Power-Efficient Beam Sweeping for Initial Synchronization in mm-Wave Wireless Networks

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***Abstract*—This paper addresses the problem of initial synchronization of users in an indoor mm-Wave scenario. With the use of a massive number of antenna elements at access nodes, the resulting beams have narrow beam width. However, the transmission of individual narrow beams may cause poor coverage in some areas as the energy is concentrated over the direction of their main lobes. To cope with that, a beam sweep procedure using phased arrays is adopted. Access nodes simultaneously transmit individual beams until a certain area of interest is thoroughly scanned. Then, distant points are covered as each beam can radiate energy using an individual transmit power. The goal is to find the minimum power setting by adjusting the individual power levels so that users over the scanned area can observe a minimum received power level. Due to the combinatorial nature and high dimensionality, this paper describes a non-iterative two-stage algorithm to perform the beam sweeping with moderate complexity, which is compared with a baseline iterative selfish algorithm. Preliminary simulation results indicate that the two-stage algorithm may perform at least equally well as the baseline approach in terms of total transmit power, consuming about 7% less power in the used simulation setup, and without suffering from any convergence issue.**

I. INTRODUCTION

The millimeter wave (mm-Wave) band [1] (e.g. 60 GHz) is expected to provide many benefits for fifth-generation (5G) systems. The accessible channel bandwidth in the mm-Wave band is potentially larger compared to commercial wireless systems at lower bands, e.g. the current long-term evolution (LTE). Also, the small wavelength allows the transceivers to have a more compact hardware due to the fact that the antenna element separation is a function of the wavelength. Consequently, a large number of antenna elements can be installed in regular-size access nodes (ANs). Many antenna elements may be used to form narrow beams and concentrate all the power in a specific/desired direction. This way, distant user equipments (UEs) can be reached without causing high interference targetting different UEs.

On the other hand, initial synchronization of UEs in a mm-Wave network may overcome some potential issues. In this context, the challenge is to make sure whenever a new UE tries to join such a network at least one AN should be able to provide it good enough signal quality to establish a connection, and using as little power as possible. However, the use of narrow beams for initial synchronization would provide good signal quality only to a small fraction of the area to be covered. If a new UE arrives in poorly-covered area, the consequence would be that it may not manage to join the network as it cannot listen to and decode satisfactorily any signal from the ANs.

Inactivating antenna elements makes it possible to create wider beam patterns with the limit being reached by a single elemental pattern. In many envisioned arrangements, say one power amplifier per antenna element, this may entail a reduction in total conducted power into the array. It is therefore more power-efficient to use all antenna elements to radiate power. Hence, the present study assumes the use of all antenna elements available. More specifically, a beam sweep [2] procedure is adopted, in which narrow beams, one at each AN, are simultaneously transmitted in contiguous transmit-time intervals, in order to radiate energy over the area where UEs may appear and try to establish connection, until the whole area is scanned. The goal is to minimize the total transmit power by adjusting the power level of all the beams in each transmit-time interval so that every UE can perform initial synchronization. Then, the problem is formulated as a joint AN-and-beam assignment and power adjustment.

To the best of the authors' knowledge, no work in the literature has proposed an efficient algorithm that jointly performs AN-and-beam assignment and power adjustment per beam in the context of UE initial synchronization. Most of the works solve only two of these aspects. For instance, beamforming techniques [3]–[7] focus on transmit beamforming, which is formulated such that the resulting beam is a linear combination of individual user-specific (precoding) weight vectors that share the total transmit power. However, they rely on a pre-defined AN assignment. Multi-cast beamforming techniques [8], [9] allow a single weight vector to serve a group of UE, but if there exist multiple groups, the total transmit power is assumed to be shared among them. But again, none of those works consider AN assignment. At last, joint power control and AN assignment was addressed in [10], but it assumes each AN has only a single transmit wide beam available, which is not suitable for the case of narrow beams.

Due to the limitations of the aforementioned works, the key contribution of this study is the proposal of algorithms for adjusting the power of each beam at each AN so that the total transmit power of all the beams in the network is as small as possible, while still ensuring that all potential UEs are satisfactorily covered. As for potential UEs, this work assumes a large record of historical received signal power observed by

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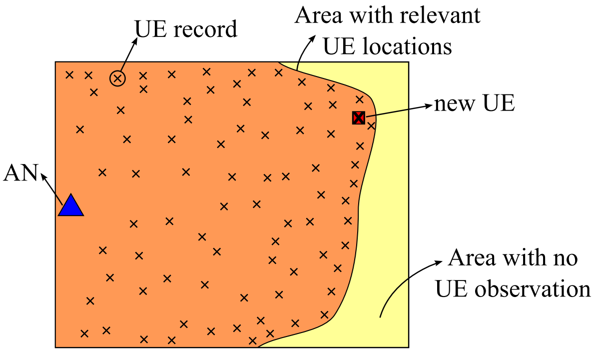

Fig. 1. Example of an indoor scenario where a new UE tries to establish connection with the network through an AN. The historical UE records (black crosses) collected by the AN comprises an area with relevant UE locations.

UEs, which is assumed to be available for all the ANs. The details of how such historical data observations are collected is beyond the scope of this work, but the collection could, for example, be performed by letting all ANs use full power for all beams for an introductory period after deployment of the network (e.g. a few months), until sufficient UE signal-to-noise ratio (SNR) feedback statistics have been obtained. This paper then describes two algorithms based on beam sweeping to cope with the UE initial synchronization problem. The first one, namely two-stage algorithm, prioritizes UE observations with low SNR and adjusts the power levels in a non-iterative manner, while the second approach iteratively finds the best-response power levels. The two algorithms are compared in terms of total transmit power over all the beams.

II. Problem Statement and System Model

Consider an indoor mm-Wave scenario where N ANs are arbitrarily placed to provide an adequate coverage to K UEs for initial synchronization. Let $\mathcal{N}$ be the set of all the ANs in the network, and let $\mathcal{K}$ be the set of UE records available in the historical data observations collected over time. Such UE records denote the received signal power per beam observed by UEs. From those records, a relevant area can be estimated where UEs are most likely to arrive and request connectivity. The ANs can then rely on the historical data and radiate energy only over the relevant area. For instance, Fig. 1 illustrates a single-room office with one AN that provides good coverage only in the relevant area from where it has collected UE records (black crosses). A new UE in the relevant area can then satisfactorily listen to and decode signals from the AN.

Each AN has a large number of $M \times M$ antenna elements, vertical and horizontally spaced by d, through the use of 2-dimensional (2D) uniform planar antenna arrays [2]. Moreover, each UE is assumed to be a single-antenna receiver, which ideally receives signals omni-directionally. The use of massive number of transmit antennas (say 64 antenna elements) at each AN results in a particular beam shape that has a main lobe with narrow beamwidth, but with high antenna gain. The direction such a main lobe points to depends on what value the relative phase excitation between the antenna elements takes on.

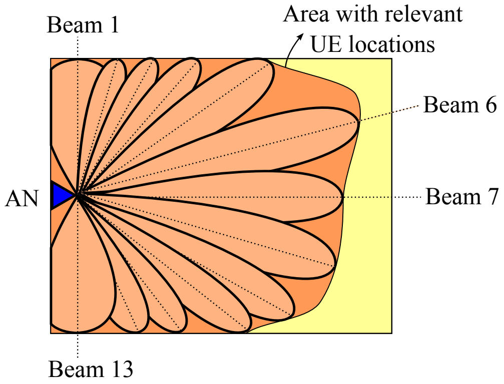

Fig. 2. Example of beam sweeping in a room with a single AN. The effective beam shape after 13 beam sweep instances coincides with the relevant area.

A. Codebook-based Beam Sweeping

An interesting feature of a phased antenna array is the possibility of changing the phase excitation between its antenna elements. Accordingly, each AN is then able to sweep (or scan) its surroundings by varying the azimuth and elevation angles associated with its antenna array. A key assumption in this work is the fact that the sets of azimuth and elevation angles are finite and pre-defined, where each ordered pair of angles defines a beam/direction. Also, all the ANs simultaneously transmit beams, but only one beam per AN is transmitted in a given transmit-time interval (namely beam sweep instance). One by one, the remaining beams are sequentially transmitted at each AN and eventually the entire relevant area is swept and properly covered. Fig. 2 illustrates an example of linear beam sweeping in azimuth executed by a single AN in a room. The AN follows a clockwise-sorted beam sequence of a total of 13 beams sweep instances. Note that beam 6 (pointing to the top-right corner) is transmitted at higher power level in order to provide adequate coverage there. For the sake of simplicity, this example ignores side lobes and reflected rays.

More specifically, let $\mathcal{L}$ be an index set enumerating the beams available at each AN. That is,

$$\mathcal{L} \triangleq \{1, 2, \ldots, L\}\,, \tag{1}$$

where L is the number of available beams. Also, let $\mathbf{u}_{n,l}$ denote a unit-norm weight vector that represents the (narrow) beam $l \in \mathcal{L}$ transmitted through the $M \times M$ transmit antenna elements at AN n. Assuming transmit phased arrays, each weight vector $\mathbf{u}_{n,l}$ is then defined as follows:

$$\mathbf{u}_{n,l} = \frac{1}{M}\begin{bmatrix} e^{-j\frac{2\pi}{\lambda}(\mathbf{x}_{n,1}-\mathbf{x}_{n,0})^T} \\ e^{-j\frac{2\pi}{\lambda}(\mathbf{x}_{n,2}-\mathbf{x}_{n,0})^T} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}(\mathbf{x}_{n,M^2}-\mathbf{x}_{n,0})^T} \end{bmatrix}\mathbf{a}_l\,, \tag{2}$$

where

$$\mathbf{a}_l = \begin{bmatrix}\cos\theta_l \sin\phi_l & \sin\theta_l \sin\phi_l & \cos\phi_l\end{bmatrix}^T\,, \tag{3}$$

vectors $\mathbf{x}_{n,m}$ and $\mathbf{x}_{n,0}$ stand for the column vectors collecting 3-dimensional (3D) Cartesian coordinates of antenna element

m at AN n and the reference point (i.e. the center point of the antenna array) of AN n, respectively, λ denotes the wavelength, and θ_l and ϕ_l are the azimuth and elevation angles that specify the relative phase excitation between antenna elements of AN n. Now let (ϕ_l, θ_l) be an ordered pair so that θ_l depends on ϕ_l, where

$$\phi_l \in \left\{0, \left(\frac{1}{2M-1}\right)\frac{\pi}{2}, \ldots, \frac{\pi}{2}\right\}. \qquad (4)$$

The angle granularity in the azimuth R_θ decreases linearly (i.e. $R_\theta = 4M-1, 4M-3, \ldots, 1$) as the angle ϕ_l increases. Eventually, the L different ordered pairs define the codebook $\mathcal{U}_n$ at each AN, that is,

$$\mathcal{U}_n \triangleq \{\mathbf{u}_{n,l} \,|\, l \in \mathcal{L}\}, \forall n \in \mathcal{N}. \qquad (5)$$

Furthermore, let $\mathbf{w}_{n,l} = \sqrt{P_{n,l}}\mathbf{u}_{n,l}$, represent the precoding weight vector where $P_{n,l}$ denotes the transmit power that AN n sets to transmit its beam l. Thus, the beam sweep instance l is defined as the transmit-time interval when AN n transmits beam l with power $P_{n,l}$, for all $n \in \mathcal{N}$. Not surprisingly, the power set per beam allows the formation of the effective beam shape shown in Fig. 2. Besides, the order each AN selects its weight vectors from the codebook in (5) to perform the beam sweeping may affect the resulting power levels. That is, the interference each receiver perceives in each beam sweep instance is dependent on such order. For the sake of simplicity, this study assumes a linear beam sweeping, where the weight vectors are evaluated so that for each value of ϕ_l, in ascending order, θ_l takes on all its possible values, also in ascending order, following the proper angle granularity R_θ.

B. Total Transmit Power Minimization

For each beam, an individual power level, subject to a maximum power constraint, is calculated so that there exist at least one precoding weight vector that provides good signal quality for all UE records in the relevant area. However, such calculations require that every UE record must be associated with one beam at one AN. Given an AN-and-beam assignment setting in a beam sweep instance, the power adjustment (a linear programming (LP) problem) can then be obtained via some classical power control technique (see [10], [11]). Thus, the challenge is to describe a methodology to efficiently obtain the AN-and-beam assignment jointly with the power setting, which is a combinatorial problem.

More precisely, let $\Gamma_{n,l,k}$ be the signal-to-interference-plus-noise ratio (SINR) observed by UE record k assigned to AN $n \in \mathcal{N}$ transmitting beam $l \in \mathcal{L}$, here defined as

$$\Gamma_{n,l,k} = \frac{P_{n,l}G_{n,l,k}}{\sum_{i \in \mathcal{N}\setminus\{n\}} P_{i,l}G_{i,l,k} + \sigma_k^2}, \quad \forall k \in \mathcal{K}, \qquad (6)$$

where $G_{n,l,k} = \mathbf{u}_{n,l}^H \mathbf{R}_{n,k}\mathbf{u}_{n,l}$ denotes the equivalent channel gain between UE record k and the AN n transmitting beam l, $\mathbf{R}_{n,k}$ is the covariance matrix of the channel response between UE record k and the AN n, and σ_k^2 is the noise power of UE record k. The first term in the denominator of (6) represents the interference caused by the other ANs transmitting beam l, i.e. in the beam sweep instance l. Note that every $G_{n,l,k}$ is drawn from the set of historial observations. Besides, each UE record k has a quality-of-service (QoS) constraint so that its $\Gamma_{n,l,k}$ must be above a certain threshold γ for successful synchronization and decoding of system control information. For this purpose, the problem formulation of the total transmit power minimization can be stated as follows:

$$\begin{aligned}
\underset{\{P_{n,l}\}_{\substack{n\in\mathcal{N}\\ l\in\mathcal{L}}}}{\text{minimize}} \quad & \sum_{n\in\mathcal{N}}\sum_{l\in\mathcal{L}} P_{n,l} \\
\text{subject to} \quad & \max_{\{n,l\}_{\substack{n\in\mathcal{N}\\ l\in\mathcal{L}}}} \Gamma_{n,l,k} \geq \gamma, \quad \forall k \in \mathcal{K}, \\
& P_{n,l} \leq P_{\text{max}}, \quad \forall n \in \mathcal{N}, \forall l \in \mathcal{L},
\end{aligned} \qquad (7)$$

where P_{max} denotes the maximum power constraint per beam. Note that each UE record has NL possible associations, which means that there are $(NL)^K$ possible AN assignment instances. Thus, the exhaustive search of the combinatorial problem in (7) grows exponentially with the number of UE records since $\Gamma_{n,l,k}$ is calculated for every assignment. Consequently, for a large set $\mathcal{K}$ and large NL, such a problem becomes intractable. Therefore, this work considers simplified, sub-optimal solutions which have moderate complexity. In what follows, two algorithms to perform the per-beam power adjustment will be presented.

III. AN-AND-BEAM ASSIGNMENT WITH PER-BEAM POWER ADJUSTMENT ALGORITHM

A two-stage methodology is proposed to provide adequate coverage to UEs in their initial synchronization phase. Here, each AN is assumed to be able to hypothesize the SNR at every UE record k as a function of AN n and beam l in a full-power transmission mode, which can be drawn from the set $\mathcal{K}$. In each beam sweep instance, UE record with the lowest SNR (evaluated at its best assignment) is prioritized. Then, a minimum power level (associated with such assignment) is set to satisfactorily cover that UE and some others that can also be satisfied. Potential interference in each beam sweep instance is neglected in order to decrease complexity. This assumption is assumed to be reasonable in the context of beam sweeping due to the narrowness of beams, which is verified in Section IV.

Specifically, let (n_k, l_k) be the assignment of UE record $k \in \mathcal{K}$ to AN $n_k \in \mathcal{N}$ transmitting beam $l_k \in \mathcal{L}$. Also, let $\bar{\mathcal{K}} \subset \mathcal{K}$ be the set of not-assigned UEs, which is assumed to contain every UE, i.e. $\bar{\mathcal{K}} = \mathcal{K}$, initially. The *first stage* of the algorithm finds the ”poorest” UE $k' \in \bar{\mathcal{K}}$, i.e. the one that has the lowest SNR if every UE were assigned to their best pair of AN and beam. It can be obtained by solving the following combinatorial problem:

$$k' = \arg\min_{k\in\bar{\mathcal{K}}} \left\{ \max_{\{n,l\}} \frac{P_{n,l}^{(r-1)}G_{n,l,k}}{\sigma_k^2} \right\}, \qquad (8)$$

where r is an iteration index. The initial condition (i.e. $r = 0$) is that of $P_{n,l}^{(0)} = P_{\text{max}}$ for all n and l. The inner maximization in (8) provides the resulting assignment $(n_{k'}, l_{k'})$ of UE k'.

Then, the *second stage* calculates the minimum amount of power to satisfy UE k' based on the threshold γ. Thus,

$$P^{(r)}_{n_{k'},l_{k'}} = \min\left[\gamma\frac{\sigma^2_{k'}}{G_{n_{k'},l_{k'},k'}}\,, P_{\max}\right]\,, \tag{9}$$

where the operator $\min[\cdot\,,\cdot]$ returns the smallest argument. Further, UEs that can be satisfied with assignment $(n_{k'}, l_{k'})$ and $P^{(r)}_{n_{k'},l_{k'}}$ are assigned to $(n_{k'}, l_{k'})$. Let $\mathcal{X}$ be the set of currently satisfied UEs, defined as

$$\mathcal{X} \triangleq \left\{ k \left| \frac{P^{(r)}_{n_{k'},l_{k'}} G_{n_{k'},l_{k'},k}}{\sigma^2_k} \geq \gamma\,, \forall k \in \bar{\mathcal{K}}^{(r-1)} \right.\right\}\,, \tag{10}$$

which can be obtained as a by-product when computing (8). Consequently, UEs in $\mathcal{X}$ must be removed from the set of not-assigned UEs. That is, $\bar{\mathcal{K}}^{(r)} = \bar{\mathcal{K}}^{(r-1)} \backslash \mathcal{X}$.

These two stages are repeated until all the UEs are satisfied and assigned to some AN and beam. The transmit power of (inactive) beams that have no UE assigned to them are set to zero. Due to the fact that only SNR is considered in (8), one could argue that it may provide a poor performance in some interference-limited scenario. To cope with this, the power levels can be readjusted afterwards taking into account interference via a classical power control step.

A. Baseline Method

For the sake of comparison, a baseline method is taken into account so that it iteratively evaluates only the inner maximization in (8), which can be seen as a best-response assignment function. In this case, for a given iteration,

$$(n_k, l_k) = \arg\max_{\{n,l\}} \frac{P^{(s-1)}_{n,l} G_{n,l,k}}{\sigma^2_k}\,, \quad \forall k\,, \tag{11}$$

where s is an iteration index and (n_k, l_k) is the resulting assignment of UE k at iteration s. Then, the power levels are adjusted, taking into account interference, to satisfy UE k assigned to (n_k, l_k), for all k. If multiple UEs are assigned to the same AN, only the QoS constraints of all but one UE would be over-satisfied. Thus,

$$P^{(s)}_{n,l} = \min\left[\max_{k\in\mathcal{X}_{n,l}} \frac{\gamma \sum\limits_{i\neq n} P^{(s-1)}_{i,l} G_{i,l,k} + \gamma\sigma^2_k}{G_{n,l,k}}\,, P_{\max}\right]\,, \tag{12}$$

where $\mathcal{X}_{n,l} \triangleq \{k \in \mathcal{K}\,|\, n_k = n, l_k = l\}$, $\forall n \in \mathcal{N}$ and $\forall l \in \mathcal{L}$. The transmit power of (inactive) beams that have no UE assigned to them are set to zero. The computations in (11) and (12) are repeated until s reaches the maximum number of iterations allowed.

IV. SIMULATION RESULTS

In this section, some preliminary numerical results are presented. The total transmit power

$$P_{\text{total}} = \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} P_{n,l}$$

is evaluated against a maximum transmit power $P_{\max}$ of 20 mW. This amount of power per beam at each AN is large enough to satisfy all the UEs. The number of simulation runs equals 100. Each threshold γ equals 0 dB. The two-stage algorithm is evaluated alone as well as followed by a power control step. The behavior of both algorithms is assessed in a iterative manner, although the two-stage one is naturally non-iterative, which was confirmed based on the numerical results. A total of 10 iterations are allowed for both algorithms.

The channel responses were obtained from the ray-tracing channel model (please refer to [12] for some brief description and explanation of this model). As for the channel characteristics, rays with up to two reflections (which includes the line-of-sight (LOS) ray) combined with random components comprising 20 random point sources were considered. Carrier frequency equals 60 GHz, reflection loss is 5.6 dB and noise figure equals 6 dB. Moreover, SNR at each receiver was measured in a single carrier with bandwidth of 1.47 GHz.

For this study, a $5\times7\times3$ cubic meter two-room studio floor was set to be the indoor scenario. Two ANs are arbitrarily placed close to the ceiling, one in each room, where $\mathbf{x}_{1,0} = \begin{bmatrix}0.5 & 5.5 & 2.8\end{bmatrix}^T$ and $\mathbf{x}_{2,0} = \begin{bmatrix}2.8 & 1. & 2.8\end{bmatrix}^T$. Each AN has an antenna array with 8×8 antenna elements with $d = \lambda/2$, pointing down to the floor with an absorptive back wall in order to block reflected rays from the ceiling. As $M = 8$, there is a total of $L = 256$ beams available. A number of 675 UEs were uniformly spread over the two rooms at constant height of 1.5 meters, one in every 0.04 square meter, to represent the historical UE records. Each UE has an ideal omni-directional receive antenna.

The two-stage algorithm outperforms the baseline one in terms of the total transmit power provided, as can be seen in Fig. 3. A percentage gain of about 2.7% is observed. For such value of $P_{\max}$, the two-stage approach seems to be invariant over the five iterations, which confirms its non-iterative aspect, whereas the baseline approach demands four iterations to converge. The percentage gain is even larger in the three first iterations, being approximately equal to 7% at the very first one. It is worth mentioning that neither approach changes performance with different values of $P_{\max}$, as long as it is large enough to guarantee that all the UEs become satisfied. Moreover, Fig. 3 also shows the two-stage algorithm provides approximately the same performance if it is followed by the additional power control step. It seems the narrow beams in such an indoor scenario with transceivers operating in mm-Wave band turns it into an almost interference-free environment. This validates the use of the two-stage methodology that takes into account only SNR observations.

Finally, Fig. 4 shows a snapshot of the resulting beam sweeping. Cyan and yellow points represent UEs records assigned to AN 1 and AN 2, respectively. Each active beam (in blue) denotes a weight vector scaled by square root of its associated power level.

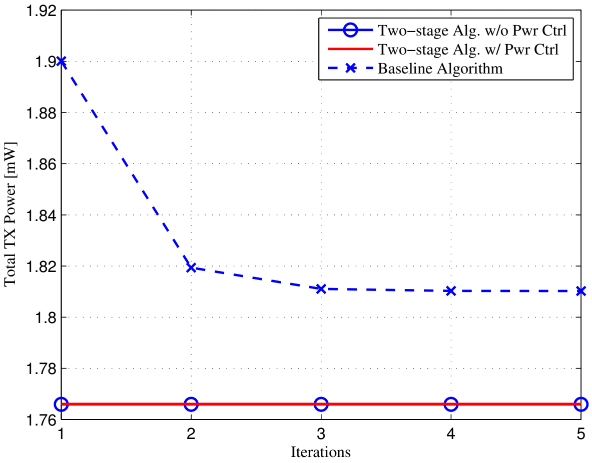

Fig. 3. Total transmit power for $P_{\max}$ set to 20 mW against iterations.

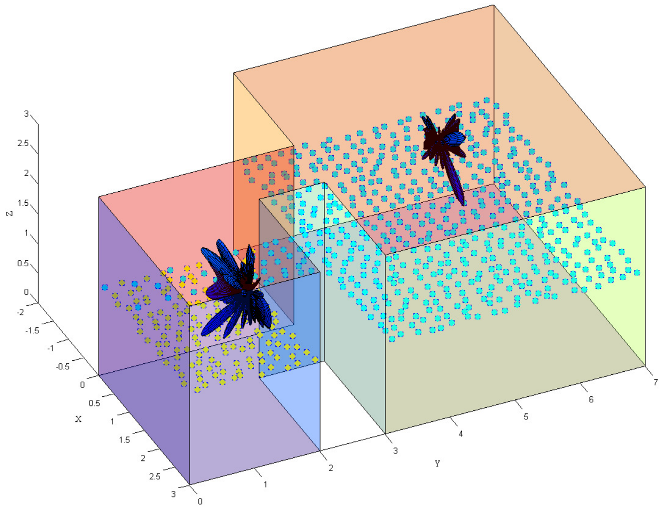

Fig. 4. Resulting beam sweeping provided by the two-stage algorithm in a given simulation run.

V. CONCLUSIONS

This work addressed the problem of initial synchronization of UEs in a mm-Wave scenario. With the use of a massive number of transmit antenna elements, the resulting transmit beams are narrow, which may provide poor coverage in some areas. To cope with that, a power adjustment per beam via a beam sweep procedure was proposed. Such an approach allows the phase excitation between antenna elements of ANs to take on different values to scan a relevant area based on historical UE observations. Each beam can radiate energy using an individual transmit power. A beam sweeping based on a non-iterative two-stage algorithm was described, which provides jointly the per-beam power setting and the AN-and-beam assignment. Simulation results showed that the two-stage algorithm performs at least equally well as the baseline iterative approach in terms of total transmit power, consuming about 7% less power in a non-iterative manner in the used simulation setup, i.e. without suffering any convergence issue.

In future, the two-stage algorithm can be evaluated in different indoor scenarios that demand more ANs. Multi-antenna UE capability can also be considered in order to allow receive beamforming. The assumption that ANs can hypothesize the SNR at each UE may be a practical issue in real systems. For example, the ANs do not know when a new UE arrives (or is turned on) in the network. One may also consider new metrics to be optimized, such as the time a new UE waits until its synchronization is established. A more challenging task would be to integrate some methodology to obtain statistics of UEs, possibly in a distributed fashion. To reach optimality, the problem formulation may be extended assuming some kind of convex relaxation over the spatial (beam) dimension. Also, the problem may be reformulated as a mixed-integer linear programming (MILP).

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