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Identification by Recursive Least Squares With Kalman Filter (RLS-KF) Applied to a Robotic Manipulator

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ABSTRACT The field of robotics has grown a lot over the years due to the increasing necessity of industrial production and the search for quality of industrialized products. The identification of a system requires that the model output be as close as possible to the real one, in order to improve the control system. Some hybrid identification methods can improve model estimation through computational intelligence techniques, mainly improving the limitations of a given linear technique. This paper presents as a main contribution a hybrid algorithm for the identification of industrial robotic manipulators based on the recursive least square (RLS) method, which has its matrix of regressors and vector of parameters optimized via the Kalman filter (KF) method (RLS-KF). It is also possible to highlight other contributions, which are the identification of a robotic joint driven by a three-phase induction motor, the comparison of the RLS-KF algorithm with RLS and extended recursive least square (ERLS) and the generation of the transfer function by each method. The results are compared with the well-known recursive least squares and extended recursive least squares considering the criteria of adjustable coefficient of determination (R_a^2) and computational cost. The RLS-KF showed better results compared to the other two algorithms (RLS and ERLS). All methods have generated their respective transfer functions.

INDEX TERMS Kalman filter, recursive least squares, optimization, systems identification, RLS-KF.

I. INTRODUCTION

The field of robotics has grown a lot over the years due to the need to increase industrial production and the search for the quality of industrialized products. As a result, the use of industrial robots has grown significantly in the context of industrial production in recent years, so investments in industrial robots continue to increase. Global sales of industrial robots doubled from 2013 to 2017 (114%) and the outlook for the coming years (2018-2021) estimates that global sales will increase by an average of 14% per year [1].

Most robots in industry perform material handling, assembly, pelleting tasks, among others. With advances in drive systems for manipulating robots, image systems and control it is possible for robots to share the same work area, perform shared tasks and interact with operators and other equipment (obstacles) [2]–[4].

In the context of advances in robotics and improvements in industrial systems, new ideas are emerging to improve robotics systems increasingly, in addition to making them more economical. The improvements concern the optimization of the model by approximations using identification techniques and computational intelligence [5], [6].

Several algorithms of system identification were proposed and discussed in literature. Some applications involve the use of the Kalman filter (KF) for identification using adaptive tracking, least squares nonlinear sequential estimation, tracking of signal variations, among others [7]. KF is a recursive Bayesian filtering technique for state estimation from noisy measurements. At each time step, a gain matrix

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is computed using a set of predefined processes and the measurement noise covariance in order to incorporate the deviation between measured and predicted output response as feedback in the system. This gain matrix, in turn, regulates the mixing between the previous belief and the information in the new data to estimate the state for the next time step. This algorithm when used for parameter identification adds the parameters to an existing state vector or considers the parameters as the only states [8].

Since the first use of KF and optimal control theory for model identification in [9], several modifications have been proposed, including the extended Kalman filter (EKF), which is widely used for nonlinear estimation [10], [11]. In [12] a variation of KF was proposed and applied to a dynamic positioning (DP) system of a ship with model uncertainties, model mismatch and slow varying drift forces. The adaptive algorithm is proposed to simultaneously adapt the process and the noise of covariance measurement adopting the principle of covariance matching. In [13] a successive Bayesian filtering structure was proposed to address the problem of estimation of the joint uncertainty of input state parameter. This uncertainty is quantitatively expressed by a vector of parameters of known functional relation with the structural matrices. An observer is consequently established, which recombines the KF.

In [14] was proposed a novel hybrid algorithm for simultaneously estimating the vehicle mass and road grade for a hybrid electric bus (HEB). The first step was the estimation of the road grade using extended Kalman filter (EKF) with the initial state including velocity and engine torque. After that, the vehicle mass was estimated twice, one with EKF and the other with recursive least square (RLS) using the estimated road grade. The proposed algorithm provided higher accuracy and faster convergence speed, compared with the results using EKF or RLS alone.

In [15] was proposed a novel method to compute the pseudo-inversion matrix in order to develop an extended recursive least-squares (ERLS) algorithm. The ERLS algorithm was proposed for solving the over-determined normal equations in the instrumental variable approaches. In [16] an algorithm was proposed based on a nonlinear function of the error, motivated by the ERLS algorithm. Simulations were performed on the problem of tracking a nonlinear Rayleigh fading multi-path channel. The results showed that the proposed algorithm can overcome the extended kernel version ones.

In [17] an identification algorithm was proposed for Hammerstein nonlinear systems with dynamic disturbances and measurement noise. By extending the parameter and information vector, an ERLS algorithm is proposed for the first time to identify recursively both the system parameters and the dynamic disturbance of a Hammerstein output error model. In order to select the optimal tracking of a fast time variation multi-path Rayleigh fading channel, [18] focused on the RLS and ERLS algorithms concluded that the ERLS was more feasible according to the comparison output of the simulation.

RLS has also been widely used to identify industrial processes [19]. Very noisy input and output signals cause great polarization with the RLS in the application of system identification. Some improved methods have been proposed to compensate the noise effect. In [20] a multi-timescale method was presented for dual estimation of SOC and capacity with an online identified battery model. The model parameter estimator and the dual estimator were fully decoupled and executed with different timescales to improve the model accuracy and stability. In [21] and [22] the compensation of the effect of noise is carried out, the online identification of the parameters and a new method of parameterization is proposed combining the estimation of instrumental variable for the equivalent circuit model applied in battery systems.

Some methodologies for estimating model parameters have been proposed using the RLS method, which updates a vector of parameters and has a lower computational cost than the method of non-recursive least squares [5]. Many researches were developed looking for an improvement of the RLS using other algorithms like neural networks, fuzzy systems, network based fuzzy inference system (ANFIS) and metaheuristics [23]–[29].

In [30] was presented a new hybrid method based on Gravity Search Algorithm (GSA) and RLS, known as GSA-RLS, to solve the harmonic estimation problems in the case of time varying power signals in presence of different noises. In [31] was presented a new hybrid method based on biogeography-based optimization (BBO) and RLS algorithms, called BBO–RLS, to solve the harmonic estimation problem. The basic BBO algorithm was combined with RLS in an adaptive way to sequentially update the unknown parameters (weights) of the harmonic signal.

In [32] two information fusion Kalman filters were proposed for time-variant multi-sensor systems with correlated measurement noises and different measurement matrices, on the basis of RLS method, least squares (LS) method and Kalman filtering theory. The theory from Cholesky factorization was used to convert the former multi-sensor systems into unrelated noise and equivalent multi-sensor pseudo-measurement and pseudo-measurement model where LS and RLS methods were respectively used to estimate the state. In [33] an improved multi-innovation least squares algorithm was proposed in order to identify the unknown model parameters of a system. The results of the model parameter identification with the steady-state fusion Kalman filter of multi-sensor system were given by using the matrix weighted fusion criterion.

In [34] was proposed a KF based least squares iterative (KF-LSI) algorithm. First, the observed data from a multi-variable system are weighted, fused and combined with the state equations to form a new system model representation. Then the KF-LSI was used in order to obtain the model parameter estimation and with addition of several subsystems estimations the fusion state is obtained. One of the challenges for control systems is to obtain the model of the process, either through first principles or using system identification techniques, because a poorly adjusted model can cause a great effort of control and consequently damage to the plant. Therefore, systems identified with greater precision are of fundamental importance for a better control performance [35]–[37]. This work proposes a hybrid system using the recursive least square method optimized with the Kalman filter method (RLS-KF) in order to weight the matrix of regressors φ and the vector of parameters θ of the recursive least squares.

The proposed identification method are compared with the classical RLS and ERLS algorithms. All methods are evaluated by the adjustable coefficient of determination (R_a^2) and its computational cost. At last, the transfer functions generated by each method are validated.

A. CONTRIBUTIONS

The main idea of the paper is to identify a robotic manipulator joint activated by a three-phase induction motor using a hybrid approach. The algorithms used for the hybrid approach are the Kalman filter and the recursive least squares, so a comparative analysis will be made with classical identification methods. In this context, it can be said that this paper presents the following contributions:

- Hybrid approach with RLS-KF, being an improvement for the RLS;
- Identification of a robotic joint driven by a three-phase induction motor;
- Comparison of the RLS-KF algorithm with RLS and ERLS;
- Identification of the transfer functions of each method.

This paper is organized as follows. In section II the characteristics of the robotic manipulator and the problem characterization are presented. In section III is discussed the formulations of the RLS, KF, ERLS algorithms. In section IV the methodology for implementing the RLS-KF method, proposed in this work, is presented. The results of the implemented algorithms and the discussions about the research are presented in section V. Finally the study conclusions and future works are commented in section VI.

II. PROBLEM CHARACTERIZATION

There are several types of manipulators used in different applications according to their geometry, however in this work a cylindrical robotic arm is used. This manipulator has 3 degrees of freedom (3-DOF), where the first one is the basis for rotating movements, the second is linear which is the trunk that makes the movements in the vertical and the third degree is the one that makes the movements in the horizontal [39].

The test bench 3-DOF manipulator is showed in Fig. 1. It should be noted that each joint is driven by three-phase squirrel cage induction motors (IM), because the robot is designed to move heavy loads. In order to activate the manipulator, a Texas Instruments digital signal processor (DSP) micro-controller is used. The main advantage in using the DSP is that, in addition to the high processing performance,

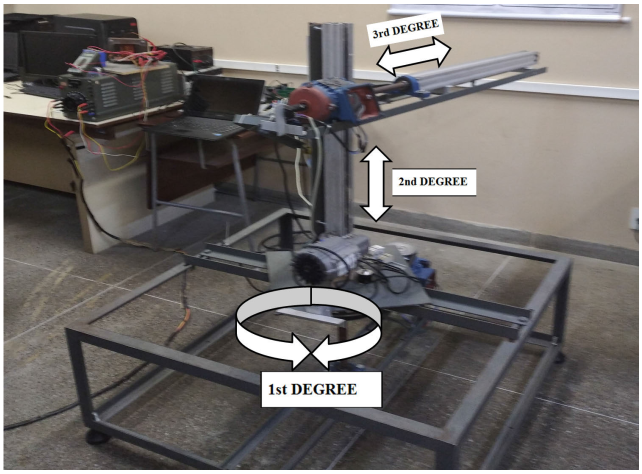

FIGURE 1. Setup of cylindrical manipulator.

with the capacity to carry out 150 *MPIS* (millions of instructions per second), a space vector pulse width modulation, SVPWM, can be implemented. The programming interface and the Code Composer Studio (CCS) are implemented in C language [40].

Although the robotic system has 3 degrees of freedom, only the data from the first joint were acquired. The goal is to show the effectiveness of the RLS-KF to model the joints, therefore signals of current and speed are collected in order to identify the mechanical behavior of one joint, which is showed in Fig. 2.

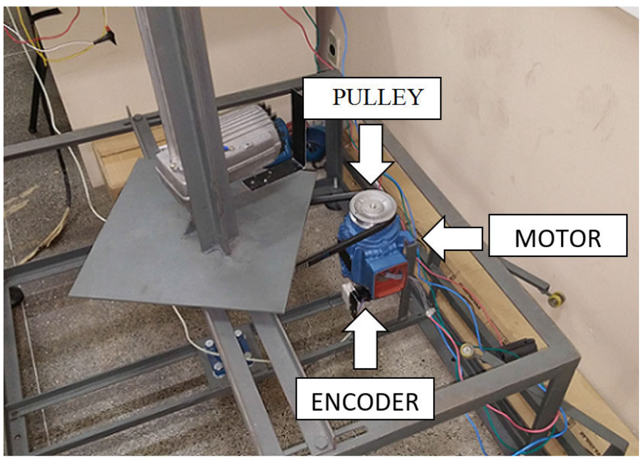

FIGURE 2. First joint motor.

The experimental setup consists of a digital signal controller (DSC) from Texas Instruments $TMS320F2812$, a Hall-effect current sensors, auxiliary voltage sources, a three-phase voltage inverter module by Semikron with a switching frequency of 2.5 kHz, a multi-turn precision potentiometer coupled to the motor shaft, with a sampling time of 0.4 ms. The movement transmission to the manipulator's first degree of freedom occurs through the use of belt and pulleys, as can be seen in the Fig. 2. In order to properly power the circuits and plates, an auxiliary voltage source capable to provide 4 levels of continuous voltage was developed. A 18 V voltage switches the triggers of the three-phase inverter, 15 V and -15 V are used for the current sensors and 5 V for the board power of signal conditioning. The electronic setup system is showed in Fig. 3.

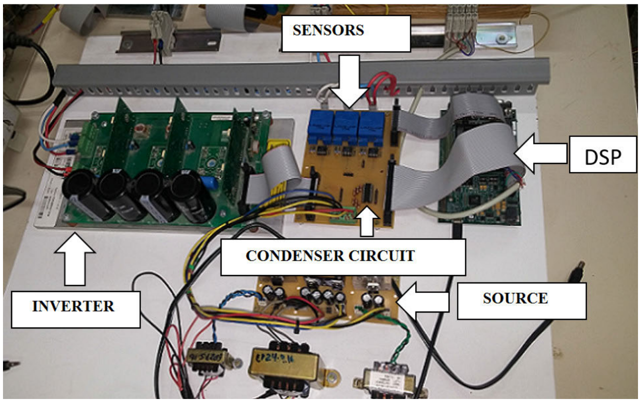

FIGURE 3. Setup of cylindrical manipulator.

The electrical and mechanical parameters of the considered IM are described in Table 1.

TABLE 1. Motor parameters.

Parameters	Value
Rated power	0.25 HP
Rated speed	1725 rpm
Rated voltage	220 V
Rated current	1.26 A
Number of poles	4
Rotor resistance (referred to the stator)	87.44Ω
Stator resistance	35.58Ω
Rotor inductance (referred to the stator)	0.16 H
Stator inductance	0.16 H
Mutual inductance	0.884 H
Inertia moment	$5 \times 10^{-4} kg \times m^2$
Viscous friction coefficient	$5.6510^{-4} kg \times m^2/s$

Two experiments are carried out in order to collect the current and speed data as showed in Figures 4 and 5. The total time of each experiment is 1001 s so that the first data set is used to estimated the model and the second one to test it. The current reference is chosen as sinusoidal and trapezoidal for more dynamic behaviors to be explored.

III. IDENTIFICATION METHODS

Methods of system identification are used in several applications such as supervision, diagnostics, filtering, prediction, signal processing, detection and variant parameter tracking for predictive control. In this section, two methods of identification are discussed: recursive least squares (RLS) and extended recursive least squares (ERLS). Furthermore, it is presented the proposed approach of recursive least squares with Kalman filter (RLS-KF) according to the literature [36], [38], [41].

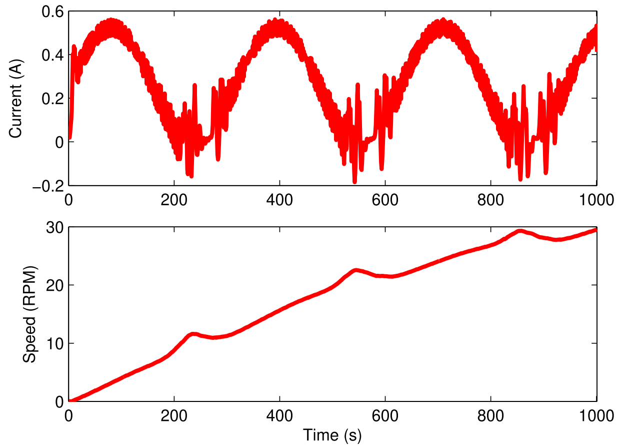

FIGURE 4. Actual input and output of the plant for training stage.

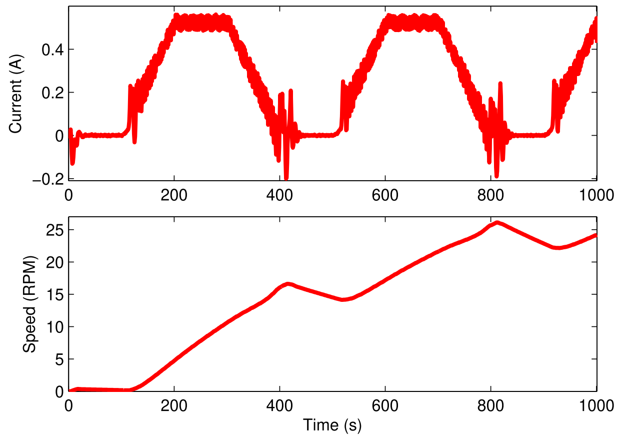

FIGURE 5. Actual input and output of the plant for test stage.

A. RECURSIVE LEAST SQUARES (RLS)

The RLS method is often applied when it is necessary to compute a model of the system in real time, while the system is in operation [36], [41]. Assuming that the model in a data set can be represented by an equation in the form of a regressor such as,

$$\hat{y} = \varphi\hat{\theta} + \xi \tag{1}$$

where φ is called the matrix of regressors and ξ is the residue. The parameters estimation is calculated as

$$\theta_{[k]} = \theta_{[k-1]} + L_{[k]}\left[y_{[k]} - \varphi_{[k]}^T\theta_{[k-1]}\right] \tag{2}$$

where

$$L_{[k]} = \frac{P_{[k-1]}\varphi_{[k]}}{\lambda_{[k]} + \varphi_{[k]}^T P_{[k-1]}\varphi_{[k]}} \tag{3}$$

and

$$P_{[k]} = \frac{1}{\lambda_{[k]}}\left[P_{[k-1]} - \frac{P_{[k-1]}\varphi_{[k]}\varphi_{[k]}^T P_{[k-1]}}{\lambda_{[k]} + \varphi_{[k]}^T P_{[k-1]}\varphi_{[k]}}\right]. \tag{4}$$

The P matrix and the regression vector are initialized with a value very close to zero. As the iterations go on, these values change and the final parameters are obtained according to a better result.

B. EXTENDED RECURSIVE LEAST SQUARES (ERLS)

The ERLS method can be applied to estimate the parameters of general nonlinear systems, as described in [38], where the regression vector $\varphi_{[k]}$ is constructed using the residual vector $\varepsilon_{[k]}$ and the regression vector written as

$$\varphi^T_{[k]} = [-y_{[k-1]} \ldots - y_{[k-N]}\ u_{[k-1]} \ldots \\ u_{[k-N]}\ \varepsilon_{[k-1]} \ldots \varepsilon_{[k-N]}]. \quad (5)$$

The recursive algorithm is given by

$$L_{[k]} = \frac{P_{[k-1]}\varphi_{[k]}}{\lambda_{[k]} + \varphi^T_{[k]}P_{[k-1]}\varphi_{[k]}}, \quad (6)$$

$$P_{[k]} = \frac{1}{\lambda_{[k]}}\left[P_{[k-1]} - \frac{P_{[k-1]}\varphi_{[k]}\varphi^T_{[k]}P_{[k-1]}}{\lambda_{[k]} + \varphi^T_{[k]}P_{[k-1]}\varphi_{[k]}}\right], \quad (7)$$

$$\theta_{[k]} = \theta_{[k-1]} + L_{[k]}\left[y_{[k]} - \varphi^T_{[k]}\theta_{[k-1]}\right] \quad (8)$$

and

$$\varepsilon_{[k]} = y_{[k]} - \varphi^T_{[k]}\hat{\theta}_{[k]}. \quad (9)$$

C. KALMAN FILTER

The Kalman filter is an instantaneous estimator of a particular state disturbed by noise, in a linear dynamic system. It is statistically efficient with respect to any quadratic function of estimate errors [42], in which its equations form a recursive process capable of reducing the sum of the squares of the differences between the measured and the estimated values. In general, it is considered a probabilistic distribution propagator, since it provides a complete characterization of the current state of a system, including past references, without actually needing the previous values.

The Kalman filter is applicable for modelling in equations of states, a common way of describing physical aspects. With the significant increase in computational power recently, the Kalman filter is widely used in control techniques, including appendices to nonlinear problems [43].

Considering a linear system modelled in state equations, according to [43],

$$x_k = A_k x_{k-1} + w_k, \quad (10)$$

$$y_k = B_k x_k + z_k, \quad (11)$$

where:

- k the current time;
- x_k the state in time k;
- w_k process noise;
- y_k the observation of the state in time k;
- z_k the noise of the measurements;
- A_k and B_k system control inputs.

It is important to note that for most applications of linear dynamic systems, including the desired one, the w_k and z_k noises are considered uncorrelated white Gaussian noises from modelling errors and measurement sensors, respectively. Thus, such noise has zero mean and covariance Q_k and R_k, respectively. In the prediction, the values of the signal of interest, $\hat{x}_{k/k-1}$, and the error covariance matrix, $P_{k/k-1}$, at time k, are based on the values of the previous step, at time $k-1$. The initial estimate, known as *a priori*, of the signal of interest, is based on the estimate of the signal itself in the previous step, $\hat{x}_{k-1}$, weighted by a control matrix of the system in question, known as the state transition matrix, A_k,

$$\hat{x}_{k/k-1} = A_k\hat{x}_{k-1}. \quad (12)$$

The covariance matrix of the *a priori* error, $P_{k/k-1}$, is based on its own value in the previous step, P_{k-1}, on the aforementioned matrix A_k, and on the noise of process inherent to the system, Q_k,

$$P_{k/k-1} = A_k P_{k/k-1} A^T_k + Q_k. \quad (13)$$

Then, the correction calculations are performed, in which the values of the signal of interest, $\hat{x}$, and the covariance matrix of the error, P_k, are again estimated, however with the updates from the Kalman gain.

The Kalman gain, K_k, is based on the estimated covariance matrix of the error, $P_{k/k-1}$, found in the prediction, on a matrix B_k, relative to the equation of states of the system and on the noise of the measurements of the system, R_k, relative to the acquisition of the measured signal,

$$K_k = P_{k/k-1}B^T_k(B_k P_{k/k-1} B^T_k + R_k)^{-1}. \quad (14)$$

Finally, a new *a posteriori* estimate of the covariance matrix of the error, P_k, is also calculated, taking into account the estimate obtained in the prediction, $P_{k/k-1}$, the matrix B_k and the Kalman gain, whose objective is to minimize this error,

$$P_k = (I - K_k B_k)P_{k/k-1}. \quad (15)$$

Based on the described equations, it is said that when the measurement noise z_k is high, the current measurement will be given little credit in the next estimate, since the measurement obtained is not reliable, making the convergence of the system slower. When the measurement noise is small, a lot of credit will be given to the current measurement in the following estimate, since the measurement has relevance, making the system to converge faster.

Considering the process noise w_k purposely inserted due to possible errors in the model, when its value is high, more credit to the current measurement is given in the next estimate, since the disturbances in the model used are relevant, making the convergence of the system faster. When the process noise is small, little credit is given to the current measurement in the next estimate, giving greater relevance to the model, and the system slowly converges.

IV. METHODOLOGY

The idea is that the KF can improve the identification of the RLS (RLS-KF). The diagram in Fig. 6 shows the general behavior of the hybrid identification system.

As the Kalman filter is a state estimator, the basic idea is to adjust some parameters of the RLS to make a good identification. Some variables are already initialized before entering

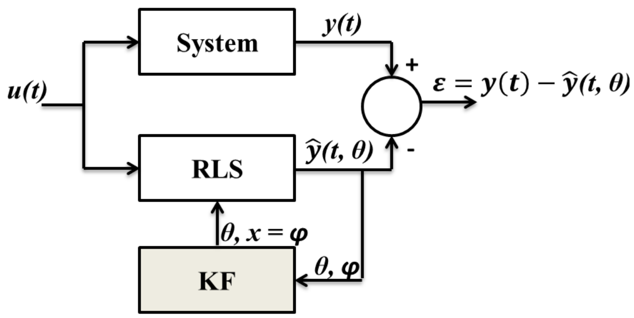

FIGURE 6. Hybrid system.

the hybrid algorithm loop. The number of delays defined for the algorithm is 2. Therefore, most of the dimension of the matrix variables are 4×4. For the execution of the algorithm, it was necessary to initialize some variables such as the matrix R_w and the variable R_v in (16) and (17):

$$R_w = \begin{bmatrix} 0.99 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 0.99 \end{bmatrix}, \quad (16)$$

$$R_v = \begin{bmatrix} 0.99 \end{bmatrix}. \quad (17)$$

The steps of the hybrid system can be seen in Algorithm 1.

Algorithm 1 Hybrid System (RLS-KF)

Initialize values from: θ, φ, x and P;
for i = delay to t, where t = length(y) **do**
$\varphi = [y_{(t-1)}; y_{(t-2)}; u_{(t-1)}; u_{(t-2)}]$ $\hat{y}_{(t)} = x'\theta$ $P = P + R_w$, propagation step
$K = \frac{Px}{R_{v+x'px}}$, correction step
$\theta_{(t)} = \theta + K(y_{(t)} - \hat{y}_{(t)})$
$P = (I - kx')P$, where I is a 4×4 dimension identity matrix
$a_1 = \theta_{(1)}$
$a_2 = \theta_{(2)}$
$b_0 = \theta_{(3)}$
$b_1 = \theta_{(4)}$
$x = \varphi$
end for

Parameters θ, φ, x and P were initialized, all with values close to zero. The matrix R_w was filled in its main diagonal 0.99 and the rest of the matrix elements by zero, also variable R_v was started by 0.99. This initialization of the parameters provides a better performance of the algorithm.

With the final model, a discrete transfer function will be obtained in the hybrid method, in the same way that it will be obtained also in the previous methods. There are some problems in the implementation of the RLS, because the method has many parameters to be adjusted, the Kalman filter solves these errors most of the time when in a hybrid system.

A. THEORETICAL COMPARISON

The RLS-KF algorithm is based entirely on the theory of RLS with KF, where the updated θ, φ, x parameters are passed to the classic RLS to do the identification. The number of instructions to discover the complexity of the algorithms is very important to have a well-explained theoretical basis.

According to Algorithm 1 that describes the RLS-KF, it has the following instruction count:

line 1 (initialize θ, φ, x and P): $1 + 1 + 1 + 1 = 4$;
line 2 (repeat): $2n + 2$;
line 3 (update of 3 variables within the repetition): $n + n + n = 3n$;
line 4 (updating a variable): n;
line 5 (updating a variable): n;
line 6 (updating a variable): n;
line 7 (updating a variable): n;
line 8 (updating a variable): n;
line 9 (updating a variable): n;
line 10 (updating a variable): n;
line 11 (updating a variable): n.
Resulting in: $C(n) = 13n + 6$.

The RLS instruction count is given by:

line 1 (initialize θ, φ and P, would be before the equations (1), (2), (3) and (4)): $1 + 1 + 1 = 3$;
line 2 (repeat, gives equation (1) to (4)): $2n + 2$;
line 3 (updating a variable, eq.(1)): n;
line 4 (updating a variable, eq.(2)): n;
line 5 (updating a variable, eq.(3)): n;
line 6 (updating a variable, eq.(4)): n;
Resulting in: $C(n) = 6n + 5$.

Finally, in the counting of ERLS instructions, we have:

line 1 (initialize θ, φ, P and ε, would be before the equations (5), (6), (7), (8) and (9)): $1 + 1 + 1 + 1 = 4$;
line 2 (repeat, gives equation (5) to (9)): $2n + 2$;
line 3 (updating a variable, eq.(5)): n;
line 4 (updating a variable, eq.(6)): n;
line 5 (updating a variable, eq.(7)): n;
line 6 (updating a variable, eq.(8)): n;
line 7 (updating a variable, eq.(9)): n;
Resulting in: $C(n) = 7n + 6$.

In a theoretical comparison, the RLS-KF algorithm maintains the same complexity when counting instructions as seen in this subsection, $C(n) = n$, although it has more instructions than the RLS and ERLS. The algorithms have the same computational complexity even though one knows that the RLS is better since it gives and equation that describes better the computational cost. In general, all algorithms have the same computational cost due to the same polynomial order of C(n).

V. EXPERIMENTAL RESULTS

In this section it is presented the experimental results with the RLS, ERLS and RLS-KF identification methods.

A. RLS RESULTS

As mentioned in Section II, a sinusoidal input current signal and a speed output signal are used for training, the identification with classic RLS presented the results in the Fig. 7.

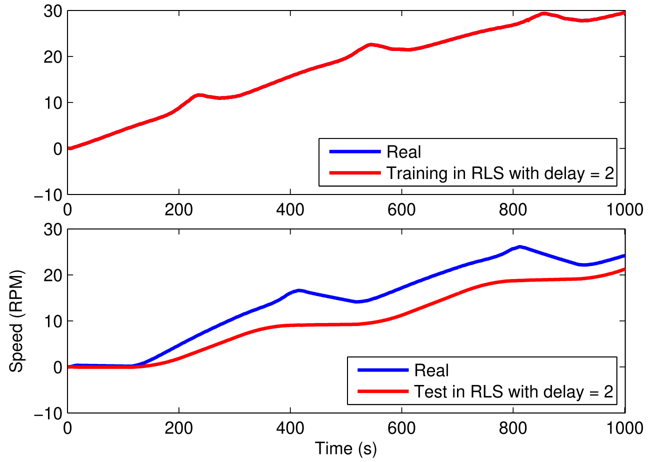

FIGURE 7. Output response: RLS.

In the top of Fig. 7 it is shown the training stage where the parameters of θ and P are obtained. After that, these test step parameters are used with the rest of the data set. The matrix P generated by exhaustive search, can be written as

$$P = \begin{bmatrix} 0.5337 & -0.533 & 0.0177 & 0.0243 \\ -0.533 & 0.5340 & -0.017 & -0.024 \\ 0.0177 & -0.017 & 0.1057 & -0.100 \\ 0.0243 & -0.024 & -0.100 & 0.1065 \end{bmatrix}, \quad (18)$$

and the regression θ vector generated in the training and used in the validation test is written as follows

$$\theta = \begin{bmatrix} -1.8279 & 0.8279 & -0.0054 & 0.0201 \end{bmatrix}. \quad (19)$$

The discrete-time transfer function generated with the training model with a sample time of 0.2 s, is presented:

$$\frac{-0.005371z + 0.02011}{z^2 - 1.828z + 0.8279}. \quad (20)$$

The RLS algorithm achieved a reasonable result with not very heavy parameters. However, the result is not yet above average, and may have problems with complex situations. It is important to note that the system has a non-linear behavior, so sometimes the RLS did not get close enough to reach the proper solution, for example in the testing stage. In more demanding situations the number of iterations can increase, thus compromising the execution time.

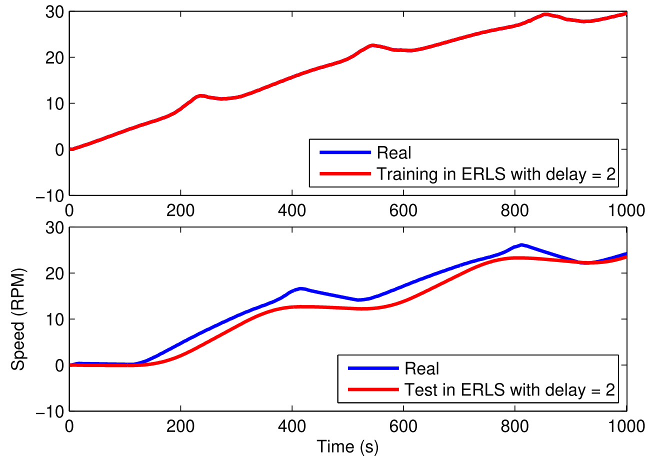

FIGURE 8. Output response: ERLS.

B. ERLS RESULTS

One of the advantages of ERLS is the identification of dynamic systems with a strong tendency to nonlinearities. Fig. 8 shows the comparison between the real and the estimated output signal ($y_{(t)}$ and $\hat{y}_{(t)}$) using the ELRS method.

The regression θ vector generated in the training stage and used in the test is written as

$$\theta = \begin{bmatrix} -1.9517 & 0.9517 & -0.0058 & 0.0121 & -0.5964 \end{bmatrix}. \quad (21)$$

The matrix P generated can be seen in the following equation (22), as shown at the bottom of the page

The discrete-time transfer function generated with the training model with a sample time of 0.2 s is formulated as follows:

$$\frac{-0.005762z + 0.0121}{z^2 - 1.952z + 0.9517}. \quad (23)$$

C. RLS-KF RESULTS

The estimation results of the proposed approach are presented in Fig. 9 for the training and the test stage, respectively, where one can see that the KF combined with the RLS has superior results compared to the classical methods. In the experiments, the matrix (16) and the scalar (17) are determined in order to verify its influence in the identification procedure.

By observing the generated curves in the training and testing steps (Fig. 9), it is possible to notice a smaller error when using the proposed algorithm compared to the other methods, however, considering that two algorithms are used,

$$P = \begin{bmatrix} 1.3189 & -1.3193 & 0.0362 & 0.0648 & 1.2030 \\ -1.3193 & 1.3196 & -0.0361 & -0.0647 & -1.2035 \\ 0.0362 & -0.0361 & 0.2114 & -0.2003 & 0.0042 \\ 0.0648 & -0.0647 & -0.2003 & 0.2140 & 0.0775 \\ 1.2030 & -1.2035 & 0.0042 & 0.0775 & 5.7300 \end{bmatrix}. \quad (22)$$

TABLE 2. Output R_a^2 and computational cost for the stages.

Training stage			Training + Test stage			Test stage		
R_a^2			Computational cost [s]			R_a^2		
RLS	ERLS	RLS-KF	RLS	ERLS	RLS-KF	RLS	ERLS	RLS-KF
0.9901	0.94024	0.9999	2.85	3.02	3.15	0.7432	0.9385	0.9773

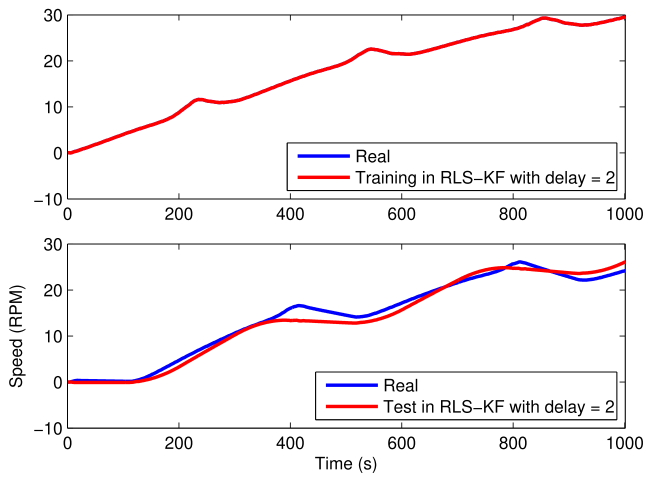

FIGURE 9. Output response: RLS-KF.

it results in a higher computational cost. It is important to emphasize that according to the Algorithm 1, the KF is used in the RLS method with the objective of estimating some parameters, therefore the KF is divided into two stages: the prediction and correction stage.

The prediction stage estimates the values of the next state, as well as the value of the covariance matrix. While in the correction stage, the state estimation values and covariance matrix are updated with the Kalman gain, which is calculated from the prediction error.

As can be seen in the training and the testing stages of the hybrid method, considering the comparison between the real output and the estimated output, the proposed method obtained better results compare to the others. The final regression vector θ obtained in the training stage and used in the test is:

$$\begin{bmatrix} -0.7292 & -0.2702 & 0.0770 & 0.0985 \end{bmatrix}. \quad (24)$$

The discrete transfer function generated from the training stage model is described as follows:

$$\frac{0.07697z + 0.09852}{z^2 - 0.7292z + 0.2702}. \quad (25)$$

D. COMPARISON OF ALGORITHMS

In this section, a more detailed comparative analysis of the different approaches is presented. A quantitative analysis of each method in identifying the speed of joint 1 of the manipulator is presented in Table 2 by means of performance indices: adjustable coefficient of determination, R_a^2 and computational cost of each algorithms. The first index, R_a^2 is

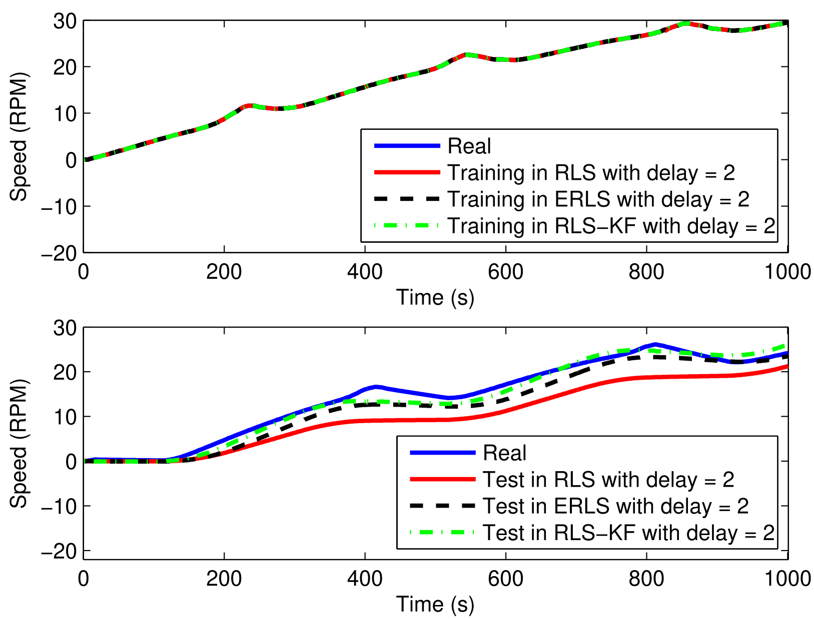

FIGURE 10. Output response: all results.

given by

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_{(i)} - \hat{y}_{(i)})^2}{\sum_{i=1}^{n}(y_{(i)} - \bar{y}_{(i)})^2},$$
$$R_a^2 = 1 - \left(\frac{n-1}{n-(p+1)}\right)(1-R^2), \quad (26)$$

where:

- $y_{(i)}$ is observation;
- $\hat{y}_{(i)}$ is prediction;
- $\bar{y}_{(i)}$ is average of observations;
  R^2 is determination coefficient;
- n is number of observations;
- p is number of parameters.

Table 2 summarizes the results of the training and testing stages for all methods. The R_a^2 of the output signal and the computational cost of each method for both training and testing stages are calculated. In all cases, the RLS-KF presents better results.

When analyzing the table above, one can see that the RLS-KF method obtained an R_a^2 of **0.9999** in the training stage and **0.9773** in the test stage. The best computational cost was obtained with the RLS method (**2.85 seconds**). For the present work the computational configuration used consists of: a mathematical simulation program (Octave 4.2.1),a 64-bit operating system (GNU Ubuntu 20.04 LTS), 8 *GB* of rapid access memory (RAM), and a Intel Core i5 processor with a 3.2 *GHz* clock speed.

The results seen in the Table 2 are presented in Fig. 10, where the identification of the proposed method is better than the others, and can be visually analyzed. Despite injection disruptions, all methods have managed to guarantee their robustness with the acceptable adjustments presented in literature.

The identification is developed in the same scenarios, in which the best delay and the same development environment are chosen for all methods. Thus, it consists of a fair comparative analysis between the proposed method and the others presented. The model generated by the RLS-KF can be used in the real system, considering the proper operating points.

VI. CONCLUSION

The hybrid method (RLS-KF) was proposed to adjust multiple parameters of the RLS method. Thus, the design and implementation are conceptually simple and easy to code. The study ensured that during the training and testing stages with input and output references (current and speed, respectively), it would obtain a better performance when compared to other strategies such as ERLS, which is an iterative method widely used in several researches. The hybrid method proposed in this work obtained an R_a^2 of **0.9999** in the training stage and **0.9773** in the test stage. The RLS method obtained an R_a^2 of 0.9901 in the training and 0.7432 in the test stages while the ERLS R_a^2 indices were 0.94024 and 0.9385 for the training and test stages, respectively. Regarding the computational cost, the RLS presented the best result with 2.85 *s*, the ERLS with 3.02 *s* and the RLS-KF with 3.15 *s*.

The comparison results show that the method proposed here presented the best result for this application, consisting in a relevant approach considering the complex scenario of the three-phase induction motor, which is not easy to model. However, the proposed method had a small limitation that can be improved in relation to the other compared methods, which is the computational cost.

As future works, it can be attempted to further reduce the coding of the hybrid method proposed by using dynamic programming techniques to reduce the complexity of the computational cost. Also, noise can be added to input and output signals to analyze the implementation of the method proposed in this scenario.

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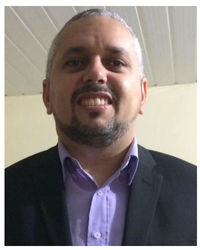

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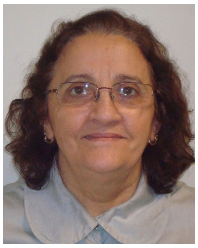

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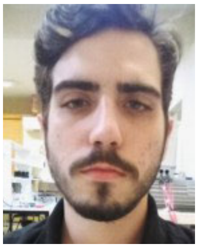

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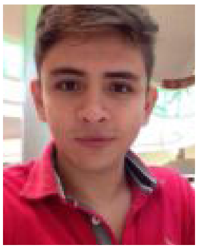

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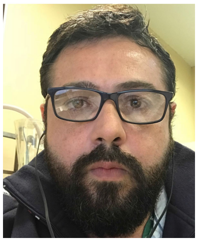

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