Transmit Power Minimization with QoS and User Satisfaction Guarantees in SC-FDMA Systems

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***Abstract*—In this article we deal with total transmit power minimization problem in SC-FDMA system constrained to Quality of Service (QoS) and user satisfaction constraints. Through algebraic manipulations we were able to convert an integer nonlinear optimization problem in an Integer Linear Problem (ILP) so as to obtain the optimal solution. Motivated by the high computational to obtain the optimal solution, we propose a low computational complexity suboptimal solution. The results obtained through computational simulations show that the proposed solution presents satisfactory results regarding both outage rate and energy consumption.**

I. Introduction

Even with the imminent advent of Fifth Generation (5G) networks, Long Term Evolution (LTE) will still play a relevant role in mobile communications. According to Ericsson, there will be 5 billion LTE subscriptions by the end of 2022 [1]. In order to reduce Peak-to-Average Power Ratio (PAPR), LTE has adopted Single Frequency Division Multiple Access (SC-FDMA) in the uplink. SC-FDMA in general requires that the frequency resource blocks assigned to each terminal should be adjacent to each other in the frequency domain.

Radio Resource Allocation (RRA) consists in an important tool to optimize the use of the available resources in the system, however, as shown in [2], the adjacency constraints of SC-FDMA is sufficient to turn RRA problems NP-Hard.

In the last years, some works have faced the challenge of conceiving RRA solutions to SC-FDMA networks. In [3] the data rate maximization problem is studied in SC-FDMA systems, however, that work ignores the adjacency constraint that is necessary to assure low PAPR. In [4], spite of the authors consider the adjacency constraint, they assumed that each user demanded the same number of frequency resources. In practice, the users have different data rate requirements and channel qualities leading to different demands regarding the number of frequency resources.

Studies on energy efficiency have been considered by other works [5], [6], [7]. In [5] an integer binary optimization problem is proposed to minimize the total transmitted power in SC-FDMA system. Although adjacent constraint is assumed in the work, Quality of Service (QoS) aspects were not considered. In [6] the authors studied the optimization of energy resources and considered QoS constraints in SC-FDMA networks, nevertheless, the adaptive power allocation was ignored. The adjacency and QoS constraints and the adaptive power allocation were considered in [7] for optimization of energy resources, nonetheless, the user satisfaction constraint was ignored. In general the traffic behavior and QoS requirements of some user applications are diverse and heterogeneous which leads to the definition of different service types. In order to assure an acceptable provision of different services, system operators could guarantee that a minimum percentage of the connections for each service should have their QoS fulfilled. In [8] the authors study the data rate maximization problem in SC-FDMA system considering the per-service minimum satisfaction, however, power allocation was not performed and energy resources were not optimized. Besides, as the majority of the previous works, the authors assume that the mapping between Signal to Noise Ratio (SNR) and data rate is given by Shannon capacity equation, nevertheless, in practical networks, a discrete function could be more representative to the mapping between SNR and data rate employed by finite Modulation and Coding Schemes (MCSs).

In summary, the main contributions of this article are: (1) Formulation and modeling of a new energy optimization problem considering a set of modeling aspects that were ignored by some of the previous works: adjacency constraint, power allocation, user satisfaction constraints and finite MCS; (2) Proposal of the optimal solution to the studied problem after a reformulation that transformed a nonlinear combinatorial problem into Integer Linear Problem (ILP); (3) Proposal of a low-complexity solution motivated by the unpractical computational complexity of the method to obtain the optimal solution; (4) Calculation of the computational complexity, in order to characterize the performance/complexity trade-off of the involved algorithms.

II. System Modeling

We assume that resource allocation should be performed in the uplink of a cell in a cellular system with a Base Station (BS) that serves the connected terminals. Both mobile terminals and BS employ single antenna transceivers. We assume that the intra-cell interference, i.e., interference between terminals of the same cell, is controlled by employing the combination of Time Division Multiple Access (TDMA) and SC-FDMA with the assignment of orthogonal resources. We define

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the minimum allocable resource or Resource Block (RB) by a time-frequency grid composed of a number of consecutive Orthogonal Frequency Division Multiplexing (OFDM) symbols in the time domain and a group of subcarriers in the frequency domain. We consider the simplifying assumption that the inter-cell interference, i.e., the interference caused by the transmission of terminals from other cells that reuse the same frequency bandwidth, is added to the thermal noise in the the SNR expression. Note that this assumption becomes more and more valid as the system load, and therefore, the number of transmitting terminals and resource usage increase [8].

We consider that J terminals are candidates to get assigned N RBs in order fulfill their QoS requirements. $\mathcal{J}$ and $\mathcal{N}$ are the terminal and RB sets. We assume that the system operator provides different services to the connected terminals e.g., Voice over IP (VoIP) and file upload. We assume that there are S services in the system and $\mathcal{S}$ is the set of services. We consider that there are J_s terminals using service $s \in \mathcal{S}$ and $\mathcal{J}_s$ is the set of these terminals. Also, $\bigcup_{s\in\mathcal{S}} \mathcal{J}_s = \mathcal{J}$ and $\sum_{s\in\mathcal{S}} J_s = J$.

We assume that the channel state remains constant during the transmission in a Transmission Time Interval (TTI) and that, at the current TTI, the terminal j has a data rate requirement equal to t_j. The minimum satisfaction constraint for each service is represented by the parameter k_s which is the minimum number of terminals from service s that should be satisfied.

The SC-FDMA multiple access imposes two constraints on the resource assignment: resource exclusivity and resource adjacency. The former constraint assures that the same RB cannot be shared by multiple terminals within a cell. The resource adjacency constraint demands that the RBs assigned to a given terminal should be adjacent to each other in the frequency domain. This condition is needed in order to obtain low PAPR.

The adjacency constraint limits the total number of possible assignments in the system. In fact, the work [9] shows that the total number of resource assignment patterns that can be build with N RBs is given by $P = N^2/2 + N/2 + 1$, e.g., with $N = 4$ the possible group of RBs or assignment patterns are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1,2\}$, $\{2,3\}$, $\{3,4\}$, $\{1,2,3\}$, $\{2,3,4\}$, $\{1,2,3,4\}$ and $\{\emptyset\}$. We assume that $\mathcal{P}$ is a set with all resource assignment pattern indices. Accordingly, instead of modeling the assignment of a specific RB to a terminal, we consider in this work the assignment of a resource assignment pattern to a terminal. So, we define the matrix $\mathbf{A}$ whose element $a_{n,p}$ (with $n \in \mathcal{N}$ and $p \in \mathcal{P}$) assumes the value 1 if the RB n belongs to the assignment pattern p, and 0 otherwise.

In this article we consider the practical assumption of discrete MCS as mentioned in section I. We consider M levels of MCSs contained in set $\mathcal{M} = \{1, 2, 3, \cdots, M\}$. Therefore, when a terminal reaches the MCS level m, and so transmits with data rate r_m, it is required that the experienced SNR be equal to γ^m, where $\gamma^m < \gamma^{m+1}$ and $r^m < r^{m+1}$. Note that SC-FDMA imposes also two other constraints in order to assure low PAPR: the same MCS and the same transmit power level should be employed in all RBs assigned to a given terminal belonging to the same assignment pattern.

The SNR experienced in the link between the BS and the terminal j when transmitting on the z^{th} subcarrier of RB n, $\gamma_{j,z,n}$, is given by

$$\gamma_{j,z,n} = ((p_{j,p,m}/(c\cdot N_p))\cdot\alpha_j\cdot||h_{j,z,n}||^2)/\sigma^2 = p_{j,p,m}\cdot\overline{g}_{j,z,n}, \tag{1}$$

where $p_{j,p,m}$ is the power required for the j^{th} terminal using the p^{th} assignment pattern to achieve the m^{th} MCS level, c is the number of subcarriers per RB, N_p is the number of RBs of the assignment pattern p, α_j represents the contributions of path gain and shadowing in the link between the BS and terminal j, σ^2 is the noise power at the BS receiver in the bandwidth of a subcarrier, $h_{j,z,n}$ is the frequency response of the channel between the BS and the terminal j on the z^{th} subcarrier of RB n with $\|\cdot\|$ returning the absolute value of its argument, and finally, $\overline{g}_{j,z,n}$ is the total channel gain in the link between the BS and terminal j on z^{th} subcarrier of RB n normalized by thermal noise power. In this article we assume the optimization of power, so, we assume that the power cannot exceed the total power available at the user j, given by P_j^{tot}.

Due to the single-carrier nature of SC-FDMA scheme, the receiver should employ frequency domain equalization in order to combat Inter Symbol Interference (ISI). Assuming that Minimum Mean Square Error (MMSE) equalizer is used, the effective SNR perceived by the receiver when data is transmitted by terminal j on the RBs that belongs to assignment pattern p, $\gamma_{j,p}^{\text{MMSE}}$, is shown in the following [10]:

$$\gamma_{j,p}^{\text{MMSE}} = \left(\left(\frac{1}{c\cdot|\mathcal{N}_p|}\sum_{n\in\mathcal{N}_p}\sum_{z=1}^{c}\frac{\gamma_{j,z,n}}{\gamma_{j,z,n}+1}\right)^{-1} - 1\right)^{-1}, \tag{2}$$

where $\mathcal{N}_p$ is the set of RBs that compose the assignment pattern p and $|\cdot|$ denotes the cardinality of a set.

Let $f(\cdot)$ be the function that represents the discrete mapping between SNR and data rate according to the employed MCSs. The transmit data rate when terminal j transmits on the RBs that belong to the assignment pattern p with MCS m is given by $r_{j,p,m} = f(\gamma_{j,p}^{\text{MMSE}})$. In order to find the transmit power level, $p_{j,p,m}$, that should be employed by terminal j in all subcarriers that belong to assignment pattern p so as to transmit on MCS m, we should replace (1) in (2) and solve the equation $\gamma_{j,p}^{\text{MMSE}} = \gamma^m$.

III. Problem Formulation

The formulated problem in this work consists in minimizing the total transmit power in the uplink of an SC-FDMA system with per-service satisfaction guarantees. Let $\mathbf{X}$ be a binary matrix whose elements $x_{j,p,m}$ assume the value 1 if the terminal j using the assignment pattern p achieves the MCS level m, and 0 otherwise. According to this, we formulate the problem as follows

$$\min_{\mathbf{X}} \left(\sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} x_{j,p,m} \cdot p_{j,p,m} \right), \tag{3a}$$

subject to

$$\sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} a_{n,p} \cdot x_{j,p,m} = 1, \ \forall n \in \mathcal{N}, \tag{3b}$$

$$\sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} x_{j,p,m} = 1, \ \forall j \ \in \ \mathcal{J}, \tag{3c}$$

$$\sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} p_{j,p,m} \cdot x_{j,p,m} \leq P_j^{\text{tot}}, \ \forall j \ \in \ \mathcal{J}, \tag{3d}$$

$$x_{j,p,m} \in \{0,1\}, \ \forall j \in \mathcal{J}, \ \forall p \in \mathcal{P} \text{ e } \forall m \in \mathcal{M}, \tag{3e}$$

$$\sum_{j\in\mathcal{J}_s} u \left(\sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} r_{j,p,m} \cdot x_{j,p,m}, \ t_j \right) \geq k_s, \ \forall s \in \mathcal{S}, \tag{3f}$$

where $u(x, b)$ is a step function at b that assumes the value 1 if $x > b$ and 0 otherwise. The objective function shown in (3a) is the total uplink power transmitted by the terminals. Constraints (3b) and (3e) assure that RBs are not reused within the cell, while constraint (3c) guarantees that only one assignment pattern is chosen by each terminal. The constraint (3d) ensures that the power used by terminal j does not exceed the maximum available power. Finally, the constraint (3f) states that a minimum number of users should be satisfied with the demanded QoS for each service.

IV. Characterization of the Optimal Solution

The problem (3) belongs to the class of nonlinear combinatorial optimization problems. A general algorithm for solving those problems is the brute force method, however, the number of combinations scale very fast as J, P and M augments. Fortunately, according to [8], the problem (3) can be transformed into an ILP problem by adding a new optimization variable and replacing the constraint (3f) by two new constraints as follows:

$$\sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} r_{j,p,m} \cdot x_{j,p,m} \geq \rho_j \cdot t_j, \ \forall j \in \mathcal{J}. \tag{4a}$$

$$\sum_{j\in\mathcal{J}} \rho_j \geq k_s, \ \forall s \ \in \mathcal{S}. \tag{4b}$$

where ρ_j is a selection variable that assumes the value 1 if terminal j is selected to be satisfied and 0 otherwise.

Based on the previous development, we have transformed the non-linear version into an integer (binary) linear optimization problem. We have accomplished that at the cost of the addition of new optimization variables and constraints. This problem can be solved by standard methods such as the Branch-and-Bound (BB) algorithm [11].

V. Low-Complexity Heuristic Solution

Motivated by the high computational complexity to obtain the optimal solution to problem (3), in this section we present a low-complexity solution. The solution is divided into two parts: *RB Assignment* and *Power Allocation*. In the assignment part, the main idea is to select the terminal with better channel quality and assign the RBs fulfilling the adjacency constraint and the minimum number of satisfied terminals per service. The power allocation part consists basically in finding the minimum power required to fulfill the data rate requirements of the system. The flowchart of the proposed solution is shown in Figure 1 and described in details below.

As the main objective is to minimize the total transmit power in the system, the number of terminals transmitting should be kept as minimum as possible while guaranteeing the minimum satisfaction guarantees. Therefore, in step 1 for each service s we select the k_s terminals with better channel gains and initialize the set $\mathcal{J}^{\text{A}}$ with the selected terminals. In step 2 we initialize the set $\mathcal{N}^{\text{A}}$ with all RBs.

In step 3 we select the terminal $j^\star$ of the set $\mathcal{J}^{\text{A}}$ with the worst average channel gain. In step 4 we assign to terminal $j^\star$ its RB with highest channel gain, $n^\star$. The main idea of steps 3 e 4 is to initially provide a fair distribution of RBs and prioritize the terminals in worst channel conditions letting them choose their best (and possibly scarce) RBs in set $\mathcal{N}^{\text{A}}$. After this, we remove $j^\star$ and $n^\star$ of sets $\mathcal{J}^{\mathcal{A}}$ and $\mathcal{N}^{\text{A}}$, respectively (step 5). In step 6 we verify if the set $\mathcal{J}^{\text{A}}$ is empty. In the positive case, in step 7 we find the first available (not assigned yet) RB $n^\dagger$ with lowest index in set $\mathcal{N}^{\text{A}}$. In negative case, we return to step 3 and repeat the process described above. At end this process, all terminals selected in step 1 have received only one RB.

In step 8 we check if there is any assigned RB, $n^\star$, allocated to a terminal $j^\star$ on the left of RB $n^\dagger$. In the positive case, we build a new assignment pattern, denominated A, with RB $n^\dagger$ and the RBs assigned to terminal $j^\star$ (step 9). In the negative case, in step 10 we find the first RB n^+ that is assigned to a terminal j^+ on the right of RB $n^\star$, and we build an assignment pattern with all contiguous RBs between $n^\dagger$ and n^+, including them. After that, we assign the assignment pattern formed to terminal j^+ and remove all assigned RBs from the set $\mathcal{N}^{A}$ in step 17. In step 11 we check if there is an RB n^+ assigned to a terminal j^+ on the right of RB $n^\star$. If there is an assigned RB on the right of RB $n^\star$, we build a new assignment pattern, denominated B, with all contiguous RBs between $n^\dagger$ and n^+, including them, and assigned to the terminal j^+ in step 12. In the negative case, we associate the assignment pattern A to terminal $j^\star$ in step 13. After this, we execute step 17. In step 14 we evaluate which one of the built assignment patterns, A or B, have the highest effective SNR that is calculated according to equation (2) assuming that the maximum transmit power is equally distributed among the RBs. If the assignment pattern A has the highest effective SNR, the RBs of the pattern A are assigned to terminal $j^\star$ (step 15), otherwise, the RBs of pattern B are assigned to terminal j^+ (step 16) and the step 17

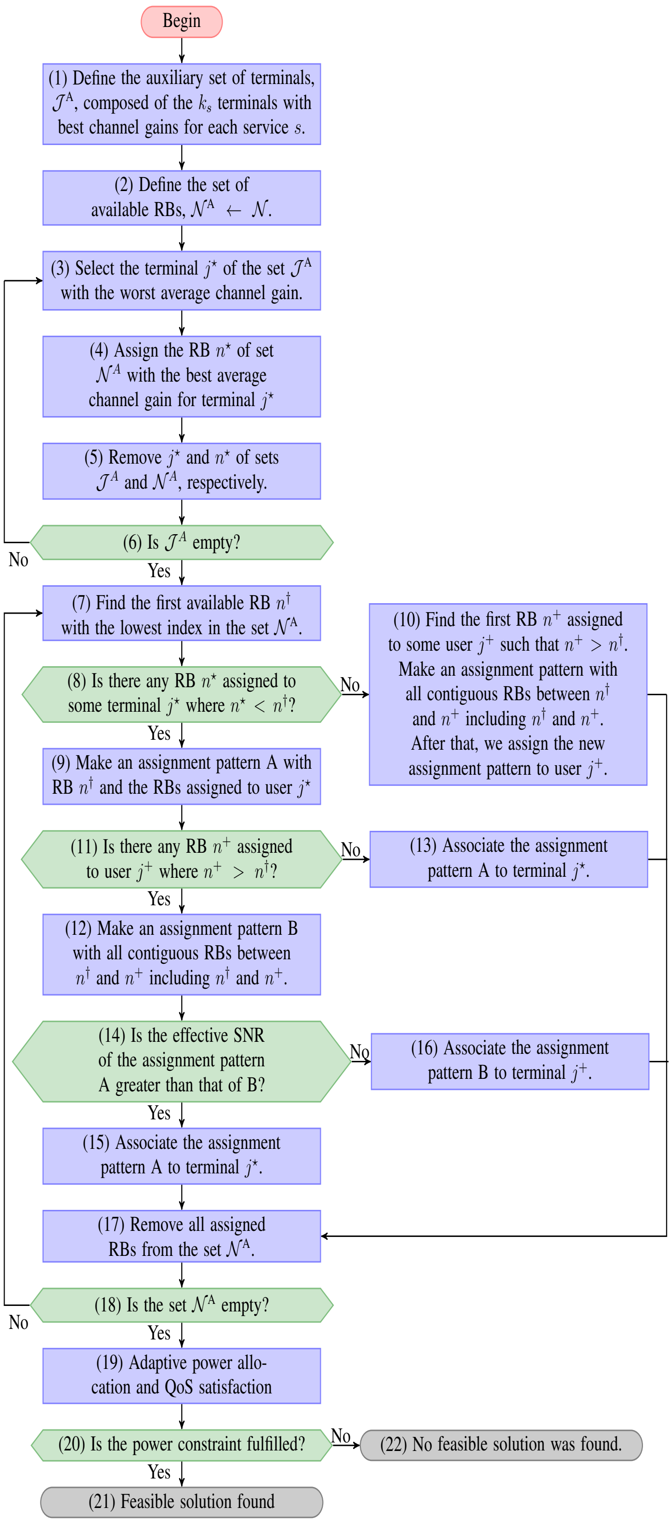


Fig. 1. Flowchart of proposed solution.

is executed. Finally, in step 18 we check if there are available RBs. If so, we return to step 7 and repeat the process again, otherwise, the assignment part is done.

In the power allocation part, we find the lowest power required to achieve the MCS level that fulfills the rate requirement of each user (step 19), and we verify if the power constraint is fulfilled (step 20). If so, the solution is found, otherwise, no solution is found (outage event). Note that an outage event can happen due to the suboptimality of the solution that is not able to explore the space of feasible solutions completely, or due to the fact that there is no feasible solution even to the optimal solution presented in section IV. Solutions to outage events are left as perspective of this study.

VI. Performance Evaluation

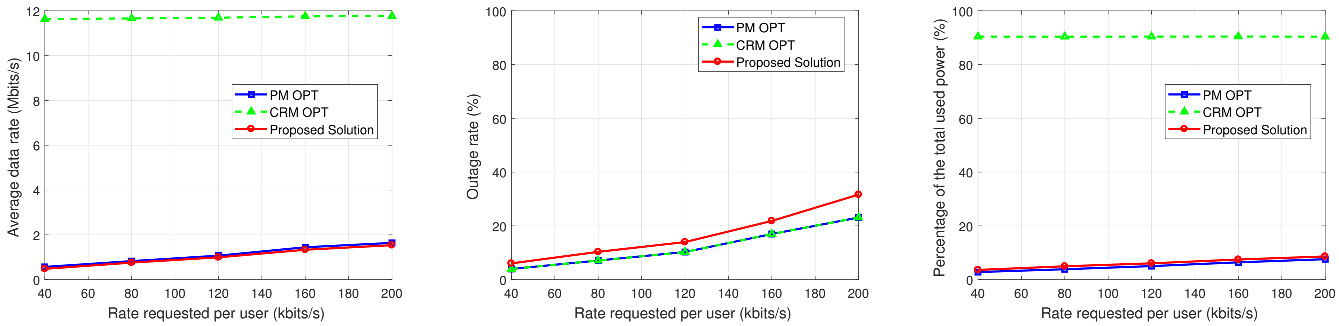
In this section we present the simulation setting and performance metrics in section VI-A, and the simulation results and analysis in section VI-B.

A. Simulation assumptions

The main aspects of an SC-FDMA uplink system presented in section II were considered in the computational simulator. We assume that an SC-FDMA RB consists of 12 adjacent subcarriers in the frequency domain and is 1 ms long in the time domain. The radio link state is modeled by the most important propagation mechanisms: distance-dependent path loss model given by $P_L(\text{dB}) = 35.3 + 36.7 \cdot log(d)$, log-normal shadowing component with standard deviation of 8 dB, and a Rayleigh-distributed fast fading component [8]. We assume that the link adaptation is realized based on the 15 indicators of Channel Quality Indicators (CQI) used by LTE [12]. The SNR thresholds for switching MCS levels were obtained from link level simulations from [13]. The available total power in the terminal is 24 dBm. The simulation methodology consists in applying the proposed solution in different realizations (or snapshots) by taking different samples of the random variables that model user positioning and channel state.

To evaluate the performance of the proposed solutions in this paper we consider a scenario where the system provides two services, i.e., $S = 2$, which has a total of eight active terminals, $J = 8$, equally distributed among the services, $J_1 = 4$ and $J_2 = 4$. We assume that four terminals should be satisfied for the service 1, $k_1 = 4$, and three terminals should be satisfied for the service 2, $k_2 = 3$. We consider that all terminals have the same data rate requirement, t_j. The system load is emulated through the variation of t_j from 40 kbps to 200 kbps. Moreover, we consider a total of 15 RBs, $N = 15$.

The algorithms simulated in this paper are the proposed solution presented in section V and identified in the plots by Proposed Solution, the optimal solution to the Power Minimization (PM) problem obtained by the method shown in section IV (identified in the plots as PM OPT) and the solution to the Constrained Rate Maximization (CRM) problem proposed in [8] which is the problem of data rate maximization with the QoS constraint present in (3f) but does not use adaptive power allocation (identified in the plots as CRM OPT). In order to solve ILP problems we used the IBM ILOG CPLEX Optimizer [14]. The choice of the values for J, N, M and S are limited by the computational complexity to obtain the optimal solutions.



(a) (b) (c)

Fig. 2. (a) Average data rate for the PM OPT and CRM OPT solutions and proposal solution. (b) Rate of *outage* for the PM OPT and CRM OPT solutions and proposal solution. (c) Percentage of the power used by the PM OPT and CRM OPT solutions and proposal solution. .

The main performance metrics used to evaluate the studied algorithms are the percentage of the total used power, which is in the ratio among the sum of the used transmit power by all terminals and the sum of the total available transmit power in all terminals ($J \cdot P_j^{tot}$); The outage rate defined as the ratio among the number of snapshots with outage events and the total number of snapshots. An outage occurs when a solution is not able to find a feasible solution which fulfills the constraints of problem (3). Finally, the last performance metric is the average data rate which is the average of the total data rate transmitted by all terminals in several snapshots.

B. Results

In Figure 2(a) we have the average total data rate versus the data rate required for all terminals for the PM OPT, CRM OPT and proposed solutions. Initially, we can see that the total data rate of PM OPT solution and proposal solution increases as the required data rate is augmented, since both solutions search only to satisfy the required data rate for each terminal in order to save transmit power. Furthermore, according Figure 2(a), we can observe that the CRM OPT solution presents the highest data rate. This result occurs because the CRM OPT solution has as objective to maximize the total data rate of the system.

In Figure 2(b) we present the outage rate versus the data rate required for all terminals to the same solutions presented in Figure 2(a). Firstly, we observe that the higher the terminal required data rate, the higher the outage rate is, as expected. Note that the outage rate of the PM OPT and CRM OPT solutions are the same for all required data rates since both solutions are obtained through ILP problems that have the same set of constraints. Finally, we see that the proposed solution presents a minimal variation of outage rate compared to the other solutions for moderated loads. When PM OPT and CRM OPT solutions present an outage rate of 10%, the proposed solution presents an outage rate of 14% resulting in a difference of only 4%.

A naive analysis of the results presented in Figure 2(a) and 2(b) can lead us to a wrong conclusion about the relevance of PM OPT and proposed solutions since CRM OPT solution presents low outage rate and higher transmit data rates than the other solutions. However, a complete analysis should consider Figure 2(c) where we show the percentage of the total used power versus the required data rate for all terminals to the PM OPT, CRM OPT and proposed solutions. Firstly, we observe that the higher the required data rate, the higher the total transmit power is, for the same reasons presented previously. While the CRM OPT solution in general allows the terminals to use almost all transmit power available in order to optimize the transmit data rate, the PM OPT and proposed solutions try to optimize the use of energy resources in order to guarantee QoS and user satisfaction. According to Figure 2(c), the CRM OPT solution presents a power consumption of approximately 90%, while the PM OPT and proposed solution present a consumption of approximately 3% and 4%, respectively, for a required data rate of 40 kbps.

A joint analysis of Figures 2(a), 2(b) and 2(c) lead us to the conclusion that the solution PM OPT is able to satisfy the QoS and minimum number of users per service with the lowest total transmit power. A low transmit power provides several advantages in wireless communications systems such as the reduction in intercellular interference in the system. Moreover, efficient power consumption is one of the pillars of 5G. However, the CRM OPT and PM OPT solutions are obtained through algorithms with exponential computational complexity, as can be seen in section A of the appendix. On the other hand, as demonstrated in section B of the appendix, the computational complexity of the proposed solution is polynomial given by $\mathcal{O}\left(\left(N - \sum_{s \in \mathcal{S}} k_s\right)^2\right)$.

VII. Conclusions and Perspective

In this article, we studied the total transmit power minimization problem considering the adjacency and QoS constraints in SC-FDMA systems such as LTE uplink. The formulated problem consists in a nonlinear combinatorial optimization problem. Due to the high computational complexity of this problem, we converted it into an ILP problem through the addition of new optimization variables and constraints. The ILP problem can be solved by conventional methods, however, at a high computational cost. Therefore, a low-complexity suboptimal solution was proposed and through computational simulations, we have shown that it presents a small performance degradation compared to the optimal solution. The optimization of other energy efficiency metrics in SC-FDMA systems is a perspective of this study.

APPENDIX
COMPUTATIONAL COMPLEXITY

The worst-case computational complexity of the ILP-based solutions CRM OPT and PM OPT, as well as the proposed solution are presented in sections A and B, respectively.

The computational complexity considered in the following is the worst-case one that gives an upper bound on the computational resources required by an algorithm and is represented by the asymptotic notation $\mathcal{O}(\cdot)$. As in [8], we consider summations, multiplications and comparisons as the most relevant and time-consuming operations.

A. CRM OPT and PM OPT

The CRM OPT and PM OPT solutions are obtained by optimally solving ILP problems with BB algorithm. For an arbitrary number of integer variables l, the number of linear programming subproblems that will be solved is $(\sqrt{2})^l$ [8]. However, the number of interations needed to solve a linear programming problem with m constraints and l variables is approximately $2(m-l)$, and each interation contains $(lm-m)$ multiplications, $(lm-m)$ sums and $(l-m)$ comparisons [8]. In the ILP problem associated to CRM OPT solution, we have $JP+J$ integer variables and $2J+N+S$ constraints, therefore, the total number of operations needed is

$$\sqrt{2}^{JP} 2(JP+J+N)(2(JP)(J+N)-3(J+N)+JP).$$

The ILP problem associated to the PM OPT solution have $JPM+J$ variables and $3J+N+S$ constraints, therefore, the total number of operations is

$$\begin{aligned}\sqrt{2}^{JPM+J} & 2(JPM+4J+N+S)(2(JPM+J) \\ & (3J+N+S)-3(3J+N+S)+JPM+J).\end{aligned}$$

Keeping only the high order terms, we have that the worst-case computational complexity of the CRM OPT and PM OPT solutions are, respectively, $\mathcal{O}(2^{JP})$ and $\mathcal{O}(2^{JPM})$.

B. Low-Complexity Heuristic Solution

So as to calculate the computational complexity of the proposed solution, we should identify the instance of the algorithm shown in Figure 1 that renders the highest number of operations. The most time-consuming step in this algorithm is the calculation of the effective SNR in step 14. The worst-case instance of the proposed algorithm is when all terminals get adjacent RBs after step 6, and the effective SNR of assignment A is always higher than the one of assignment B. Therefore, the number of interations of the main loop is $N - \sum_{s\in\mathcal{S}} k_s$. Moreover, in the end, a single receiver terminal will get a set of contiguous RBs of size $N - \sum_{s\in\mathcal{S}} k_s + 1$ while the other terminals would stay with only one RB.

According to this, the number total of operations of the proposed solution is

$$\left(N - \sum_{s\in\mathcal{S}} k_s\right)\left(2cN + 2\sum_{s\in\mathcal{S}} k_s + N + 3\right) + \sum_{s\in\mathcal{S}} k_s$$

summations,

$$\left(N - \sum_{s\in\mathcal{S}} k_s\right)\left(2cN + 20\sum_{s\in\mathcal{S}} k_s - 20\right)$$

multiplications and

$$\begin{aligned}\left(N - \sum_{s\in\mathcal{S}} k_s\right)&\left(2\left(N - \sum_{s\in\mathcal{S}} k_s\right) + \sum_{s\in\mathcal{S}} k_s + 2N + 3\right) \\ &+ \sum_{s\in\mathcal{S}} k_s\,(2N + M + J) + JS\end{aligned}$$

comparisons. Retaining the term of higher order, we get a computational complexity given by $\mathcal{O}\left(\left(N - \sum_{s\in\mathcal{S}} k_s\right)^2\right)$.

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REFERENCES

[1] Ericsson, "Ericsson mobility report: on the pulse of the networked society," whitepaper, June 2017.

[2] H. Yang, F. Ren, C. Lin, and J. Zhang, "Frequency-domain packet scheduling for 3GPP LTE uplink," in *INFOCOM, 2010 Proceedings IEEE*. IEEE, 2010, pp. 1–9.

[3] J. Lim, H. G. Myung, K. Oh, and D. J. Goodman, "Channel-dependent scheduling of uplink single carrier FDMA systems," in *IEEE Vehicular technology conference*. IEEE, 2006, pp. 1–5.

[4] O. Nwamadi, X. Zhu, and A. Nandi, "Dynamic subcarrier allocation for single carrier-FDMA systems," in *Signal Processing Conference, 2008 16th European*. IEEE, 2008, pp. 1–5.

[5] A. Ahmad and M. Assaad, "Power efficient resource allocation in uplink SC-FDMA systems," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2011 IEEE 22nd International Symposium on*. IEEE, 2011, pp. 1351–1355.

[6] D. Triantafyllopoulou, K. Kollias, and K. Moessner, "QoS and energy efficient resource allocation in uplink SC-FDMA systems," *IEEE Transactions on Wireless Communications*, vol. 14, no. 6, pp. 3033–3045, 2015.

[7] F. A. Orakzai, A. Ahmad, M. T. R. Khan, and M. Iqbal, "Optimal energy-efficient resource allocation in uplink sc-fdma networks," *Transactions on Emerging Telecommunications Technologies*, 2017.

[8] F. R. M. Lima, T. F. Maciel, and F. R. P. Cavalcanti, "Radio resource allocation in SC-FDMA uplink with resource adjacency constraints," *Journal of Communication and Information Systems*, vol. 31, no. 1, 2016.

[9] I. C. Wong, O. Oteri, and W. McCoy, "Optimal resource allocation in uplink sc-fdma systems," *IEEE Transactions on Wireless communications*, vol. 8, no. 5, 2009.

[10] 3GPP, "Simulation methodology for EUTRA UL: IFDMA and DFT-Spread-OFDMA," Third Generation Partnership Project, Tech. Rep. R1 050718, August 2005.

[11] G. L. Nemhauser and L. A. Wolsey, "Integer programming and combinatorial optimization," 1988.

[12] Z. Bai, "Evolved universal terrestrial radio access (e-utra); physical layer procedures," *3GPP, Sophia Antipolis, Technical Specification 36.213 v. 11.4. 0*, 2013.

[13] C. Mehlführer, M. Wrulich, J. C. Ikuno, D. Bosanska, and M. Rupp, "Simulating the long term evolution physical layer," in *Signal Processing Conference, 2009 17th European*. IEEE, 2009, pp. 1471–1478.

[14] I. ILOG, "Cplex optimization studio," *URL: http://www-01. ibm. com/software/commerce/optimization/cplex-optimizer*, 2014.