# A Simplified Model for Wind Turbine Based on Doubly Fed Induction Generator


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*Abstract*— **Improving the Fault Ride Through capability (FRT) of Doubly Fed Induction Generators (DFIG) in wind power applications is a very important challenge for wind power industry. The mathematical models of such generators permit to analyze their response under generic conditions. However, their mathematical complexity does not contribute to simplify the analysis of the system under transient conditions, and hence do not help to find straight-forward solutions to enhance their FRT. This paper presents a simplified model of the DFIG, that has been extracted from the classical fifth order model, which is able to estimate accurately the behavior of the system while reduces significantly its complexity. In this work, the mathematical deduction of this model will be presented and simulations and experimental results will be shown, in order to demonstrate the accuracy and reliability of the proposed algorithm.**



***Keywords- Wind Power Generation, DFIG Vector Control, Low Voltage Ride Through.***


## I. INTRODUCTION

The increasing capacity of the installed wind power generation facilities, as well as the high scale penetration of such systems in the future, is a new challenge for the electric system operators. This new scenario forced to renew the existing grid codes (GC), that now include specific requirements regarding the operation of wind power generators and farms. Among these new requirements, those concerning the capability of wind power generators to remain connected to the grid, in case of grid voltage sags, have gained great importance.

This feature, known as low voltage ride through capability (LVRT), states the fault condition boundaries among the ones a wind turbine (WT) should not get disconnected from the grid and in some cases, give also the operation pattern for the system under such conditions.

At the present time WTs based on - Doubly Fed Induction Generator - DFIG (DFIG-WT), controlled by means of back-to-back converters in the rotor circuit, constitute 50% of the installed WTs worldwide [1]. In such systems, fulfilling the new GC requirements is not a simple job, as due to its connection topology, they are especially sensitive to any voltage sag. A block diagram of DFIG-WT is shown in Fig. 1.

In a DFIG when the voltage at the stator windings drops a sudden overcurrent appears in the stator that in turn, is induced to the rotor windings. This current peak may damage the machine and the rotor side converter, and as a consequence, the system must be protected. Within this field several studies focused on improving the LVRT capability of DFIG-WTs have been presented in [2]–[9].

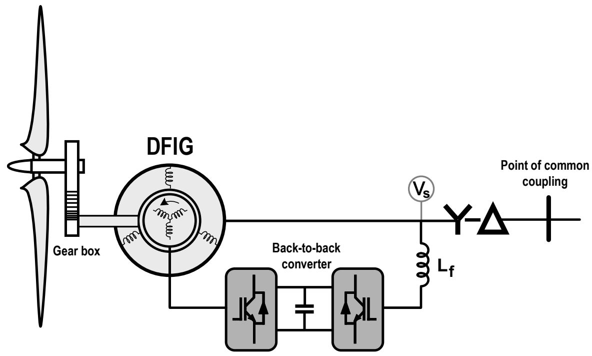



Figure 1. Wind turbine based on doubly-fed induction generator.

On the other hand, currently the search for accurate models for wind turbines based on DFIGs able to represent them adequately is of great importance. When it is necessary to study the impact of wind farms (especially large wind farms containing hundreds of wind turbines), in the power system, the computational effort associated with this study is a fundamental issue that sometimes represents an unreachable limit. In this context, the aim of this paper is to propose a new analytical simplified model for DFIG driven by wind turbines. This would guarantee an accurate representation of the generator in both normal and grid-fault conditions.

This issue was already addressed in another work of the same authors [10]. However, the present paper presents different results from those previously presented in [10]. The experimental results for new experiments are presented in this paper, and they take into account both balanced and unbalanced voltage sags.

The following sections will show the theoretical development, as well as simulation and experimental results to validate the proposed theory.

## II. CLASSICAL MODELING OF A DFIG

The objective of this section is focused on finding a simple relationship between the state-space variables that could permit to predict the behavior of DFIG under fault conditions. Later this analysis will constitute the basis for the design of the control system for the rotor side converter.

The mathematical model of the DFIG, that will be later simplified in this paper, is presented here considering the generator's variables in the *dq* synchronous reference frame. The equations for the stator and rotor windings can be written as:

$$v_{sd} = r_s i_{sd} + \frac{d\psi_{sd}}{dt} - \omega_s \psi_{sq}, \tag{1}$$

$$v_{sq} = r_s i_{sq} + \frac{d\psi_{sq}}{dt} + \omega_s \psi_{sd}, \tag{2}$$

$$v_{rd} = r_r i_{rd} + \frac{d\psi_{rd}}{dt} - (\omega_s - \omega_r)\psi_{rq}, \tag{3}$$

$$v_{rq} = r_r i_{rq} + \frac{d\psi_{rq}}{dt} + (\omega_s - \omega_r)\psi_{rd}. \tag{4}$$

The d-q synchronous reference frame equations of the stator flux and rotor may be written also as:

$$\psi_{sd} = L_s i_{sd} + L_m i_{rd}, \tag{5}$$

$$\psi_{sq} = L_s i_{sq} + L_m i_{rq}, \tag{6}$$

$$\psi_{rd} = L_r i_{rd} + L_m i_{sd}, \tag{7}$$

$$\psi_{rq} = L_r i_{rq} + L_m i_{sq}. \tag{8}$$

By substituting (5)-(8) in (1)-(4) it is possible to obtain a state-space model based on the current components.

Once the rotor and stator currents are found, the electromagnetic torque and the active/reactive power at the stator windings can be calculated as:

$$T_e = \frac{3}{2}\frac{L_m}{L_s} p\left(\psi_{sq} i_{sd} - \psi_{sd} i_{sq}\right), \tag{9}$$

$$P_s = v_{sd} i_{sd} + v_{sq} i_{sq}, \tag{10}$$

$$Q_s = v_{sd} i_{sq} - v_{sq} i_{sd}. \tag{11}$$

Finally, the mechanical dynamics of the system, which is given by:

$$J\frac{d\omega_r}{dt} + B\omega_r = T_{mec} - T_e. \tag{12}$$

Equations (1)-(12) constitute the fifth order model of the DFIG that will be considered in this paper.

## III. Control of a DFIG using a Field Oriented Control algorithm

In wind power systems, based on DFIG-WTs, the most widely used method for controlling the injection of P and Q into the electrical network is based on the field oriented control (FOC) principle [7]. This algorithm permits to perform a decoupled control of P and Q by means of regulating the *dq* components of the rotor currents that are being injected to the DFIG [11].

In this control strategy the rotor-side converter acts as a current-controlled power converter, whose variables are referred to a rotating reference frame that is oriented alongside the stator magnetic flux vector position [7],[11], as it is shown in Fig. 2.

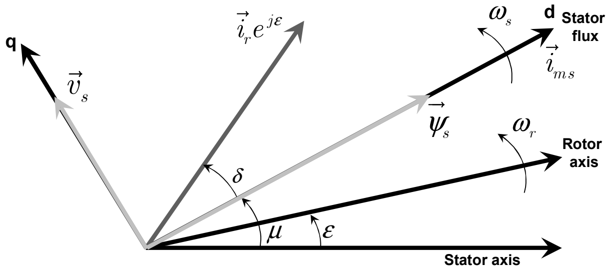



Figure 2. Vector diagram in the *dq* reference frame considering a field-oriented control.

As a consequence of alignment of the stator flux vector with d-axis, and considering that the stator resistance is very low, the following assumptions can be made:

*i) the stator voltage has a single component in the q axis, hence* $v_{sd} = 0$,

*ii) the stator flux is totally aligned with the d axis, then* $\psi_{sq} = 0$.

If both conditions are mixed with (10) and (11), the active/reactive power injected through the stator can be written as:

$$P_s = v_{sq} i_{sq}, \tag{13}$$

$$Q_s = -v_{sq} i_{sd}. \tag{14}$$

The stator currents in the last equation can be written as function of the rotor currents as:

$$i_{sq} = -\frac{L_m}{L_s} i_{rq}, \tag{15}$$

$$i_{sd} = i_m - \frac{L_m}{L_s} i_{rd}. \tag{16}$$

In this last expression $i_m$ corresponds to the magnetizing current of the generator. From (15) and (16), together with (13) and (14), it can be concluded that the in-quadrature current component of the rotor, $i_{rq}$, can be used to regulate the active power delivered by the stator, while the direct current $i_{rd}$, is responsible of controlling the stator reactive power.

This relationship between the rotor currents and the active/reactive power generated has been used in many control applications, showing its good response and reliability.

Considering the features of this control and the fifth order model of the DFIG, that were previously presented, a simplified model of the whole system will be derived in the following section.

## IV. Simplified Modeling of a DFIG Controlled with a FOC Algorithm

The objective of this section will be focused on finding a simple relationship between the state space variables and the outputs of a DFIG that could permit to predict easily its behavior under transient and fault conditions.

As shown in many publications, the fifth order model represents very well the behavior of the generator. However, considering that the DFIG is connected to an electrical system that has other components, such as the power converters, that are responsible of controlling their operation, as well as the electrical networks where it is connected, simplifying the model helps its use.

Taking into account that the objective of a DFIG is to deliver a certain power to the network, and therefore to control the injected currents, and assuming that the rotor side converter behaves as a current source, the response of the DFIG depends on only three variables*: i) Network voltage, ii) Rotor currents and iii) Rotor speed.*

### *A. Simplified model analysis*

Considering that the system described by (1)-(8) is linear, and after applying the Laplace transform it is possible to obtain the following stator currents in the synchronous reference frame [11]:

$$i_{sd} = \frac{\left(L_s s + R_s\right) v_{sd} + \omega_s L_s v_{sq}}{\left(L_s^2 s^2 + 2L_s R_s s + R_s^2 + \omega_s^2 L_s^2\right)} - \frac{\left(L_s s^2 + R_s s + \omega_s^2 L_s\right) L_m i_{rd} - R_s \omega_s L_m i_{rq}}{\left(L_s^2 s^2 + 2L_s R_s s + R_s^2 + \omega_s^2 L_s^2\right)}, \quad (17)$$

$$i_{sq} = \frac{-\omega_s L_s v_{sd} + \left(L_s s + R_s\right) v_{sq}}{\left(L_s^2 s^2 + 2L_s R_s s + R_s^2 + \omega_s^2 L_s^2\right)} - \frac{R_s \omega_s L_m i_{rd} + \left(L_s s^2 + R_s s + \omega_s^2 L_s\right) L_m i_{rq}}{\left(L_s^2 s^2 + 2L_s R_s s + R_s^2 + \omega_s^2 L_s^2\right)}. \quad (18)$$

The equations (17) and (18) can be simplified considering the conditions indicated in the third section. Neglecting the in-quadrature flux of the stator, and considering that $v_{sd}$ is null they can be rewritten as:

$$i_{sd} = \frac{\omega_s L_s v_{sq}}{\left(L_s s^2 + 2R_s s + \omega_s^2 L_s\right) L_s} - \frac{\left(L_s s^2 + R_s s + \omega_s^2 L_s\right) L_m i_{rd} - R_s \omega_s L_m i_{rq}}{\left(L_s s^2 + 2R_s s + \omega_s^2 L_s\right) L_s}, \quad (19)$$

$$i_{sq} = \frac{\left(L_s s + R_s\right) v_{sq}}{\left(L_s s^2 + 2R_s s + \omega_s^2 L_s\right) L_s} - \frac{R_s \omega_s L_m i_{rd} + \left(L_s s^2 + R_s s + \omega_s^2 L_s\right) L_m i_{rq}}{\left(L_s s^2 + 2R_s s + \omega_s^2 L_s\right) L_s}. \quad (20)$$

However, these last equations can be simplified even more. Taking into account that in the second term of both expressions the crossed terms of the rotor current, $i_{rd}$ and $i_{rq}$, are negligible, due to the low value of the $R_s\omega_s L_m$ coefficient if compared with the multiplicative factor $(L_s s^2 + R_s s + \omega_s^2 L_s)L_m$, and considering that

$$\frac{\left(L_s s^2 + R_s s + \omega_s^2 L_s\right)}{\left(L_s s^2 + 2R_s s + \omega_s^2 L_s\right)} \cong 1. \quad (21)$$

The final simplified model can be obtained as detailed in:

$$i_{sd} = \frac{1}{L_s} \frac{\omega_s}{s^2 + 2\left(R_s / L_s\right) s + \omega_s^2} v_{sq} - \frac{L_m}{L_s} i_{rd}, \quad (22)$$

$$i_{sq} = \frac{1}{L_s} \frac{s + R_s / L_s}{s^2 + 2\left(R_s / L_s\right) s + \omega_s^2} v_{sq} - \frac{L_m}{L_s} i_{rq}. \quad (23)$$

In both equations, the rotor current and the stator voltage appear as the input variables, as the first one is fixed by the rotor side converter while $v_{sq}$ depends on the grid behavior.

As it can be concluded from (22) and (23), any variation in the stator voltage results in oscillatory response in the *dq* components of the stator currents in the synchronous reference frame. The frequency of such oscillation, in the *dq* axis, is equal to the grid frequency and its damping is very poor, due to the low value of the stator resistance, $R_s$ (generally around 0.005 pu). In the *abc* reference frame this transient response corresponds to dc level.

This phenomenon can be specially noticed when voltage sags occur. If it is a balanced sag, the stator currents in *dq* coordinates oscillate at $\omega_s$. Additionally, if the voltage sag is unbalanced, the negative-sequence components that appear forces oscillations with a frequency equal to $2\omega_s$ in $v_{sd}$ and $v_{sq}$.

The steady-state equation of the simplified model described in (22) and (23) are given by

$$i_{sd} = \frac{1}{L_s \omega_s} v_{sq} - \frac{L_m}{L_s} i_{rd} \quad (24)$$

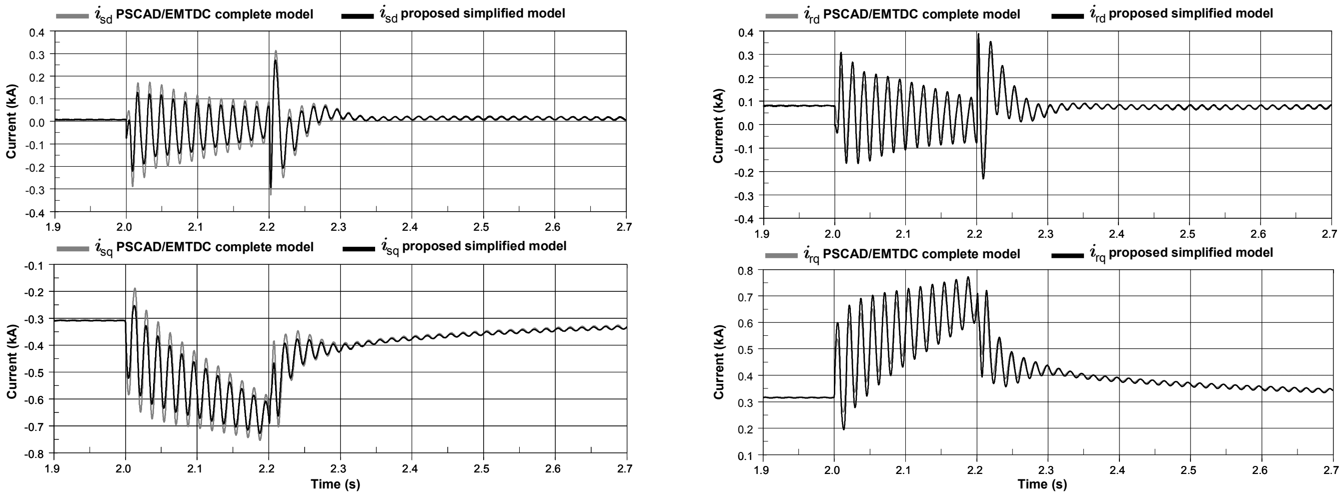
$$i_{sq} = \frac{R_s}{L_s^2 \omega_s^2} v_{sq} - \frac{L_m}{L_s} i_{rq} \quad (25)$$

By means of analyzing (25), it can be concluded that the multiplicative factor of the q-axis component of the stator voltage tends to zero and considering that $R_s << \omega_s^2 L_s^2$, this equation can be reduced to:

$$i_{sq} = -\frac{L_m}{L_s} i_{rq}. \quad (26)$$

In equation (26), $i_{sq}$ and $i_{rq}$ are linearly dependent.

On the other hand, the final value of $i_{sd}$ in (24) depends on two terms. The first one, considering the steady-state conditions, describes its relationship with the magnetizing




(a) Stator currents.

(b) Rotor currents.

Figure 3. Transient response of the stator and rotor currents with the proposed simplified model in front of the traditional fourth order model, when a 70% depth three-phase balanced fault is applied at t = 2.0s and cleared after 200ms.

current, while the second depends on the rotor's direct current component.

## V. SIMULATION RESULTS

The performance of the DFIG can be easily predicted when using only the simplified model, if compared with the full model. This is a very interesting issue, as the behavior of the generator under transient and unbalanced conditions is of great interest nowadays. Therefore, if a simplified model is able to represent the system under such conditions proposing possible solutions in order to improve its response will be easier.

In this subsection, the simplified DFIG model, presented in (22) and (23), has been quick validated through simulations carried out in PSCAD/EMTDC, for the complete and simplified model.

The simulation results, depicted in Fig. 3(a) and Fig. 3(b), show the response of the stator and rotor currents in the *dq* axes when a 70% depth balanced voltage sag at the stator winding occurs at t = 2s and last for 200ms.

In both figures the output of a classical fifth order model and the proposed simplified model are shown together, in order to prove their relationship.

As it can be easily realized, these results permit to conclude that the proposed simplified model describes the DFIG's behavior accurately. In this point it is worth to mention that the current waveforms in Fig. 3(a) and Fig. 3(b) show clearly the current peaks that appear in the rotor and in the stator windings when the fault occurs, as well as when it is cleared. Although its magnitude depends on other parameters, the simulations show peaks that exceed two times the rated current of the rotor side converter, something that would damage seriously this device in a real wind power plant.

## VI. EXPERIMENTAL RESULTS

The dynamical performance of the simplified model was experimentally analyzed, considering different operating points of the generation plant, during steady-state and transient conditions of the network's voltage.

In this experiment, the measured stator voltage during the balanced voltage sag and the injected currents are displayed in Fig. 4(a) to Fig. 4(f). As shown in Fig. 4(a) and Fig. 4(b), the produced sag gives rise to a 63% voltage drop. The stator active and reactive powers and the stator currents are shown in Fig. 4(c) and Fig. 4(d), respectively.

In this experiment, the comparison between the estimated values of the model and the real performance of the stator currents can be carried out based on Fig. 4(e) and Fig. 4(f). The comparison between the estimated and real values of $i_{sd}$ and $i_{sq}$ that can be carried out through Fig. 4(e) and 4(f) show a good response of the simplified model.

As for the stator currents, as shown in the figure, their magnitudes change during and after the sag. This effect is produced by the control strategy, which is applied to the rotor side converter (RSC), which is oriented to support the grid by injecting reactive power.

On the other hand, another experiment was conducted, this time the system was subjected to unbalanced voltage sag as can be seen in Fig. 5(a) and Fig. 5(b).

Figure 5(b) shows a oscillation at twice the grid frequency (100Hz in this case) in stator voltage *dq*-components.

Figure 5(c) shows the behavior of stator active and reactive powers during the unbalanced voltage sag. The oscillation at twice the grid frequency occurs in these waveforms too.

The current waveforms in *abc* coordinates are shown in Fig. 5(d). This figure also shows strong current peaks at the beginning of the voltage sag.

Finally, the Figures 5(e) and 5(f) show the comparison between the measured and estimated *dq*-components of the stator currents. From this figures it is possible to conclude the good response of the proposed model presented in this work.

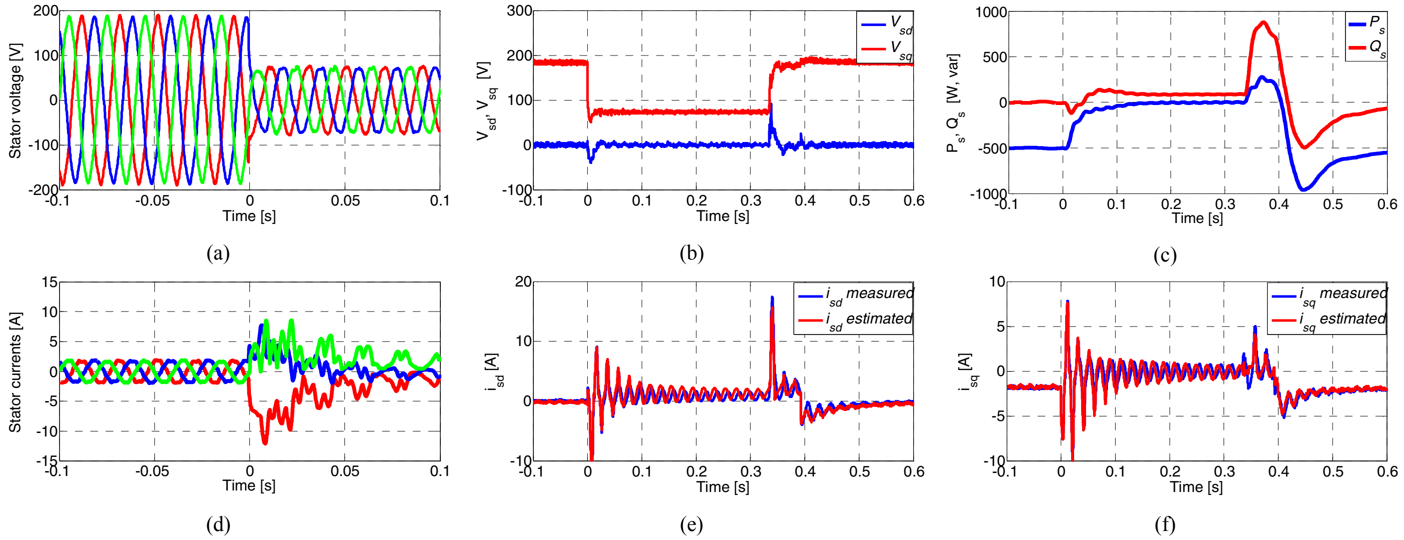



Figure 4. Experimental results: Behavior of the DIFG during balanced voltage sag. (a) Stator voltage in *abc* coordinates; (b) Stator voltage in *dq* coordinates; (c) Stator active and reactive powers; (d) Stator currents in *abc* coordinates; (e) Measured and estimated $i_{sd}$ current; (f) Measured and estimated $i_{sq}$ current.

The model proved very accurate both during steady state and transient grid voltage.

From the figures 4(e), 5(e), 4(f) and 5(f) it can be seen the phenomenon described in Section IV, i.e. when a balanced voltage sag occurs, the stator currents in *dq* coordinates oscillate at $\omega_s$. On the other hand, if it is unbalanced, in the stator currents in *dq* coordinates it will appear oscillations which will be the result of a composition between both $\omega_s$ and $2\omega_s$. This behavior is due to the superposition of the natural and forced responses of the system to the unbalanced voltage sag.

## VII. Conclusion

The simplified model presented in this work permits to carry out an accurate analysis of the DFIG's performance when transients in the rotor current or in the stator voltage occur.

The reliability of this model has been tested by means of simulations, using PSCAD/EMTDC but also through experiments performed in a scaled prototype, giving rise to good results in both cases.

This model can be useful when simulating large scale wind

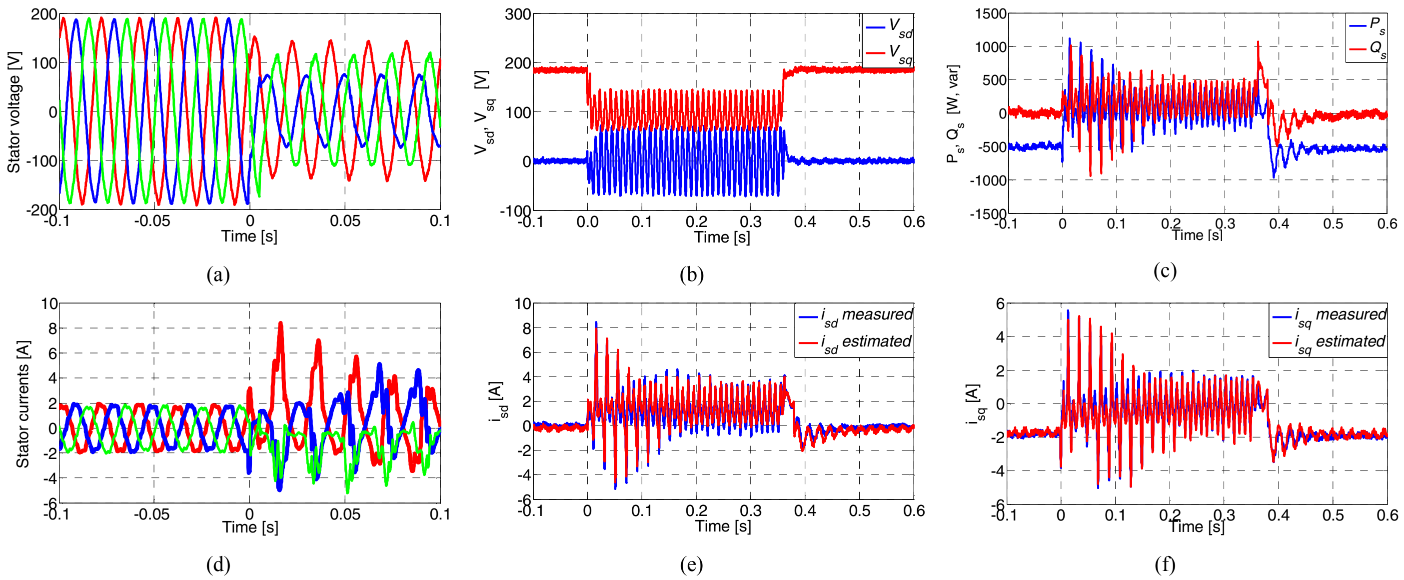



Figure 5. Experimental results: Behavior of the DIFG during unbalanced voltage sag. (a) Stator voltage in *abc* coordinates; (b) Stator voltage in *dq* coordinates; (c) Stator active and reactive powers; (d) Stator currents in *abc* coordinates; (e) Measured and estimated $i_{sd}$ current; (f) Measured and estimated $i_{sq}$ current.

power, although this theme is not treated in this paper, as it is possible to represent the behavior of a DFIG by means of two simple second order transfer functions. Therefore simulating the behavior of several DFIG-WTs would not take as much computational time as when using the fifth order model.

In addition, the simplified model permits to design simple strategies oriented to enhance the performance of DFIGs under sag conditions.

Although the same conclusions could be reached using a more complex model this simplified version enables to conduct a more intuitive estimation about the behavior of the system.

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## REFERENCES

[1] F. Blaabjerg, Z. *Chen, Power Electronics for Modern Wind Turbines*, 1st. ed., USA: Morgan & Claypool, 2006.

[2] A. Petersson, S. Lundberg, T. Thiringer, “A DFIG wind-turbine ride-through system influence on the energy production”, *In: Nordic Wind Power conference*, Göteborg, March 2004.

[3] I. Erlich, H. Wrede, C. Feltes, “Dynamic Behavior of DFIG-base Wind Turbines during Grid Faults”, *In: Power Conversion Conference*, pp. 1195-1200, Nagoya, Abril 2007.

[4] J. Morren, S. W. H. De Haan, “Short-circuit current of wind turbines with doubly fed induction generator”, *IEEE Transaction on Energy Conversion*, vol. 22, nº 1, pp. 174-180, March 2007.

[5] D. Xiang, L. Ran, P. J. Tavner et al., Control of a doubly-fed induction generator in a wind turbine during grid fault ride-through. IEEE Transaction on Energy Conversion, vol. 21, nº 3, pp. 652-662, Set. 2006.

[6] C. Abbey, G. Joos, “Effect of low voltage ride through (LVRT) characteristic on voltage stability”, *In: IEEE Power Engineering Society General Meeting*, vol. 2, pp. 1901-1907, San Francisco, June 2005.

[7] R. W. De Doncker, D. W. Novotny, “The universal field oriented controller”, *IEEE Transactions on Industry Applications*, vol. 30, nº. 1, pp. 92-100, 1994.

[8] W. Leonhard, *Control of Electrical Drives*, 3rd ed., Springer: New York, 2006.

[9] E. Levi, “Multiphase Electric Machines for Variable-Speed Applications”, *IEEE Transaction on Industrial Electronics*, vol. 55, nº. 5, pp. 1893-1909, May 2008.

[10] A. Luna, F. K. A. Lima, D. Santos, P. Rodríguez, E. H. Watanabe, S. Arnaltes, “Simplified Modeling of a DFIG for Transient Studies Wind Power Applications”, *IEEE Transactions on Industrial Electronics*, vol. 58, nº. 1, pp. 9-20, January 2011.

[11] F. K. A. Lima, A. Luna, P. Rodríguez, E. H. Watanabe, F. Blaabjerg, “Rotor Voltage Dynamics in the Doubly-Fed Induction Generator during Grid Faults”, *IEEE Transactions on Power Electronics*, vol. 25, nº. 1, pp. 118-130, January 2010.