Graph-based Power-Efficient Beam Sweep for Initial Synchronization

Igor M. Guerreiro*, Johan Axnäs†, Dennis Hui‡ and Charles C. Cavalcante*
*Wireless Telecomm. Research Group (GTEL), Universidade Federal do Ceará (UFC), Fortaleza, Brazil
†Ericsson Research, Stockholm, Sweden
‡Ericsson Research, San José, California, USA

***Abstract*—This paper addresses the problem of initial synchronization of users in an indoor mm-Wave scenario. By using a massive number of antenna elements at access nodes, the resulting beams have narrow beamwidth. However, the transmission of individual narrow beams may cause poor coverage in some areas as the energy is concentrated over the direction of their main lobes. To cope with that, a beam sweep procedure using phased arrays is adopted. Access nodes simultaneously transmit individual beams until a certain area of interest is thoroughly scanned. The goal is to find the minimum power setting by adjusting the individual power levels so that users over the scanned area can observe a minimum received power level. The problem is formulated as a total consumed power minimization, suitably modeled for the standard min-sum algorithm. The proposed graph-based algorithm features some modifications in the message computation to decrease computational complexity, and adopts a random message-passing scheduling to deal with convergence issues. Simulation results indicate that the proposed algorithm usually outperforms a baseline iterative one, consuming about 13% less power in a typical simulation setup adopted.**

I. Introduction

The millimeter wave (mm-Wave) band [1] (e.g. 60 GHz) is expected to provide many benefits for fifth-generation (5G) systems. The small wavelength allows the transceivers to have a more compact hardware due to the fact that the antenna element separation is a function of the wavelength. Many antenna elements may then be used to form narrow beams and concentrate all the power in a specific/desired direction. On the other hand, initial synchronization of users in a mm-Wave network has to overcome some potential issues. The use of narrow beams in this context may provide good signal quality only to a small fraction of the area to be covered. If a new user equipment (UE) arrives in poorly-covered area, it may not manage to join the network as it cannot detect and decode satisfactorily any signal from access nodes (ANs). For example, the left-hand side of Fig. 1 shows an AN transmitting a narrow beam and two UEs waiting for synchronization signals. By chance, the narrow beam is transmitted towards one of the UEs, while the other may not listen to the transmitted signal. The challenge is to make sure that whenever a new UE tries to join such a network at least one AN should be able to provide it good enough signal quality to establish a connection, and using as little power as possible.

Assuming the use of all antenna elements available, the idea is to minimize the total consumed power by adjusting the beam power levels in each transmit-time interval so that every UE can perform initial synchronization. A beam sweep procedure is then adopted, in which narrow beams, one at each AN, are simultaneously transmitted in contiguous transmit-time intervals, namely beam sweep instances, in order to radiate energy over the area where UEs may appear and try to establish connection, until the whole area is scanned.

In order to minimize the total consumed power during initial synchronization, an optimization problem is formulated so that the objective is to minimize the sum of all the transmit power levels to be used during the beam scan. The objective is subject to a set of signal-to-interference-plus-noise ratio (SINR) constraints and a set of transmit power constraints. From the resulting beam power setting, the beam sweep pattern is determined. By nature, this problem is combinatorial with high computational complexity for large-scale networks. The exhaustive search of it grows exponentially with the number of SINR constraints. Thus, its optimal solution is hard to be found. One low-complexity approach is to find the beam sweep pattern at random with no power optimization, as in [2]. However, it would make ANs to transmit synchronization signals to areas where UEs rarely appears. Then, unnecessary power is consumed and UEs may experience high interference levels. The beam pattern and the beam power setting can be designed and optimized, respectively, based on historical statistics of UEs provided by the system, as in [3]. From such a data set, ANs find a power setting sufficient to provide good synchronization signal quality for UEs to synchronize. However, the algorithms proposed in [3] are performed centralized and provide solutions usually far from the optimum.

The key contribution of this work is the proposal of a low-complexity message-passing (MP) algorithm to solve the underlying problem. MP algorithms [4] have been widely used in probabilistic modeling of the relationship of interdependent parameters. For instance, it has been successfully applied for low-density parity-check (LDPC) decoding [5]. Beyond that, variations of the sum-product algorithm based on the generalized distributive law [6] have also been used to solve many problems in wireless communications. For example, the min-sum algorithm has been applied for the problem of downlink precoding selection [7] and for power control in very-large scale networks [8]. Here, the proposed MP is based on the min-sum algorithm. To apply it to the underlying problem, the constrained total consumed power minimization is

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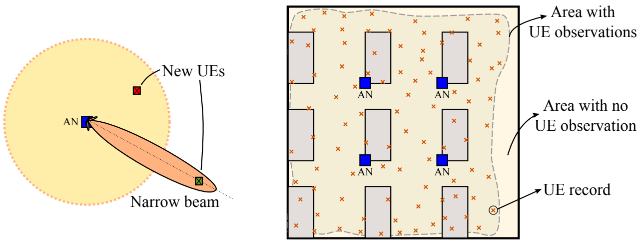

Fig. 1. Example of a mm-Wave indoor scenario. On the left, new UEs try to synchronize with an AN transmitting a narrow beam. On the right, UE records collected by ANs determine an area with relevant UE locations.

reformulated in order to plug constraints into the objective. A convenient factorization of the resulting objective is provided so that the standard MP framework can be directly applied. To deal with continuous variables in the message computation, interference is neglected, which is eventually shown to be a reasonable assumption. Consequently, computed messages are step functions, which facilitates the message exchange via parameterization or encoding. The proposed MP algorithm is compared with a baseline algorithm presented in [3].

II. System Model

Consider an indoor mm-Wave network where ANs are arbitrarily placed to provide an adequate coverage to UEs for initial synchronization. Let $\mathcal{N}$ be the set of N ANs, and let $\mathcal{K}$ be the set of K UE records, collected over time as historical statistics and available to ANs, as in [3]. Such UE records denote the received signal power per beam direction reported by synchronized UEs. From those records, a relevant area is estimated where UEs are most likely to arrive and request connectivity. The ANs can then rely on the historical data and radiate energy only over the relevant area. For instance, the right-hand side of Fig. 1 illustrates an open-plan office with four ANs that provide good coverage only in the relevant area from where it has collected UE records (orange crosses). A new UE in the relevant area can then satisfactorily detect and decode signals from the ANs. Each AN has $M \times M$ antenna elements, vertical and horizontally spaced by d, through the use of 2-dimensional (2D) uniform planar antenna arrays [9]. Moreover, each UE is assumed to be a single-antenna receiver, which ideally receives signals omni-directionally. Also, M^2 is assumed to be large (e.g., 64).

Accordingly, each AN is able to sweep its surroundings by varying the azimuth and elevation angles associated with its antenna array. The sets of azimuth and elevation angles are pre-determined and discrete in $[0\ 2\pi]$, where each ordered pair of angles defines a beam direction. Also, all the ANs simultaneously transmit beams, but only one beam per AN is transmitted in a given beam sweep instance. One by one, beams are sequentially transmitted at each AN and eventually the entire relevant area is swept and properly covered.

For simplicity, beams are ordered as a linear sequence. Let $\mathcal{L}$ be an index set enumerating the beams available at each AN. That is, $\mathcal{L} \triangleq \{1, 2, \ldots, L\}$, where L is the number of beam directions. Also, let (n, l) be the ordered pair that refers to the AN $n \in \mathcal{N}$ and its beam direction $l \in \mathcal{L}$. Each (n, l) is mapped as $(n, l) \to m \in \mathcal{M}$, so that

$$m = L(n-1) + l\,, \quad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}\,, \tag{1}$$

and, reversely,

$$n = \lceil m/L \rceil\,, \tag{2a}$$

$$l = \mathrm{mod}[m-1, L] + 1\,, \tag{2b}$$

where $\lceil \cdot \rceil$ and $\mathrm{mod}[\cdot,\cdot]$ stand for the ceiling function and the modulo operation, respectively, and $\mathcal{M} \triangleq \{1, 2, \ldots, NL\}$. It eases the representation of beams through their corresponding power levels in a factor graph. Eventually, when a UE is assigned to a beam indexed by m, it can easily map m back into (n, l). Further, a pre-determined discrete codebook of precoding weight vectors $\mathbf{w}_{n,l}$ is considered, defined as follow:

$$\mathbf{w}_{n,l} = \frac{\sqrt{P_{n,l}}}{M}\left[e^{-j\frac{2\pi}{\lambda}\mathbf{x}_{n,1}^T\mathbf{a}_l} \ldots e^{-j\frac{2\pi}{\lambda}\mathbf{x}_{n,M^2}^T\mathbf{a}_l}\right]^T, \tag{3}$$

where $P_{n,l}$ denotes the transmit power that AN n sets to transmit its beam l, $\mathbf{a}_l = \left[\cos\theta_l \sin\phi_l \quad \sin\theta_l \sin\phi_l \quad \cos\phi_l\right]^T$, column vector $\mathbf{x}_{n,\{1,\ldots,M^2\}}$ collects the 3-dimensional (3D) Cartesian coordinates of antenna elements at AN n with respect to the center point of the array, λ denotes the wavelength, and θ_l and ϕ_l are the azimuth and elevation angles that specify the relative phase excitation between antenna elements of AN n. The lth beam sweep instance is thus defined as the transmit-time interval when AN n transmits $\mathbf{w}_{n,l}$, for all n. Note that power levels $P_{n,l}$ and vectors $\mathbf{w}_{n,l}$ can also be mapped into P_m and $\mathbf{w}_m$, according to (1) and (2).

A. Optimization problem formulation

Each UE record is treated as a virtual UE that emulates a UE in the optimization problem, to be described in the following, with SINR as a function of spatial directions synchronization signals may be transmitted in.

For each virtual UE k, let $\mathcal{A}_k$ be an index set of beams that virtual UE k is capable of listening to during synchronization and reported data of. For each beam m, let $\mathcal{B}_m$ be an index set of virtual UEs that the system is capable of receiving data from regarding beam m transmitted from its associated AN. Moreover, for each beam m, an individual power level P_m, subject to a maximum power constraint $P_{\max}$, is calculated so that there exists at least one vector $\mathbf{w}_m$ that provides good synchronization signal quality for all the virtual UEs. Let $G_{m,k}$ denote the equivalent channel gain between virtual UE k and AN transmitting beam m. The SINR observed by virtual UE $k \in \mathcal{K}$ listening to beam $m \in \mathcal{M}$ is then defined as

$$\Gamma_{m,k} = \frac{P_m G_{m,k}}{\sum_{m' \in \mathcal{M}_m} P_{m'} G_{m',k} + \sigma_k^2}\,, \tag{4}$$

where $\mathcal{M}_m$ is the set of beams that interfere beam m, defined as $\mathcal{M}_m = \{m' \in \mathcal{M} \mid \mathrm{mod}[|m'-m|, L] = 0\,,\, m' \neq m\}$. The interfering beams of beam m are those in the same beam

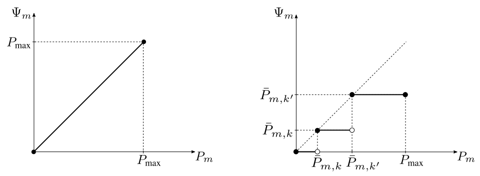

Fig. 2. Examples of power consumption model function Ψ_m vs. P_m . On the right side, $\Psi_m(P_m)$ is a linear relation, while on the right it is based on *a priori* information. Variable node m knows the power levels $\bar{P}_{m,k}$ and $\bar{P}_{m,k'}$ to individually satisfy virtual UE k and UE k' , respectively.

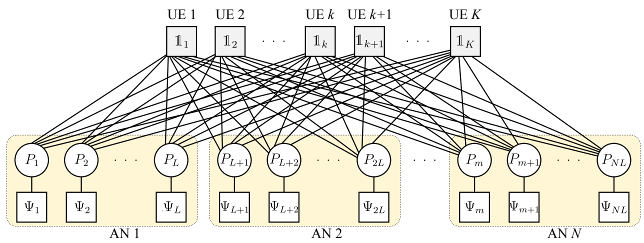

Fig. 3. Factor graph with check nodes, variable nodes and factor nodes. Curved rectangles delimit the local beams at ANs. Squares in gray denote check nodes, while squares in white denote factor nodes.

sweep instance from neighboring ANs. To deal with the SINR constraints, an indicator function $\mathbb{1}_k$ is defined as

$$\mathbb{1}_k\left[\Gamma_{m,k} \geq \gamma\right] = \begin{cases} 0 & \exists m\,, \Gamma_{m,k} \geq \gamma\,, \\ +\infty & \text{otherwise,} \end{cases} \tag{5}$$

where γ is a SINR threshold of what every $\Gamma_{m,k}$ must be above for successful synchronization and robust decoding of synchronization signals. The indicator function penalizes any infeasible power setting by returning $+\infty$.

A power consumption model function $\Psi_m(P_m)$ is introduced to account for the relation between the transmit power and the consumed power at each AN. In general, it can have any shape to take into account nonlinearities and power dissipation, but it can be simply a linear relation so that the power consumption depends only on the transmit power. Another aspect is that some *a priori* information ANs obtain from the historical statistics can be incorporated into each Ψ_m. As an example, virtual UEs may inform to the system what power ANs should transmit with in order to satisfy every SINR constraint. Let $\bar{P}_{m,k}$ denote the minimum power level that beam m should be transmitted with in order to satisfy the SINR constraint of virtual UE $k \in \mathcal{K}$, defined as

$$\bar{P}_{m,k} = \frac{\gamma}{G_{m,k}} \left(\sum_{j \in \mathcal{M}_m} P_j G_{j,k} + \sigma_k^2 \right) , \forall m \in \mathcal{M}\,. \tag{6}$$

Note that each $\bar{P}_{m,k}$ depends on its set of interfering beams $\mathcal{M}_m$. If such information is taken into account, then the search space of the best power setting becomes a function of every $\bar{P}_{m,k}$. In other words, ANs do not have any incentive to transmit a beam with any power different from the minimum power levels $\bar{P}_{m,k}$. Consequently, each function Ψ_m looks like a sum of step functions, each step being a function of an individual minimum power level. Fig. 2 shows two examples of power consumption model function $\Psi_m(P_m)$, where, on the left side, Ψ_m is a linear relation, while on the right it is based on *a priori* information from two neighboring UEs.

The total consumed power function f is formulated as an objective that comprises univariate factors, represented by the functions Ψ_m , and multivariate checks, represented by the functions $\mathbb{1}_k$, as follows:

$$f\left(\{P_m\}_{m \in \mathcal{M}}\right) = \sum_{m \in \mathcal{M}} \Psi_m + \sum_{k \in \mathcal{K}} \mathbb{1}_k\,. \tag{7}$$

Note that f factorizes into a sum of NL factor terms and into a sum of K check terms, each set $\mathcal{A}_k$ determining the interdependency of them. Finally, the problem of minimizing the total consumed power can then be stated as

$$\underset{\{P_m \in [0\; P_{\max}]\}_{m \in \mathcal{M}}}{\text{minimize}} \quad f\left(\{P_m\}_{m \in \mathcal{M}}\right)\,. \tag{8}$$

The historical statistics are assumed to be rich enough so that every beam in $\mathcal{M}$ has at least one UE in $\mathcal{K}$ that listens to it.

III. MP Framework for Initial Synchronization

Due to the structure of Eq. (8), the min-sum algorithm is applied to find the beam power setting that minimizes the total consumed power in the network. The function f defined in (7) is graphically modeled by a bipartite graph, namely factor graph, comprising factor nodes, check nodes and variable nodes. Each check node represents a virtual UE, while each variable node represents a beam direction, at a particular AN, where the transmit power level for the beam is the variable. Besides, each factor node is attached to the corresponding variable node so that it represents the power consumption model function of the respective beam. More precisely, check node k, with neighboring variable nodes in $\mathcal{A}_k$, acts as the constraint checker of virtual UEs k, while variable node m , with neighboring check nodes in $\mathcal{B}_m$, acts to establish the transmit power levels of their respective beams. Fig. 3 shows an example of factor graph modeling a network with N ANs, L beams per AN, and K virtual UEs, which means that there are NL variable nodes, NL factor nodes, and K check nodes.

A. Granular MP algorithm

To decrease computational complexity, the interference terms in (4) and in (6) are neglected, as narrow beams can be assumed to cause very low interference to one another, and also that the SINR threshold γ is usually small (e.g., -10 dB) for robust decoding and detection. Thus, $\Gamma_{k,m}$ and $\bar{P}_{m,k}$ are approximated as

$$\Gamma_{k,m} \approx \frac{P_m G_{m,k}}{\sigma_k^2} \tag{9}$$

and

$$\bar{P}_{m,k} \approx \frac{\gamma \sigma_k^2}{G_{m,k}}\,, \quad \forall m \in \mathcal{M}\,, k \in \mathcal{B}_m\,. \tag{10}$$

Further, let D be the degree of every check node, i.e., the number of incident edges to each check node. The value of D

is determined so that each set $\mathcal{A}_k$ comprises only the variable nodes whose corresponding beams are those providing the D largest equivalent channel gains.

In general, a user does not need to be served by more than one beam. Thus, a message from a virtual UE k to a beam m can be defined as a piecewise function. A threshold is then defined as the minimum power level needed for that beam m to serve virtual UE k, i.e., $\bar{P}_{m,k}$. Above such a threshold, the power of the other neighboring beams are hypothesized so that they consume as little power as possible. Otherwise, virtual UE k hypothesizes the minimum power consumed by the other beams if at most one of them would serve. With this in mind, let $\alpha_{k\to m}$ denote the message to be passed from check node k to variable node m, and let $\beta_{m\to k}$ denote the normalized message to be passed from variable node m to check node k. The min-sum algorithm then simply iterates between the following two kinds of message computations and exchanges:

- *Summary message*, from check node to variable node:

$$\alpha_{k\to m}(P_m) = \begin{cases} 0 & \text{if } P_m \geq \bar{P}_{m,k}\,, \\ \min\limits_{j\in\mathcal{A}_k\setminus\{m\}} \left(\min\limits_{P_j \geq \bar{P}_{j,k}} \beta_{j\to k}(P_j) \right) & \text{otherwise.} \end{cases} \tag{11}$$

- *Aggregate message*, from variable node to check node:

$$\beta_{m\to k}(P_m) = \Psi_m(P_m) + \sum_{j\in\mathcal{B}_m\setminus\{k\}} \alpha_{j\to m}(P_m) - \kappa\,, \tag{12}$$

  where κ is a normalizing constant to prevent messages from increasing endlessly, herein defined as the minimum of each corresponding message.

The expression of summary messages above has a low computational complexity. Each check node k has to do only a few number of checks to compute its summary messages.

The algorithm begins with variable node m computing outgoing aggregate message $\beta_{m\to k}$ to check node k, for $k \in \mathcal{B}_m$, assuming all the incoming summary messages initialized to zero. That is, the aggregate messages initially equal their respective power consumption model functions. Upon receipt of messages, check node k computes outgoing summary message $\alpha_{k\to m}$ to variable node m, for $m \in \mathcal{A}_k$. Then, summary messages are sent to variable nodes. This back-and-forth message exchange defines an iteration of the MP algorithm. At the end of a MP iteration, each variable node computes its approximate min-marginal ψ_m , herein defined as

$$\psi_m\,(P_m) = \Psi_m(P_m) + \sum_{j\in\mathcal{B}_m} \alpha_{j\to m}(P_m)\,, \quad m\in\mathcal{M}\,, \tag{13}$$

and then determine its power level that minimizes it by

$$P_m^* = \arg\min_{P_m} \{\psi_m\,(P_m)\}\,, \quad m\in\mathcal{M}\,. \tag{14}$$

The algorithm then iterates until a stopping criterion is reached, either a predetermined maximum number of iterations or when the beam power setting computed at the end of each iteration converges to a fixed state.

Eventually, UEs can observe some interference since multiple ANs can be transmitting in the same beam sweep instance. However, interference can be treated after running the MP algorithm via some power control technique. This is reasonable due to the fact that, after finding the best power setting via message passing, UE assignment can be straightforwardly performed. Then, at each beam sweep procedure, any power control function can readjust the power setting to satisfy every constraint in terms of SINR. Regarding convergence issues, a random MP scheduling is adopted, where on average only some percentage of check nodes computes and passes messages over MP iterations.

IV. Simulation Results

The total consumed power and some convergence aspects are evaluated. The granular MP algorithm is assessed and compared with the iterative baseline algorithm present in [3]. The number of simulation runs was 100. In each run, P_{max} was set to 1 mW and γ to -10 dB. The channel responses in 60 GHz frequency band were obtained from the ray-tracing channel model (please refer to [10] for some brief description and to [3] for the channel characteristics setup adopted). Each AN has an array with 8×8 antenna elements, $d = \lambda/2$, pointing down to the floor, with $L = 256$ beams available per AN. UE records are assumed to be measured by UEs equipped with an ideal omni-directional receive antenna each. Random channel components and UE positions varies from one simulation run to another. The indoor scenario was a $9\times9\times3$ cubic meter open-plan office containing nine 1×2 square meter desks at height of 0.75 meters, conveniently placed, and $N = 4$ ANs symmetrically placed close to the ceiling. $K = 81$ virtual UEs were uniformly spread over the office, such that there was one UE per square meter at a constant height of 0.75 meter. D was equal to 3, 5, 7 or 9, while random MP scheduling percentage was 100% or 80%.

The result provided by the baseline algorithm is repeated for every value of D and random scheduling percentage. Regarding the granular MP, as it is not guaranteed to converge, it is evaluated at every MP iteration. If it converges, the result provided at the last iteration is considered, labeled as "MP (conv.)". Otherwise, every power setting found over the iterations that satisfies the constraints are taken. Then, the minimum one is considered as the result, labeled as "MP (no conv.)". Both are individually shown in terms of total consumed power. At last, the convergence rate (percentage of convergent runs) for both baseline and granular MP algorithms is measured. Also, the average number of iterations demanded until convergence is calculated. Convergence is considered when the power setting stabilizes for 10 consecutive iterations with an arbitrary numerical precision of 10^{-5}.

The granular MP algorithm, when it converges, outperforms the baseline algorithm for any D and any random scheduling percentage. Otherwise, it provides a worse performance for D larger than 5 and for 100% random scheduling, what can be seen in Fig. 4, due to the presence of cycles in the graph. For 80% random scheduling, it has approximately the same performance independently of convergence, which indicates that every power setting calculated over the MP iterations

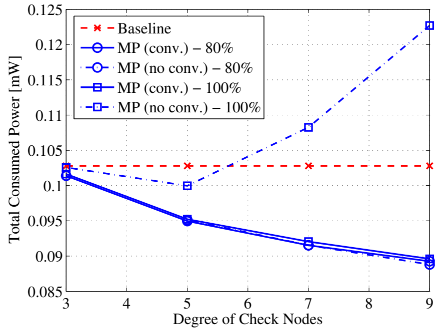

Fig. 4. Total consumed power vs. D for 100% and 80% random scheduling.

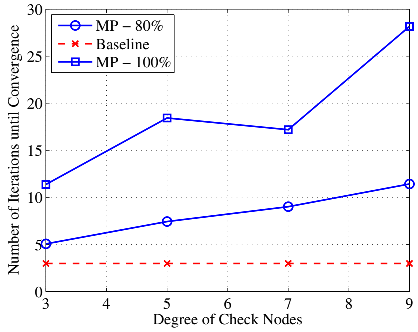

Fig. 6. Average number of iterations until convergence vs. D for 100% and 80% random scheduling.

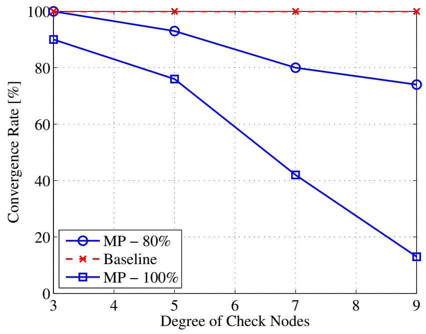

Fig. 5. Convergence rate vs. D for 100% and 80% random scheduling.

should be checked. For $D = 9$ and 80% random scheduling, the average total consumed power of granular MP algorithm is 0.08926 mW when it converges, while the baseline algorithm consumes 0.1028 mW, around 15% more power. With respect to convergence, for 100% random scheduling, the granular MP algorithm converges in 90% of the cases for $D = 3$ and 14% for $D = 9$, as shown in Fig. 5. For 80% random scheduling percentage, it tends to converge less often as D increases, but converges in 100% for $D = 3$. For $D = 9$, it converges in 74% of the cases, which can also be seen in Fig. 5. However, it demands more iterations to converge as D increases. For instance, it converges within approximately 12 iterations for $D = 3$ and 28 for $D = 9$, for 100% random scheduling, as shown in Fig. 6. Besides, it converges within approximately 5 iterations for $D = 3$ and 12 for $D = 9$ for 80% random scheduling, as also shown in Fig. 6. The baseline solution converges in 100% within 3 iterations on average. At last, the maximum square error of SINR when it is below γ was observed to be approximately $4 \cdot 10^{-5}$, which can justify the interference neglect in the MP formulation.

V. Conclusion

This paper addressed the UE initial synchronization problem in a mm-Wave indoor scenario. Narrow beams, due to antenna arrays with a massive number of elements at ANs, may provide poor coverage in some areas. To cope with that, the adjustment of beam power levels along with a beam sweep procedure was described. Relying on historical statistics, the beam sweep in the indoor scenario can then be optimized. To decrease complexity, interference was neglected. A graph-based model for beam sweep was proposed, which comprises a low-complexity MP algorithm, namely granular MP, based on the min-sum algorithm, featuring a low-complexity message computation. Simulation results showed that the granular MP algorithm usually outperforms an iterative baseline algorithm, consuming about 13% less power in a particular simulation setup. At last, interference neglect was shown to be a reasonable assumption due to narrowness of beams and low SINR threshold.

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