Analysis of Interference Alignment for Wireless Communication with External Interference

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Abstract—**In order to better cope with the increasing levels of interference in wireless cellular systems, interference mitigation techniques that consider a certain degree of coordination/cooperation among cells have been recently employed, such as Joint Processing (JP) and Interference Alignment (IA). In this paper we evaluate the performance of different IA-based algorithms in the presence of uncoordinated external interference, with JP schemes used as a benchmark. Sum rate and Bit Error Rate (BER) results are presented for different external interference scenarios and it is shown that there is trade-off between IA and JP, and that for high external interference levels IA algorithms modified to handle this external interference can achieve better BER performances.**

Index Terms—**Interference Alignment, Cooperative Precoding, External Interference.**

I. INTRODUCTION

In the context of aggressive throughput requirements of the current wireless network, Interference Alignment (IA) arose as a promising technique to mitigate the interference and to approach the capacity limits of these systems [1]. This technique consists on aligning, at each receiver, multiple interference signals in a subspace with dimension smaller than the number of interferers. IA can also be viewed as a cooperative and altruistic approach since the transmitters may neglect the performance of their own link to allow other users to perfectly cancel interference.

Several precoding design algorithms have been proposed to perform IA. A closed-form solution, such as the one for the three user case [1], cannot be found for most situations. This motivated the proposal of more flexible iterative algorithms, such as the one based on alternating minimization [2]. Another relevant algorithm is the Minimum Mean Square Error (MMSE) based on IA, which takes into account the direct gain and relaxes the perfect alignment constraint [3].

Nevertheless, the scenarios or system configurations on which the alignment is possible or feasible were not entirely clear until [4]. This work addressed the IA feasibility conditions and derived a relation between the number of users, antennas and transmitted streams necessary to accomplish the IA technique. For example, we now know that considering two available antennas per node, the technique can be applied for at most three transmitter-receiver pairs, in order to achieve the perfect alignment.

Therefore, in practical scenarios the IA technique can just be applied among few transmitter-receiver pairs, since there is a limitation on the number of antennas available, especially at the receiver side. Thus, the consideration of an uncoordinated interference burdening the communication of the cooperating group may be a reasonable assumption, which might represent a more realistic scenario. This external interference can also be mitigated, assuming that its covariance matrix is known [5]. This work therefore aims to perform a more detailed evaluation of how IA behaves in a cellular network scenario under the presence of an external interference source. To carry out this evaluation we resort to the algorithms presented in [5] and propose a simple modification on them to better mitigate the external interference.

This paper is divided as follows. Initially, the system model is presented in Section II. Then, Section III describes the employed IA algorithms. Section IV presents the simulation parameters and discusses the impact of an external interference source on the performance of the IA technique. Finally, conclusions and perspectives are presented in Section V.

II. SYSTEM MODEL

Consider a K-user MIMO Interference Channel (MIMO-IC), where transmitters and receivers are respectively equipped with N_T and N_R antennas. The mapping of S transmitted symbols into N_T antennas is performed via the precoding matrix $\mathbf{V}_k \in \mathbb{C}^{N_T \times S}$. The precoded data of transmitter j is sent to receiver k through a complex Gaussian channel $\mathbf{H}_{kj} \in \mathbb{C}^{N_R \times N_T}$ with i.i.d. entries.

Thus, the receiver signal is given by

$$\mathbf{y}_k = \underbrace{\mathbf{H}_{kk}\mathbf{V}_k\mathbf{d}_k}_{\text{desired signal}} + \sum_{j=1,j\neq k}^{K} \mathbf{H}_{kj}\mathbf{V}_j\mathbf{d}_j + \underbrace{\mathbf{e}_k}_{\text{external interference}}, \quad (1)$$

where $\mathbf{d}_k \in \mathbb{C}^{S\times 1}$ denotes the vector of transmitted symbols and $\mathbf{e}_k \in \mathbb{C}^{N_R\times 1}$ is a random vector that comprises the external interference plus the thermal noise in receiver k. In order to recover the original data streams at the receiver side, the received signal $\mathbf{y}_k \in \mathbb{C}^{N_R\times 1}$ is processed by applying a receiver filter $\mathbf{U}_k^H \in \mathbb{C}^{S\times N_R}$.

In this work, the interference is composed by just one dominant interferer that applies a precoder $\mathbf{V}_E$, whose channel to receiver k is denoted as $\mathbf{H}_{kE} \in \mathbb{C}^{N_R\times N_T}$. Hence, the

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interference plus noise covariance matrix is given by $\mathbf{R}_k = \mathbf{H}_{kE}\mathbf{V}_E\mathbf{V}_E^H\mathbf{H}_{kE}^H + \sigma_n^2\mathbf{I}$, where σ_n^2 denotes the variance of the noise. Note that the rank of the covariance matrix is related to the dimension of the precoder applied in the external interferer and although $\mathbf{H}_{kE}^H$ and $\mathbf{V}_E$ are not known, $\mathbf{R}_k$ can be estimated.

III. INTERFERENCE ALIGNMENT ALGORITHMS

This current section describes the employed IA algorithms.

A. *Interference Alignment via Alternating Minimization*

The first considered algorithm is based on alternating minimization approach [5] which can be applied in a wide range of network configurations. Naturally, this configuration must still abide to the feasibility conditions. The algorithms' idea consists on aligning the interference at each receiver by adjusting their interference subspace via alternating optimization, while seeking to minimize the interference leaked to the desired subspace. Its optimization function can be written as:

$$J_{IA} = \min_{\substack{\mathbf{V}_j^H\mathbf{V}_j=\mathbf{I},\forall j \\ \mathbf{\Phi}_k^H\mathbf{\Phi}_k=\mathbf{I},\forall k}} \sum_{k=1}^{K}\sum_{\substack{j=1 \\ j\neq k}}^{K} \mathrm{tr}\left(\mathbf{\Phi}_k^H\left(\mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{kj}^H + \mathbf{R}_k\right)\mathbf{\Phi}_k\right), \tag{2}$$

where $\mathbf{\Phi}_k$ is an orthonormal basis for the desired subspace, and $\mathrm{tr}(\mathbf{A})$ is the trace of matrix $\mathbf{A}$. This objective function can be minimized through an alternating optimization approach by choosing the desired subspace and the precoders as:

$$\mathbf{\Phi}_k^{opt} = \nu_{min}^S\left(\sum_{\substack{j=1 \\ j\neq k}}^{K}\mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{kj}^H + \mathbf{R}_k\right) \tag{3}$$

and

$$\mathbf{V}_k^{opt} = \nu_{min}^S\left(\sum_{\substack{j=1 \\ j\neq k}}^{K}\mathbf{H}_{kj}^H\mathbf{\Phi}_j\mathbf{\Phi}_j^H\mathbf{H}_{kj}\right), \tag{4}$$

where $\nu_{min}^S(\cdot)$ is a function that returns a matrix whose columns are the eigenvectors corresponding to the S smallest eigenvalues of the input matrix. After the precoding calculation the receivers can apply a Zero-Forcing (ZF) filter in order to cancel the interference.

It is important to highlight that the original form of the algorithm does not consider the external interference in the $\mathbf{R}_k$ covariance matrix [6]. Nevertheless, by taking the external interference into account we still can minimize the cost function, but the perfect alignment can not be guaranteed [5]. In this work, both implementations are considered, thus when the external interference is taken into account we name the algorithm as "enhanced alternating", otherwise we call it as "conventional alternating".

Furthermore, we propose a modification on the enhanced algorithm in order to bring it closer to the feasibility and then improve the quality of the interference alignment. In this modification, we suppress one stream for the Tx-Rx pair perceiving the strongest external interference. So, instead of all transmitters sending the maximum allowed streams for IA to be feasible, one of them will send one stream less. By using this approach the IA problem becomes easier to solve, since the internal interference may fit better onto the space generated by the external interference.

B. *IA-MMSE*

In this section, we present another approach for the IA algorithm that uses the MMSE criterion. Its solution tries to balance the goals of aligning and eliminating the interference at the receivers with the need of keeping the signal level well above the thermal noise [5]. Similarly to the IA alternating algorithm, the IA MMSE uses an alternating optimization framework to find the precoders and receiver filters. The solution of the precoders is given by:

$$\mathbf{V}_k = \left(\sum_{j=1}^{K}\mathbf{H}_{jk}^H\mathbf{U}_j\mathbf{U}_j^H\mathbf{H}_{jk} + \lambda_k\mathbf{I}\right)^{-1}\mathbf{H}_{kk}^H\mathbf{U}_k, \tag{5}$$

where λ_k is the Lagrangian multiplier and can be solved by replacing (5) in the power constraint, $\mathrm{tr}\left(\mathbf{V}_k\mathbf{V}_k^H\right) \leq 1$, which provides a monotonically decreasing function that can be solved through the bisection method [5]. On the other hand, the optimal receivers are written as:

$$\mathbf{U}_k = \left(\sum_{j=1}^{K}\mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{kj}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}_{kk}\mathbf{V}_k. \tag{6}$$

In this work, particularly for this algorithm, we consider to initialize the precoders with the closed-form solution [1], since it is well-known that this approach provides a sensible improvement in the algorithm performance. This algorithm can also take into account the external interference as well as the alternating approach. We just need to substitute the noise variance term with the interference plus noise covariance matrix of the external interference [5]. Thus, we refer to these two algorithms as "conventional" and "enhanced" IA MMSE, respectively.

Besides the IA algorithms, we resorted to a Joint Processing (JP) approach to support our analyses. So, the Block Diagonalization (BD) algorithm, proposed in [7], is adopted here as a benchmark. It is worth pointing out that BD requires the knowledge of the data to be transmitted to all users, which increases the infrastructure requirements when compared to IA. On the other hand, by having the complete network knowledge the BD is able to transmit up to N_R streams per users. Therefore, we expect that it will be an upper bound for IA regarding the achievable data rates.

IV. EXTERNAL INTERFERENCE ANALYSES

In this section, we discuss the impact of the external interference on the performance of the IA techniques. In the following, we present the simulation scenario and two sets of

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results. First, transmitters and receivers are equipped with two antennas, then this number is increased to four antennas. These two configurations allow us to model the external interference covariance matrices with a different ranks, and consequently emulate different kinds of external interference.

A. Simulation Scenario

All simulations considered a scenario with only one cluster composed of three cells with one user per cell, forming, this way, a three user interference channel. The number of antennas in each node varies from 2 to 4 depending on the set of simulations.

The transmission power of each cell is calculated in order to match a Signal to Noise Ratio (SNR) value at the border of the cell, parameter which will be varied in the analyses. The dominant interferer is located at the border of the cluster on the closest location to each receiver. Also, to assess a cases from low to high external interference power, this value is varied from 0 to 20 dBm. The simulation parameters are summarized in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Cell Radius	1 km
Cluster Radius	2 km
Antennas (each Base)	2 or 4
Antennas (each user)	2 or 4
Modulation	QPSK
Path Loss Model	$128.1 + 37.6\log_{10}(d)$ (in dB with d in km)
Noise Power	$N_0 = -116.4\,\text{dBm}$
Transmission Power	Adjust to match SNR at the border of the cell

It is assumed that the users are randomly placed inside the cells, and two cases are considered: users can be distributed over all the cluster or closer to the border, respecting a distance of 2/3 of the cluster radius. The simulation scenario is illustrated in Figure 1.

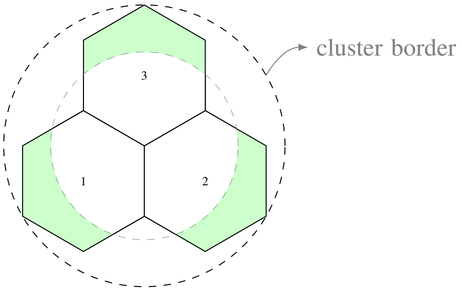


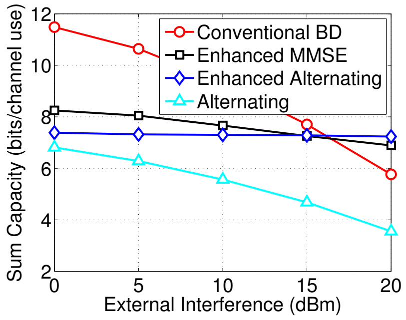
Fig. 1. Simulation scenario. The highlighted area refers to the case where the users are placed close to the cluster edge.

The analyses were mainly based on the sum of the Shannon capacity [8] achieved by each link, henceforth referred as sum capacity. However, the average Bit Error Rate (BER) achieved by the system is also analyzed. The first metric can be translated as an upper bound of the system throughput, while the BER gives us the sense of how much the algorithms are robust to errors in the transmissions. Thus, these two metrics may provide a complimentary view of the actual performance of each algorithm. Finally, the simulations were carried out through the Monte Carlo method.

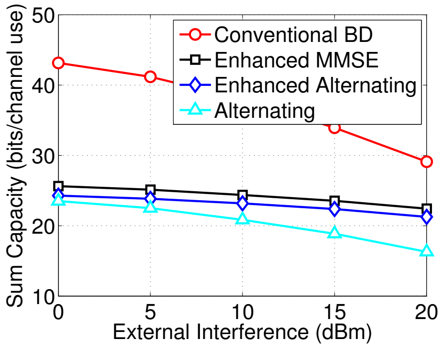
B. Nodes with Two Antennas

When assuming nodes with two antennas, the joint precoding BD algorithm is able to transmit 2 streams per user, while the IA strategies can transmit at most 1 stream per user. Figure 2 shows the average sum capacity achieved by the different algorithms as a function of the external interference power, when the users are uniformly placed over all cells for different values of SNR. Analyzing this result, it is possible to verify, especially for the low SNR case, that the presence of an uncoordinated interference causes a great impact on the algorithms that do not try to mitigate this interference, namely conventional BD and alternating IA.

On the other hand, for the IA algorithms that mitigate the external interference, Enhanced Alternating and MMSE, the impact is not too severe. These enhanced IA-based algorithms try to adapt, as much as possible, the interference subspace to the external interference direction. Nevertheless, when the external interference is strong and its directions can not be adjustable, then perfect alignment is almost surely not possible [5], resulting in performance loss. It is also interesting to note that, for low values of SNR, such as in Figure 2(a), these IA algorithms are able to outperform BD for high external interference, even though they transmit less streams.



(a) SNR of 0 dB at the border of the cell.



(b) SNR of 20 dB at the border of the cell.

Fig. 2. Sum Rate achieved by the algorithms versus External Interference level for different SNR values at the border of the cell. Nodes with two antennas.

C. Nodes with Four Antennas

We also consider nodes equipped with four antennas. In that case, we can vary the rank of the external interference covariance matrix up to four. At this configuration, the BD algorithm is able to transmit up to four streams per user while IA users are able to transmit at most two streams each. Also, to enhance the external interference effect the users were randomly distributed respecting a distance of 1.33 km (2/3 of cluster radius) of the cluster center.

For simplicity, we choose to perform just the alternating algorithm to assess the IA technique. Besides, an additional modified version of this algorithm is considered, in which the transmitter/receiver pair that perceives more interference will send one stream less. So, for this modified IA, two users will receive two data streams while the other user will receive just one stream. In order to provide a fairer benchmark, we also employed a BD algorithm that handles the external interference by whitening it [9].

Figures 3(a), 3(b), 3(c) present the average sum capacity achieved by the algorithms when an external interference with a single dominant direction burdens the communication. Figure 3(a) shows that for low values of external interference all BD algorithms achieve a higher performance than IA-based algorithms at all SNR cases. It can be also noticed a slight gain for the whitening BD algorithm when compared to the conventional BD. As the external interference level increases, in Figures 3(b) and 3(c), the whitening BD algorithm that explicitly handles the external interference outperforms the other algorithms by a larger gap, especially the conventional BD algorithm.

Regarding the IA algorithms, we remember that they transmit less streams than the BD algorithms, but IA does not require the knowledge of all network data at every transmission, which leads to a lighter backhaul requirement. This explains why IA algorithms usually obtain sum capacities lower than the BD algorithms. Performing the comparison only among IA algorithms, we can notice that the enhanced one achieves a slightly better sum capacity than the modified one, since it sends one stream more than the modified IA.

However sum capacity is only one side of the coin. Figures 3(d), 3(e) and 3(f) present the BER achieved by the network when the different algorithms are applied. The conclusions we arrived for sum capacity results are inverted now, the IA algorithms provide a much lower BER than the BD algorithms. Hence, these results can translate the trade-off of sending more streams to achieve higher sum capacities or transmitting less streams to provide a transmission more robust to errors.

Finally, we increased the rank of the external interference covariance matrix. Figure 4 illustrates the results obtained with a rank-2 external interference. Similar conclusions to the ones obtained from Figure 3 can be drawn. Increasing the dimension of the external interference moves the enhanced IA algorithm further away from alignment feasibility. This can be attenuated by the modified IA, where a stream of a single user is sacrificed, which moves it again close to alignment feasibility. In fact, the modified IA algorithm is able to achieve sum capacities very similar to the enhanced IA, but with better BERs.

V. CONCLUSIONS

This work discussed the application of the Interference Alignment technique in a cellular network under external interference. The performance of the algorithms was evaluated through the sum capacity and BER achieved by the three cell network burdened by an external interference source. The analysis showed that when the IA algorithms are modified to handle the interference they could, in certain cases, provide higher sum rates than the conventional BD. Another interesting result occurred when the modification on the enhanced alternating algorithm, which chooses to suppress one stream, was applied. Due to this new approach, the modified IA could provide similar capacities and better BER results than the enhanced IA, for different ranks of the external interference covariance matrix. This happens since the external interference can better fit onto the available interference space.

Regarding the IA and BD comparison, it could be noticed that BD-based algorithms could provide better sum capacities than IA, however IA is more reliable in relation to the BER. Also, in further networks deployments, this result can steer the choice of which technique is better to be applied. For instance, if we consider a network where the Quality of Service (QoS) is very important, then it may be better to apply an IA algorithm.

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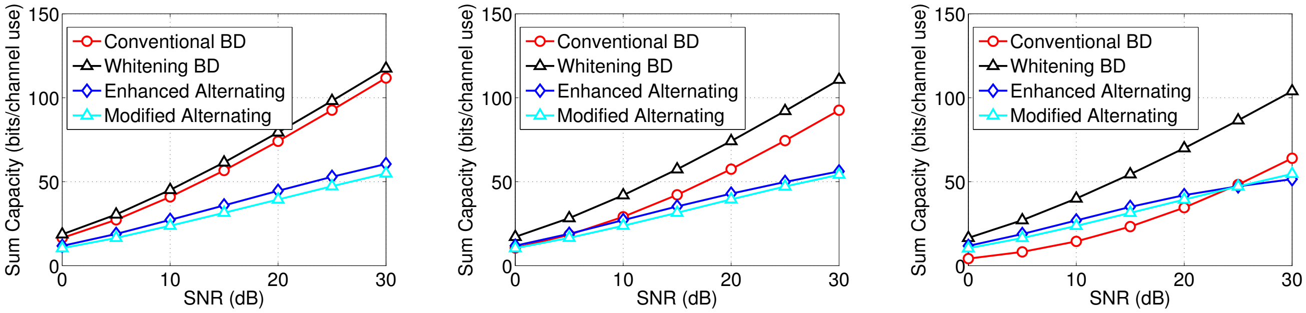
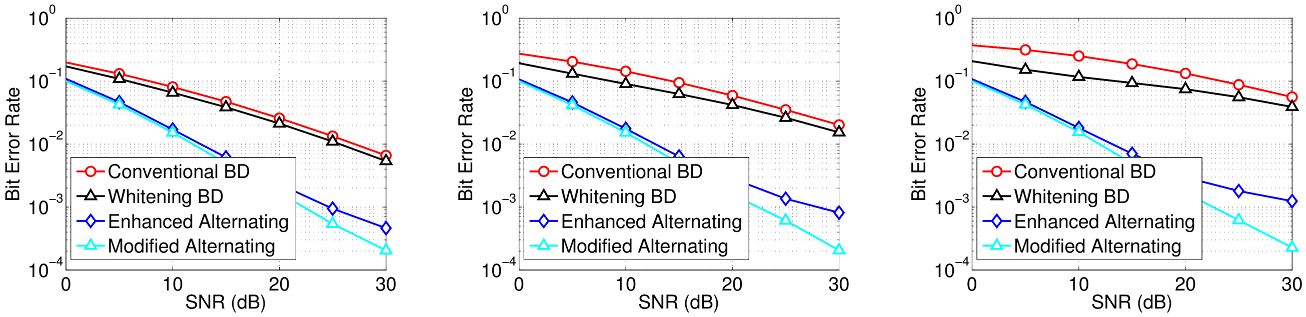
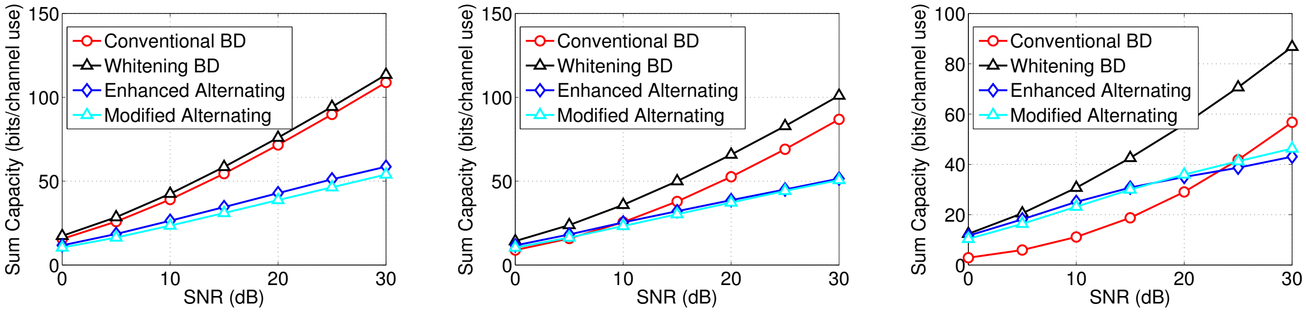


(a) External Interference of 0 dBm at the cluster edge. (b) External Interference of 10 dBm at the cluster edge. (c) External Interference of 20 dBm at the cluster edge.

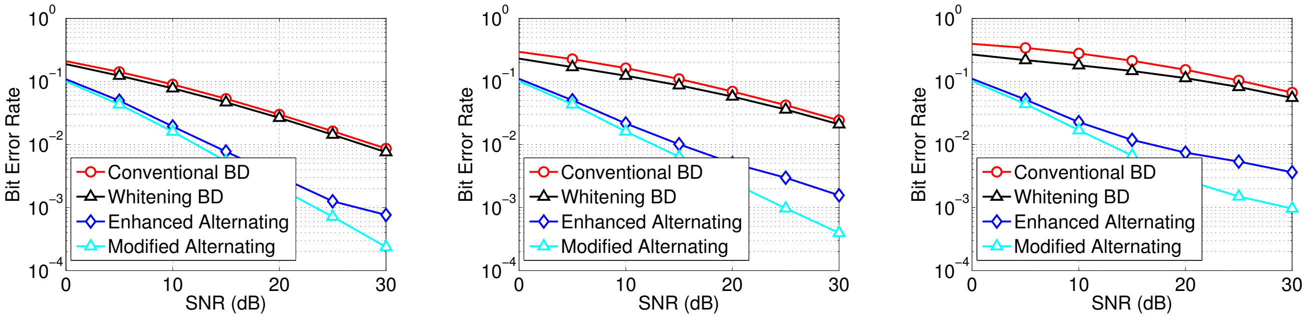


(d) External Interference of 0 dBm at the cluster edge. (e) External Interference of 10 dBm at the cluster edge. (f) External Interference of 20 dBm at the cluster edge.

Fig. 3. Sum Rate, at the top, and BER, at the bottom, versus SNR for different external interference values at the border of the cluster. Users are distributed respecting a distance of 2/3 of the cluster radius. Simulation with a Rank-1 external interference and nodes equipped with 4 antennas.



(a) External Interference of 0 dBm at the cluster edge. (b) External Interference of 10 dBm at the cluster edge. (c) External Interference of 20 dBm at the cluster edge.



(d) External Interference of 0 dBm at the cluster edge. (e) External Interference of 10 dBm at the cluster edge. (f) External Interference of 20 dBm at the cluster edge.

Fig. 4. Sum Rate, at the top, and BER, at the left, versus SNR for different external interference values at the border of the cluster. Users are distributed respecting a distance of 2/3 of the cluster radius. Simulation with a Rank-2 external interference and nodes equipped with 4 antennas.

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