 5° step, from 0° to 360°, thus forming 73 samples for every image. Again, those corresponding to 0° have been used for training, and the others, for testing.

3.4 Dataset E (Brodatz Database)

Finally, we considered using a texture database having very different background intensities. The Brodatz album available in (Safia, 2013) has 112 texture images, which have been resized from 640 x 640 to 128 x 128 pixels.

Here again we rotated images with 5° step, from 0° to 360°, thus forming 73 samples for every image. Once more, those corresponding to 0° have been used for training, and the others, for testing.

Table 1 exhibits, for each dataset, information about the experiments. The classifier has been trained and tested 50 times for each dataset.

Table 1: Parameters for every experiment done.

Image set (pixels)	Number of Coefficients	Samples of Training	Samples of Testing
Set A (1024×1024)	12	12	4320
Set B (512×512)	7	7	2520
Set C (50×64)	18	18	150
Set D (128×128)	77	77	5544
Set E (128×128)	112	112	8064

4 RESULTS

Figure 4 displays the mean recognition rates for the experiments using the proposed ordering method. Limits of x-axis indicate ranging from non-ordering (hence, traditional ICA approach) to full-ordering. Ordering rate appearing in x-axis indicates the amount of elements of a given input vector undergoing the ordering transformation **λ**.

This result is impressive because it shows that our approach shifts the performance of the ICA-based classification system from as low as 5% to near 100% after the full-ordering of the input vectors.

A less remarkable but not a negligible result has been obtained with the low-resolution dataset C, which showed an increasing performance on the recognition rates from 70.00% to about 93.33% after the ordering transformation.

Overall, clearly an ascendant trend comes out from this analysis, i.e., ordering images has the positive effect of making the classification accuracy higher.

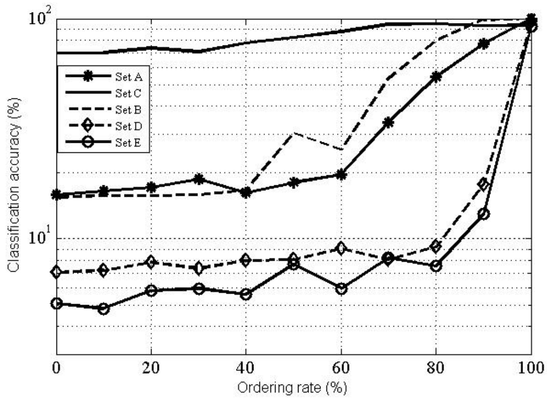

Figure 4: Mean recognition rates obtained for classification experiments with the various datasets.

We associate the above effect on the results with the increase of non-gaussianity in the data, leading to better ICA representation. We explain that on the basis of the improved non-gaussianity achieved on the independent components when ordering transformation is applied and when it is omitted.

Indeed, as explained in section 4.2 of (Hyvärinen et al., 2001), the estimation of the independent components of the ICA model relies on the maximization of non-gaussianity of a linear transformation of the observed data, $\boldsymbol{x}$. If the data is presented in such a way to increase non-gaussianity *a priori*, the esimation of the ICA model is favoured. That is our claiming.

In order to provide support for this claiming, we proceeded to measure the non-gaussianity for the datasets D and E, only. Since the transformation $\boldsymbol{\lambda}$ changes the way the input data is presented to the ICA algorithm, thus modifying the ICs, one should verify changes in the non-ordered case as compared to the full-ordering scenario.

One should also compare the independent components provided by the ICA representation in both scenarios. We emphasize that the next results will not consider partial ordering scenario, as in Figure 4, but only null or full-ordering instead.

Entropy is calculated by performing 150 rounds to extract and provide average values. This is due to the random initialization of the mixing matrix $\boldsymbol{A}$ as calculated from the FastICA algorithm.

By following this procedure, the algorithm prevents the calculated non-gaussianity to be dependent on any initial condition. Figures 5 and 6 summarize the non-gaussianity measurements for datasets D and E, respectively.

Figures 5a and 6a show that the non-gaussianity increases in the whitened data after full-ordering (to see this, note that the lower the entropy, the lower the gaussianity. The scenario of this augmented non-gaussianity also occurred in the calculated ICs. Figures 5b and 6b display this trend.

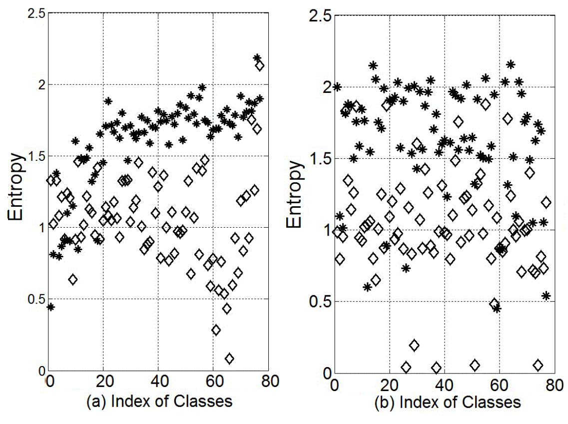

Figure 5: Entropy of the variables for set D: a) whitened $\boldsymbol{z}$ (asterisk) and $\boldsymbol{z}_{order}$ (diamond); b) independent components $\boldsymbol{s}$ (asterisk) and $\boldsymbol{s}_{order}$ (diamond).

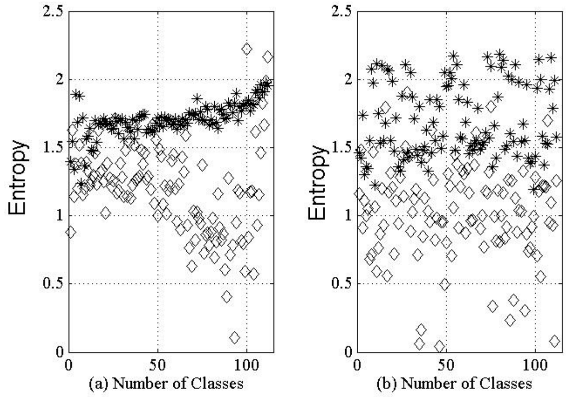

Figure 6: Entropy of the variables for set E: a) whitened $\boldsymbol{z}$ (asterisk) and $\boldsymbol{z}_{order}$ (diamond); b) independent components $\boldsymbol{s}$ (asterisk) and $\boldsymbol{s}_{order}$ (diamond).

Altogether, Figures 5 and 6 reveal a separation between diamonds (ordered case) and asterisks (non-ordered case), pretty like a “frontier”, which is more evident for the datasets A and B (not shown here for brevity).

4 CONCLUSIONS

In this paper, a preprocess for extracting rotation-invariant features using independent component analysis is proposed. Although ICA can be directly used for feature extraction, it often requires data preprocessing. Thus, our approach may be thought as a preprocessing, in which the input data undergo full or partial ordering. Experiments performed on four different image datasets showed how the ordering transformation improved the representation of feature vectors, which are inputs to the classifier, i.e., the mixing matrix of the ICA model.

Tests were carried out on rotated images to evaluate the efficiency of the method. The increased classification accuracy rate ranging from 5% to near 100% (in high-resolution images) and from 70.00% to 93.33% (in low-resolution images) suggests the use of the proposed technique as a useful input data preprocessing. The entropy and kurtosis measures confirmed that the increased non-gaussianity of the estimated independent components improved the representation of the feature vectors provided by the ICA model.

Summing up, although the ordering of pixels of an image does not affect its histogram, we showed that it is helpful in making the feature extraction from ICA a good alternative for rotation-invariant image recognition. As a future work, other approaches for rotation-invariant feature descriptors will be studied and compared to the alternative here discussed.

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