

# Different horizons of the application of Weibull distribution for Wind resource assessment: a case study for the Brazilian Northeast region

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## Abstract

This paper aims to apply the Weibull distribution in the annual and semiannual horizons to contribute to the assessment of wind energy potential in three cities located in Northeastern Brazil (BRNER) region. The potential for wind power in a period can be evaluated by a probability density function such as the Weibull distribution. Different methods for calculating these parameters, including the method of equivalent energy, developed to optimize the application of this frequency distribution to the data collected in northeastern Brazil, were tested. This research seeks to optimize the fit to the data of wind speed and contribute to the reduction of errors in the analysis of wind potential of cities located in the Brazilian Northeast. According to the analysis, it was found that the reduction of the horizon of application provides better efficiency in the adjustment of calculated and observed frequencies. It was also observed that the Weibull distribution applies satisfactorily to the data collected in coastal cities and its efficiency is reduced when applied to distant cities on the coast. Among all analyzed methods for calculating the parameters ( $k$  and  $c$ ) of Weibull distribution, the method of equivalent energy was the one which performed better.

## 1. Introduction

The use of frequency distributions as a tool for data representation of local wind speed is a very common practice in the wind industry, because they can aggregate the numerous variants that influence the behavior of the wind. According to Silva (2003), a good measure to evaluate the wind energy potential of a site is through the use of frequency distributions.

As noted in the literature, the function of the Weibull distribution is the most used in the description of the behavior of the values of wind speed at a given location (Maceira*et al.*, 2011). The Weibull distribution has been shown as a convenient method of characterizing wind resources. With the two factors ( $k$  and  $c$ ) and the average air density is usually possible to estimate the annual output of a wind turbine with good accuracy.

Northeastern Brazil (BRNER) is a region that has a well-defined seasonal pattern in relation to the behavior of wind speed data, with a first half of the year with low values of speed and a second half that features high velocity values.

Apart from analyzing the distribution parameters and the way to calculate them, it is necessary to analyze the horizons of adjustment. When apply in the Weibull distribution function for annual data, the curve should be adjusted every observed month, therefore, the estimations of ( $k$  and  $c$ ) must represent the low-speed data (first semester) and also the high-speed data (second semester). Besides the improvement in getting the Weibull parameters, the application of the Weibull distribution at different horizons could improve the fit to the data analysis and contribute to the reduction of errors in time to estimate the produced electricity and the analysis of potential of wind city in BRNERlocated cities.

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This paper aims to apply the Weibull distribution of the annual and half-yearly horizons, using different numerical methods for the calculation of this distribution. This application seeks to improve the data fit and reduce the estimation errors of wind power generation in the analyzed cities.

## 2. Applications and Results

### 2.1. Study Area and Data Description

The BRNER has an area of 1.5 million km<sup>2</sup> (18% of Brazilian territory), equivalent to the size of France, Italy, United Kingdom and Germany combined, with an estimated population of 53 million inhabitants, or in other words, 28% of the Brazilian population (IBGE, 2010). Wind speed data was collected by 80 metres high measurement towers, installed on three sites in three separate states of the BRNER: Parnaíba (state of Piauí), Maracanaú (state of Ceará) and Petrolina (state of Pernambuco) (Figure 1). The measurement periods for the three sites were: February 2012 to January 2013 for Maracanaú, August 2012 to July 2013 for Parnaíba and May 2012 to March 2013 for Petrolina.

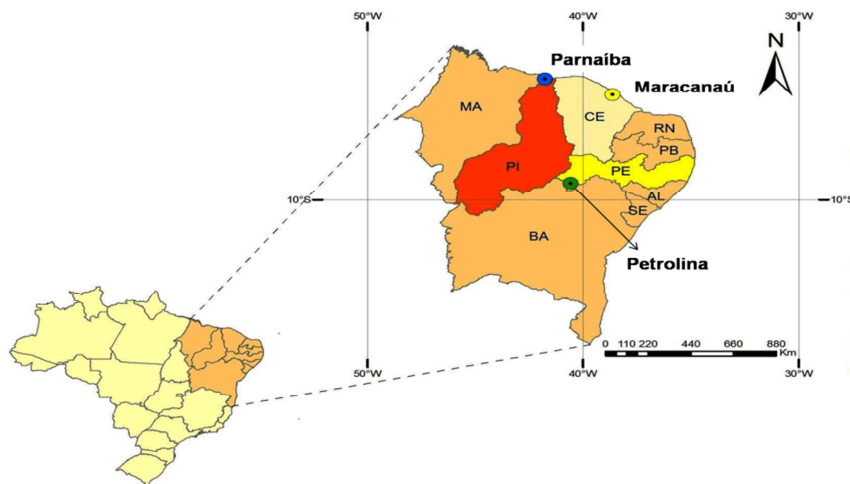


Figure 1: Localization of the measurement towers in the states of Piauí (PI), Ceará (CE) and Pernambuco (PE).

### 2.2. Investigation of numerical methods for estimation of Weibull parameters

The wind energy potential in a period may be evaluated by a probability density function, such as the Weibull distribution. This distribution is characterized by two variables:  $k$ , the dimensionless shape parameter, and  $c$ , the scale parameter, both having the same wind speed unit (Borges; Carvalho, 2012). Several numerical methods for the calculation of the Weibull parameters, observed from a wind speed set, are found in the relevant literature. Thus, in this way, investigations into the accuracy of the numerical methods for a specific site should be considered.

Six variants of numerical methods for estimating Weibull parameters are viewed in Chang (2011), and are categorized as: moment (M), empirical (E), graphical (G), maximum likelihood (ML), modified maximum likelihood (MML) and energy pattern factor (EPF). The ML method performed the best fit and the G had the lowest performance.

A new method for the estimation of Weibull parameters, called the equivalent energy (EE) method, is considered in Silva et al. (2004). It was found that errors between 2% and 7% in the energy

content could occur on half of the tested sites. Therefore, the research objective was to develop a method that should present a mean error of 1% in the energy content of the wind analysis.

An analysis and comparison of the seven aforementioned numerical methods, using wind speed data collected on two sites along the coastline of Ceará, Brazil, can be found in Rocha et al. (2012). The EE method had the best performance.

Five numerical methods for the calculation of Weibull parameters are utilized in Abdel-Hady et al. (2012). These methods are categorized as: Mean Wind Speed (MWS), G, ML, MML and Power Density (PD). The best performances are found for MWS and ML methods.

Based upon the observed wind speed data, the Weibull distribution can be described as

$$f_{Weibull}(v) = \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \quad (1)$$

$k$  and  $c$  are the shape and scale parameters of the Weibull distribution and  $v$  is the wind speed.

To estimate the Weibull distribution parameters, four numerical methods are used in the present research: Moment Method (M), Empirical Method (E), Energy Pattern Factor Method (EPF) and Equivalent Energy Method (EE).

The M method determines the  $k$  and  $c$  parameters with the use of Eqs.(2)and(3)(Chang, 2011).

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \quad (2)$$

$$\sigma = c \left[ \Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{2}{k}\right) \right]^{1/2} \quad (3)$$

$\bar{v}$  and  $\sigma$  are the mean wind speed and the standard deviation of the observed data, respectively.

$\Gamma$  represents the gamma function.

The E method is considered a special case of the M method, determined by using Eqs (4) and (5) (Chang, 2011).

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1,086} \quad (4)$$

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \quad (5)$$

The EPF Method is related to the mean wind speed, and is defined by Eqs.(6), (7) and (8) (Akdag;Dinler, 2009).

$$E_{pf} = \frac{\bar{v}^3}{(\bar{v})^3} \quad (6)$$

$$k = 1 + \frac{3,69}{(E_{pf})^2} \quad (7)$$

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \quad (8)$$

$E_{pf}$  is the Energy Pattern Factor.

The EE method was developed by Silva (2003), aiming to identify a methodology for estimation of Weibull parameters with an adequate adjustment to the wind resource found in the BRNER (wind data with relatively high shape factor values). The name comes from the fact that it is based upon the equivalence between the energy density of the Weibull curve and the energy density of the observed data. The Weibull parameters are determined with the use of Eqs.(9) and (10).

$$\sum_{i=1}^n \left[ W_{v_i} - e^{-\left\{ \frac{(v_i-1)\left[\Gamma\left(1+\frac{3}{k}\right)\right]^{1/3}}{v^3} \right\}^k} + e^{-\left\{ \frac{(v_i)\left[\Gamma\left(1+\frac{3}{k}\right)\right]^{1/3}}{v^3} \right\}^k} \right]^2 = \sum_{i=1}^n \varepsilon_{v_i}^2 \quad (9)$$

$$c = \frac{(\bar{v})^3}{\Gamma\left(1 + \frac{3}{k}\right)} \quad (10)$$

$W_{v_i}$  is the frequency of occurrence of each interval,  $\bar{v}^3$  the mean of the cubic wind speed and  $\varepsilon_{v_i}$  the approximation error.

The method efficiency is determined by using the statistical tests: RMSE (root mean square error),  $X^2$  (chi-square) and  $R^2$  (analysis of variance or efficiency of the method), as described in Eqs.(11), (12) and (14).

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (X_i - Y_i)^2}{N}} \quad (11)$$

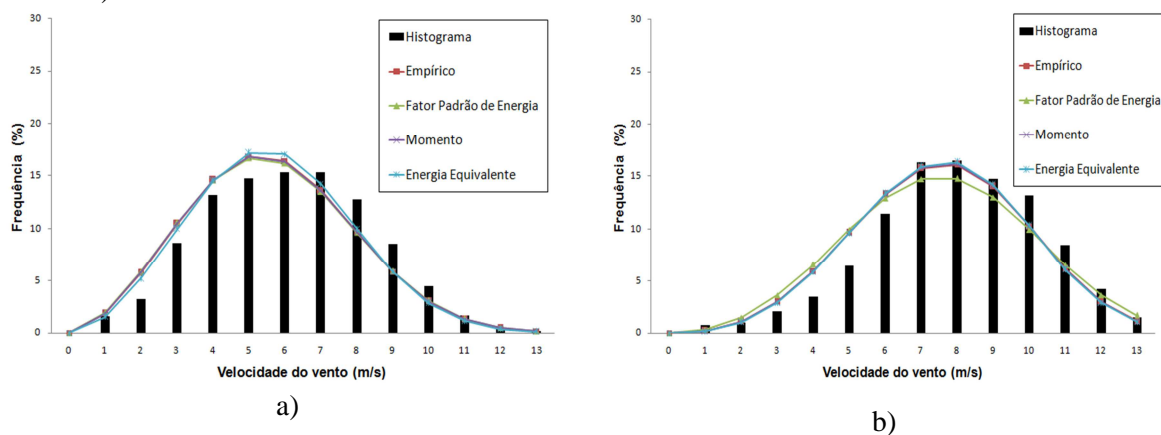
$$X^2 = \sqrt{\frac{\sum_{i=1}^N (Y_i - X_i)^2}{N - n}} \quad (12)$$

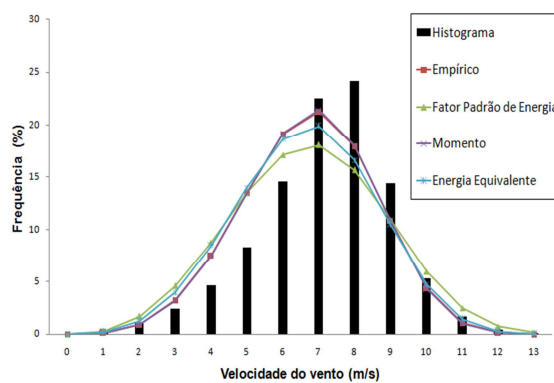
$$R^2 = \frac{\sum_{i=1}^N (Y_i - Z_i)^2 - \sum_{i=1}^N (Y_i - X_i)^2}{\sum_{i=1}^N (Y_i - Z_i)^2} \quad (13)$$

$N$  is the number of observations,  $Y_i$  the frequency of observations,  $X_i$  the frequency of Weibull and  $Z_i$  the mean wind speed.

### 2.3. Results

Figure 2, 3 e 4 illustrate Weibull curves for each of the four utilized numerical methods for Maracanaú, Parnaíba and Petrolina, the different horizons (annual, first semester and second semester).





c)

Figure 2: a) Weibull distribution –Maracanaú; b) Weibull distribution –Parnaíba; c) Weibull distribution –Petrolina.

According to Figure 2, Weibull curves for each of the four numerical methods considered in the analysis have different coincidence levels with the histograms. The best adjustments are found in Figures 2a and 2b, representing Weibull adjustment to the wind data obtained on coastal sites. Lower adjustments are found in Figure 2c, representing wind data from a site far from the coast. Tables 1, 2 and 3 show the application of the statistical tests (RMSE,  $X^2$  e  $R^2$ ) for Maracanaú, Parnaíba and Petrolina, respectively, using a measurement height of 78 metres.

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.008714	0.000075	0.929075
<b>M</b>	0.018224	0.000439	0.925308
<b>E</b>	0.017216	0.000296	0.925795
<b>EPF</b>	0.020956	0.000439	0.925308

Table 1. Application of statistical tests for Maracanaú – CE.

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.094722	0.008972	0.921334
<b>M</b>	0.107416	0.011538	0.926805
<b>E</b>	0.111639	0.012463	0.926314
<b>EPF</b>	0.247164	0.06109	0.900961

Table 2. Application of statistical tests for Parnaíba – PI.

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.000125	0.000156	0.833267
<b>M</b>	0.001272	0.000160	0.880779
<b>E</b>	0.001232	0.000152	0.879844
<b>EPF</b>	0.006303	0.003972	0.824999

Table 3. Application of statistical tests for Petrolina - PE.

According to the statistical tests, the EE method has a good performance, since it shows the best fit in tables 1 and 2, and the second best in table 3. Considering the variations in the calculation of the parameters  $k$  and  $c$  of the Weibull distribution, the same four numeric values for both semesters of the year was implemented methods, as shown in Figures 3 and 4.

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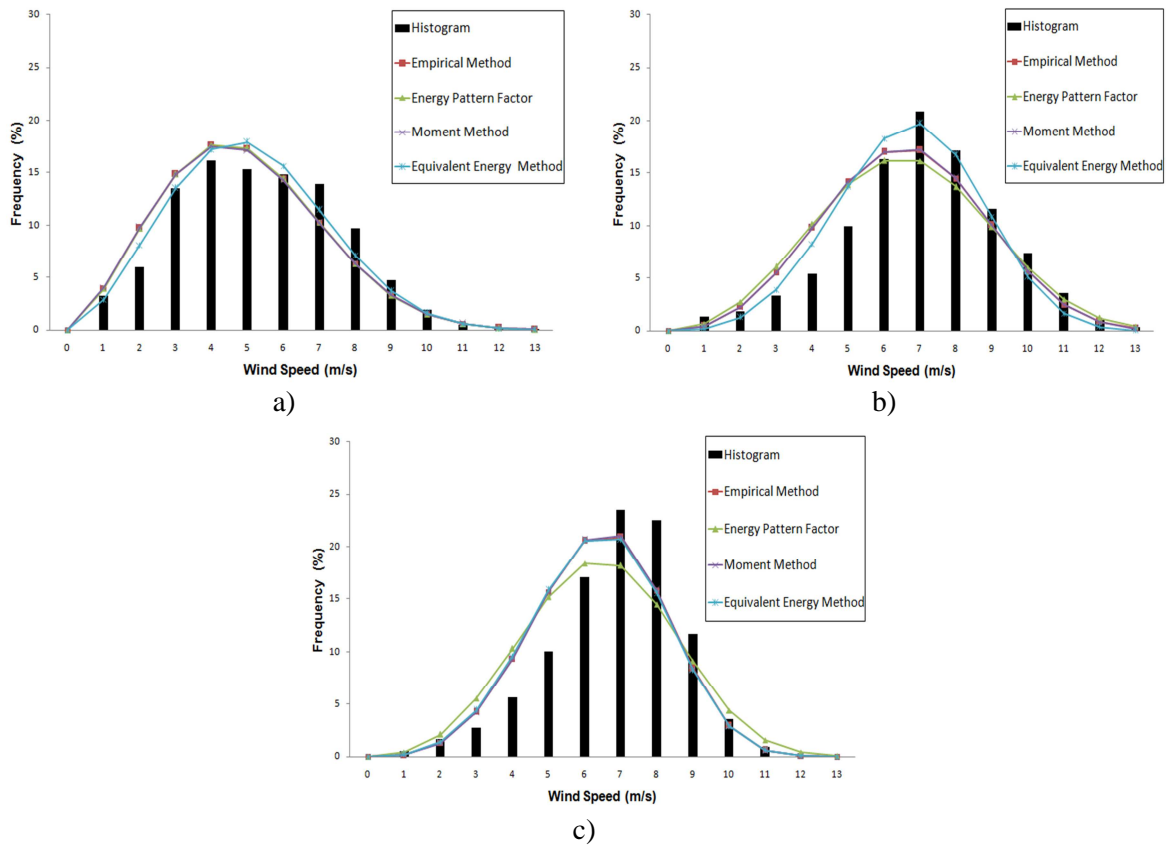


Figure 3: a) Weibull distribution –Maracanaú(first semester); b) Weibull distribution - Parnaíba (first semester); c) Weibull distribution - Petrolina (first semester).

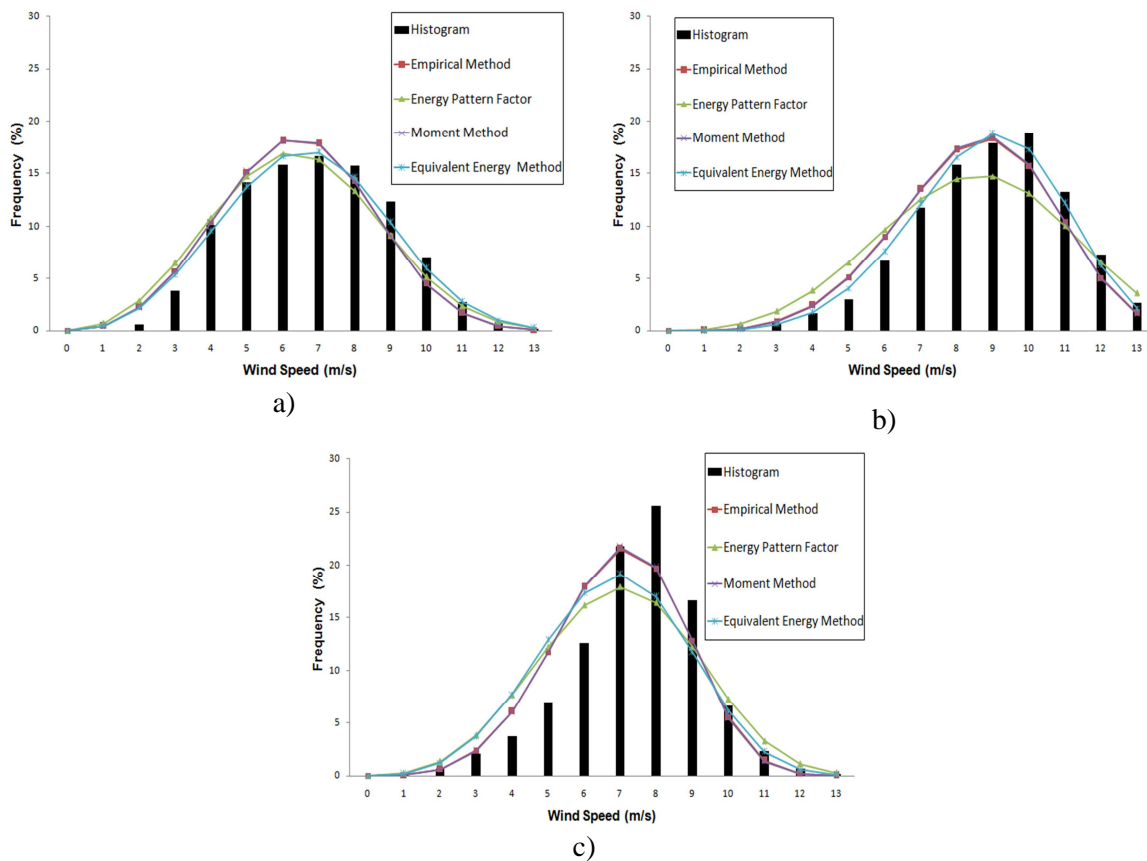


Figure 4: a) Weibull distribution –Maracanaú(second semester); b) Weibull distribution - Parnaíba (second semester); c) Weibull distribution - Petrolina (second semester).

According to Figures 3 and 4, the optimal settings were found when applied to Maracanaú and Parnaíba (coastal zones) and a lower setting has been found to Petrolina (city far from the coast). In Tables 4 to 9 show the statistical tests (RMSE,  $X^2$  and  $R^2$ ) to evaluate the implementation of different numerical methods for semesters.

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.018758	0.000351	0.954162
<b>M</b>	0.043527	0.001895	0.918983
<b>E</b>	0.040858	0.001669	0.918661
<b>EPF</b>	0.038651	0.001494	0.918836

Table 4. Applications of statistical tests for Maracanaú .CE (first semester).

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.000441	0.000019	0.934093
<b>M</b>	0.011233	0.000126	0.889658
<b>E</b>	0.011687	0.000137	0.889023
<b>EPF</b>	0.032806	0.001076	0.863718

Table 5. Applications of statistical tests for Parnaíba-PI (first semester).

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.000388	0.000015	0.851544
<b>M</b>	0.000334	0.000011	0.869615
<b>E</b>	0.000348	0.000012	0.869059
<b>EPF</b>	0.001408	0.000198	0.830615

Table 6. Applications of statistical tests for Petrolina - PE (first semester).

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.001680	0.000282	0.976802
<b>M</b>	0.002330	0.000543	0.952098
<b>E</b>	0.002516	0.000633	0.942526
<b>EPF</b>	0.016639	0.000268	0.941909

Table 7. Applications of statistical tests for Maracanaú .CE (second semester).

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.013626	0.018567	0.986711
<b>M</b>	0.097462	0.009498	0.940495
<b>E</b>	0.106931	0.011434	0.940675
<b>EPF</b>	0.628612	0.395152	0.873212

Table 8. Applications of statistical tests for Parnaíba-PI (second semester).

Methods	Statistical tests		
	RMSE	$X^2$	$R^2$
<b>EE</b>	0.002160	0.004664	0.803604
<b>M</b>	0.000156	0.000243	0.889602
<b>E</b>	0.000371	0.000138	0.888433
<b>EPF</b>	0.012454	0.001551	0.815431

Table 9. Applications of statistical tests for Petrolina – PE (second semester).

Analyzing Tables 4 - 9, it can be noticed that in the two semesters, the method of equivalent energy shows the best fit adjustment to Maracanaú and Parnaíba (coastal cities); the moment method and empirical method have the best adjustment to Petrolina (far from the coast).

Considering the statistical tests values for Maracanaú and Parnaíba, it was observed that the reduction of the Weibull application horizon provides better adjustment efficiency of the calculated

and observed frequencies ( $R^2$  - over 97% and RMSE – lower 0.00168, for a 6 months period). However, using the Weibull distribution fit for Petrolina no significant improvement was observed.

### 3. Conclusion

Taking into account the four numerical methods used for the estimation of the Weibull distribution parameters, the Equivalent Energy Method (EE) shows the best results for sites near the coast. For Petrolina, ca. 650 km from the coast, Moment Method (M) and Empirical Method (E) show the best results.

For all the considered sites,  $k$  and  $c$  monthly values vary considerably throughout the year, with a peak occurring in the second semester. Taking this local characteristic into account, namely the difference between monthly and annual  $k$  and  $c$  parameters, the wind speed potential analysis based only on annual values can lead to false estimations.

Independent of the used numerical method, the Weibull distribution analysis for a 6 months period has a better performance than the analysis for a year, especially for the second semester of Maracanau and Parnaíba, with adjustment above 97% and RMSE lower than 0.00168.

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