Spectral and Energy Efficiency with Satisfaction Constraints

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***Abstract*—In this article we revisit the Radio Resource Allocation (RRA) problem of maximizing the total transmit data rate subject to satisfaction guarantees. It was previously studied on the perspective of Resource Block (RB) assignment (only) considering Equal Power Allocation (EPA) and, herein, we study this problem assuming adaptive power allocation. Two power allocation strategies are proposed and shown to be able to achieve energy and/or spectral efficiency gains compared to the original solution proposed in the literature. While the first power allocation strategy is capable of saving a considerable amount of transmit power maintaining the same transmit data rate of EPA, thus obtaining energy efficiency gains, the second strategy is able to convert this energy efficiency gain in spectral efficiency.**

***Index Terms*—Radio Resource Allocation, Power Allocation, Spectral Efficiency and Energy Efficiency.**

I. Introduction

Mobile networks have experienced huge developments since the analog 1st Generation (1G) in the 1980's until the modern 4th Generation (4G) [1]. This evolution has been largely motivated by the need of improved spectral and energy efficiency in order to support higher data rates with limited power budget and frequency bandwidth, as well as by the demand for multimedia services with specific Quality of Service (QoS). These developments have been achieved by multidisciplinary efforts involving intense research on network communication protocols, signal processing, and optimization, among others.

In order to further optimize modern networks, a specific functionality is highly desired: Radio Resource Allocation (RRA). RRA algorithms are responsible for managing the scarce radio resources such as power, time slots, spatial channels and frequency chunks [2]. The rational and efficient use of these resources in the radio access interface has the potential to optimize even further the aforementioned objectives such as spectral and energy efficiency, and QoS.

In this article we deal with the problem of frequency resource assignment and power allocation for improved spectral and energy efficiency subject to QoS and satisfaction constraints. It is organized as follows. In section II we review some related works and also show our main contributions whereas in section III we present the system model. The proposed RRA strategies are presented in section IV and their performance is evaluated in section V. Finally, in section VI we conclude this work.

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II. Literature Review and Main Contributions

In Orthogonal Frequency Division Multiplexing (OFDM) point-to-point connections, one of the most important objectives is the transmit power optimization. The classic solution for data rate maximization in this scenario is the well-known *water filling* solution that can be obtained by convex analysis [3]. An *water filling* downside is that it relies on a continuous log-shaped mapping between channel quality and data rate. In practice, discrete Modulation and Coding Schemes (MCSs) are used and, therefore, the achieved transmit data rates and transmit power levels are discrete. In this context, it has been shown that the Hughes Hartogs (HH) bit-loading algorithm achieves optimality [3]. The main idea of HH is to allocate transmit power to the frequency resources that require less power in order to increase their MCS level.

For Orthogonal Frequency Division Multiple Access (OFDMA) point-to-multipoint connections (e.g., the downlink of mobile networks) not only the transmit power allocation is important but also the frequency resource assignment. Generally, the complexity of the joint frequency resource assignment and power allocation is much higher than in point-to-point connections. In most cases, methods to obtain the optimal solution are not feasible for practical deployments due to the high computational complexity.

Several objectives can be pursued in point-to-multipoint connections: total data rate maximization (spectral efficiency), power minimization (energy efficiency) and/or satisfaction of QoS constraints. In [4], the problem of frequency resource assignment and power allocation to maximize the total data rate is studied. Therein, the original problem is shown to be optimally solved by splitting it into two distinct sub-problems: frequency resource assignment and power allocation. We call this technique as *two-step splitting* approach. Note that in this specific case, splitting the original problem into two new sub-problems does not impose loss in optimality. The solution of the first sub-problem consists in assigning each frequency resource to the terminal in better channel conditions. Then, the solution of the second sub-problem consists in distributing the total available power to the assigned frequency resources following the *water filling* or HH solutions.

One of the first works on RRA for power minimization with QoS constraints was [5]. Therein, the objective was to assign resources so as to guarantee minimum user QoS constraints while minimizing the total transmit power. Therein, the *two-step splitting* approach was used and the frequency resource assignment was solved using a Lagrangian-based algorithm

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and, then, transmit power allocation was performed for each user in order to fulfill QoS requirements. In [6], the *two-step splitting* approach was applied again and the frequency resource assignment was further split into two other subproblems that answered, firstly, how many, and secondly, which subcarriers to assign to each user. After that, a point-to-point power allocation strategy was applied on the frequency resources of each user. In [7], the performance of the solution of [5] was improved for some scenarios, but with the same computational complexity. In general, energy efficiency is a key research topic nowadays and has received considerable interest due to environmental issues and the increased demand for extended battery lifetime in mobile nodes [8].

In [9] we have studied a new RRA problem of maximizing the overall data rate subject to minimum satisfaction constraints per service in OFDMA-based systems. Therein, we assume that the system operators require a certain fraction of the connected terminals (pre-defined number of terminals) of each service to be satisfied, i.e., to attain a target QoS (minimum rate). In [9] we considered only the frequency resource assignment and assumed that the transmit power was equally distributed among frequency resources. In this work, we revisit that problem to evaluate the possible performance gains that can be achieved with adaptive transmit power allocation. The main contributions of this article are: generalization the problem defined in [9] in order to take into account the adaptive power allocation; proposal of an RRA strategy that gains in energy efficiency compared to [9] and uses the *two-step splitting* approach; and conception, based on the previous proposal, of another solution that achieves both spectral and energy efficiency gains compared to [9].

III. System Modeling

We consider the downlink of a cellular system composed of a number of sectored cells. For a given sector of a cell, there is a group of terminals connected to cell's Base Station (BS). The available resources are arranged in an OFDMA frequency-time grid, therefore, the minimum allocable resource or Resource Block (RB) is defined as a group of one or more adjacent subcarriers and a number of consecutive OFDM symbols representing one Transmission Time Interval (TTI). In order to avoid intra-cell interference, each RB is assigned to at most one terminal within a sector. As in [9], we assume for simplicity that the inter-cell interference is added to the noise in the Signal-to-Noise Ratio (SNR) expression.

In a given TTI, we assume that there are J active terminals. Each terminal is a candidate to get assigned some of the N available RBs. Moreover, $\mathcal{J}$ and $\mathcal{N}$ are the sets of terminals and RBs, respectively. We also assume that the system operator provides S different multimedia services. Each candidate terminal has an end-user connected to one of the S services of the set $\mathcal{S}$ of all services. The set of terminals using service $s \in \mathcal{S}$ is $\mathcal{J}_s$ and $|\mathcal{J}_s| = J_s$, where the operator $|\cdot|$ denotes the cardinality of a set[1]. Notice that $\bigcup\limits_{s\in\mathcal{S}} \mathcal{J}_s = \mathcal{J}$ and $\sum\limits_{s\in\mathcal{S}} J_s = J$.

[1]When the operator $|\cdot|$ is used in a scalar it denotes its absolute value.

We define $\mathbf{X}$ as a $J \times N$ assignment matrix with elements $x_{j,n}$ that assume the value 1 if RB n is assigned to the terminal j, and 0 otherwise. If RB n is assigned to terminal j, the received SNR $\gamma_{j,n}$ of terminal j on RB n is $\gamma_{j,n} = \left(\alpha_j \cdot p_n \cdot |h_{j,n}|^2\right)/\sigma_j^2$ where α_j models the joint effect of path gain and long-term fading experienced in the link between the BS and the terminal j, $h_{j,n}$ is the short-term frequency response of the channel experienced by terminal j on RB n, σ_j^2 is the noise power at terminal j, and p_n is the transmit power allocated to the RB n. We assume that $\mathbf{p}$ is a $N \times 1$ vector with elements p_n. The vector $\mathbf{p}$ together with the assignment matrix $\mathbf{X}$ are the optimization variables of the studied problem. We also assume that P^{tot} is the total available power at the BS.

Using link adaptation, a terminal can receive at different data rates according to its channel state, allocated power and perceived noise/interference. Herein, the mapping between the achieved SNR and the transmit data rate is performed by the function $f(\cdot)$. The transmit data rate $r_{j,n}$ when the RB n is assigned to terminal j is given by $r_{j,n} = f(\gamma_{j,n})$.

Without loss of generality, we assume a Bit Error Rate (BER)-based link adaptation where for a given SNR, the chosen MCS level is the one with the highest transmit data rate that assures a BER lower than a given fixed BER target. Different data rates can be achieved depending on the experienced SNR, as shown in Table I. There, we assume M possible MCSs levels and, therefore, M possible non-zero transmit data rates per RB, where v^m represents the transmit data rate corresponding to the m^{th} MCS level. Notice that the m^{th} MCS level is employed when the estimated SNR is between γ^m and γ^{m+1}, i.e., $f(\gamma_{j,n}) = v^m$ for $\gamma^m \leq \gamma_{j,n} < \gamma^{m+1}$.

Table I: Mapping from SNR to transmit data rate per RB.

SNR range	Transmit data rate per RB
$\gamma_{j,n} < \gamma^1$	0
$\gamma^1 \leq \gamma_{j,n} < \gamma^2$	v^1
...	...
$\gamma^{M-1} \leq \gamma_{j,n} < \gamma^M$	v^{M-1}
$\gamma_{j,n} \geq \gamma^M$	v^M

IV. Algorithmic Contributions

A. Problem description and RB assignment solution

In the following we present the studied problem presented in [9] considering not only the RB assignment but also the power allocation[2]:

$$\max_{\mathbf{X},\mathbf{p}} \sum_{j\in\mathcal{J}} \sum_{n\in\mathcal{N}} r_{j,n} \cdot x_{j,n}, \tag{1a}$$

$$\text{subject to} \sum_{j\in\mathcal{J}} x_{j,n} \leq 1, \; \forall n \in \mathcal{N}, \tag{1b}$$

$$x_{j,n} \in \{0,1\}, \; \forall j \in \mathcal{J} \text{ and } \forall n \in \mathcal{N}, \tag{1c}$$

$$\sum_{j\in\mathcal{J}_s} u\left(\sum_{n\in\mathcal{N}} r_{j,n} \cdot x_{j,n}, \; t_j\right) \geq k_s, \; \forall s \in \mathcal{S}, \tag{1d}$$

[2]Note that the transmit power is present in the definition of $r_{j,n}$.

where we assume that terminal j requires at least a transmit data rate t_j at current TTI, and that k_s terminals of service s should have their data rate requirement fulfilled. $u(x, b)$ is a step function at b that assumes 1 if $x \geq b$ and 0 otherwise. Basically, the objective is to maximize the total transmit data rate subject to minimum satisfaction guarantees for each service. Therefore, the system operator can define which provided multimedia services should be prioritized by setting high values for k_s.

Problem (1) is a non-linear combinatorial optimization problem that in general is hard to solve optimally. Motivated by this we employ here the heuristic (suboptimal) *two-step splitting* approach previously presented. Firstly, we solve the RB assignment and then the transmit power is distributed to the RBs. In order to perform RB assignment we use the low-complexity solution proposed in [9] called Reallocation-based Assignment for Improved Spectral Efficiency and Satisfaction (RAISES) assuming Equal Power Allocation (EPA). RAISES is split into two parts: Unconstrained Maximization and Reallocation. The main idea of the first part is to get an RB assignment with high aggregated data rate with no regard to QoS or satisfaction constraints. In general, it is expected that most of the RBs are assigned to terminals in better channel conditions that we call "donor terminals", i.e., terminals that can donate RBs to other terminals. The other terminals are the "receiver terminals" in the sense that they need RBs to become satisfied. In the second part of RAISES, RBs are exchanged between donor and receiver terminals until the unsatisfied terminals get the required data rate. More details on the RAISES algorithm can be found in [9]. After performing RB assignment we employ the adaptive power allocation according to one of the two following strategies.

B. Proposal 1

Based on the RB assignment found by RAISES with EPA, we apply the HH adaptive power allocation solution on the RBs of each terminal individually. According to EPA, the allocated power per RB is P^{tot}/N. Therefore, we consider that the total available power to terminal j is $(N_j P^{\text{tot}})/N$ where N_j is the number of RBs assigned to terminal j after applying RAISES. In summary, proposal 1 consists in applying the HH algorithm on the RBs of each terminal j individually assuming $(N_j P^{\text{tot}})/N$ as the available power.

Notice that HH will always adjust the transmit power to achieve the lowest SNR that supports a specific MCS. For example, in order to transmit with data rate v^m in RB n, HH allocates the transmit power so as to achieve γ^m (see Table I) since this SNR suffices to guarantee the desired BER performance of this MCS. Therefore, if at a specific stage of the HH algorithm it is concluded that the remaining power for terminal j is not sufficient to move to the next MCS on any RB of this terminal, this power is saved.

C. Proposal 2

The power allocation of proposal 2 is employed over the power distribution obtained by proposal 1. The main idea is to collect all the power saved after applying proposal 1, and then redistribute this power to the RBs that can use it most efficiently. More specifically, assume that after applying proposal 1 the saved power for terminal j is P_j^{saved}. Thus, the total saved power is $\sum_{j=1}^{J} P_j^{\text{saved}}$. The main idea in proposal 2 is to run HH power allocation on all RBs in order to redistribute the total saved power. Differently of proposal 1, the HH algorithm is employed here not on the RBs of each terminal individually but on all RBs. Figure 1 presents an illustration of both RRA strategies using power allocation according to proposals 1 and 2.

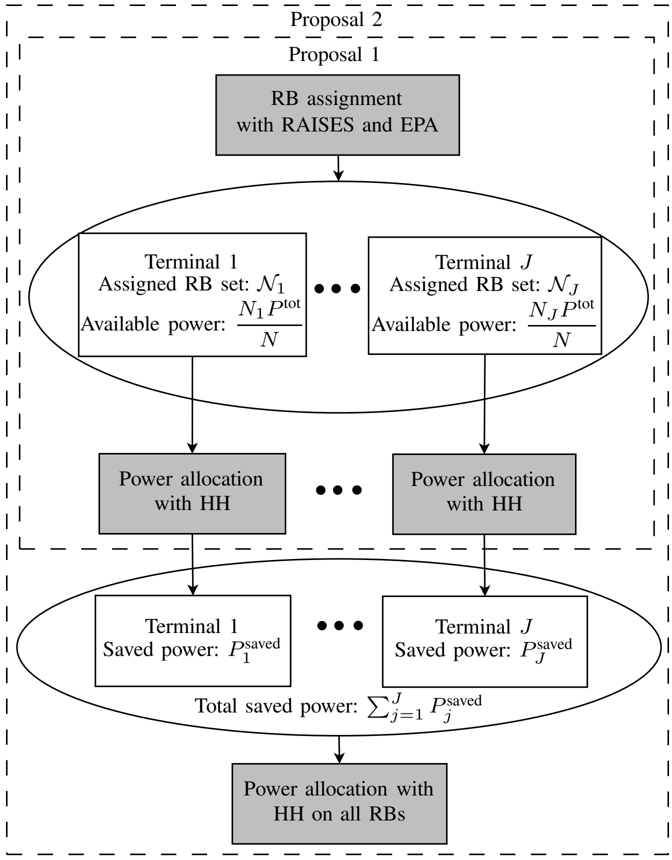

Figure 1: Illustration of the RB assignment using RAISES and power allocation strategies according to proposals 1 and 2.

Notice that the terminals' data rates in proposal 2 should be greater than or equal to the ones in proposal 1, since the allocated power on each RB in proposal 1 is either unchanged or increased after applying proposal 2. Therefore, proposal 2 is an alternative solution to proposal 1 that is capable of converting the energy efficiency gain from proposal 1 into an spectral efficiency gain.

V. Performance Results

A. Simulation parameters

The main assumptions stated in section III and in [9] were implemented in a computational simulator. The results were obtained by performing several independent snapshots in order to get valid results in a statistical sense. In each snapshot, the terminals are uniformly distributed within each sector, whose

BS is placed at its corner. We consider resources arranged in a time-frequency grid with each RB composed of a group of 12 adjacent subcarriers in the frequency dimension and 14 consecutive OFDM symbols in the time dimension [10].

The propagation model includes a distance-dependent path loss model (with the distance d in meters), a log-normal shadowing component and a Rayleigh-distributed fast fading component. The link adaptation is performed based on the report of 15 discrete Channel Quality Indicators (CQIs) used by the Long Term Evolution (LTE) system [10]. SNRs thresholds for switching among MCS were obtained from the link level simulations of [11]. The main simulation parameters are shown in Table II.

Table II: Main simulation parameters.

Parameter	Value	Unit
Cell radius	334	m
Total transmit power	5.25	W
Number of subcarriers per RB	12	-
Shadowing standard deviation	8	dB
Path loss	$35.3 + 37.6 \cdot \log_{10}(d)$	dB
Noise spectral density	$3.16 \cdot 10^{-20}$	W/Hz
Number of snapshots	3000	-
Number of services	2	-
Number of terminals per service	4	-
Number of RBs	15	-
Min. number of satisfied terminals	Service 1: 3; Service 2: 3	-

For comparison, we include in the simulation results the optimal solution of the studied problem (labeled OPT + EPA) considering only the RB assignment (assuming EPA). As shown in [9], the RB assignment problem can be formulated as an Integer Linear Program, which can be solved by using specific algorithms based on Branch and Bound method [12]. The other RRA strategies included in the simulations are the RAISES solution for RB assignment and the proposals 1 and 2 for power allocation (labeled RAISES + Proposal 1 and RAISES + Proposal 2, respectively), and also with EPA (labeled RAISES + EPA). The choice of the number of terminals, RBs and services in the simulations is limited by the computational complexity to obtain the optimal solution.

As performance metrics we consider the *total data rate* that consists is the sum of the data rates obtained by all the terminals in the sector in a given snapshot, and the *percentage of saved power* that consists in the fraction of the total available power that was saved.

B. *Results for proposal 1*

In Figure 2 we show the Cumulative Distribution Function (CDF) of the total transmit data rate for RAISES + EPA and RAISES + Proposal 1, as well as OPT + EPA when all terminals require a data rate of 900 kbps[3]. Firstly, we can see that the OPT + EPA solution presents higher total data rates than the RAISES solution with and without power allocation. However, we should highlight here that this better performance comes at a price: exponential worst case complexity of the OPT solution. This is in contrast to the polynomial worst case complexity of RAISES with and without power allocation.

[3]Similar relative performance among algorithms is observed in other required data rates.

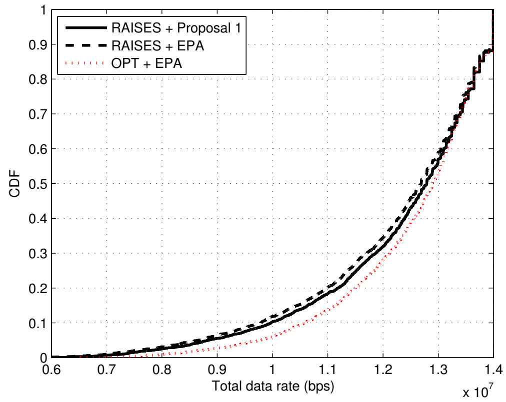

Figure 2: CDF of the total data rate for OPT + EPA and RAISES with proposal 1 and EPA strategies at the required data rate of 900 kbps.

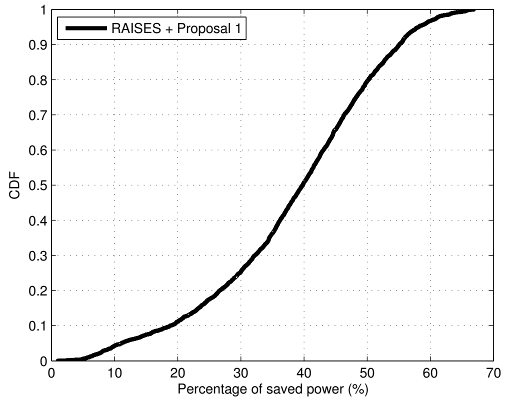

Figure 3: CDF of the total saved transmit power for RAISES with proposal 1 at the required data rate of 900 kbps.

Secondly, we can see a general behavior that is present in other loads: the gains in total data rate due to the use of adaptive power allocation with proposal 1 are not significant. For the presented scenario, the gain of proposal 1 over EPA for RAISES is of only 0.7%. The main reason for that is the fact that the adaptive power allocation is performed on the RBs of each terminal individually. Consequently, there is a "trunking" loss due to the fragmented RB set and total available power.

Figure 3 presents another standpoint when the performance of the proposal 1 is concerned. This figure shows the CDF of the percentage of saved power when proposal 1 is employed for RAISES at the required data rate of 900 kbps. The 50^{th} percentile of the percentage of the total saved power is of 39.8%. As the data rate demands of the terminals increase, it is expected a higher power consumption. All in all, the significant power saving gain comes from the optimization of the target SNR that is set as the minimum within the SNR region of the chosen MCS, as explained in section IV-B.

The small performance gain in the total data rate criterion and the significant power saving corroborates to the conclusion that the proposal 1 applied to the studied problem overcomes the EPA strategy by a small amount in the total data rate criterion, but leads to significant power saving gains.

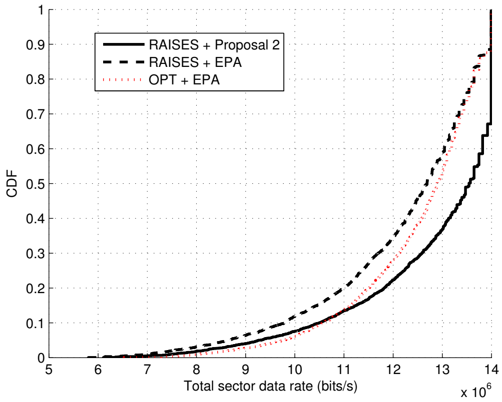

Figure 4: CDF of the total data rate for OPT + EPA and RAISES with proposal 2 and EPA strategies at the required data rate of 900 kbps.

C. Results for proposal 2

In Figure 4 we show the CDF of the total transmit data rate for RAISES + EPA and RAISES + Proposal 2, as well as the OPT + EPA when all terminals require a data rate of 900 kbps. Therein, we can see that the RAISES + Proposal 2 achieves much higher total data rates than RAISES + EPA. In fact, the joint use of RAISES and proposal 2 overcomes also the optimal solution of the studied problem considering only RB assignment. This is achieved with much lower computational complexity than OPT solution. The gains of the RAISES + proposal 2 over the RAISES + EPA and OPT + EPA are of 7.1% and 5.2% at the 50^{th} percentile of the total data rate, respectively. These gains result from redistributing the saved power of proposal 1 among RBs with proposal 2.

The use of proposal 2 still results in unused transmit power. The reason for that is the fact that some RBs get allocated a transmit power sufficient to achieve the highest MCS level and therefore it is not worth allocating more power to them. This unused power sometimes is not sufficient to increase the MCS levels of other RBs in poor channel conditions. Therefore, some fraction of the transmit power is saved. Figure 5 presents the CDF of the percentage of saved transmit power of RAISES + Proposal 2 at the required data rate of 900 kbps. The 50^{th} percentile of the percentage of the total unused transmit power is of 13.1% and, thus, less than the one achieved when proposal 1 is applied. Finally, through the analysis of the present simulation results we can see that proposal 2 is an alternative to trade the energy efficiency gains of proposal 1 for spectral efficiency gains.

VI. Conclusions and Perspectives

In this article we studied the total data rate maximization problem subjected to QoS and satisfaction constraint with adaptive power allocation. We solved this problem using *two-step splitting* approach dividing the original problem into a RB assignment and a power allocation subproblems. For the RB assignment, we used the low complexity solution of [9] while for the power allocation we proposed two strategies capable of achieving energy and/or spectral efficiency gains. From simulation results, we could see that the first proposal is capable of maintaining the same total transmit data rate of the original strategy with EPA but using only about 60% of the available transmit power. The second proposed power allocation strategy provided a gain of 7.1% in the total data rate compared to the first one and converted the energy efficiency gains of the first proposal into spectral efficiency gains. As perspectives, we point out the optimal analysis of the joint RB assignment and power allocation problem.

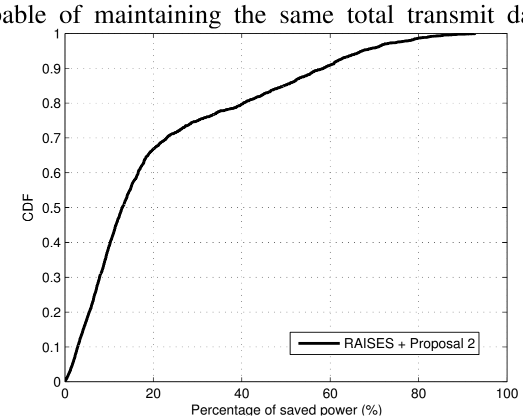

Figure 5: CDF of the total saved transmit power for RAISES with proposal 2 at the required data rate of 900 kbps.

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