Clustering-based Assignment within CoMP Systems

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***Abstract*—More and more Coordinated Multi-Point (CoMP) systems are attracting interest as a means to improve the performance of conventional cellular networks, particularly thanks to the joint processing approach. However, the large amount of signaling and computer operations required deserves special attention regarding both backhaul and radio resources constraints. Partitioning the set of transmission points into clusters is likely to reduce this coordination cost, but the search for the best formations of clusters may lead to expressive further computational efforts. Herein we introduce a novel approach, the Clustering-based Assignment Algorithm (CbAA), to cluster dynamically the transmission points on the downlink in order to transmit coherently to multiple users. When compared to static clustering, the proposed CbAA provided expressive gains in terms of system spectral efficiency. Compared to state-of-the-art dynamic clustering approaches, CbAA can waive the exhaustive search over all possible formations as well as the repetitive precoder calculations during the clustering procedure.**

***Index Terms*—Clustering. Coordinated Multi-Point System. Scheduling.**

I. Introduction

In the last years, advanced multiantenna systems have been attracting a lot of interest as a means to improve the spectral efficiency of conventional cellular networks. This is the case of Coordinated Multi-Point (CoMP) systems in the context of 3rd Generation Partnership Project (3GPP) Long Term Evolution - Advanced (LTE-A). By using Joint Processing (JP) [1] in CoMP systems, multiple Base Stations (BSs) work together as a distributed antenna array such that spatial multiplexing techniques can be employed for transmitting to multiple User Equipments (UEs).

Nonetheless, there is a significant processing and signaling cost associated to the coordination among BSs. Reliable estimates of all complex channels between every Transmission Point (TP) and every UE are required for each CoMP-cell.In addition, the large channel matrices will burden the central processing unit (CPU) on calculating the precoders, given the usual dependence on expensive matrix inversion operations.

A known approach to reduce this coordination cost consists on decreasing the number of TPs to be coordinated [2]. Through clustering, the set of all available TPs belonging to a CoMP-cell can be partitioned into non-overlapping clusters. Therefrom, this cluster of TPs is assigned to a group of UEs. Thus, each cluster establishes a kind of macroscopic Multiple Input Multiple Output (MIMO) scheme, servicing the UEs within its coverage area.

Increasingly, clustering has been seen as a friendly approach in overcoming some practical challenges on exploiting CoMP [3]. The clustering approaches can be classified into static [4] and dynamic [5], depending on how often the TP coordination is updated. The dynamic approach is preferable especially when temporal variations of the radio channel are considered, however, the instantaneous adaptation to multiple links and interference changes relies on feasibility issues. The greedy algorithm introduced in [5] depends on a broad and sequential combination of candidate TPs, favoring the clusters formed earlier. Furthermore, at each combination tested, the precoders have to be computed in order to predict the sum-rate that is possible to be achieved. As a different approach, the algorithm proposed in [2] is based on a long-term Channel State Information (CSI), requiring a search over all possible cluster formations. Baracca et al. [6] proposed to restrict the search space in order to shorten the search for the solution, even so, the algorithm is still considered greedy.

Based on the mathematical concept of cluster analysis [7], we herein propose the Clustering-based Assignment Algorithm (CbAA), which makes UE-centered choices for determining a common set of TPs to service a group of UEs. By employing CbAA, the exhaustive search over all possible cluster formations and the repetitive calculation of precoders during the clustering procedure are waived.

The rest of this paper is organized as follows: section II describes the adopted system model; section III introduces the clustering concept applied to the assignment problem; section IV presents the simulation approach and analyzes the obtained results; finally, in section V, some remarks and conclusions are drawn.

II. System Model

The CoMP system herein investigated has a number C of CoMP-cells, indicated by $c = 1, 2, \ldots, C$, each of them with a CPU controlling a number B of BSs, through a fast backhaul. In its turn, the area associated to a BS is referenced as BS-cell, which is tri-sectored. Each hexagonal sector is serviced by its own single-antenna TP; the TPs related to a same BS are at the corner shared by the three sectors. Thus, a total of M TPs is geographically distributed in each CoMP-cell, indicated by $m = 1, 2, \ldots, M$.

We focus on the downlink transmission, wherein each CoMP-cell services a number J of single-antenna UEs, indicated by $j = 1, 2, \ldots, J$, which are uniformly distributed over the coverage area. In addition, each of K clusters comprises a disjoint subset of all available TPs.

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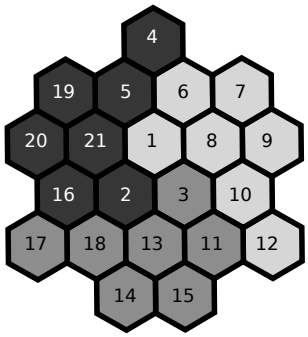

Figure 1. CoMP-cell and the referenced configuration of three static clusters.

Without loss of generality, let us consider $B = 7$ BSs and $K = 3$ clusters. On the conventional case for CoMP systems, the coordination is established among all the $M = 21$ available TPs. Alternatively, the M TPs can be partitioned into three disjoint subsets. Figure 1 exemplifies a single CoMP-cell with a configuration of these three clusters distinguished by the gray tones, where all TPs are enabled. That snapshot will be taken as our static formation of clusters. Later on, we will introduce our approach for the dynamic formation of clusters.

The considered downlink employs Orthogonal Frequency Division Multiple Access (OFDMA), with equal power allocated among the S subcarriers. The subcarriers are grouped in blocks of $\check{S}$ adjacent subcarriers as the Physical Resource Blocks (PRBs) [8]. The PRBs are indicated by $n = 1, 2, \ldots, N$, and each of them might be assigned to one or more $\langle$TP, UE$\rangle$ pairs within each CoMP-cell.

The channel gain, $G_{j,m,c}$ in dB, from TP m of CoMP-cell c to UE j is composed of average path loss, shadowing, antenna gain and short-term fading components. The channel coherence bandwidth is assumed larger than the PRB bandwidth.

The link adaptation searches for the modulation scheme – among Binary Phase-Shift Keying (BPSK), 4-, 16- and 64-Quadrature Amplitude Modulation (QAM) – that yields the maximum throughput under the current Signal to Interference-plus-Noise Ratio (SINR) value, by resorting to the Gaussian approximation for the total received interference. Furthermore, based on the Calvo's approximation [9], TPs do not transmit to those UEs under an SINR value below 5.57 dB, since the packet error rate would be too high. We assume that SINR estimations are available to the CoMP-cells, and that each CPU has perfect channel knowledge about all UEs associated with the TPs it controls.

III. Cluster Analysis

Clustering is an unsupervised method for assigning a set of observations into mutually exclusive subsets or clusters [7]. Observations assigned to a same cluster have some kind of similarity among themselves; observations belonging to different clusters should be as dissimilar as possible.

Let $\mathcal{M} = \{1, 2, \cdots, M\}$ be the active set comprising all the TPs belonging to a CoMP-cell c. As the following formulation is restricted to a single CoMP-cell c and PRB n, henceforth we omit the indexes c and n for simplicity of notation. Let $\mathbf{v}_j$ be the strength vector with length M associated to UE j, whose elements are given by the channel gain $G_{j,m}$. The strength vector of each UE is taken as a simple observation in the $\mathbb{R}^M$ space, and, therefore, we have J observations to be partitioned into K clusters per CoMP-cell.

Let $\mathcal{V}$ be the set with the strength vectors v_j for every $j \in \mathcal{J}$. Every clustering will yield subsets of these observations, $\mathcal{V}_k$, for $k = 1, 2, \cdots, K$, disjoint of each other, so that $\mathcal{V} = \bigcup_{k=1}^{K} \mathcal{V}_k$ and $\mathcal{V}_{k_1} \cap \mathcal{V}_{k_2} = \varnothing$, for all $\mathcal{V}_{k_1}, \mathcal{V}_{k_2} \in \mathcal{V} \mid k_1 \neq k_2$. In addition, we can restrict the number of clusters to remain intact throughout the procedure, by assuring that $\mathcal{V}_k \neq \varnothing,\ \forall k \in \mathcal{K}$. A cluster formation instance is denoted as $\mathcal{S} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_K\}$. The cardinalities $|\mathcal{V}_k|$ are unrelated.

We based on the *k-means clustering problem*, wherein the clusters are formed to minimize the within-cluster sum of squares of the distances to their centroids:

$$\min_{\mathcal{S}} \sum_{k} \sum_{\mathbf{v}_j \in \mathcal{V}_k} \left\| \mathbf{v}_j - \bar{\mathbf{v}}_k \right\|_2^2, \tag{1}$$

in which $\| \cdot \|_2$ denotes Euclidean norm, and $\bar{\mathbf{v}}_k$ is the kth prototype vector obtained from the calculation of the centroid over all the strength vectors associated to cluster k, i.e.

$$\bar{\mathbf{v}}_k = \frac{1}{|\mathcal{V}_k|} \sum_{\mathbf{v}_j \in \mathcal{V}_k} \mathbf{v}_j. \tag{2}$$

Each prototype vector $\bar{\mathbf{v}}_k$ is an approximate strength vector valid to all the related UEs. The closer the strength vectors of all UEs of a cluster are, the more representative for them the prototype vector will be. The algorithm employed to form a given cluster k has to look for UEs that can be serviced with an adequate level of quality by this same subset of TPs.

Steps (1) and (2) are iteratively performed to solve that problem. A stop criterion is met when a maximum number of iterations is achieved. Once the clusters are properly formed, we compare the K prototype vectors and assign each TP to the cluster in which it is strongest, i.e.

$$k_m = \arg\max_{k} \left\{ \bar{\mathbf{v}}_k(m) \right\}, \tag{3}$$

thereby the subset $\mathcal{M}_k$ will be given by $\mathcal{M}_k = \{m : k_m = k\}$ for every $k \in \mathcal{K}$. The number M_k of TPs assigned to each cluster k is independent and time variant, but in the average it tends to M/K.

Once the subsets $\mathcal{M}_k$ and $\mathcal{J}_k$ are properly determined, we may opt to enable all clusters or parts thereof, defining the *full selection* and *partial selection*, respectively. The notation $\check{K}/K$ clusters expresses the partial selection of clusters. This means that the 21 available TPs in a CoMP-cell are partitioned into K clusters, but just $\check{K} \leq K$ clusters will be enabled simultaneously. Also note that 2 and 3 clusters might be alternatively expressed as $^2/_2$ and $^3/_3$ clusters, respectively. The way we choose the $\check{K}$ clusters is a strategy apart. Let us initially assume that the choice of $\check{K}$ among the K clusters is made at random.

The M_k TPs of cluster k will be employed to perform linear Zero-Forcing (ZF) precoding and service M_k UEs,

as the largest supported population size. Because of that, we run into the subproblem of choosing which M_k UEs will be preferentially serviced. That subproblem was already addressed in [10], where the Best Fit (BF) algorithm proved to be an adequate approach, albeit suboptimal. Based on the CSI available at the CPU, we select first the UE with the highest channel gain for each PRB. The BF algorithm finds the most spatially compatible UE with respect to the previously admitted ones. This is performed successively until the group of M_k UEs is completed. The CbAA is summarized in Algorithm 1, wherein the kmeans function takes all available strength vectors $\mathbf{v}_j$ and the number K of clusters to perform the k-means algorithm; it returns the prototype vectors $\bar{\mathbf{v}}_k, k = 1, 2, \cdots, K$, as well as c_j for $j = 1, 2, \cdots, J$, which represents the index of the cluster associated to UE j. The cluster q_m supposed to use the mth TP is that one whose mth prototype entry has the largest value, which is achieved by finding the position of the largest value row-by-row in $[\bar{\mathbf{v}}_1\ \bar{\mathbf{v}}_2 \cdots \bar{\mathbf{v}}_K]$ through the $\arg\max_{\mathrm{r}}$ operator. Now the set $\mathcal{M}_k$ of TPs and the set $\mathcal{J}_k$ of UEs assigned to each cluster $k = 1, 2, \cdots, K$ are known. The $\mathrm{bestfit}$ function takes the complex channel coefficients related to all possible connections between TPs in $\mathcal{M}_k$ and UEs in $\mathcal{J}_k$ comprised in the kth cluster as well as the number J_k of UEs to be scheduled; $\mathrm{bestfit}$ returns the $J_k^\star$ most spatially compatible UEs found, assuming $J_k^\star = |\mathcal{M}_k|$.

Algorithm 1. Clustering-based Assignment.

$([\bar{\mathbf{v}}_1\ \bar{\mathbf{v}}_2 \cdots \bar{\mathbf{v}}_K], [c_1\ c_2 \cdots c_J]) \leftarrow \mathrm{kmeans}([\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_J], K)$
$[q_1\ q_2 \cdots q_M] \leftarrow \arg\max_r \{[\bar{\mathbf{v}}_1\ \bar{\mathbf{v}}_2 \cdots \bar{\mathbf{v}}_K]\}$
for $k = 1$ to K **do**
  $\mathcal{M}_k \leftarrow \mathrm{find}([q_1\ q_2 \cdots q_M] = k)$
  $\mathcal{J}_k \leftarrow \mathrm{find}([c_1\ c_2 \cdots c_J] = k)$
  $\mathcal{J}_k^\star \leftarrow \mathrm{bestfit}(h_{j,m,c,n}, J_k), \quad \forall m \in \mathcal{M}_k, \forall j \in \mathcal{J}_k$
end for

By employing clustering in this way, just the absolute gains must be estimated. Once clustering is processed, a decrease in the amount of the required signaling is perceived, if compared to the conventional assignment. In general, it is required to report channel estimates for all the involved UEs, leading to a complex channel matrix with dimensions $J_k \times M_k$. Therefore, the relative amount of signaling required in relation to the conventional case, Ψ, is given by:

$$\Psi = \frac{\psi + \sum_{k \in \mathcal{K}} 2 J_k M_k}{2JM}, \tag{4}$$

in which ψ refers to the amount of signaling that may be required for the formation of clusters. Let use assume that these TPs are equally partitioned, i.e. $M_k = M/K$ and $J_k = J/K$, as well as all absolute gains are required to form the clusters, i.e. $\psi = JM$; therefore, the relative amount of signaling is approximated as $\Psi \approx \frac{K+2}{2K}$. Any information of any link *crossing* the clusters – i.e. any link between a given TP and a UE assigned to another TP – can be simply neglected.

Similar benefits are obtained regarding the processing demanded to apply some precoding technique within every cluster. Let us assume that computing the ZF precoder involves a number of matrix inversion operations, whose complexity scales up cubically with the cluster size [11]. Considering a value for maximum number of iterations for the k-means algorithm as low as 100, the time complexity due to (1) and (2) can be neglected on the overall time complexity triggered by the CbAA. So, the alleviation of the computational complexity promoted through CbAA, again in relation to the conventional case, can be expressed as:

$$\kappa = \frac{\sum_{k \in \mathcal{K}} M_k^3}{M^3}, \tag{5}$$

which can further be approximated as $\kappa \approx \frac{1}{K^2}$, for $M_k = M/K$. Under partial selection, the alleviation on the computational efforts can even reach $\kappa \approx \frac{\check{K}}{K^3}$.

IV. Results and Analyses

System-level simulations have been executed based on the models before described in order to evaluate the CbAA performance. A set of $B = 7$ BS-cells composes a CoMP-cell, and a set of $C = 7$ CoMP-cells composes the system. Thus, a total of $M = 21$ TPs is available to each CoMP-cell. Furthermore, a wrap-around approach is used to avoid border effects. The maximal diameter D of the hexagon representing each sector is equal to 334 m. The link budget is designed such that the UEs at the cell edge experience at least 5.57 dB SNRs, since the interference is neglected at this point. On top of that, it is assumed that UEs make use of a non-real time service – which neglects strict packet delay requirements – and always have data to receive.

The models adopted for the channel components are specified as follows: the power-delay profile is TU from [12]; the average path loss is $35.3 + 37.6 \log_{10}(d)$ [13], where the variable d is the distance in meters of a UE from a given TP; the antenna gain is $14 - \min\left\{12\left(\frac{\theta}{70}\right)^2, 20\right\}$ [14], where θ is the azimuth in degrees; the shadowing is simulated through a log-normal random variable; and the short-term fading is modeled by the Jakes' method [15].

For the shadowing, a standard deviation of $\sigma_{\mathrm{sh}} = 8$ dB is considered. Short-term fading assumes an average UE speed of 3 km/h. The CoMP system considers a carrier frequency of 2 GHz and $N = 25$ PRBs, each composed of $\tilde{S} = 12$ subcarriers spaced of 15 kHz. On each subcarrier, 14 symbols are transmitted per Transmission Time Interval (TTI) (of 1 ms). Moreover, we assume perfect knowledge at the CPU about the channels of all links within the CoMP-cell.

Simulations are organized in snapshots, each taking 1 s of the system behavior. In each snapshot, path loss and shadowing are assumed to remain constant, but the temporal variations of short-term fading are considered. The results over several snapshots are taken into account such that the confidence interval at a 90% level can be estimated. The spectral efficiency is the main performance metric to be analyzed, which does not take into account those UEs left out on the scheduling, presented as a function of the offered load, which in its turn means the number of UEs physically present, regardless of whether they are scheduled or not.

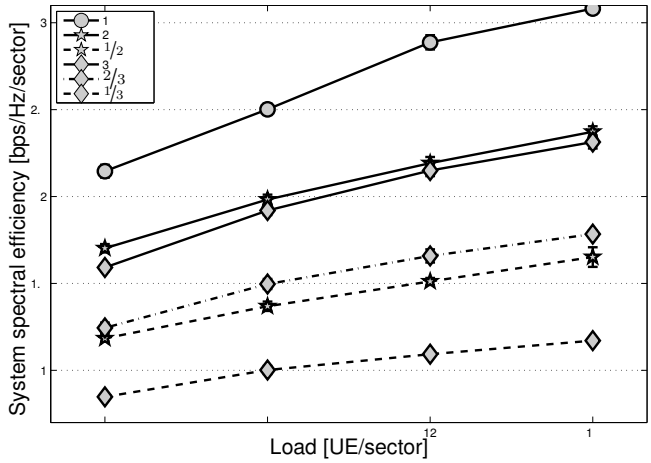

Figure 2. System spectral efficiency for conventional case, 2 or 3 clusters and their variants of partial selection.

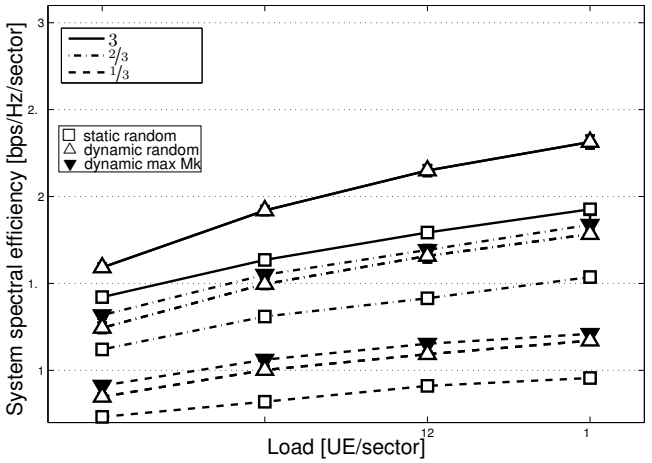

Figure 3. System spectral efficiency for 3-cluster configurations, full and partial selections, under different UE assignment approaches.

Figure 2 shows the system spectral efficiency versus offered load for conventional assignment alongside several clustering configurations: 2 or 3 clusters and their variants named $1/2$, $1/3$ and $2/3$ clusters. The system spectral efficiency is reduced as we increase the number of clusters. We note a decrease of about 22.1% in system spectral efficiency when comparing the conventional case to the 2 clusters case for all evaluated loads, and about 24.9% when comparing the conventional case with the 3 clusters case. From 2 clusters to 3, we see a decrease of about 3.5% in system spectral efficiency. This degradation is mainly due to the generation of more interference for each additional cluster. In fact, ZF mitigates the cluster-internal interference, but not the interference of one cluster on another. This way, clustering ends up reinforcing the system interference, which is the main cost from reducing processing and signaling. Table I gathers the average values of the total interference suffered by the system at a load of six UEs per sector.

The larger the number of clusters formed in full configuration, the higher the interference. We also observe that the interference can be alleviated through the partial selection of clusters. As the clusters are dropped, interference power can reach even lower levels than in full configurations with less clusters (including the conventional case). However, the benefit from diminishing the interference shows itself to be inefficient on overtaking the loss in coverage and capacity for each cluster dropped, leading to a further reduction in the system spectral efficiency, as also noted in Figure 2: from 2 clusters to $1/2$ configuration, the system spectral efficiency decreases of about 30.7%; from 3 clusters to $1/3$, about 48.3%; and from 3 clusters to $2/3$, about 22.5%.

Table I
AVERAGE INTERFERENCE SUFFERED BY THE SYSTEM FOR SIX UES PER SECTOR AND SEVERAL CLUSTERING CONFIGURATIONS.

Clustering configuration	Power (dBmW)
conventional	−85
2	−79
$1/2$	−86
3	−77
$2/3$	−79
$1/3$	−88

Another interesting aspect is the performance gain due to dynamic clustering. The system spectral efficiency of the static configuration of Figure 1 was calculated and compared with the one of dynamic clustering, regarding full and partial selections of 3-cluster configurations with random choice of clusters. These results are presented in Figure 3 as the curves with unfilled markers. The performance gain due to dynamic clustering is notorious for all the evaluated loads and cluster configurations, ranging from 11% to 20%.

In addition, we can rate the impact of another criterion for choosing the clusters on the partial selection. The curves with filled triangle markers represent the performance for the *max Mk* criterion, which prioritizes the selection of those clusters with the largest number M_k of TPs available for coordinated transmission. The gain in terms of spectral efficiency is perceived for all the loads and cluster configurations evaluated. At $1/3$ configuration the average gain is of 5.6%; at $2/3$ this is roughly 3.8%.

In spite of the performance loss, CbAA allows to reduce the signaling costs and joint processing demands. For 3-cluster configurations, one has to estimate three 7×7 matrices in place of a single 21×21 matrix; thus clustering reduces the signaling amount to $\psi \approx 83\%$ of all estimates. Furthermore, the number of floating-point operations required for matrix inversions through employing CbAA will be decreased to $\kappa \approx \frac{1}{9}$ in the case of full selection, or $\frac{1}{21}$ for the $1/3$ configuration.

The stack bar chart in Figure 4 shows the number of times each TP participates in the formation of each cluster for the configuration of 3 clusters with full selection. After storing the formation of all three clusters at every observation – TTI and PRB –, wherein each formation is represented as a vector $\mathbf{d} \in \mathbb{R}^M$, the k-means algorithm was employed again, now to group similar formations into clusters α, β and γ. The elements of $\mathbf{d}$ are set to 0 or 1 depending on whether the respective TP was enabled at that observation: $\mathbf{d}(m) = 1$ if $m \in \bigcup_{k=1}^{K} \mathcal{M}_k$,

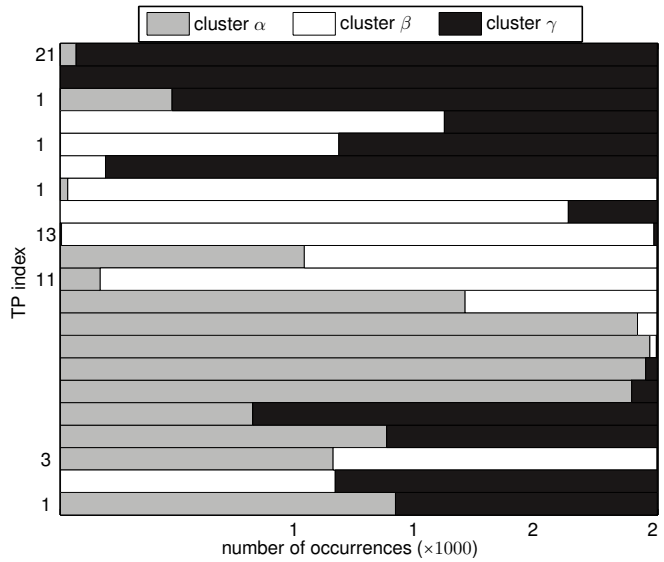

Figure 4. Occurrence of every TP forming every cluster in the configuration of 3 clusters with full selection.

otherwise $\mathbf{d}(m) = 0$. Every TP is always employed, however it is allocated more often to one cluster than to others, as well as each cluster is predominantly composed by a given subset of TPs. At least in 60% of the observations, cluster α was composed of the TPs whose indexes are 6 to 10; cluster β of the TPs 11, 13, 14, 15 and 18; and cluster γ of the TPs 5, 16, 19, 20 and 21; the remaining TPs indexes are 1 to 4, 12 and 17, which indistinctly and eventually take part of any one among two or three clusters.

V. Conclusions and Final Remarks

In this work, we investigated the CoMP-system performance with a constrained set of coordination. We introduced the Clustering-based Assignment Algorithm (CbAA) as a dynamic strategy for forming the clusters. The set of all available TPs was partitioned into mutually exclusive clusters and, later, linear precoding was applied to each of them. We employed a typical cluster analysis tool – the *k-means* algorithm – to incorporate an assignment strategy among TPs and UEs.

Compared to conventional assignment for CoMP systems, CbAA proved to be attractive in terms of reduction of both required signaling and computational effort. After clusters are formed, each transmission filter will need channel knowledge only with regard to the respective cluster. Any information about other clusters or even the links *crossing* the clusters can be disregarded. When compared to static clustering, our computer results showed that the performance gains obtained from CbAA are expressive in terms of system spectral efficiency. Compared to state-of-the-art dynamic clustering approaches, CbAA is attractive since it does not require an exhaustive search over all cluster formations as well as repetitive precoder calculations.

On the other hand, the clustering strategy tends to increase the interference within the system and so jeopardize the system spectral efficiency. The formation and placement of the clusters becomes a crucial issue for dictating whether the interference reinforcement will be large or small, whereas the signaling reduction can be obtained just by the sizes of the clusters. We explored the partial selection of clusters, which may be more suitable for interference-limited scenarios. Even though the system spectral efficiency is degraded even more, the interference is drastically reduced.

All in all, CbAA shows itself as an adequate approach to make use of TPs coordination under constraints on the backhaul and/or on the computing power of the CPU, if the loss in performance is an affordable cost. The final decision whether its adoption can actually be advantageous is tightly dependent on the characteristics of each intended application.

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