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An ADMM Approach to Distributed Coordinated Beamforming in Dynamic TDD Networks

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***Abstract*—We consider a dynamic time division duplexing wireless network and propose a distributed coordinated beamforming algorithm based on Alternating Direction Method of Multipliers (ADMM) technique assuming the availability of perfect channel state information. Our design objective is to minimize the sum transmit power at the base stations subject to minimum signal-to-interference-plus-noise ratio (SINR) constraints for downlink mobile stations and a maximum interference power threshold for uplink mobile stations. First, we propose a centralized algorithm based on the relaxed Semidefinite Programming (SDP) technique. To obtain the beamforming solution in a distributed way, we further propose a distributed coordinated beamforming algorithm using the ADMM technique. Detailed simulation results are presented to examine the effectiveness of the proposed algorithms. It is shown that the proposed algorithm achieves better performance in terms of the design objective and converges faster than the reference algorithm based on primal decomposition.**

***Index Terms*—Dynamic TDD, ADMM, distributed algorithm.**

I. Introduction

Recently, the 3rd Generation Partnership Project has introduced the dynamic time division duplexing (DTDD) for Long Term Evolution (LTE) systems [1] to cope with traffic fluctuations between uplink and downlink, which is mostly experienced in small cell deployments [2]. In DTDD, each base station (BS) is allowed to dynamically reconfigure its time division duplexing (TDD) pattern based on its instantaneous traffic demand and/or interference status [3]. Earlier studies [1], [4] have evaluated the DTDD system performance in terms of various performance metrics and found that DTDD systems provide significant improvement in throughput as compared to statically configured TDD systems.

However, cross-link interference arises in DTDD systems, and therefore, there is a need for a proper inter-cell interference management to realize the DTDD advantages. Earlier studies [5], [6] have considered interference avoidance schemes. In [5], a cell-clustering scheme was proposed by grouping a number of cells into a cluster according to some metric(s), where cells in the same cluster adopt the same TDD configuration. The Authors of [6] proposed dynamic time slot allocation for an adaptive and flexible interference avoidance scheme. Adaptive power control techniques have been used as well, such as in [7], to reduce and compensate for the cross-link interference.

Moreover, Multiple-Input Multiple-Output (MIMO) based-systems provide more degrees of freedom to suppress the interference. A notable scheme in this area is the coordinated beamforming (CBF) technique [8], where each BS communicates with its served users while minimizing the interference leakage to users in adjacent cells. However, In DTDD systems, the interference situations are more complicated than those in static TDD systems since the uplink and downlink users coexist at the same time among neighboring cells. Therefore, interference management becomes more challenging and requires special consideration from the optimization view point. A possible approach to interference management is to formulate the optimization problem in DTDD systems similarly to how it is advantageously formulated in Cognitive Radio (CR) networks [9]. That is, by assuming that the uplink cells are the *primary* cells and the downlink cells are the *secondary* cells allows to include a threshold on the maximum cross-link interference power from the downlink to uplink cells. With this approach, the required uplink performance can be guaranteed.

The distributed algorithm in [9] uses the primal decomposition technique, where the problem is decoupled by introducing auxiliary variables that are updated later using the sub-gradient technique. However, the update of the variables problem is unbounded, which often leads to infeasible solutions. Our preliminarily tests show that this undesired situation often happens, especially with large scale problems.

To resolve this issue, we propose a novel distributed algorithm using the Alternating Direction Method of Multipliers (ADMM) [10] technique for DTDD wireless networks. ADMM is a powerful decomposition technique that blends the superior convergence properties of dual decomposition and the numerical robustness of augmented Lagrangian methods [10]. Therefore, ADMM is proved to be numerically stable and faster in convergence [10]. Our design objective is to minimize the total transmit power of downlink BSs, while satisfying the minimum signal-to-

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interference-plus-noise ratio (SINR) target on downlink mobile station (MS) and maximum interference threshold on uplink MS. Simulation results show that the proposed algorithm outperforms the algorithm from [9], where it has faster and more numerically stable convergence behavior.

II. System Model and Problem Formulation

We consider a DTDD system consisting of M cells. In each cell there is one BS equipped with N antennas and K MSs, each equipped with a single-antenna. We denote the nth BS by BS_n and the kth MS in each cell by MS_k. Let $\mathcal{M} \overset{\text{def}}{=} \{1,\ldots,M\}$ and $\mathcal{K} \overset{\text{def}}{=} \{\mathcal{K}_1,\ldots,\mathcal{K}_M\}$ denote the sets of all BSs and MSs, respectively, whereas $\mathcal{K}_n$ denotes the set of MSs associated with BS_n. At each time instant, we assume that there are $|\mathcal{M}^{dl}|$ cells in the downlink direction and $|\mathcal{M}^{ul}|$ cells in the uplink direction, whereas $\mathcal{M}^{dl}$ ($\mathcal{M}^{ul}$) denotes the set of BSs in the downlink (uplink) direction, such that $|\mathcal{M}^{dl}| + |\mathcal{M}^{ul}| = M$ and $\mathcal{M}^{dl} \bigcap \mathcal{M}^{ul} = \emptyset$. Similarly, we assume that $\mathcal{K}^{dl}$ ($\mathcal{K}^{ul}$) denotes the set of MSs in the downlink (uplink) direction, such that $|\mathcal{K}^{dl}| + |\mathcal{K}^{ul}| = MK$ and $\mathcal{K}^{dl} \bigcap \mathcal{K}^{ul} = \emptyset$.

In this paper [1], we consider minimizing the sum power of downlink BSs, while satisfying (i) the downlink MSs minimum SINR targets $\gamma_k, \forall k \in \mathcal{K}^{dl}$, and (ii) the maximum BS to BS interference power thresholds $\omega_k, \forall k \in \mathcal{K}^{ul}$. The SINR of MS_k, $k \in \mathcal{K}_n^{dl}$, is given as

$$\Gamma_k = \frac{|\mathbf{h}_{nk}^{\mathrm{H}}\mathbf{t}_k|^2}{\displaystyle\sum_{i\in\mathcal{K}_n^{dl}\setminus k}|\mathbf{h}_{nk}^{\mathrm{H}}\mathbf{t}_i|^2 + \sum_{m\in\mathcal{M}^{dl}\setminus n}\sum_{i\in\mathcal{K}_m^{dl}}|\mathbf{h}_{mk}^{\mathrm{H}}\mathbf{t}_i|^2 + \varphi_k}, \tag{1}$$

where $\mathbf{h}_{mk} \in \mathbb{C}^N$ denotes the channel vector from BS_m to MS_k and $\mathbf{t}_k \in \mathbb{C}^N$ denotes the transmit beamforming vector. Further, $\varphi_k = \sum_{m\in\mathcal{M}^{ul}}\sum_{i\in\mathcal{K}_m^{ul}}|h_{ik}|^2 q_i + \sigma_k^2$ denotes the MS to MS interference power plus noise, whereas $h_{ik} \in \mathbb{C}$ denotes the channel from MS_i to MS_k, q_i denotes the uplink transmit power, and σ_k^2 denotes the noise power. Here we assume that the uplink power $q_i, \forall i \in \mathcal{K}^{ul}$, are fixed. Thus, $\varphi_k, \forall k \in \mathcal{K}^{dl}$, are fixed for the considered transmission time.

The BS to BS interference power that is received by BS_m, $m \in \mathcal{M}^{ul}$, is given as

$$\varpi_m = \sum_{n\in\mathcal{M}^{dl}}\sum_{k\in\mathcal{K}_n^{dl}} \mathrm{tr}\left[\mathbf{H}_{nm}\mathbf{t}_k\mathbf{t}_k^{\mathrm{H}}\mathbf{H}_{nm}^{\mathrm{H}}\right]. \tag{2}$$

Note that each BS_m, $m \in \mathcal{M}^{ul}$, serves K MSs, where each $\mathrm{MS}_j, j \in \mathcal{K}_m^{ul}$, has an independent interference threshold ω_j. Therefore, to take into account all the thresholds, the downlink BSs should consider the minimum one as $\omega_m = \arg\min_j\{\omega_j, \forall j \in \mathcal{K}_m^{ul}\}$.

[1]Throughout this paper, letters n and m are used to index cells/BSs, while letters k, i and j are used to index users/MSs.

From the above, the system-wide optimization problem is given as

$$\min_{\mathbf{t}_k,\forall k\in\mathcal{K}^{dl}} \sum_{n\in\mathcal{M}^{dl}}\sum_{k\in\mathcal{K}_n^{dl}} \|\mathbf{t}_k\|^2 \tag{3a}$$

$$\text{s.t.} \quad \Gamma_k \geq \gamma_k, \forall k \in \mathcal{K}^{dl}, \tag{3b}$$

$$\varpi_m \leq \omega_m, \forall m \in \mathcal{M}^{ul}. \tag{3c}$$

Note that, for some values of ω_m, constraints (3c) might be inactive. Thus, removing them do not change the solution. In the rest of this paper, we assume that the constraints (3c) are active for at least one uplink BS.

III. Proposed Solution

First, let χ_{mk} and ϕ_{mn} denote the inter-cell and BS to BS interference power from BS_m to MS_k and from BS_m to BS_n, respectively, which are defined as

$$\chi_{mk} \overset{\text{def}}{=} \mathbf{h}_{mk}^{\mathrm{H}}\widetilde{\mathbf{T}}_m\mathbf{h}_{mk}, \tag{4}$$

$$\phi_{mn} \overset{\text{def}}{=} \mathrm{tr}\left[\mathbf{H}_{mn}\widetilde{\mathbf{T}}_m\mathbf{H}_{mn}^{\mathrm{H}}\right], \tag{5}$$

where $\widetilde{\mathbf{T}}_m = \sum_{i\in\mathcal{K}_m^{dl}}\mathbf{T}_i \succeq 0$ and $\mathbf{T}_k = \mathbf{t}_k\mathbf{t}_k^{\mathrm{H}} \succeq 0$. Further, let $\boldsymbol{\chi}$ and $\boldsymbol{\phi}$ denote the two vectors that collect all the inter-cell and BS to BS interference power variables, respectively, i.e.,

$$\boldsymbol{\chi} \overset{\text{def}}{=} [\{\chi_{nj}\}, \forall n \in \mathcal{M}^{dl}, \forall j \in \mathcal{K}^{dl}\setminus\mathcal{K}_n^{dl}]^{\mathrm{T}}, \tag{6}$$

$$\boldsymbol{\phi} \overset{\text{def}}{=} [\{\phi_{nm}\}, \forall n \in \mathcal{M}^{dl}, \forall m \in \mathcal{M}^{ul}]^{\mathrm{T}}. \tag{7}$$

Furthermore, let $\boldsymbol{\chi}_n$ and $\boldsymbol{\phi}_n$ denote the two vectors that collect all the inter-cell and BS to BS interference variables, respectively, that are relevant only to BS_n, i.e.,

$$\boldsymbol{\chi}_n \overset{\text{def}}{=} [\{\chi_k\}, \forall k \in \mathcal{K}_n^{dl}, \quad \{\chi_{nk}\}, \forall k \in \mathcal{K}^{dl}\setminus\mathcal{K}_n^{dl}]^{\mathrm{T}}, \tag{8}$$

$$\boldsymbol{\phi}_n \overset{\text{def}}{=} [\{\phi_{nm}\}, \forall m \in \mathcal{M}^{ul}]^{\mathrm{T}}, \tag{9}$$

where $\chi_k \overset{\text{def}}{=} \sum_{m\in\mathcal{M}^{dl}\setminus n}\chi_{mk}$. From the above, it can be shown that there exist permutation matrices Ξ_n and Π_n such that $\boldsymbol{\chi}_n = \Xi_n\boldsymbol{\chi}$ and $\boldsymbol{\phi}_n = \Pi_n\boldsymbol{\phi}$. Let $\mathsf{P}_n = \mathrm{tr}[\widetilde{\mathbf{T}}_n]$, then problem (3) can be written as

$$\min_{\substack{\{\boldsymbol{\chi}_n\},\{\boldsymbol{\phi}_n\}\\ \{\mathsf{P}_n\},\{\mathbf{T}_k\}}} \sum_{n\in\mathcal{M}^{dl}} \mathsf{P}_n \tag{10a}$$

$$\text{s.t.} \quad \{\boldsymbol{\chi}_n, \boldsymbol{\phi}_n, \mathsf{P}_n, \{\mathbf{T}_k, \forall k \in \mathcal{K}_n^{dl}\}\} \in \mathcal{S}_n, \forall n \in \mathcal{M}^{dl}, \tag{10b}$$

$$\boldsymbol{\chi}_n = \Xi_n\boldsymbol{\chi}, \forall n \in \mathcal{M}^{dl}, \tag{10c}$$

$$\boldsymbol{\phi}_n = \Pi_n\boldsymbol{\phi}, \forall n \in \mathcal{M}^{dl}, \tag{10d}$$

$$\widetilde{\Pi}\boldsymbol{\phi}_n \leq \boldsymbol{\omega}, \tag{10e}$$

where $\widetilde{\Pi} = \sum_{n\in\mathcal{M}^{dl}}\Pi_n$, $\boldsymbol{\omega} = [\{\omega_m\}, \forall m \in \mathcal{M}^{ul}]^{\mathrm{T}}$, which collects all the BS to BS interference power thresholds, and $\mathcal{S}_n$ is the convex set of $\mathrm{BS}_n, n \in \mathcal{M}^{dl}$, defined as

$$\mathcal{S}_n = \Big\{ \left(\boldsymbol{\chi}_n, \boldsymbol{\phi}_n, \mathsf{P}_n, \{\mathbf{T}_k, \forall k \in \mathcal{K}_n^{dl}\}\right) \Big|$$

$$\mathbf{h}_{nk}^{\mathrm{H}}\mathbf{D}_k\mathbf{h}_{nk} \geq \chi_k + \varphi_k, \forall k \in \mathcal{K}_n^{dl}, \tag{11a}$$

$$\mathbf{h}_{ni}^{\mathrm{H}}\widetilde{\mathbf{T}}_n\mathbf{h}_{ni} \leq \chi_{ni}, \forall i \in \mathcal{K}^{dl}\setminus\mathcal{K}_n^{dl}, \tag{11b}$$

$$\mathrm{tr}\left[\mathbf{H}_{nm}\widetilde{\mathbf{T}}_n\mathbf{H}_{nm}^{\mathrm{H}}\right] \leq \phi_{nm}, \forall m \in \mathcal{M}^{ul}, \tag{11c}$$

$$\mathrm{tr}[\widetilde{\mathbf{T}}_n] = \mathsf{P}_n \Big\}, \forall n \in \mathcal{M}^{dl}, \tag{11d}$$

where $\mathbf{D}_k = \frac{1}{\gamma_k}\mathbf{T}_k - \sum_{j\in\mathcal{K}_n^{dl}\setminus k}\mathbf{T}_j \succeq 0$. Problem (10) is a convex relaxed Semidefinite Programming (SDP) problem, which can be efficiently solved using the interior point methods [11]. To this end, several convex solvers are freely available, e.g. CVX [12].

A. Distributed Algorithm via the Alternating Direction Method of Multipliers

In this section, we will apply the ADMM concepts [10] to solve problem (10) in a distributed fashion. The ADMM solves problems in the form [10]

$$\min_{\mathbf{x}\in\mathbf{R}^n,\mathbf{z}\in\mathbf{R}^m} \quad f(\mathbf{x}) + g(\mathbf{z}), \quad \text{s.t. } \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} = \mathbf{c}, \tag{12}$$

where f and g are convex, $\mathbf{A} \in \mathbf{R}^{p\times n}$, $\mathbf{B} \in \mathbf{R}^{p\times m}$, and $\mathbf{c} \in \mathbf{R}^p$. The augmented Lagrangian of the above problem is given as

$$\mathscr{J}(\mathbf{x},\mathbf{z},\mathbf{y}) = f(\mathbf{x}) + g(\mathbf{z}) + \mathbf{y}^{\mathrm{T}}(\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{z}-\mathbf{c}) + \tag{13}$$
$$\frac{\rho}{2}\|\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{z}-\mathbf{c}\|_2^2, \tag{14}$$

where $\rho > 0$ is the penalty parameter and $\mathbf{y}$ is the dual vector. Then, the ADMM consists of the following iterations:

$$\mathbf{x}^{(t+1)} = \min_{\mathbf{x}} \quad \mathscr{J}(\mathbf{x},\mathbf{z}^{(t)},\mathbf{y}^{(t)}) \tag{15}$$
$$\mathbf{z}^{(t+1)} = \min_{\mathbf{z}} \quad \mathscr{J}(\mathbf{x}^{(t+1)},\mathbf{z},\mathbf{y}^{(t)}) \tag{16}$$
$$\mathbf{y}^{(t+1)} = \mathbf{y}^{(t)} + \rho(\mathbf{A}\mathbf{x}^{(t+1)} + \mathbf{B}\mathbf{z}^{(t+1)} - \mathbf{c}). \tag{17}$$

where (t) denotes the iteration index. Note that $\mathbf{x}$ and $\mathbf{z}$ are updated in an alternating fashion, which accounts for the term alternating direction. This is in contrast to the original method of multipliers [10] that considers updating both variables jointly. Separating the minimization over $\mathbf{x}$ and $\mathbf{z}$ into two steps allows for decomposition when f or g are separable. According to [13, Lemma 2], a sufficient condition for convergence of the above steps is that $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ and $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ are invertible. When this sufficient condition is met, then the sequence generated by the above steps is bounded, and every limit point is an optimal solution of the original problem.

According to the ADMM concepts, the augmented-form of problem (10) is given as

$$\min_{\substack{\{\boldsymbol{\chi}_n\},\{\boldsymbol{\phi}_n\}\\ \{\mathsf{P}_n\},\{\mathbf{T}_k\}}} \sum_{n\in\mathcal{M}^{dl}} \Big(\mathsf{P}_n + \frac{\rho}{2}\|\Xi_n\boldsymbol{\chi} - \boldsymbol{\chi}_n\|^2 + \frac{\rho}{2}\|\Pi_n\boldsymbol{\phi} - \boldsymbol{\phi}_n\|^2 + \frac{\rho}{2}(\tilde{\mathsf{P}}_n - \mathsf{P}_n)^2\Big) \tag{18a}$$
$$\text{s.t.} \quad \{\boldsymbol{\chi}_n,\boldsymbol{\phi}_n,\mathsf{P}_n,\{\mathbf{T}_k,\forall k\in\mathcal{K}_n^{dl}\}\} \in \mathcal{S}_n, \forall n\in\mathcal{M}^{dl}, \tag{18b}$$
$$\boldsymbol{\chi}_n = \Xi_n\boldsymbol{\chi}, \forall n\in\mathcal{M}^{dl}, \tag{18c}$$
$$\boldsymbol{\phi}_n = \Pi_n\boldsymbol{\phi}, \forall n\in\mathcal{M}^{dl}, \tag{18d}$$
$$\tilde{\mathsf{P}}_n = \mathsf{P}_n, \forall n\in\mathcal{M}^{dl}, \tag{18e}$$

where $\tilde{\mathsf{P}}_n \geq 0, \forall n \in \mathcal{M}^{dl}$, are slack variables. Note that problem (18) is clearly equivalent to problem (10) (after excluding constraint (10e)), since for any feasible solution, the added terms to the objective are zero. However, the added penalty terms bring numerical stability and faster convergence [10], [13]. The augmented Lagrangian of problem (18) is given by

$$\mathscr{L}(\boldsymbol{\chi},\boldsymbol{\phi},\{\tilde{\mathsf{P}}_n\},\{\boldsymbol{\eta}_n\},\{\boldsymbol{\xi}_n\},\{\mu_n\}) = \sum_{n\in\mathcal{M}^{dl}} \Big(\mathsf{P}_n + \frac{\rho}{2}\|\Xi_n\boldsymbol{\chi} - \boldsymbol{\chi}_n\|^2 + \frac{\rho}{2}\|\Pi_n\boldsymbol{\phi} - \boldsymbol{\phi}_n\|^2 + \frac{\rho}{2}(\tilde{\mathsf{P}}_n - \mathsf{P}_n)^2 + \boldsymbol{\eta}_n^{\mathrm{T}}(\Xi_n\boldsymbol{\chi} - \boldsymbol{\chi}_n) + \boldsymbol{\xi}_n^{\mathrm{T}}(\Pi_n\boldsymbol{\phi} - \boldsymbol{\phi}_n) + \mu_n(\tilde{\mathsf{P}}_n - \mathsf{P}_n)\Big), \tag{19}$$

where $\boldsymbol{\eta}_n, \boldsymbol{\xi}_n$, and $\mu_n, \forall n \in \mathcal{M}^{dl}$, are the dual vectors associated with constraints (18c), (18d) and (18e), respectively. Note that the Lagrangian function (19) is separable between the downlink BSs. Thus, we can write (19) as

$$\mathscr{L}(\boldsymbol{\chi},\boldsymbol{\phi},\{\tilde{\mathsf{P}}_n\},\{\boldsymbol{\eta}_n\},\{\boldsymbol{\xi}_n\},\{\mu_n\}) = \sum_{n\in\mathcal{M}^{dl}} \mathscr{L}_n(\boldsymbol{\chi},\boldsymbol{\phi},\tilde{\mathsf{P}}_n,\boldsymbol{\eta}_n,\boldsymbol{\xi}_n,\mu_n). \tag{20}$$

From the above, the $\mathrm{BS}_n, \forall n \in \mathcal{M}^{dl}$, augmented dual problem based ADMM is given as

$$\min_{\substack{\boldsymbol{\chi}_n,\boldsymbol{\phi}_n\\ \mathsf{P}_n,\{\mathbf{T}_k\}}} \mathscr{L}_n(\boldsymbol{\chi},\boldsymbol{\phi},\tilde{\mathsf{P}}_n,\boldsymbol{\eta}_n,\boldsymbol{\xi}_n,\mu_n) \tag{21a}$$
$$\text{s.t.} \quad \{\boldsymbol{\chi}_n,\boldsymbol{\phi}_n,\mathsf{P}_n,\{\mathbf{T}_k,\forall k\in\mathcal{K}_n^{dl}\}\} \in \mathcal{S}_n. \tag{21b}$$

Next, each $\mathrm{BS}_n, \forall n \in \mathcal{M}^{dl}$, updates the global variables $\boldsymbol{\chi}^{(t+1)}$ and $\boldsymbol{\phi}^{(t+1)}$ and the local variable $\tilde{\mathsf{P}}_n^{(t+1)}$ as a solution to the following problems[2]:

$$\boldsymbol{\chi}^{(t+1)} = \min_{\boldsymbol{\chi}} \mathscr{L}(\boldsymbol{\chi},\boldsymbol{\phi},\{\tilde{\mathsf{P}}_n\},\{\boldsymbol{\eta}_n\},\{\boldsymbol{\xi}_n\},\{\mu_n\}). \tag{22}$$
$$\boldsymbol{\phi}^{(t+1)} = \min_{\substack{\boldsymbol{\phi}\\ \text{s.t. } \widetilde{\Pi}\boldsymbol{\phi}=\boldsymbol{\omega}}} \mathscr{L}(\boldsymbol{\chi},\boldsymbol{\phi},\{\tilde{\mathsf{P}}_n\},\{\boldsymbol{\eta}_n\},\{\boldsymbol{\xi}_n\},\{\mu_n\}). \tag{23}$$
$$\tilde{\mathsf{P}}_n^{(t+1)} = \min_{\tilde{\mathsf{P}}_n} \mathscr{L}_n(\boldsymbol{\chi},\boldsymbol{\phi},\{\tilde{\mathsf{P}}_n\},\{\boldsymbol{\eta}_n\},\{\boldsymbol{\xi}_n\},\{\mu_n\}). \tag{24}$$

Problems (22), (23) and (24) are convex quadratic problems. The closed-form solutions are given, respectively, as

$$\boldsymbol{\chi}^{(t+1)} = \Xi^{\dagger}\Big(\tilde{\boldsymbol{\chi}}^{(t+1)} - \frac{1}{\rho}\tilde{\boldsymbol{\eta}}(t)\Big), \tag{25}$$
$$\boldsymbol{\phi}^{(t+1)} = \Pi^{\dagger}\Big(\tilde{\boldsymbol{\phi}}^{(t+1)} - \frac{1}{\rho}\tilde{\boldsymbol{\xi}}(t)\Big) + \frac{1}{\rho}(\Pi^{\mathrm{T}}\Pi)^{-1}\widetilde{\Pi}^{\mathrm{T}}\boldsymbol{\zeta}, \tag{26}$$
$$\tilde{\mathsf{P}}_n^{(t+1)} = \mathsf{P}_n^{(t+1)} - \frac{1}{\rho}\mu_n^{(t)}, \tag{27}$$

where $\boldsymbol{\zeta}$ denotes the dual vector associated with problem (23) constraint, which has a closed-form solution given as $\boldsymbol{\zeta} = \Big(\frac{1}{\rho}\widetilde{\Pi}(\Pi^{\mathrm{T}}\Pi)^{-1}\widetilde{\Pi}^{\mathrm{T}}\Big)^{-1}\Big(\boldsymbol{\omega} - \widetilde{\Pi}\Pi^{\dagger}\Big(\tilde{\boldsymbol{\phi}}^{(t+1)} - \frac{1}{\rho}\tilde{\boldsymbol{\xi}}^{(t)}\Big)\Big)$. Moreover, $\Xi, \Pi, \tilde{\boldsymbol{\chi}}^{(t+1)}, \tilde{\boldsymbol{\eta}}^{(t)}, \tilde{\boldsymbol{\phi}}^{(t+1)}$, and $\tilde{\boldsymbol{\xi}}^{(t)}$ are defined as

$$\Xi = [\Xi^{\mathrm{T}}_{\mathcal{M}^{dl}(1)} \dots \Xi^{\mathrm{T}}_{\mathcal{M}^{dl}(|\mathcal{M}^{dl}|)}]^{\mathrm{T}}, \tag{28}$$

where $\Pi, \tilde{\boldsymbol{\chi}}^{(t+1)}, \tilde{\boldsymbol{\eta}}^{(t)}, \tilde{\boldsymbol{\phi}}^{(t+1)}$, and $\tilde{\boldsymbol{\xi}}^{(t)}$ are defined in a similar way. Note that $\Xi^{\mathrm{T}}\Xi \succ 0$ and $\Pi^{\mathrm{T}}\Pi \succ 0$. Thus, both

[2]Note that we included constraint (10e) in problem (23).

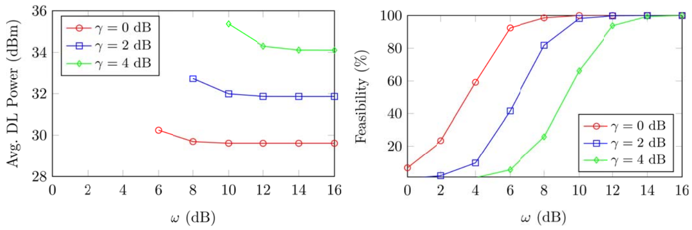

Fig. 1. Algorithm 1 performance and feasibility with range of γ and ω values.

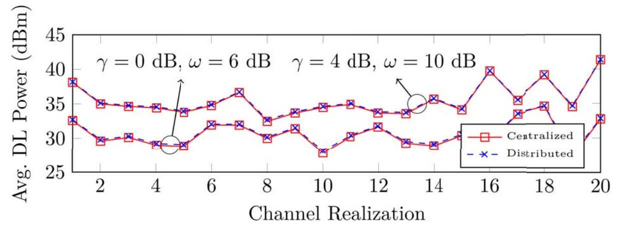

Fig. 2. Comparison between centralized and distributed approaches with number of channel realizations [$t_{max} = 10$].

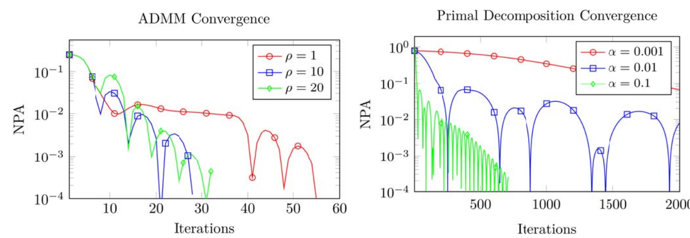

Fig. 3. Convergence comparison between algorithm 1 and algorithm from [9] [$\gamma = 4$ dB, $\omega = 10$ dB].

matrices are invertible, which is a sufficient condition for convergence of the above steps according to [13, Lemma 2].

Finally, each $\text{BS}_n, n \in \mathcal{M}^{dl}$, updates the dual vectors $\boldsymbol{\eta}_n^{(t+1)}, \boldsymbol{\xi}_n^{(t+1)}$ and $\mu_n^{(t+1)}$ as

$$\boldsymbol{\eta}_n^{(t+1)} = \boldsymbol{\eta}_n^{(t)} + \rho\Big(\Xi_n \boldsymbol{\chi}^{(t+1)} - \boldsymbol{\chi}_n^{(t+1)}\Big), \tag{29}$$

$$\boldsymbol{\xi}_n^{(t+1)} = \boldsymbol{\xi}_n^{(t)} + \rho\Big(\Pi_n \boldsymbol{\phi}^{(t+1)} - \boldsymbol{\phi}_n^{(t+1)}\Big), \tag{30}$$

$$\mu_n^{(t+1)} = \mu_n^{(t)} + \rho\Big(\tilde{\mathsf{P}}_n^{(t+1)} - \mathsf{P}_n^{(t+1)}\Big). \tag{31}$$

The distributed algorithm based ADMM technique for solving problem (3) is summarized in Algorithm 1.

Algorithm 1 Distributed algorithm via ADMM.

1: Initialize: $\boldsymbol{\chi}, \boldsymbol{\phi}, \tilde{\mathsf{P}}_n, \boldsymbol{\eta}_n, \boldsymbol{\xi}_n, \mu_n$, and ρ. Set $t = 1$.
2: **while** not converged or $t < t_{max}$ **do**
3: Each $\text{BS}_n, n \in \mathcal{M}^{dl}$, solves problem (21) for $\boldsymbol{\chi}_n^{(t+1)}$, $\boldsymbol{\phi}_n^{(t+1)}$, $\tilde{\mathsf{P}}_n^{(t+1)}$.
4: Update $\boldsymbol{\chi}^{(t+1)}$, $\boldsymbol{\phi}^{(t+1)}$, and $\tilde{\mathsf{P}}_n^{(t+1)}$ using (25), (26) and (27), respectively.
5: Update $\boldsymbol{\eta}_n^{(t+1)}, \boldsymbol{\xi}_n^{(t+1)}$ and $\mu_n^{(t+1)}$ using (29), (30) and (31), respectively.
6: **end while**

IV. Numerical Results

We consider a flat Rayleigh fading scenario with uncorrelated channels between antennas. We assume $\gamma_k = \gamma$. $\omega_k = \omega, \forall k \in \mathcal{K}$, $q_k = 23$ dBm$, \forall k \in \mathcal{K}^{ul}$, and $\sigma^2 = 1$. We consider a system of $[M, K, N] = [3, 2, 8]$, such that $|\mathcal{M}^{dl}| = 2$ and $|\mathcal{M}^{ul}| = 1$.

Fig. 1 shows the system performance and feasibility with a range of input parameters averaged over 500 channel realizations. Fig. 2 shows a performance comparison between the centralized and distributed algorithms using 20 feasible channel realization and assuming $t_{max} = 10$.

As expected, we can see from Fig. 1 that the downlink power increases and the feasibility rate decreases when γ increases and/or ω decreases. Moreover, we can see from Fig. 2 that the proposed distributed algorithm converges to *near-optimal* solution after few iterations for low-to-moderate convergence tolerance threshold. Similar results to Fig. 2 can be obtained using the distributed algorithm based primal decomposition technique used in [9], where the main difference is in the convergence behavior. In Fig. 3, the convergence behavior of both algorithms is shown for one channel realization (rather than the average to expose the convergence behavior of each algorithm) in terms of normalized power accuracy (NPA), which is defined as NPA $= \frac{|\mathsf{P}^{(t)} - \mathsf{P}^{\star}|}{\mathsf{P}^{\star}}$, where $\mathsf{P}^{(t)} = \sum_{n \in \mathcal{M}^{dl}} \mathsf{P}_n^{(t)}$ is the sum of downlink power at iteration (t) with the distributed algorithm and $\mathsf{P}^{\star}$ is the sum of downlink power with the centralized algorithm.

From Fig. 2, we can see that the convergence behavior of either algorithm depends on the selection of ρ and α parameters, where α denotes the variables' update step-size used in [9]. Unfortunately, the optimal value of either parameter is not known, and in general it is dependent on the system-scale and input parameter values. However, it can be observed that both algorithms converge to a *near-optimal* solution after few iterations. Nevertheless, it can be seen that the proposed algorithm based ADMM has a faster convergence rate.

V. Conclusions

In this paper we have considered dynamic time division duplexing (DTDD) wireless networks and proposed a distributed coordinated beamforming (CBF) algorithm based on the Alternating Direction Method of Multipliers (ADMM) technique. Simulation results have shown that the proposed algorithm converges to a *near-optimal* solution after a few iterations and has a faster convergence rate, as compared to the primal decomposition technique.

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References

[1] 3GPP, "Evolved Universal Terrestrial Radio Access (E-UTRA): Further enhancements to LTE Time Division Duplex (TDD) for Downlink-Uplink (DL-UL) interference management and traffic adaptation," Tech. Rep. 36.828, Jun. 2012.

[2] O. Stanze and A. Weber, "Heterogeneous networks with LTE-advanced technologies," *Bell Labs Technical Journal*, vol. 18, no. 1, pp. 41–58, 2013.

[3] C. Wang, X. Hou, A. Harada, S. Yasukawa, and H. Jiang, "HARQ signalling design for dynamic TDD system." in *Proc. IEEE Vehicular Technology Conference (VTC)*, Sep. 2014, pp. 1–5.

[4] B. Yu, H. Ishii, and L. Yang, "System level performance evaluation of dynamic TDD and interference coordination in enhanced local area architecture," in *Proc. IEEE 77th Vehicular Technology Conference (VTC Spring)*, Jun. 2013, pp. 1–6.

[5] Y.-T. Lin, C.-C. Chao, and H.-Y. Wei, "Dynamic TDD interference mitigation by using soft reconfiguration," in *Proc. 11th International Conference on Heterogeneous Networking for Quality, Reliability, Security and Robustness (QSHINE)*, Aug. 2015, pp. 352–357.

[6] Y. S. Choi, I. Sohn, and K. B. Lee, "A novel decentralized time slot allocation algorithm in dynamic TDD system," in *CCNC 2006. 2006 3rd IEEE Consumer Communications and Networking Conference, 2006.*, vol. 2, Jan. 2006, pp. 1268–1272.

[7] H. Lee and D.-H. Cho, "Combination of dynamic-TDD and static-TDD based on adaptive power control," in *Proc. IEEE Vehicular Technology Conference (VTC)*, Sep. 2008, pp. 1–5.

[8] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1748–1759, May 2010.

[9] H. Pennanen, A. Tölli, and M. Latva-aho, "Multi-cell beamforming with decentralized coordination in cognitive and cellular networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 295–308, Jan. 2014.

[10] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

[11] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*. CRC Press, 2001.

[12] M. Grant, S. Boyd, and Y. Ye, *CVX: MATLAB software for disciplined convex programming*, 2008.

[13] C. Shen, T. H. Chang, K. Y. Wang, Z. Qiu, and C. Y. Chi, "Distributed robust multicell coordinated beamforming with imperfect CSI: An ADMM approach," *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 2988–3003, Jun. 2012.