Block diagonalization with subspace based external interference mitigation

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***Abstract*—In this work we propose a variation to the Block Diagonalization (BD) algorithm described that mitigates the interference from an external interference source which does not participate in the joint transmission inherent to the BD algorithm. The idea consists of precoding the signal at the transmission side so that the received signal at each receiver lies in a subspace orthogonal to the subspace containing the external interference. It is similar to how interference alignment algorithms work, but here instead of limiting the caused interference at the unintended receivers to a specific subspace, we are limiting the desired signal at the intended receivers. Since this "limiting" aspect reduces the dimensions available for the useful signal, a trade-off is involved between how many useful streams can be transmitted and how much of the external interference energy can be canceled. The proposed algorithm is analyzed and we show different metrics that can be used to determine the "good point" in this trade-off.**

***Index Terms*—Block Diagonalization, Other Cell Interference, Subspace.**

I. Introduction

Due to the nature of wireless communication systems, interference has always been an important concern to some extent. There are many different conventional interference management approaches to deal with it, such as [1]:

- Treating the interference as noise;
- Decoding the interference;
- Orthogonalizing the desired signal and the interference.

If the interference signal is weak, then introducing structure does not help [2], and simply treating the interference as noise might be enough. If the interference is strong, it could then be decoded and subtracted from the received signal thus allowing the receiver to decode the desired information. However, this method is less common in practice due to complexity [2] as well as being harder to generalize for more than two users. In practice, the most common way to handle interference is to simply avoid it by orthogonalizing the desired signal and the interference signal in some dimension (time, frequency, code, space, etc). This method is simple and practical, but this division of resources does not scale well with the number of users, since the more users the lower the efficiency.

Notice, however, that in the end the system capacity is still limited by interference when using these methods, regardless of the number of users. On the other hand, there are more interesting alternatives that can be employed. If the transmitters can be coordinated to perform a joint transmission, then the system model becomes similar to a single user Multiple Input Multiple Output (MIMO) transmission, which may then scale linearly with the number of users. A very well known algorithm for such case is the BD algorithm described in [3]. The coordination of the transmitters allows to extract the full multiplexing gain. Even though there is orthogonalization in the space domain, the joint transmission nature of the BD algorithm increases the total number of Degrees of Freedom (DoF) in the network. The system model for BD is described in Section II-A.

In this work we extend the original BD algorithm proposed in [3] to account for an external interference source. We address this modification as *enhanced BD* and the idea is to separate the receiving subspace into a *desired subspace* and an *interference subspace*, while limiting the transmit signal to span the desired subspace at each receiver. This is similar to the idea behind subspace based Interference Alignment (IA) algorithms [4, 5], but while in IA the transmitters seek to limit their signals to the *interference subspace* at the *unintended* receivers, here we limit the transmit signals to a *desired subspace* at the *intended* receivers. This desired subspace is designed to be orthogonal to the interference subspace that contains the external interference energy, thus allowing the receiver to mitigate the external interference by eliminating anything outside the desired subspace.

The system model is described in Section II while our proposed method is described in Section III. Section IV provides some simulation results and describes the simulated scenario, while Section V presents some conclusions.

II. System Model

Figure 1 depicts a system where there are K transmitters and K receivers, where each transmitter only wants to send data to its own receiver. This scenario is known as the K-user $n_{T_i} \times n_{R_i}$ model, where n_{T_i} is the number of antennas at the i-th transmitter and n_{R_i} is the number of antennas at i-th receiver. Figure 1 also shows an external interference source, which will be addressed later.

Even when we do not consider the external interference source, in the general case each receiver cannot decode the

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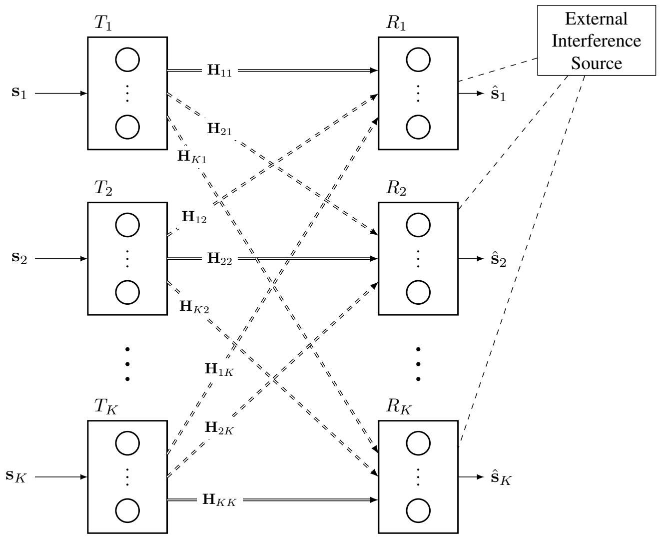

Fig. 1. MIMO Interference Channel with K users.

information of its own transmitter due to the interference from the other transmitters. There are different approaches in the literature to solve this problem. One example is the BD algorithm, described in Section II-A.

The rest of this section is divided into two parts, where II-A describes the system model for the original BD, and II-B extends the model to introduce the "external interference source" shown in Figure 1.

As a common notation for the multiple approaches, $\mathbf{d}_i$ is an M_i-dimensional vector with the information intended for user i, $\mathbf{n}_i$ is the (additive white Gaussian) noise vector, n_{T_i} is the number of antennas at transmitter i and n_{R_i} is the number of antennas at receiver i.

A. Block Diagonalization

The BD algorithm in a coordinated scenario corresponds to a joint processing where the multiple transmitters act as a single transmitter that sends information to multiple receivers. It requires a communication link between the transmitters to allow the joint transmission and can be thought of as a generalized version of channel inversion for situations with multiple antennas at each receiver [3]. Since there is no cooperation or any kind of communication between the receivers, the receivers are only capable of decoding their information because of the special structure created by the BD algorithm. This structures ensures that each receiver does not perceive the signal intended for the other receivers by canceling this interference at the transmitter side.

The received signal at the i-th receiver can be written as [3]

$$\begin{aligned}\mathbf{y}_i &= \sum_{j=1}^{K} \mathbf{H}_i \mathbf{M}_j \mathbf{d}_j + \mathbf{n}_i \\ &= \mathbf{H}_i \mathbf{M}_i \mathbf{d}_i + \mathbf{H}_i \tilde{\mathbf{M}}_i \tilde{\mathbf{d}}_i + \mathbf{n}_i\end{aligned} \quad (1)$$

where $\mathbf{H}_i$ is the ($n_{R_i} \times n_T$ dimensional) channel between all transmitters and receiver i (with n_T being the total number of transmit antennas), $\mathbf{M}_i$ is the precoder matrix for the i-th user, $\tilde{\mathbf{M}}_i = [\mathbf{M}_1, \ldots, \mathbf{M}_{i-1}, \mathbf{M}_{i+1}, \ldots, \mathbf{M}_K]$ and $\tilde{\mathbf{d}}_i = [\mathbf{d}_1^T, \ldots, \mathbf{d}_{i-1}^T, \mathbf{d}_{i+1}^T, ; \ldots, \mathbf{d}_K^T]^T$.

The precoder matrix $\mathbf{M}_i$ has dimension equal to $n_T \times M_i$ (M_i being the dimension of the data vector $\mathbf{d}_i$) and maps $\mathbf{d}_i$ to all the transmit antennas. To eliminate the interference from the unintended transmitters, the BD algorithm forces that $\mathbf{H}_i \mathbf{M}_j = 0$ for $i \neq j$. That is, if we define $\tilde{\mathbf{H}}_j$ as

$$\tilde{\mathbf{H}}_j = [\mathbf{H}_1^T, \ldots, \mathbf{H}_{j-1}^T, \mathbf{H}_{j+1}^T, \ldots, \mathbf{H}_K^T]^T, \quad (2)$$

then the zero-interference constraint sought by the BD algorithm forces $\mathbf{M}_j$ to be in the null space of $\tilde{\mathbf{H}}_j$ [3]. Thus, one possible value for $\mathbf{M}_j$ is given by the M_i least significant right singular vectors of $\tilde{\mathbf{H}}_j$ [3] (which is valid as long as $n_T - \mathrm{rank}(\tilde{\mathbf{H}}_j) \geq M_i$). Let us address this value as $\mathbf{M}_j^{\mathrm{init}}$, since we can do better than this.

It is interesting to note that $\mathbf{M}_j^{\mathrm{init}}$ can still be improved in a number of ways. More specifically, if we right multiply this $\mathbf{M}_j^{\mathrm{init}}$ matrix by another matrix, then the condition $\mathbf{H}_i \mathbf{M}_j^{\mathrm{init}} = 0$ for $i \neq j$ is still valid. In fact, in [3] the matrix $\mathbf{H}_j \mathbf{M}_j^{\mathrm{init}}$ is decomposed through a Singular Value Decomposition (SVD) and the value of the modulation matrix $\mathbf{M}_j$ is equal to $\mathbf{M}_j^{\mathrm{init}} \mathbf{V}_j^{(1)} \Lambda_j^{1/2}$, where $\mathbf{V}_j^{(1)}$ corresponds to the M_i most significant right singular vectors of $\mathbf{H}_j \mathbf{M}_j^{\mathrm{init}}$ and $\Lambda_j^{1/2}$ is a diagonal matrix accounting for the different power loading strategies. The power loading strategies will not be considered here and thus we use $\mathbf{M}_j = \mathbf{M}_j^{\mathrm{init}} \mathbf{V}_j^{(1)}$.

B. Dealing with Interference

Here we are considering that the interference is not negligible to be considered as noise, but decoding the interference in order to cancel it is also not an option. That leaves us with the option of orthogonalizing the desired signal and the interference. This is precisely what the BD algorithm does in order to cancel the *internal interference*. Also, because of the joint transmission, it does not have the drawback of not scaling with the number of users.

However, the interference canceled with the BD algorithm is only the *internal interference*. That is, interference is caused by transmitters that *participate in the joint transmission* thus not including the "external interference source" shown in Figure 1, and which is approached for example in [6].

When we consider an external interference, the received signal in (1) is now summed with an $\mathbf{e}_i$ term corresponding to the external interference perceived by receiver i. Section III describes the model including this external interference term, as well as the proposed method to handle it.

III. Method Description

The method consists of projecting the desired signal space onto a subspace orthogonal to the subspace where the external interference lies. That is, since we do not control the external interference source, we limit the transmit signal to be as orthogonal to the external interference energy as possible by multiplying the equivalent channel $\mathbf{H}_j \mathbf{M}_j$ by a $\mathbf{P}_j$ matrix, where the number of columns of $\mathbf{P}_j$ will determine the dimension of the useful subspace. We address the dimensions *not contained* in the useful subspace as "sacrificed streams" to emphasize the fact that we are giving up sending useful information through these dimensions as the original BD algorithm would. Equation (1) then becomes

$$\begin{aligned} \mathbf{y}_i &= \sum_{j=1}^{K} \mathbf{H}_i \mathbf{M}_j \mathbf{P}_j \mathbf{d}_j + \mathbf{n}_i + \mathbf{e}_i \\ &= \underbrace{\mathbf{H}_i \mathbf{M}_i \mathbf{P}_i}_{\mathbf{H}_{i_{eq}}} \mathbf{d}_i + \sum_{\substack{j=1 \\ j \neq i}}^{K} \mathbf{H}_i \mathbf{M}_j \mathbf{P}_j \mathbf{d}_j + \mathbf{n}_i + \mathbf{e}_i, \end{aligned} \tag{3}$$

where $\mathbf{P}_i$ is an orthonormal matrix corresponding to a linear transformation from the original space to a lower dimensional subspace of dimension m_i (number of streams that user i will effectively transmit with the reduction, with $m_i < M_i$ and $\mathbf{d}_i$ now also having dimension m_i). This lower dimensional subspace is orthogonal to the interference subspace, reserved to contain only the external interference.[1] This linear transformation is addressed here as the "projection step" and, since it reduces the dimension of the equivalent channel the receiver will see, the method also involves determining if this dimension reduction is worth. The proposed method can be summarized into the following steps:

1) Estimate the External Interference → Each receiver estimates the covariance matrix $\mathbf{R}_{e_i}$ of the external interference (plus noise) it perceives. In this work we will assume full knowledge of $\mathbf{R}_{e_i}$;
2) Perform the BD algorithm as usual to determine $\mathbf{M}_j$;
3) Perform the projection step → For each user, the matrix $\mathbf{P}_i$ is given by the m_i least significant right singular vectors of the external interference covariance matrix $\mathbf{R}_{e_i}$ (note that $\mathbf{P}_i$ will have orthonormal columns);
4) Cancel the external interference → At the receiver, the signal $\mathbf{y}_i$ in (3) is projected onto the subspace spanned by the columns of $\mathbf{P}_i$. This projection is performed by (left) multiplying the received signal by [7]

$$\overline{\mathbf{P}}_i = \mathbf{P}_i \mathbf{P}_i^H; \tag{4}$$

5) Perform the usual BD reception → Since $\mathbf{H}_i \mathbf{M}_j = 0$ for $i \neq j$, due to the BD algorithm, then the received signal after external interference cancellation, $\overline{\mathbf{y}}_i$, is given by

$$\overline{\mathbf{y}}_i = \overline{\mathbf{P}}_i \mathbf{H}_{i_{eq}} \mathbf{d}_i + \overline{\mathbf{P}}_i \mathbf{n}_i + \overline{\mathbf{P}}_i \mathbf{e}_i, \tag{5}$$

where $\overline{\mathbf{P}}_i \mathbf{e}_i$ will be equal to zero if the reserved interference subspace completely contains the external interference. Considering a zero-forcing criterion and incorporating the external interference cancelation in the receive filter $\mathbf{W}_i$, the original symbols can be recovered from (3) by the corresponding filter $\mathbf{W}_i$ given by

$$\mathbf{W}_i = (\overline{\mathbf{P}}_i \mathbf{H}_{i_{eq}})^{\dagger} \overline{\mathbf{P}}_i, \tag{6}$$

where $(\cdot)^{\dagger}$ denotes the Moore–Penrose pseudo-inverse.

[1]Or part of its energy if the rank of the external interference is greater than the dimension of the interference subspace.

Note that each user can individually determine how many streams should be sacrificed, or even completely avoid the projection step if it is not worth it. In order to do this we need some metric to determine the best value for m_i. One suitable metric consists of choosing the value of m_i that maximizes the data rate. This metric is explained in Section III-A.

A. Decision Metric

Since we are concerned with increasing the data rates, the decision metric should be related to some sort of rate expression. When the multiple dimensions of the desired subspace are employed to transmit independent streams as in the Bell Labs Layered Space-Time (BLAST) MIMO scheme, the expected data rate of the i-th user, R_i, is given by

$$R_i = \sum_{k=1}^{m_i} \log_2 \left(1 + \mathrm{SINR}_{ik}\right), \tag{7}$$

with the Signal-to-Interference-plus-Noise Ratio (SINR) of the k-th stream of user i given by

$$\mathrm{SINR}_{ik} = \frac{1}{\left(\mathbf{W}_i \mathbf{W}_i^H N_0 + \mathbf{W}_i \mathbf{R}_{ei} \mathbf{W}_i^H\right)_{kk}}, \tag{8}$$

where $(\cdot)_{kk}$ is the k-th element in the diagonal of a square matrix. By calculating equations (7) and (8) for different values of m_i, and thus different dimensions reserved for the external interference subspace, each user can determine the best number of streams to transmit.

Employing equations (7) and (8) as the decision metric corresponds to maximizing the capacity for each user, and thus will be addressed as the "capacity metric". However, it does not consider practical aspects such as a limited number of modulation cardinality, for instance. As a different metric taking this into account, we can calculate the effective SINR of each stream as in equation (8), determine the expected Bit Error Rate (BER) for the chosen modulation from the literature [8], and then calculate the effective throughput with

$$R_i = (1 - (1 - \mathrm{BER})^{\gamma}) \cdot k, \tag{9}$$

where γ is the length of the package (in bits) and k is the number of bits in each modulated symbol. We will address this metric as the "Effective Throughput Metric". The metric in (9) assumes that any bit error will lead to a packet error, which is how we have performed our simulations. However, the main idea is to map the SINR from (8) to an effective throughput, and in case coding is used we could simply use a different equation for assuming the role of (9).

IV. Simulation Results

In order to illustrate the performance of the different metrics in our proposed enhanced BD algorithm, we have simulated a cluster of 3 cells, each cell containing a mobile user, as shown in Figure 2. Each user is positioned at a distance from the cell center equivalent to 70% of the cell radius in what we call the "Symmetric far Scenario" and each (transmit and receive) node has two antennas. The advantage of using this scenario is that the external interference is more significant than, say,

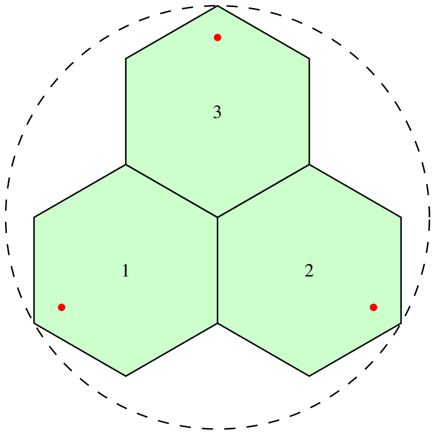

Fig. 2. Symmetric Far Scenario.

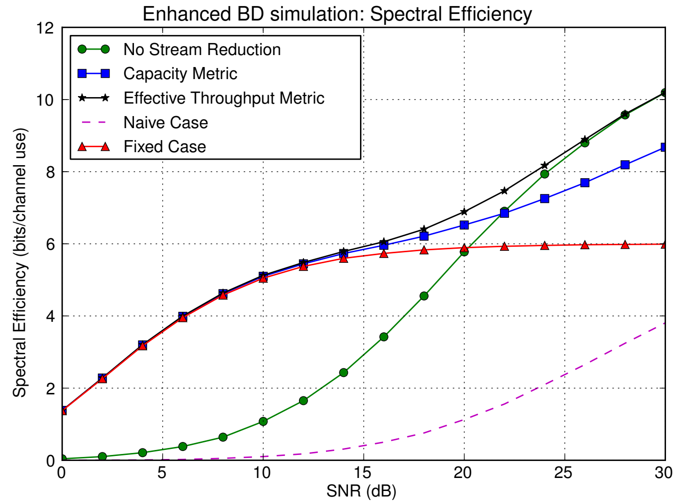

Fig. 3. Spectral Efficiency (external interference with 10 dBm Tx power).

a "random placing" scenario. This makes it clearer to see the performance differences of the several simulated cases, but the conclusions are still expected to be valid for the more general case. The Base Station (BS) transmission power is adapted to match the desired expected mean Signal-to-Noise Ratio (SNR) at the border of the cell. The external interference source has a rank equal to one, and its transmission power was set to 10 dBm at the cluster border (the dashed line). The simulation parameters are summarized in Table I.

For comparison, we have simulated 5 different cases: *no stream reduction* case, which corresponds to a regular BD algorithm; a *fixed* and *naive* cases, which always sacrifice a single stream; and the *capacity metric* and *effective throughput metric* cases which choose the number of streams according to the capacity and effective throughput metrics previously described. Since each user has two antennas, then at most one stream can be sacrificed per user, but since the external interference has a rank equal to one, this is enough to allow canceling all the interference at the receiver. Therefore, the capacity and effective throughput metric cases will choose either not to sacrifice any stream or to sacrifice a single stream, while the fixed and naive cases will always sacrifice a single stream. Nevertheless, while in the fixed case the matrix $\mathbf{P}_i$ is chosen so that the transmit signal is orthogonal to the external interference, nothing special is performed for the naive case. That means that when the receiver cancels the external interference part of the desired signal energy might be also canceled. This illustrates the need to actively limit the transmitted signal at the transmitter side to fall into a subspace (at the receiver) orthogonal to the subspace containing the external interference.

TABLE I
SIMULATION PARAMETERS.

Parameter	Value
Cell Radius	1 km
Number of antennas	2 at each node
User position	70% cell radius from center
Modulation	4-PSK
Path Loss Model (in dB with d in km)	$128.1 + 37.6 \log_{10}(d)$
Noise Power	$N_0 = -116.4$ dBm
Transmission Power	Adjusted to match SNR at the border of the cell
External Interference Power	10 dBm
Packet Length	60 bits

Figure 3 shows the spectral efficiency for each case from a simulation with the parameters described in Table I. As we can see, for low SNR values (corresponding to a lower transmission power) the external interference is a limiting factor in the achieved spectral efficiency. That is, the cases that sacrifice a stream to cancel the external interference achieve a much better throughput than the "No stream reduction" case. The only exception is the "naive" case, since even though it also sacrifices a stream, it does not seek to send its signal through a subspace orthogonal to the external interference (thus achieving the lower packet error rates pay off from the stream reduction).

As the SNR value increases the "No stream reduction" case starts to *catch up* since the (fixed) external interference becomes comparatively weaker, but more interesting is the behavior of the other cases. The fixed case achieves an upper bound of 6 (bits per channel use) when the packet error becomes negligible, since it has 3 users, each one transmitting one stream with 4-Phase-Shift Keying (PSK) modulation. The surprise is the "capacity metric" case, which at some point is surpassed by the no stream reduction case. Since one of the main points in the enhanced BD algorithm is to choose the best number of streams to transmit, it should not "lose" to the regular BD algorithm with no stream reduction. The reason for this result is that the capacity metric does not take into account the fact that the modulation is limited to 4-PSK here (or at least to a set of possible options in the general case)[2]. Therefore, it might decide to reduce the number of streams because this would achieve a higher throughput in conjunction with a slightly higher modulation order. However,

[2]The same argument may be used for other link level parameters.

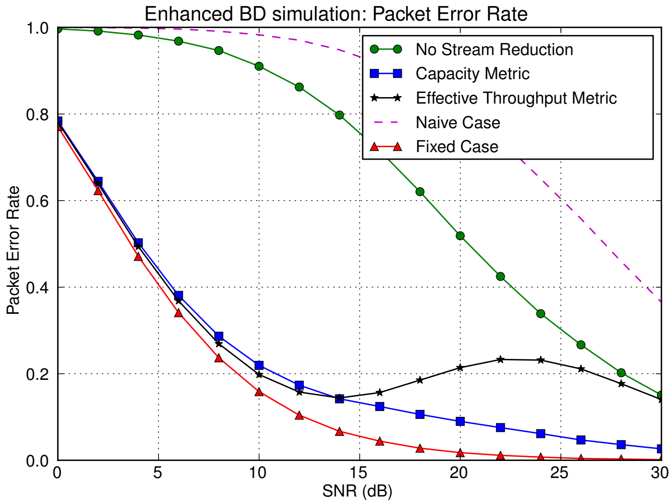

Fig. 4. Packet Error Rate (external interference with 10 dBm Tx power).

since the modulation is fixed this does not happen. On the other hand, the "Effective Throughput" metric is able to take this into account and always achieve a throughput higher than or equal to the regular BD algorithm.

Having a closer look at Figure 3 we can see that at some point between SNR values of 15 and 20 the slope of the effective throughput curve changes slightly. In order to better understand this behavior, Figure 4 shows the Packet Error Rate (PER) for all metrics. As we can see in Figure 4, between the SNR values of 15 and 20 the PER for the effective throughput case actually increases instead of decreasing. This means that around this region the enhanced BD algorithm with the effective throughput metric is starting to favor keeping both streams instead of sacrificing one of them. This results in higher PER, since the external interference is not mitigated, but the effective throughput metric is able to balance a higher PER with a better throughput as we see in Figure 3. If it were possible to change the modulation order at will for the best value (considering even non integer orders), it is expected that the capacity metric could achieve similar behavior. However, in the practical case where the possibilities are limited, this confirms that the effective throughput metric is a better choice for the proposed enhanced BD algorithm.

V. Conclusions

Using the proposed enhanced BD algorithm an external interference can be mitigated or even eliminated, but this involves reducing the number of transmit streams in order to reserve dimensions at each receiver that will only contain external interference. Since this involves a trade-off between interference reduction and number of transmit streams we have analyzed different decision metrics with the objective of achieving a higher throughput. We have shown that for this goal the effective throughput metric is able to account for practical aspects and always achieve a higher throughput than the regular BD algorithm.

On the other hand, if a different objective is desired instead of maximizing the effective obtained throughput, then a different and more appropriated metric would be required. However, the overall idea of stream reduction and projection of the received signal onto a desired subspace is still the same and, provided that this metric is appropriated for the desired objective, the algorithm should again perform better than or at least equal to the regular BD algorithm.

In this paper we have assumed perfect knowledge of the channel state information as well as the $\mathbf{R}_{e_i}$ covariance matrix. An interesting topic for future research consists of analyzing the impact of errors on the estimation of the channel state information and the $\mathbf{R}_{e_i}$ matrix, as well as how the algorithm could be modified in order to make it more robust to such estimation errors.

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References

[1] S. A. Jafar, *Interference Alignment - A New Look at Signal Dimensions in a Communication Network*, ser. Foundations and Trends in Communications and Information Theory. Now Publishers Inc, 2011, vol. 7.

[2] V. R. Cadambe and S. A. Jafar, "Interference Alignment and Degrees of Freedom of the K User Interference Channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, pp. 461–471, Feb. 2004.

[4] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the Capacity of Wireless Networks through Distributed Interference Alignment," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, Dec. 2008, pp. 1–6.

[5] S. W. Peters and R. W. Heath, "Interference Alignment via Alternating Minimization," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Apr. 2009, pp. 2445–2448.

[6] S. W. Peters and R. W. Heath, "Cooperative algorithms for MIMO interference channels," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 1, pp. 206–218, Jan. 2011.

[7] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd edition. The Johns Hopkins University Press, 1996.

[8] J. G. Proakis, *Digital Communications 4th Edition*. McGrawHill, 2000.