DISSERTATION

A PROCEDURE FOR RESERVOIR SIZING ON INTERMITTENT RIVERS UNDER HIGH EVAPORATION RATE

Submitted by

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY JOSÉ NILSON BESERRA CAMPOS ENTITLED A PROCEDURE FOR RESERVOIR SIZING ON INTERMITTENT RIVERS UNDER HIGH EVAPORATION RATE BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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ABSTRACT

A PROCEDURE FOR RESERVOIR SIZING ON INTERMITTENT RIVERS UNDER HIGH EVAPORATION RATES

Most of the rivers located in arid and semi-arid areas at low latitudes have two well-defined seasons: a wet season where all flows occur, and a dry season with no flows. Another characteristic of these areas is the high evaporation rates -- ranging from two to three meters per year in some places -- reducing significantly the controllable release from surface reservoirs.

The present research develops a procedure for sizing the storage capacity required for surface reservoirs located in areas with similar hydrologic conditions. The procedure has its theoretical support in the Stochastic Theory of Reservoirs, and consists of a collection of graphics linking the following three variables: the reservoir capacity, the annual release and the reservoir probability of failure. Others variables included in the sizing procedure are: the mean value of annual inflows, the coefficient of variation of annual inflows, the annual evaporation depth, and a parameter to take into account the shape of the lake. The main objective is to provide, quickly and with a certain accuracy, a tool for estimate the required reservoir capacity.

As additional and related topics the research investigates: the reservoir behavior during the transient phase where the probability of emptiness depends on the initial storage as well as the relationship between the required capacity and the reservoir horizon life for several values of the initial storage, and the effect of annual and seasonal intermittence of inflows in the reservoir required capacity.

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CHAPTER I

INTRODUCTION

A great expanse of tropical land lies under climatic conditions characterized by a precipitation regimen involving a concentrated rainy season, usually from three to four months, and a high annual variability. These conditions often come together with an intense evaporation rate and, sometimes, with low permeability soil. That combination generates intermittent rivers that have a season of no flows that can last from six to nine months or even the whole year when severe droughts occur.

Some authors think that only the abnormal pluviometric regimen and the intense evaporation are enough to generate that kind of river. For example, Pereira (1979), studying the African climate, stated that even the most favorable conditions of heavy forest and deep porous soil could not succeed in regulating streamflow with that acute concentration of rainfall. This point of view is shared by Trewartha (1981), who in his studies about African climate stated that 'B' climates -- prevalent in a large part of that continent -- cannot originate permanent rivers.

The worst condition occurs when the soil is crystalline. In such a situation, during the dry season, if no surface reservoir is available, the only source of fresh water is alluvial aquifers, and the potential of most of these aquifers is insufficient to permit irrigation practice. In addition to that, when a drought occurs, and they often occur, the aquifers get depleted and the situation, obviously, becomes critical. As a conclusion, surface reservoirs, despite their low efficiency under high evaporation, are a "sine qua non" survival condition for inhabitants of similar areas.

Another point is that most of the research in reservoir sizing is based on perennial rivers. No one can expect that a sizing model, with some simplifications embodied in it, gives results as accurate for a dissimilar rivers regimen as those shown in figure 1.1. When a scientist simplifies a model, he has two main objectives. First, to make the mathematical computation easier. Second, not to stray too far from real-life. However, the real-life for each person comes from his environment, and as long as most of the existing models come from temperate environments most of them are appropriate only for temperate, or better, for perennial rivers.

The state-of-the-art of data generation models supports the prior statement. If one intends to generate monthly flows for perennial rivers, applicable models abound in the literature, e.g. AR, ARMA, ARIMA, FGN, BL, SL. On the other hand, for intermittent rivers, except for an adaptation of the Thomas Fiering model by R.T. Clarke (1973), almost nothing exists.

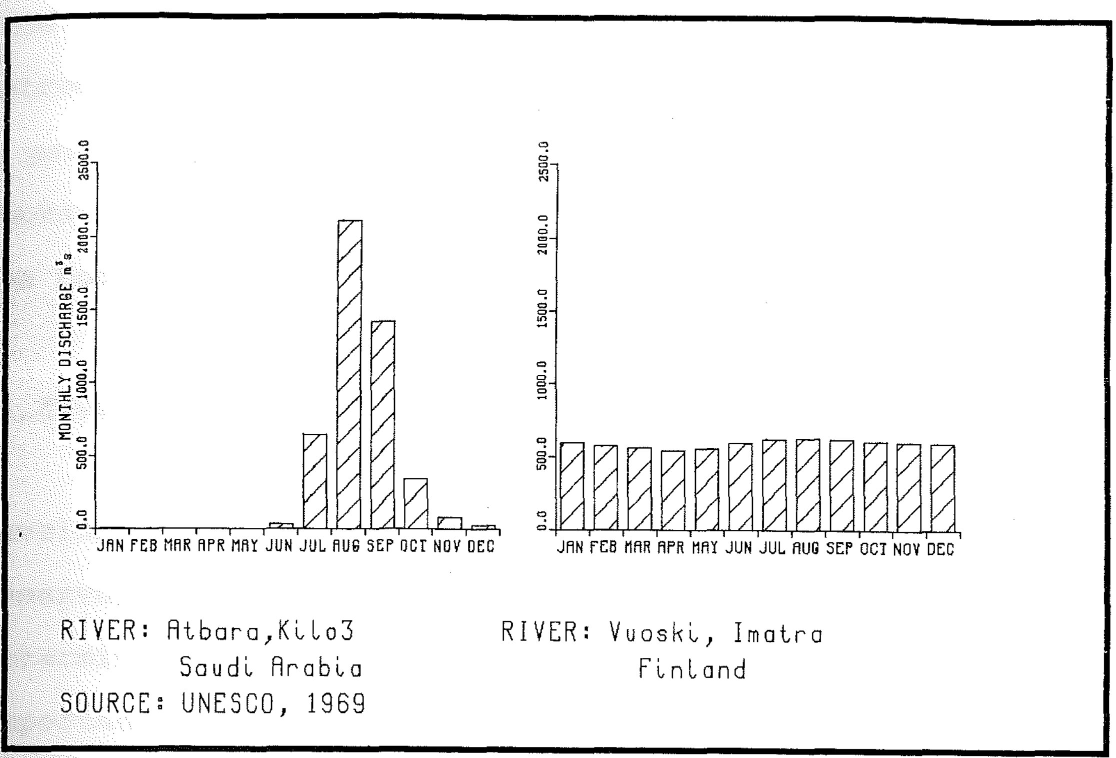

Figure 1.1 Annual hydrograph pattern for Atbara River, Saudi Arabia, and Vuoski River, Finland.

A. Statement of the Problem

Applying sizing procedures that are based on perennial rivers to intermittent rivers tends to undersize the required capacity. As an example, assume a river with a variance of inflow equal to zero. In the case of a perennial river the required capacity to regulate the mean inflow is zero. On the other hand, in the case of an intermittent river, that capacity would be equal to the mean inflow.

Another point is in regard to the evaporation effect. For example, in Brazil's Northeast some reservoirs regulate only 20 percent of the mean inflow. In such a case the evaporation consumes more than the net release. Besides, some sizing procedures deal with evaporation only by introducing a correction factor. It seems obvious that such a procedure does not yield good results. All in all, it looks that for similar situations the evaporation must be in the body of the procedure.

B. Study Objectives and Scope.

The main objective of the present research is to develop a sizing procedure for preliminary design of reservoir capacity. The procedure covers the case of intermittent rivers where the evaporation effect is important.

To build a good model one needs to understand the physical system as well as the parameters that affect its output. Keeping this in mind and knowing about the lack of studies for the reservoir sizing process for tropical semi-arid conditions, we decided to study the effect of evaporation, initial conditions and intermittence separately. Therefore, some other objectives are:

- evaluating the effect of evaporation on storage capacity need.
- studying the transient phase and evaluating the effect of initial storage.
- studying the effect of annual and seasonal intermittence.

Moran's theory , which in its original form is suitable for two-seasoned rivers, provides the theoretical base for the model development. However, the introduction of evaporation effect required some modifications in that theory. The final model will be appropriate for sizing reservoirs on rivers with approximately these conditions:

- hydrological regimen with two well-defined seasons-- a wet season where all inflow occurs, and a dry season where all release occurs;
- annual inflows are independent;
- annual release is constant;
- inflows are gamma-II distributed;
- evaporation losses are important.

Rivers following these conditions exist in Africa, the North of Australia, and in Brazil's Northeast. However, to develop a graphical model where a wider range of the input parameters implies a larger number of graphics,it seems important to put limits on these parameters. In other words, it seems important to choose a focus area for the model. The logical criteria to choose the focus area are the author's experience and the availability of data. As a result, the Northeast of Brazil was the area selected to provide the data for the model test and to put the limits on input parameters.

C. Research Organization

Chapter II presents a survey of relevant literature in reservoir sizing processes. The presentation follows the common classification of approaches used for the storage problem, namely the empirical, the analytical and the experimental.

Chapter III introduces a review of the transition matrix and the Markovian chain theory. It also presents the definition of the hydrological system and the mathematical treatment for the theoretical model.

Chapter IV covers the procedure for reservoir sizing. First, it analyzes the way that the existing procedures deal with the evaporation. Then it introduces the dimensionless evaporation factor as a sizing parameter. Second, it investigates the effect of the annual intermittence on the required capacity. Finally, it presents the graphical model and a test thereof.

Chapter V presents some additional studies on the initial storage. It also presents a succinct study on the effect of the seasonal intermittence.

Last, Chapter VI contains the conclusions drawn in this study and some recommendations for further studies.

CHAPTER II

LITERATURE REVIEW

Sizing a reservoir is a decision process. Someone, or some group of people, with the knowledge about the hydrologic, economic, political and social factors involved, must choose the appropriate reservoir size. A set of data, which after analysis becomes information, provides the support for the right decision. This set of data and procedures for analyses together make a Decision Support System. To illustrate, Figure 2.1 shows an example of a framework for a Decision Support System.

From the engineering standpoint the sizing process has two main and interlinked steps. First, the hydrological step examines the relationship among inflow, storage capacity and dependable output or yield. Second, the decision step mixes the results from the first step with economic data to provide information to the decision-maker.

The hydrological step has three different approaches namely the empirical or critical period-related methods, the experimental, which uses simulations, and the analytical,which uses range, deficit analysis and transition matrix. The decision step has its main support in Operational Research techniques, such as deterministic or stochastic optimization. In recent works the trend is toward the use of stochastic optimization procedures and decision criteria, such as max-min, maximum expected value, the E-V(expected value-variance) efficient frontier, and the utility function theory.

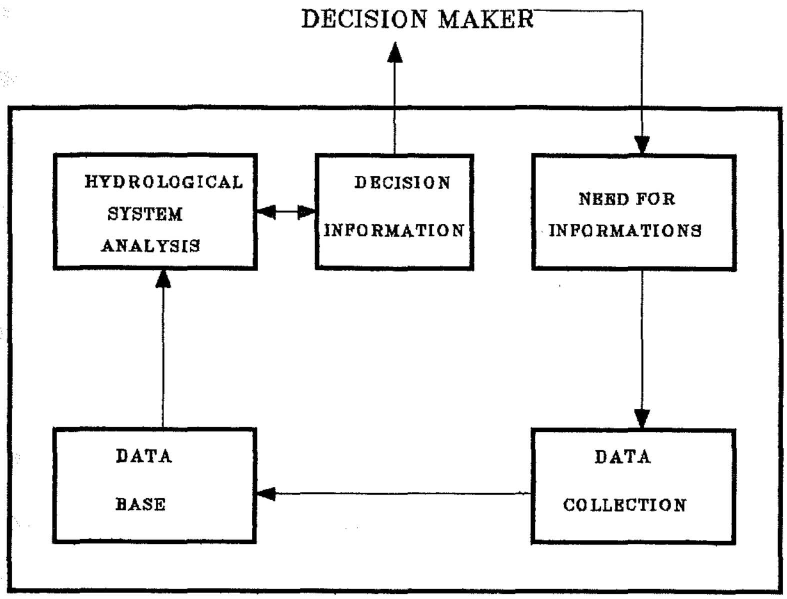

Figure 2.1. Decision Support System framework.

This literature review has three parts. The first consists of a brief summary of early important works in the storage design theory and a more detailed presentation of methods using range and deficit analysis. The second part deals with works following Moran's transition matrix theory. Finally, the third part presents a few important works using the Monte Carlo method associated with some decision criteria.

A. Methods Using Range and Deficit Analysis

The modern methods of reservoir sizing have their origin in Rippl's (1883) contribution of mass curve analysis. Rippl's method is based on the assumption that inflow and outflow are known functions of the time, and its purpose is to compute the storage capacity needed to provide a 'safe' yield .

Hazen (1914), using the mass diagram, introduced the concept of semi-infinite reservoir and dependable water. This was, possibly, the foundation for the storage-yield-reliability curbs used currently.

Following Hazen's work, Sudler (1924) generated 100 years of synthetic inflows to size a reservoir for water supply. He drew cards to put the chance effect in the generated values.

Hurst (1950), when sizing the Great Lakes of the Nile River Basin, introduced the concept of range and deficit analysis. This was the beginning of analytical procedures in the sizing process and the basis for a great number of research efforts to follow. For a better understanding of Hurst's work some definitions are necessary.

Let us assume a sequence of random variable X_i, where :

X_i=net input into the reservoir at time step i, with $E[X_i]=0.$, $i=1,2,....,n$.

Define:

$$S_i=\Sigma X_i,$$

for a sample of size n,

$$S_n=\Sigma Xi, \quad i=1,2,...,n,$$

$$M_n=\max(0,S_1,S_2,....S_n),$$

$$m_n=\min(0,S_1,S_2,....S_n),$$

$$R_n=M_n-m_n.$$

These random variables, S_i, S_n, M_n, m_n, R_n, stand for, respectively, accumulated net input, accumulated net input for a sample of size n, maximum accumulated surplus, maximum accumulated deficit, and range.

Sometimes each component of partial sums S_i is corrected for the sample mean $\bar{X}$. Then the set of equations becomes:

$$S_i^*=S_i-\bar{X},$$

$$M_n^*=\max(0,S_1^*,S_2^*,\ldots S_n^*),$$

$$m_n^*=\min(0,S_1^*,S_2^*,\ldots S_n^*),$$

$$R_n^*=M_n^*-m_n^*.$$

These new random variables stand for, respectively, adjusted partial sum adjusted maximum surplus, adjusted maximum deficit, and adjusted range. Hurst took the adjusted range, which he called "just range," as the reservoir capacity needed to maintain a constant release from the reservoir equal to the mean inflow. Using combinatorial analysis and binomial approximation, he showed that the asymptotic expected value of the range could be computed by the relation:

$$E\{R_n\}=1.25\sqrt{n}.$$

Feller (1951), using a more precise mathematical treatment, derived the asymptotic distribution of the range for independent normal inputs. He got:

$$E\{R_n\}=2(2n/\pi)^{1/2},$$

$$\mathrm{Var}\{R_n\}=4n(\log 2-2/\pi),$$

Melentijevich (1965) studied the range for outflow depending on the reservoir storage. However, in his study the expected value of the releases depending on storage is zero, so this is not a good representation for the evaporation process.

Yevjevich (1965), using three different approaches--empirical, analytical, and data generation -- analyzed the application of surplus, deficit and range in hydrology. Assuming inflow as normal, and using data generation, he computed the distribution of surplus, adjusted surplus, range, and adjusted range for n up to 50 and for six different values of autocorrelation lag-one.

Salas (1972) derived the exact expected value of the range for n=1,2 and 3, considering the joint distribution of the sequence of partial sum as a multivariate normal. For the particular case of autoregressive lag-one normal input, with $\mathrm{corr}[X_{t+s},X_t]=r^s$, $\mathrm{var}(x)=1$ and $E\{x\}=0$, he got the following equations:

$$E\{R_1\} = (2/\pi)^{1/2}$$

$$E\{R_2\} = (2/\pi)^{1/2}[1+(1/\sqrt{2})(1+r)^{1/2}]$$

$$E\{R_3\} = (2/\ )^{1/2}\{(3/4) + A +$$

$$+ (2+2r)^{1/2}(1/4 + B) +$$

$$+ (3+4r+2r^2)^{1/2}(1/4+C)\};$$

$$A = 1/\pi \arctan(1+r),$$

$$B = 1/(2\pi) \arctan\{(2+2r-r\ )/[2r(2+2r)^{1/2}]\},$$

$$C = (1/2\pi) \arctan [(1+r)^2/(3+4r+2r^2)^{1/2}.$$

Based on the foregoing exact results for n=1, 2 and 3, and using computer simulation, he obtained approximated equations of the expected range for large values of n. Using these approximations, he developed a method to determine the storage capacity for the case when the inputs follow a Markovian model with mean input equal to mean output.

Gomide (1976) using deficit analysis developed an algorithm to compute analytically the reservoir storage capacity. The problem formulation follows:

Let X_i, i=1,2,...,n be a sequence of independent identically distributed random variable, and let's take the partial sums,

$$S_1=0;\ S_2=0;$$
$$S_i=X_1+X_2+\ldots+X_i;$$

in addition the transformation is applied:

$$\begin{aligned} S_i &= 0 && \text{if } S_{i-1}+X_i>0 \\ &= S_{i-1} && \text{otherwise;} \end{aligned}$$

then the maximum accumulated deficit is defined as,

$$D_n= -\min\{0,S_i\},\ i=0,1,2,\ldots,n.$$

For a reservoir of size K, the probability of emptiness in the first n steps is $P_r\{D_n>K\}$. It is not difficult to see that this formulation is only valid for the case where the reservoir starts full. If one intends to compute the probability of emptiness for an arbitrary initial volume s_0, the formulation should be:

$$P_r\{S_i=0|S_0=s_0\} = P_r\{D_n<K\}+P_r\{m_n<s_0\}-P_r\{D_n<K,m_n<s_0\}$$

The solution of this latter equation presents a lot of mathematical complication, and apparently no procedure to solve it exists.

Lansingan (1982), assuming input and storage as discrete variables, extended Gomide's work to include seasonality in inputs. He defined total storage as the maximum deficit of seasonal inputs, and annual storage as the maximum deficit of annual series. For seasonal storage he figured the difference between total and annual storage. After that, Lansingan developed a general numerical procedure for the cases when the seasonal structure of inflows is an independent process, an AR(1) process, or an approximate ARMA(1,1) process. According to him the major drawback is the large computational effort required when the system state space increases.

Pegram, Salas, Boes and Yevjevich (1980) made an exhaustive compilation on the state-of-the-art of range and deficit analysis. It is worthwhile to transcribe here an interesting remark from them about the role of range and deficit analysis on storage sizing process:

> "Notwithstanding their intellectual appeal because of the interesting(but often difficult) problems associated with range and deficit analysis, and their undoubted influence on time series analysis, in hydrology, their practical application in the sizing of reservoirs may leave a lot to be desired, if only because storage analysis is far richer in results than the former, and often yields unequivocal answers to a wide variety of problems with relatively simple algorithms there are other considerations that may speak against the use of range and deficit analysis in storage problem:
> i) They are each approximations of storage analysis

which yield over-conservative reservoir sizes unless adjustments are made.
ii) The simplicity of the expression for the range may once have been a justification for its use when storage analysis was in its infancy. ......"

B. Methods Using Transition Matrix

Moran (1954) developed the first model to deal with finite reservoirs in an analytical way. He assumed that the reservoir storage at time t, Z_t, follows a Markovian chain, and he applied, for the first time, the concept of transition matrix to storage problems. To allow a simple analytical treatment he made the following simplifications:

- time is discrete;
- the reservoir is filled in the wet season and the withdrawal is made instantaneously at the end of the year;
- the reservoir volume is discretized in N layers;
- inflows are discrete and uncorrelated;
- losses from evaporation and seepage are neglected.

Moran's model, in brief (more detailed explanation on chapter III), consists of discretizing the reservoir volume into N slices, computing the annual transition matrix [Q], and solving an equation system NxN to find the storage probability function at steady state conditions.

The assumptions of independent inflows and no losses restricted the application of Moran's model to annual flows of rivers with low autocorrelation coefficients in places where the evaporation rate is not so important. In addition to that, the model did not provide a means to compute the within-the-year probability of failure.

Subsequently, Moran (1955) divided the annual transition matrix [Q] into two matrices: the first [A] had all, and only, information on input, while the second [B] had all, and only, information on the output. In that case the annual matrix [Q] is equal to the product [B][A]. The main advantage introduced by Moran was to save effort when one was looking at more complex release rules.

Gould (1961) modified Moran's work to use monthly data and allow the computation of the within-the-year frequency of failure. Like Moran, he assumed discrete volumes and independent annual flows. To compute the annual transition matrix, Gould followed the steps listed here: i) dividing the reservoir into N slices of equal volume, each one representing a state; ii)assuming the reservoir at state j at the beginning of the year; iii)routing through the reservoir, one at a time, all years of recorded streamflow(Ny) and computing the number of times the reservoir reaches the i state at the end of the year(nj); iv)making the observed frequency nj/Ny equal to the probability of the reservoir going from j to i, and forming the annual transition matrix[Q]; v)computing the storage probability function for steady state conditions, as in Moran's approach; vi)computing the within-the -year probabilities of failure from the empirical values of the routing process in step iii. The main points in Gould's procedure are: to make easier the introduction of more flexible rules of release, to provide information on within-the-year probability of failure, and to include the effect of evaporation as well as the monthly autocorrelation coefficients. The drawback, according to Kottegoda (1980), is the assumption that the observed streamflow sequence represents the streamflow distribution better than a fitted theoretical function. As the records may be biased, this hypothesis can generate biased results as well.

Lloyd (1963) expanded Moran's theory to deal with Markovian's inputs. He used a bi-variate Markovian chain(Z_t,X_t) where Z_t stands for storage, and X_t stands for inflow. In such a case, Z_t and X_t are discretized and as a result the transition matrix size increases exponentially. For example, if Z_t and X_t were discretized in N and M steps respectively, the size of the matrix would be M*N. In fact, a matrix this large could require an enormous computational effort and this is the major drawback of Lloyd's formulation.

Later, Lloyd (1979), assuming input as ARMA(1,1) incorporated also the Hurst-like inflow process in the theory. The process equation is:

$$X_t = b*X_{t-1}+U_t+g*U_{t-1}.$$

Then,although X_t is not a Markovian process the vector (X_t,U_t) is, and as a result so is the vector (Z_t,X_t,U_t). In that situation, the tri-variate Markovian chain (Z_t,X_t,U_t) could be used to compute the transition matrix. As in the prior formulation, the main drawback is the computational effort required.

Lloyd and Odoom (1964) suggested the division of the year into K seasons to take into account seasonality of inputs. In that formulation each season has its own transition matrix$[Q_i]$, and the annual matrix would come from the multiplication of K's seasonal matrix in the proper order, that is the matrix $[Q_{1-k}]=[Q_1]*[Q_2]*.....*[Q_k]$ would represent the annual matrix for the year beginning at season 1 and ending at season K.

White (1966) offered a different approach to deal with seasonality. He developed a variable season model based on the mass diagram wave-like form. He divided the year into two seasons of variable length. The first, with positive net input, he named "increment". The second, with negative net input, he called "decrement." The procedure consisted on fitting theoretical distributions to increments and decrements to build two transition matrices. Using some logical and theoretical reasoning, as well as results of the model application to two British rivers, he drew the following conclusions: i)the correlation between increments and decrements is usually low (this condition is necessary for validation of uni-variate seasonal models); ii)dividing the reservoir volume in 10-20 layers produces accurate results for practical application; iii)the frequency distributions of increments and decrements are skewed and this skewness increases when draft decreases; iv)further studies were necessary for the choice of increments and decrements theoretical distribution function.

Klemes (1970) proposed a different way to decompose the annual transition matrix into an input matrix and a release matrix. According to his study this formulation is appropriate for situations where the input transitions probabilities come from a continuous distribution function. Klemes argued that the fragmentation done by Moran represented just a partial separation because the input matrix also depends on the release matrix.

More recently, Klemes (1981) did an exhaustive review on the state-of-the-art of the stochastic theory of storage. He covered since Saverinsky's work in 1940 up to Phatarfod's in 1979. It is worthwhile to transcribe here some of his words about the application of the theory:

> "The history of applied stochastic theory of storage is interesting in many other respects. One of them is the peculiar influence of the computer on its development. The computer brought the theory into the realm of practical applicability by removing the originally insurmountable computational burden involved in the handling of large matrices , which inevitably arise if reasonably accurate discrete approximations of the storage distribution are to be obtained. However, the same computer is now pushing the theory out of the practical use by making it much more convenient for the user generate synthetic realizations of the inflow process model and perform on them the traditional storage-yield analysis mentioned earlier. This technique gives the engineer a much better insight into the reservoir performance than can be obtained from the compact but more abstract direct formulations of the matrix methods of storage theory to be discussed in the following sections...."

C. Methods Using Monte Carlo Analysis

Loucks, Stedinger and Haith (1981) define "simulation" as a solution of a management problem by trial and error. According to them, simulation and stochastic simulation are, perhaps, the most preferred technique in evaluating alternative water resources systems. In the case of complex stochastic systems, as a reservoir system, simulation can provide the decision maker with insight into the expected system performance that would be inaccessible by the analytical method. In fact, for most practical situations, a good water analyst can develop a mathematical model that fits the case.

On the other hand,the drawback with simulation is the considerable computational effort often required. This is particularly true when the inputs have high variability, thus requiring a very large number of synthetics traces. For example, to overcome the sampling error when sizing a reservoir it is necessary to run from 300 to 1000 different traces of inflow -- each trace with a length equal to the reservoir design life -- Salas and Obeysekera (1985).-- Nevertheless, as long as computer power keeps increasing concurrent with decreasing costs, simulation will probably attract more and more engineers.

Another point is the number of alternatives usually generated by simulation models to be handled by the decision maker. The question is, among these alternatives, which one to choose. This problem resulted in the link, often found in practice, between the combination of simulation and optimization, or the combination of screening, simulation and optimization.

As a result of simulation flexibility and popularity, one can find in the literature a number of simulation models applied to water resources and reservoir design. So, we will restrict this topic review to a few important models.

Fiering (1962) applied the theory of the queues and the Monte Carlo technique to selecting the optimum size for a single multipurpose reservoir. As optimality criteria he used the maximum expected value of gross benefits generated by irrigation, hydropower and flood control, under no budgetary constraint. He assumed inflows following a truncated normal distribution and draft depending only on the water available each year. The procedure involves routing a long synthetic series through a design reservoir to compute the probability distribution of the annual draft. From the draft distribution function Fiering got the condition which yielded the optimum benefit.

Fontane (1982), intending to ease the water resource planners' task, developed a computational system linking simulation and optimization to get the maximum of these approaches. Three different and complementary models form the system. The first model, WATPOW, performs the simulation for each potential site for a reservoir and determines the relationship between firm yield and reservoir size. The second model, SCREEN, evaluates different demand scenarios.

This model use dynamic programming to find, for each scenario, the optimal storage level that is strategic for a given construction cost. The third model, OPTRES, combines optimization and simulation to improve the results from SCREEN. OPTRES uses HEC-3 simulation capabilities and the Powell algorithm to pursue the optimization. The main advantage of these models is their simple conversational language which make them accessible to a decision-maker with low computational skills.

The conclusion of this chapter comes from the clever analysis done by Phatarfod (1976) about the trends on reservoir sizing processes, and the confrontation between analytical and simulation methods. He said:

> "... In fact, in the last few years, mathematicians have ventured into more and more esoteric areas using only the imagery of the reservoir problem. The other drawback is that the final product of their efforts is expressed in terms of Laplace transforms , thus preventing both the engineer and the mathematician from getting an insight into the problem.
>
> It is not surprising, therefore that engineers and hydrologists have more or less ignored this literature, and have relied very heavily on simulation techniques for solving the second problem. Usually this takes the form of generating a synthetic sequence of riverflows and running them trough the reservoir,(together with the draft) of various sizes to determine the reservoir size for a certain fixed probability of failure of the reservoir. Sometimes the same technique has been used for studying the effects of the various parameters of inflows- mean, variance, skewness and serial correlation etc.- on the storage size estimates.
>
> The simulation method has a few drawbacks. First, one usually considers inflow models which are easy for data generation. Unfortunately, models which are convenient for data generation may not always fit the historical data very well. Secondly, the final result(storage size, say) is very much subject to sampling errors. Lastly, the computational effort required is enormous.

The present writer takes the view that the two approaches(analytical and simulation) should complement each other. Since in any practical situation the problem is usually very complicated, the final solution must involve some simulation. The analytical solution would give a first approximation which should help the engineer in selecting reservoir sizes for simulation purposes, and an insight into the effects of various parameters of the inflow distribution on the reservoir size for a given reliability...."

CHAPTER III

THEORETICAL MODEL

The primary purpose of a reservoir is to store water during surplus periods for use during periods of deficits. In statistical language a reservoir is a system to change a usually highly variable input provided by nature into a less variable and more dependable water supply to meet the demands of man. The aim of the reservoir sizing process is to answer the question: "For a given stream, what size reservoir should be built to meet the required demand with a certain risk?"

In other words, a reservoir is a system, as in Figure 3.1, designed to meet the goal of matching supply to demand. For a certain site, the input function is defined by nature. Hence, for a given release rule, the output function is a result. That is, for each site and release policy exists a function linking the release to the reservoir capacity and to the probability of failure. The reservoir reliability--equal to one minus the probability of failure -- is sometimes used to measure the reservoir performance. Klemes (1963) named this function as "regulation regimen function" or simply "regimen function."

The regimen function is a surface in the release x capacity x risk space. Its solution consists of choosing two

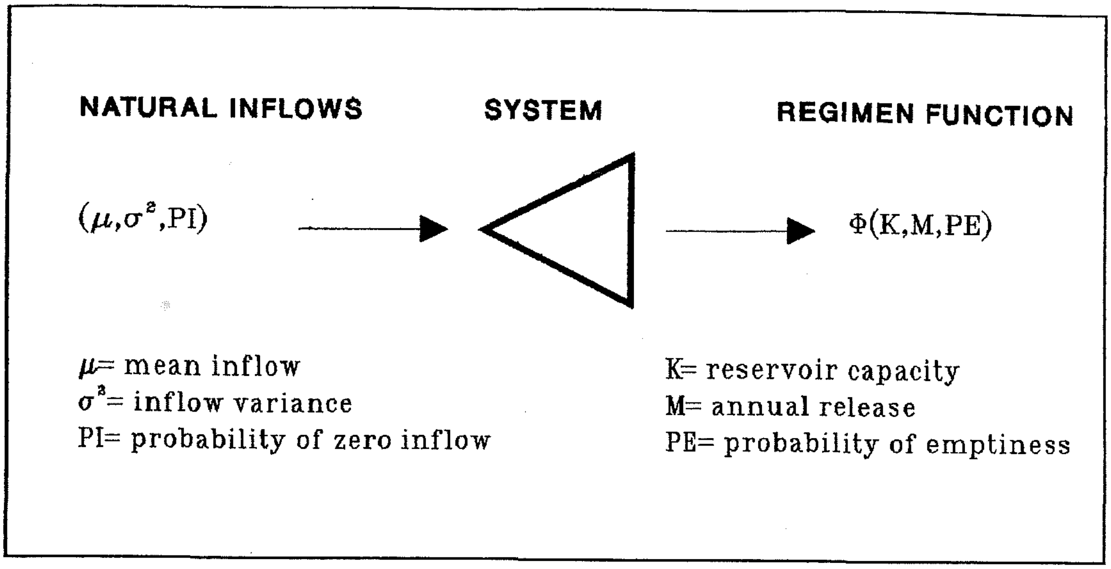

Figure 3.1. Reservoir System representation.

of the three variables and computing the other. As an example, if the capacity and the release are chosen, the probability of failure is a consequence. Its mathematical representation is:

$$\Phi = \Phi(K, M, R_a) \qquad [3.1]$$

where,

K= Reservoir capacity;

M= Annual release;

R_a= Reservoir reliability.

According to the time step used in its computation, the reservoir capacity can be classified as follows:

K_1= long-term storage, or carryover storage, when the time step is a year;

K = total storage capacity, when computed considering the within-the-year fluctuations of inflow, and

K_s= seasonal storage, which is the difference between K and K_1.

In the present work we deal with the regimen function as follows: i) one year is used as the time step to compute the reservoir capacity, but for the sake of simplicity the subscript '1' was dropped; ii) the release is constant; iii) the evaporation occurs only in the dry season; iv) the probability of failure or the reservoir reliability measure the reservoir performance.

The release rule for the system is:

$$
\begin{aligned}
Z_{t+1} &= 0 && \text{when} \quad Z_t+X_t \ O_t \\
&= Z_t+X_t-O_t && \phantom{\text{when}} \quad O_t<Z_t+X_t \ K \\
&= K-O_t && \phantom{\text{when}} \quad Z_t+X_t>K
\end{aligned}
\qquad [3.2]
$$

where,

O_t = total output during the period t,t+1; (It is equal to the release(M) plus the evaporated volume V_{EV}.)

Z_t = reservoir storage at time t; and

X_t = Total input during period t,t+1.

A. Review on Markovian Process

Consider Y_t to be a random variable in discrete time--that is Y_t is defined at time steps {t=1,2,..}. In addition consider that Y_t has a countable, or finite, state space. In other words, Y_t can reach states {0,1,2,..,N}. Now assume that in one realization of the process with n time steps we get for Y_t the following random sequence: $\{i_1,i_2,i_3,...,i_n\}$, where i_t is the state reached by Y_t at time step t. The stochastic process $\{Y_t\}$, t=1,2,...,n, follows a Markovian process if for every n and all sequences $\{i_1, i_2, \ldots i_n\}$ it is true that:

$$P_r(Y_t = i_n | Y_{t-1} = i_{n-1}, Y_{t-2} = i_{n-2}, \ldots, Y_1 = i_1) = P_r(Y_t = i_n | Y_{t-1} = i_{n-1}) \quad [3.3]$$

In words Equation 3.3 means that the probability of the system reaching the state i_n at time t depends only on the system state at time t-1 and the states reached prior to the time t-1 do not influence the state at time t. Or, putting it in another way, if we consider the time t-1 as the present time, and the time t as the future, then to predict the system behavior we do not need to know about its past. In brief, the system has no memory. In conclusion, the essence of a Markovian process is that all information on the prior system behavior is embodied in the present system state.

The conditional probability $P_r\{Y_t=i | Y_{t-1}=j\}$, which is the probability of Y being at state i at time t given that it was at state j at time t-1, is called th "one-step transition probability" and is represented by p_{ij}. When this conditional probability does not depend on the time step t, as in equation 3.4, the sequence $\{Y_t\}$ has stationary transition probability and follows a Markovian chain:

$$p_{ij} = P_r\{Y_t = i | Y_{t-1} = j\} = P_r\{Y_k = i | Y_{k-1} = j\} \text{ for all t and k} \quad [3.4]$$

For the case of annual inflows with zero serial correlation feeding a reservoir, the storage at time t, Z_t, follows a Markovian chain(Moran, 1954).

B. Review on Moran's Theory

This section briefly reviews Moran's theory of storage. This theory provides the basis to develop the sizing procedure considering intermittent inflows as well as reservoir evaporation. Moran's theory solves the above referred regimen function for the case when the reservoir capacity and the annual releases are known and one is searching for the reservoir reliability. In order to see how Moran's theory actually works we will start by defining the storage state space.

1. Reservoir state space. To define the reservoir state space let us use the Lloyd's (1964) approach. The procedure is as follows.

- Firstly, divide the total volume K of the reservoir by N. Take a=K/N as the the unit of volume for discrete storages.
- The reservoir is said to be at state z=0 when $Z<a/2$. This state represents the emptiness condition.
- The reservoir is said to be at states z=j -- j = 1,2,3,..and N-1 -- when $(j-.5)a \leq Z < (j+.5)a$.
- The reservoir is said to be at state z=N when $Z \geq (N-.5)a$. This state represents the fullness condition.

The foregoing definitions are depicted in Figure 3.2 and Table 3.1 below.

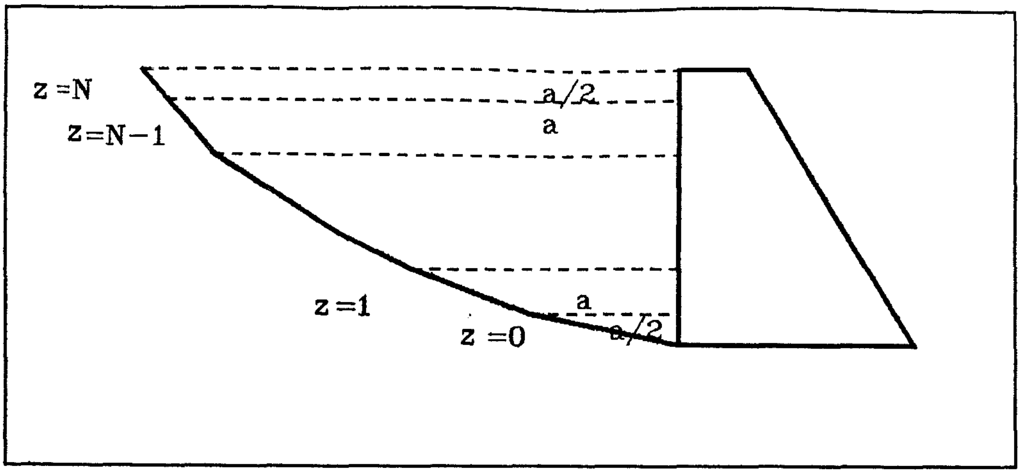

Figure 3.2. Reservoir state space representation.

Table 3.1 Storage boundaries for reservoir states.

state storage	Storage Boundaries	state number
Z=empty	$0 \le Z < (1/2)a$	z = 0
Z=a	$(1/2)a < Z \le (3/2)a$	z = 1
Z=2a	$(3/2)a < Z \le (5/2)a$	z = 2
...	...	.
...	...	.
...	...	.
Z=(N-1)a	$(N-3/2)a < Z \le (N-1/2)a$	z = N-1
Z=full	$Z > (N-1/2)a$	z = N

<u>2. Storage probability vector</u> Moran's theory of storage studies the probability of the reservoir level, or storage, being in a certain state at a given time. So let us define:

$p_{i,t} = \Pr\{z_t = i\}$ $i=0,1,2,\ldots,N$ as the probability of the reservoir storage being at state i at time t.

The vector which has as entries the $p_{i,t}$'s elements is called the Storage Probability Vector. It is represented as:

$$\underline{p}_t = \begin{vmatrix} P_r\{z_t=0\} \\ P_r\{z_t=1\} \\ P_r\{z_t=2\} \\ \cdots \\ \cdots \\ Pr\{z_t=N\} \end{vmatrix} = \begin{vmatrix} p_{0,t} \\ p_{1,t} \\ p_{2,t} \\ \cdots \\ \cdots \\ p_{N,t} \end{vmatrix} \qquad [3.5]$$

Since at a given time the reservoir storage must be in some state, the summation of the elements of this vector must be equal to one.

3. One-step transition matrix. For purposes here the one-step transition probability is defined as:

q_{ij}= Probability of the reservoir reaching the state i from state j in one time step. Or:

$$q_{ij} = P_r\{z_t = i \mid z_{t-1} = j\}. \qquad [3.6]$$

To make it clear let us present an example. Assume that the reservoir is at state 5, Z=5a, at time t. Assume also, for simplicity, a release equal to 'a.' Then, compute the probability of the reservoir being at state '6' at time t+1 as follows:

$$q_{65} = Pr\{z_{t+1} = 6 \mid z_t = 5\}.$$

From the state space table we have:

$$q_{65} = Pr\{5.5a \leq Z_{t+1} < 6.5a \mid Z_t = 5a\}.$$

Now, using the release rule we have:

$$Z_{t+1}=Z_t+X_t-O_t,$$

$$Z_{t+1}=5a+X_t-a=X_t+4a.$$

Finally,

$$q_{65} = \Pr\{5.5a \leq X_t+4a < 6.5a\},$$

$$q_{65} = \Pr\{1.5a \leq X_t < 2.5a\}.$$

The matrix which has as entries the one-step transitions probabilities, q_{ij}'s, is called One-step Transition Matrix. Its representation is in equation [3.7].

$$[Q] = \begin{vmatrix} q_{00} & q_{01} & q_{03} & \cdots & q_{0N-1} & q_{0N} \\ q_{10} & q_{11} & q_{12} & \cdots & q_{1N-1} & q_{1N} \\ q_{20} & q_{21} & q_{22} & \cdots & q_{2N-1} & q_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{N0} & q_{N1} & q_{N2} & \cdots & q_{NN-1} & q_{NN} \end{vmatrix} \qquad [3.7]$$

4. Definition of steady state. For the case where the inflows are independent and identically distributed the storage probability vector, $\underline{P}_t$, follows the relationship:

$$\underline{P}_{t+1} = [Q]\underline{P}_t \qquad [3.7]$$

It means that if we know the storage probability vector at time t and the one-step transition matrix, we can predict, in a probabilistic sense, the reservoir state at time t+1.

By recurrence one can extend this concept to time steps 2,3,4,...,T as follows:

$$\underline{P}_2 = [Q]\underline{P}_1,$$

$$\underline{P}_2 = [Q][Q]\underline{P}_0,$$

$$\underline{P}_2 = [Q]^2\underline{P}_0,$$

or,

$$\underline{P}_T = [Q]^T\underline{P}_0, \qquad [3.8]$$

As a consequence of [3.8], one can find the storage probability vector at any time step from the knowledge of the initial condition and the one-step transition matrix.

When the time step increases indefinitely, the matrix $[Q]^T$ tends to a constant matrix that has all elements in a given row equal to one another. Futhermore, the vector $\underline{P}_t$ tends to a constant vector.It means that when T is large $\underline{P}_{t+1}=\underline{P}_t$ for any initial vector $\underline{P}_0$. This condition is called steady state or equilibrium state. In this case the equations [3.5] and [3.6] become:

$$\lim_{t \to \infty} \underline{P}_t = \underline{\Pi} = \begin{vmatrix} \Pi_0 \\ \Pi_1 \\ \Pi_2 \\ \cdot\cdot \\ \cdot\cdot \\ \Pi_N \end{vmatrix} \qquad [3.9]$$

and,

$$\lim_{T \to \infty} [Q]^T = \begin{vmatrix} \Pi_0 & \Pi_0 & \Pi_0 & \cdots & \Pi_0 \\ \Pi_1 & \Pi_1 & \Pi_1 & \cdots & \Pi_1 \\ \Pi_2 & \Pi_2 & \Pi_2 & \cdots & \Pi_2 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \Pi_N & \Pi_N & \Pi_N & \cdots & \Pi_N \end{vmatrix} \qquad [3.10]$$

One interesting property of the steady state is that it does not depend on the initial storage. In other words, for any storage at time '0,' the $\underline{P}_t$ vector converges to the steady state vector $\underline{\Pi}$. In this property lies the reason for using the steady state probability in designing reservoir sizes.

5. Computing the storage vector at steady state

The transition matrix approach in reservoir sizing pursues the solution of the regimen function at steady state conditions. There are two common ways to achieve that. The first involves squaring the matrix [Q] to $[Q]^2, [Q]^4, [Q]^8, \ldots, [Q]^{2n}$ until the result is a constant matrix, and, thus we can get the vector $\underline{\Pi}$. The second approach uses the equation 3.7 as follows:

$$\underline{\Pi} = [Q] * \underline{\Pi} \qquad [3.11]$$

The relation [3.11] provides a system with N+1 equations to N+1 unknowns, but the system is not homogeneous. Then to make it homogeneous we should substitute one of the equations with the condition that the summation of Π's must be one. The system of equations becomes:

$$\begin{aligned}
q_{00}\Pi_0 + q_{01}\Pi_1 + \ldots + q_{0N}\Pi_N &= 0 \\
q_{10}\Pi_0 + q_{11}\Pi_1 + \ldots + q_{1N}\Pi_N &= 0 \\
\ldots\ldots & \qquad [3.12] \\
q_{N0}\Pi_0 + q_{N-10}\Pi_1 + . + q_{NN}\Pi_N &= 0 \\
\Pi_0 + \Pi_1 + .. + \Pi_N &= 1
\end{aligned}$$

The solution of this system yields the information on the steady state storage vector used in the sizing process.

C. Model Development

The basic assumptions we used to develop the sizing model were:

- time is discrete and time step is a year;
- reservoir volume is discrete, and the unit volume is 1/20 of the reservoir capacity;
- serial correlation of annual inflow is zero;
- all inflows occur in a wet season and all output in a dry season;
- the output occurs in a three part sequence-- first half of evaporation, then all release, finally the remaining evaporation;
- inflows come from a gamma-II distribution.

The first assumption, taking the time step as one year, has its justification in the fact that we are pursuing a model for preliminary design, and this time unit is used in most, if not in all, of these models.

The second assumption, discrete volumes, does not itself constitute a problem. The question is, how fine should we divide the reservoir volume to get results accurate enough for engineering practice without consuming too much computer time. Gould (1961) and White (1966) agree that dividing the reservoir into 10-20 layers is enough, while Kottegoda (1980) talks about 5-30 layers.

The third assumption rests on the fact that most rivers in the study area have a long season -- from six to nine months -- with zero discharge. So, there is no significant transportation of water from one year to the next by the soil. As a consequence, the effect of the watershed on annual streamflow autocorrelation becomes too small. Under these circumstances the annual inflow autocorrelation is fed only by rainfall autocorrelation which is disregarded in most hydrological studies. Aside from that, the adoption of correlated flows implies two undesirable effects: first by using a bi-variate Markovian chain the matrix size is increased exponentially; second the number of model parameters is increased.

The fourth assumption comes from Moran's original model. As has been pointed out in the literature that model is appropriate for intermittent rivers. In the study area the conditions are very similar to Moran's assumptions. During the usual wet season all rainfall and run-off occur. The use of stored water at this time is very low, except for occasional very dry years. During the dry season the stored water is used for irrigation or other purposes.

The fifth assumption is intended to provide a mean value for evaporation losses. When the first half of evaporation happens, the lake is supposed to be at a higher level with a greater area A_1. When the remaining evaporation occurs, after the release, the lake must be at a lower level with a smaller area A_2. Thus, the procedure tends to make the evaporated volume closely proportional to the average of A_1 and A_2.

The matricial representation of these assumptions is:

$$\underline{p}_{t+1} = [E][R][E][W]\underline{p}_t, \qquad [3.13]$$

where,

$\underline{p}_t$ = storage distribution at time t;

[E] = evaporation transition matrix due to half of dry season evaporation;

[R] = release transition matrix due to yearly release;

[W] = input transition matrix due to natural inflows.

The product [E][R][E]=[O] is called output transition matrix, while [O][W]=[Q] is the one-step transition matrix. The procedure used to compute each of these matrices is described hereunder.

<u>1. Input matrix</u> To compute the input matrix we assumed that the inflow process $\{X_t\}$ follows a mixed distribution with a probability mass at X=0 and a probability density, gamma-II, for inflows greater than zero.(Figure 3.4).

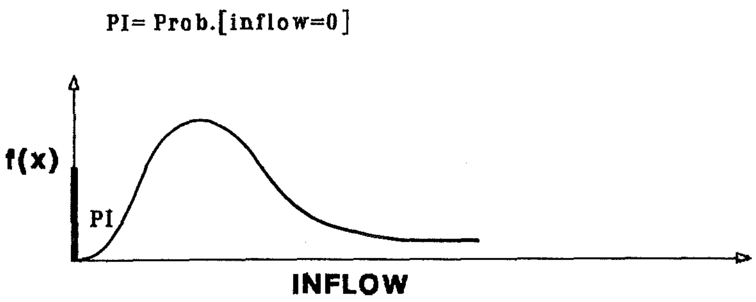

Figure 3.4 Probability distribution function for inflow process.

The distribution parameters are:

PI = Probability mass for annual flow zero;

μ = Expected value of overall distribution, include zero values;

μ' = Expected value of the marginal gamma-II;

$\sigma 2$ = variance of the overall distribution;

$\sigma 2'$ = variance of the marginal gamma-II.

These values are linked by the relation

$$\mu = (1-PI)\,\mu', \tag{3.14}$$

$$\sigma^2 = (1-PI)\,\sigma^{2\prime} + PI\,\mu'^2 (1-PI). \tag{3.15}$$

Now let us take w_{ij} as the probability of the reservoir changing from state j to state i after the annual flow has happened, and [W] as the matrix which has the w_{ij}'s as entries. As the reservoir does not have any outflow in this transition step, the matrix will be lower band. It means that the storage level can rise but never drop. This matrix can be represented as:

$$[W] = \begin{vmatrix} p_0 & 0 & 0 & 0 & \dots & 0 & 0 \\ p_1 & p_0 & 0 & 0 & \dots & 0 & 0 \\ p_2 & p_1 & p_0 & 0 & \dots & 0 & 0 \\ p_3 & p_2 & p_1 & p_0 & \dots & 0 & 0 \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ p_{N-1} & p_{N-2} & p_{N-3} & p_{N-4} & \dots & p_0 & 0 \\ f_1 & f_2 & f_3 & f_4 & \dots & f_{N+1} & 1 \end{vmatrix} \tag{3.16}$$

where,

p_1 = probability of reservoir water rising 1 states;

f_l = probability of reservoir water to rising l or more states.

For a reservoir with capacity K divided into N+1 states, with a unit volume equal to a=K/N, fed by an inflow $\{X_t\}$, these values are computed as follows.

$$p_0 = P_r\{X \leq (1/2)a\}$$

$$p_l = P_r\{(l-1/2)a < X \leq (l+1/2)a\} \quad l=1,2,...,N-1 \qquad [3.17]$$

$$f_l = P_r\{X > (l+1/2)a\} \quad l=1,2,....,N.$$

Now let us present an example. Assume that the inflows are normally distributed with mean μ =10 and a coefficient of variation C_V=1.0. Take a=10 and compute the probability of the storage rising one state p_1.

Using equation 3.17 we have:

$$p_1 = P_r\{0.5a < X \leq 1.5a\}$$

$$p1 = P_r\{5 < X \leq 15\}.$$

From a normal table we got:

$$p_1 = 0.38$$

<u>2. Evaporation matrix</u> The volume evaporated from a reservoir during a given period of time can be estimated by the product between the evaporation depth and the mean water surface. It means that this volume depends on local climatic conditions and on the reservoir geometry. To lump these two factors we assumed that the reservoir storage(Z), area(A) and height(H) follow the relationship:

$$Z(H) = \alpha H^3 \qquad [3.18]$$

$$A(H) = \frac{dZ(H)}{dH} = 3\alpha H^2, \qquad [3.19]$$

where α is a parameter of shape for the lake. It can be obtained using least squares with the data from the storage-height table. For Z=0 we must have H=0.

The evaporated volume from the reservoir when it is at level H is:

$$V_{EV} = A_{(H)} * E_V, \qquad [3.20]$$

where E_V stands for the evaporation depth during the specified time period.

Using [3.18] and [3.19] we get:

$$V_{EV} = 3\,\alpha^{1/3} E_V Z(H)^{2/3} \qquad [3.21]$$

The factor $XEV=3\,\alpha^{1/3}E_V$ will be used to introduce the evaporation effect on the sizing model.

Now let us define e_{ij} as the probability of the reservoir changing from j to i due to half of the annual evaporation and [E] as the matrix which has as entries the e_{ij}'s. The procedure to compute this matrix using Moran's approach is:

- compute the reservoir storage at state j,

$$Z_j = ja;$$

- compute the volume evaporated from the reservoir at that state using [3.21];
- compute the number of states that the reservoir drop due to that release,

$$N_L = \text{INTEGER}(V_{EV}/a);$$

- then, for a column j we get --

$$\text{if } j>N_L \qquad e_{ij} = 1 \text{ for } i=j-N.$$
$$= 0 \text{ otherwise}$$

$$\text{if } j \leq N_L \qquad e_{ij} = 1 \text{ for } i=0,$$
$$= 0 \text{ otherwise.}$$

One important point in the evaporation matrix is the bias introduced for assuming discrete volumes. The explanation for this comes from the fact that as long as the water level falls to lower stages, the storage Z_j decreases and so does the evaporated volume V_{EV}. As a consequence, there is a stage j_{min} where the evaporated volume comes out to be less than the unit volume 'a.' Under these circumstances, N_L becomes zero, and the system works as if the evaporation had ceased.

We have visualized two possible ways to correct the bias. The first is based on increasing the number of reservoir states to make j_{min} as low as we want. The second involves introducing a correction in the matrix computation.

To evaluate the first approach, assume that you want keep the evaporation occurring up to the lowest reservoir state, that is $j_{min}=1$. To achieve that you have to have a unit volume 'a' at least equal to the volume evaporated from the first layer. Or:

$$a=XEV(ja)^{2/3}$$

Making $a=K/N$ and $j=1$ you have,

$$N=K/XEV^3$$

For K=1000 and XEV=1.0, common figures for the study area, N is 1000. Such a size for the matrix will skyrocket the computer time and make this approach unfeasible.

Besides, if you assume $j_{min}=N/10$ it does not improve matters much.

The second approach consists of computing the e_{ij} as follows:

- compute $X_L = (V_{EV}/a)$,

$N_L = \text{INTEGER}(X_L)$,

for $X_L \geq j$ $\quad e_{ij} = 1.0 \quad$ if $i=0$

$= 0. \quad$ otherwise;

For $X_L<j$ $\quad e_{ij} = 1-(X_L-N_L) \quad$ if $i= j+1-N_L$

$= X_L-N_L \quad$ if $i= j-N_L$

$= 0. \quad$ otherwise.

The expected value of V_{EV} at state j will be:

$E\{V_{EV}\} = [1-(X_L-N_L)][j-(j-N_L)]a+ (X_L-N_L)[j-(j-N_L-1)]a$

$E\{V_{EV}\}=(1-X_L+N_L)(N_L)a+(X_L-N_L)(N_L+1)a$

$E\{V_{EV}\}=X_L.a=(V_{EV}/a)a=V_{EV}.$

3. Release matrix Let us start presenting the way the release matrix was computed in Moran's model.

First define:

r_{ij} = probability of reservoir reaching state i from state j due to release M;

[R] = release matrix which has the r_{ij}'s as entries.

The procedure to compute [R] is:

-make the unit volume a=K/N, and take the state space for N as; 0,a,2a,... .

-compute the number of states that the reservoir drop after the release M --

$N_L=M/a$.

-compute the r_{ij}'s by:

for a column j, j=0,1,2,..,N,

if $j>N_L$ $\quad r_{ij} = 1$ for $i=j-N$,

$= 0$ otherwise;

if $j \leq N$ $\quad r_{ij} = 1$ for $i=0$,

$= 0$ otherwise.

The release matrix will look like the one in Figure 3.5.

$$\begin{vmatrix} 1 & 1 & 1 & & & & \\ & & & 1 & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \\ & & & & & & \\ \end{vmatrix}$$

Figure 3.5 Release matrix according to Moran's model.

To permit release M to take values from the set of positive real numbers, we introduced corrections in the same way we did for the evaporation matrix, that is:

- compute the real number,

$X_L = M/a;$

-take the integer part of X_L,

$N_L = \text{integer}(X_L);$

-then for a column j, j=0,1,2,...,N, we get --

For $X_L \geq j$

$r_{ij} = 1.0$ $\quad$ if $i=0$,

$= 0.0$ $\quad$ otherwise;

For $X_L < j$

$$
\begin{aligned}
r_{ij} &= 1-(X_L-N_L) && \text{if } i = j-N_L, \\
&= X_L-N_L && \text{if } i = j-N_L-1, \\
&= 0 && \text{otherwise.}
\end{aligned}
$$

This matrix will look like Figure 3.6.

$$
\left|\begin{array}{ccccccc}
1. & 1. & 1. & .8 & & & \\
 & & & .2 & .4 & & \\
 & & & & .6 & .7 & \\
 & & & & & .3 & \\
 & & & & & & .9 \\
 & & & & & & .1 \\
 & & & & & & \\
 & & & & & &
\end{array}\right|
$$

Figure 3.6 Release matrix after corrections

4. <u>Model review</u> In brief the theoretical model consists of:

i) computing the input matrix [W];

ii) computing the evaporation matrix due to the half of dry season evaporation [E];

iii) computing the release matrix [R];

iv) computing the annual matrix by the product,

$$[Q]=[E][R][E][W].$$

v) forming the equation system for steady state,

$$\underline{\Pi} = [Q]\underline{\Pi}$$

and substituting one of these equations with

$$\Sigma \Pi_i = 1;$$

vi) solving the equation system for Π_0, that is, for the probability of emptiness.

CHAPTER IV

DEVELOPMENT OF THE SIZING PROCEDURE

This chapter covers the development of a procedure to size reservoirs on two-seasoned rivers with independent annual inflows in places where the evaporation losses are important. The procedure, in graphical form, is based on results from the theoretical model developed in Chapter III. The objective is to provide, quickly and with an acceptable accuracy, the solution of the function linking reservoir capacity, annual release and probability of failure.

Before developing the procedure, we studied the effects of evaporation on the required capacity. We demonstrated that the introduction of a correction factor on the reservoir capacity is not appropriate for situations with a high evaporation rate. We also introduced the dimensionless evaporation factor as a better way to deal with that phenomenon. In addition, we investigated the effect of the annual intermittence on the required capacity.

A. Evaporation

When Hurst (1950) first developed his theory for reservoir sizing, he did not include any factor to take into account the evaporation losses. He argued that unless these losses were small the site was not suitable for a over-year storage.

Later, Hurst, Black and Simaika (1965) recognized that the high evaporation rates occur, usually, in semi-arid/arid regions, and in such places the water stored on the surface can be the most important source of fresh water. In fact, the point is that there are a lot of places around the world where the evaporation is high and the over-year storage is the most convenient, and sometimes the only, source of fresh water for the inhabitants. Besides, the evaporation effects on a reservoir's efficiency depend on three main factors, namely the evaporation depth, the lake shape and the mean inflow.

In the last cited work, Hurst,Black and Simaika tried to introduce the evaporation losses in the theory, assuming the relation among inflow(Q), outflow(B) and reservoir capacity(C) given by:

$$dC = (Q-B-L)dt \qquad [4.1]$$

Taking a geometrical figure for the lake they made the losses $L = lC^{2/3}$ (l is a constant). Hence they tried to combine the prior equation and the equation $R = \sigma(N/2)^K$. Their attempt resulted in nothing of much value. Next, they computed the range numerically for 20 cases, six with no losses and 14 with losses. From the results they concluded that the evaporation can increase or decrease the storage required to give the maximum draft. Nevertheless, their results looks strange. In fact, for a finite reservoir one can expect the evaporation always acting in the same direction, that is, decreasing the usable water for a given reservoir capacity or increasing the required storage for a given release.

Most Sizing Procedures, exception maybe to McMahon (1976), simply disregard the evaporation losses. Even McMahon's procedure only introduces a correction factor in the required capacity. This factor is a empirical one based on results from 156 Australian Rivers. His procedure is as follows.

- Compute the storage capacity by,

$$S/\bar{x} = (0.57C_V^{1.82})/[p^{0.52}(1-D)^{1.34}], \quad [4.2]$$

where:

S = required storage;

C_V = inflow coefficient of variation;

p = reservoir probability of failure;

D = draft;

$\bar{x}$ = mean annual inflow at site.

- Using a graphic, correct the storage capacity to include the effect of serial correlation of yearly inflows.
- Compute the required increment in storage to include the evaporation effect, by

$$\Delta S_E = 0.7A\Delta EC_p, \quad [4.3]$$

where:

A = surface area of full reservoir(km^2);

C_p = critical drawdown period(years) from Alexander equation(1962);

ΔS_E = increase in storage to account for net evaporation losses(millions of cubic meters);

ΔE = net annual evaporative losses(m).

The range for that equation according to the author is:

$$2\% \leq p \leq 10\%$$

$$0.3 \leq D \leq 0.9$$

Aside from the logic of correcting the required storage to include the evaporation effect, some danger is embodied in this procedure, mainly in the case of high drafts and a small probability of emptiness. To justify this statement assume that you are sizing a reservoir for a given probability of failure, f_1. Now, assume that when the reservoir has an infinite capacity the annual release is M_1, and the total evaporation losses in volume is E_1. It means that for that river, M_1 is the maximum amount of water that the reservoir yields with a probability of emptiness f_1. Now, assume no evaporation in the process. Of course, for a certain finite reservoir size you can find a release M_2 greater than M_1, but as M_1 represents the maximum release for the design risk there is no reservoir size that is able to yield a release M_2. As the evaporation represents a part of the output, it is likely that the introduction of a correction on the release would work better.

<u>Definition of the dimensionless evaporation factor</u> Let us see how to lump all the factors affecting the evaporation

losses on a reservoir into one dimensionless evaporation factor. Let us take the continuity equation:

$$Z_{t+1} = Z_t+X_t-E_V(A_{t+1}+A_t)/2-M_t-S_{pt}, \qquad [4.4]$$

where,

Z_{t+1}, Z_t = Storage at beginning of year t+1 and t;

X_t = Inflow into the reservoir during year t;

E_V = Net annual evaporation depth;

A_{t+1}, A_t = Lake area at beginning of year t+1 and t;

M_t = Release from the reservoir during year t;

S_{pt} = Spill from the reservoir during year t.

Using equations 3.18 and 3.19 to make $A_t=3\alpha^{1/3}Z_t^{2/3}$ and putting this relation in 4.4 we have:

$$Z_{t+1} = Z_t+X_t - 3\,\alpha^{1/3}E_V(Z_{t+1}^{2/3}+Z_t^{2/3})/2 - M_t - S_{pt} \qquad [4.5]$$

Dividing all terms in equation 4.5 by the mean inflow and rearranging we get:

$$Z_{t+1}/\mu = Z_t/\mu + X_t/\mu - 3\,\alpha^{1/3}E_V/\mu^{1/3}(Z_{t+1}^{2/3}+Z_t^{2/3})/(2\,\mu^{2/3}) - $$
$$- M_t/\mu - S_{pt}/\mu \qquad [4.6]$$

The equation 4.6 is the dimensionless form of the continuity equation, and the term $f_E = (3\,\alpha^{1/3}E_V/\mu^{1/3})$ is the dimensionless evaporation factor. It is not hard to see that equal values of f_E affect the reservoir in the same degree. In other words, if you preserve the dimensionless evaporation factor, you also preserve evaporation losses.

Another way to reach the dimensionless evaporation factor, with a better physical understanding, is to use similitude of geometrical figures. In this case we assumed that the relative effect of evaporation on two bodies of water is the same when the ratios between the evaporated volume and the total volume are equal. We imagine two reservoirs with the following data:

Reservoir #1- μ_1; $K_1 = a\mu_1$, α_1; E_{V1};

Reservoir #2- μ_2; $K_2 = a\mu_2$; α_2; E_{V2}.

Now, using equations 3.18 and 3.19 and equating the ratios volume evaporated/reservoir capacity for both reservoirs we get:

$$\frac{\alpha_1(H_1)^3 - \alpha_1(H_1-E_{V1})^3}{\alpha_1(H_1)^3} = \frac{\alpha_2(H_2)^3 - \alpha_2(H_2-E_{V2})^3}{\alpha_2(H_2)^3} \text{ or,}$$

$$1 - \frac{(H_1-E_{V1})^3}{(H_1)^3} = 1 - \frac{(H_2-E_{V2})^3}{(H_2)^3}$$

After simplifications,

$$\frac{E_{V1}}{H_1} = \frac{E_{V2}}{H_2},$$

$$\frac{E_{V1}}{(K_1/\alpha_1)^{1/3}} = \frac{E_{V2}}{(K_2/\alpha_2)^{1/3}},$$

Finally,

$$\frac{E_{V1}(\alpha_1)^{1/3}}{(\mu_1)^{1/3}} = \frac{E_{V2}(\alpha_2)^{1/3}}{(\mu_2)^{1/3}}.$$

One can draw the conclusion from using the dimensionless evaporation factor that, even though it is the strongest term in the factor, the evaporation depth alone cannot determine if the place is appropriate or not to build a reservoir.

B. Intermittence of Annual Inflows

The actual probability distribution of inflows for the study area is mixed with a probability density for inflow greater than zero and a probability mass for inflow equal to zero(PI). It means that the real distribution function has three parameters, and, as a consequence, it also means problems in developing the sizing procedure. A way to simplify that function is to use an equivalent continuous probability density function (p.d.f.) which preserves the mean and standard deviation from the real distribution function. The question is: how does that simplification affect the required storage capacity?

The sensitivity analysis of R_K -- defined as the ratio between the storage capacity resulting from use of the real distribution function and the storage capacity resulting from use of the equivalent p.d.f.- - provided the answer to the question. So, we performed this sensitivity analysis using the procedure parameters f_M, f_K, f_E, C_V, PE and PI for the five cases below --

$f_M = M/\mu$ dimensionless release,

$f_K = K/\mu$ dimensionless capacity.--

CASE 1

Procedure- Fix PE and PI and compute the expected value of R_K for $f_M = 0.10, 0.15, 0.20, \ldots, 0.60$.

Conclusion- There is no apparent trend linking R_K to f_M.(see table 4.1).

Table 4.1. Expected value of the ratio between the storage capacity computed using a mixed p.d.f and a continuous p.d.f. for inflows- $E\{R_K\}$- for several values for the dimensionless release- f_M.

	PE=0.02		PE=0.05		PE=0.10		PE=0.15	
f_M	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10
0.10	1.10	1.23	1.04	1.09	1.05	*	*	*
0.15	1.10	1.23	1.09	1.19	1.05	1.10	1.03	1.07
0.20	1.11	1.22	1.07	1.14	1.05	1.11	1.03	1.06
0.25	1.09	1.20	1.08	1.16	1.04	1.09	1.04	1.08
0.30	1.11	1.21	1.07	1.12	1.04	1.08	1.03	1.06
0.35	1.11	1.20	1.07	1.14	1.04	1.08	1.03	1.06
0.40	1.13	1.19	1.07	1.13	1.04	1.08	1.02	1.05
0.45	1.10	1.20	1.06	1.12	1.03	1.07	1.02	1.05
0.50	1.09	1.18	1.05	1.11	1.02	1.05	1.01	1.04
0.55	1.07	1.16	1.04	1.09	1.02	1.05	1.07	1.16
0.60	1.11	1.20	1.06	1.13	1.04	1.07	1.02	1.04

*- No value of storage capacity in this range.

CASE 2

Procedure- Fix PE and PI and compute the expected value of R_K for f_E = 0.05, 0.10, 0.15,....,0.40.

Conclusion- R_K tends to increase slowly when f_E increases. It means that the error tends to be greater for larger evaporation ratio, but in the range studied the variation of R_K does not have practical significance(see table 4.2).

CASE 3

Procedure- fix PE and PI and compute the expected value of R_K for C_V = 0.6, 0.7,...1.4.

Conclusion- There is no apparent trend in R_K due to C_V variations.(see table 4.3).

CASE 4

Procedure- Fix PE and compute the expected value of R_K for PI = 0.05, 0.10.

Conclusion- R_K tends to increase when PI increases (see table 4.4).

CASE 5

Procedure- Fix PI and compute the expected value of R_K for PE = 0.02, 0.03, 0.05, 0.08, 0.10, 0.15 and 0.20.

Conclusion- R_K tends to increase when PE increases. In that case the trend looks important. As an example for PE = 0.02 and PI = 0.10 if you size a reservoir assuming a continuous p.d.f. you expect to get a reservoir capacity around 83% of

Table 4.2. Expected value of the ratio between the storage capacity computed using a mixed p.d.f and a continuous p.d.f. for inflows- $E\{R_K\}$- for several values for the coefficient of variation of inflows-C_V.

	PE=0.02		PE=0.05		PE=0.10		PE=0.15	
C_V	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10
0.60	1.13	1.28	1.07	1.13	1.03	1.07	1.02	1.04
0.70	1.11	1.16	1.06	1.13	1.04	1.08	1.02	1.04
0.80	1.14	1.24	1.08	1.18	1.04	1.08	1.02	1.04
0.90	1.09	1.18	1.05	1.10	1.03	1.06	1.02	1.04
1.00	1.13	1.23	1.08	1.15	1.04	1.08	1.02	1.04
1.10	1.08	1.17	1.05	1.09	1.03	1.07	1.02	1.04
1.20	1.08	1.18	1.06	1.13	1.05	1.09	1.02	1.05
1.30	1.11	1.25	1.06	1.13	1.03	1.06	1.02	1.04
1.40	1.09	1.16	1.07	1.13	1.04	1.08	1.02	1.05

Table 4.3. Expected value of the ratio between the storage capacity computed using a mixed p.d.f and a continuous p.d.f. for inflows- $E\{R_K\}$- for several values of the dimensionless evaporation factor f_E.

	PE=0.02		PE=0.05		PE=0.10		PE=0.15	
f_E	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10	PI=0.05	PI=0.10
0.05	1.061	1.117	1.042	1.080	1.027	1.053	1.019	1.040
0.10	1.066	1.129	1.044	1.086	1.026	1.055	1.017	1.036
0.15	1.077	1.148	1.048	1.096	1.028	1.058	1.017	1.037
0.20	1.078	1.155	1.050	1.099	1.030	1.062	1.018	1.039
0.25	1.083	1.165	1.054	1.108	1.031	1.063	1.018	1.039
0.30	1.092	1.182	1.058	1.115	1.033	1.069	1.019	1.042
0.35	1.102	1.193	1.062	1.125	1.035	1.073	1.019	1.043
0.40	1.105	1.200	1.064	1.128	1.035	1.074	1.019	1.043

the real required capacity(see table 4.4 and Figure A1 in appendix A).

As a conclusion we can state that substituting the real mixed distribution function by a continuous p.d.f. tends to introduce a bias in the sizing process. This bias increases when the reservoir reliability increases -- PE decreases-- and when the probability of annual inflow being zero increases.

Table 4.4. Expected value of the ratio between the storage capacity computed using the real distribution function and the equivalent p.d.f. for several values of PE.

PE \ PI	0.05	0.10
0.02	1.11	1.20
0.03	1.08	1.17
0.05	1.06	1.13
0.08	1.04	1.09
0.10	1.04	1.07
0.15	1.02	1.04
0.20	1.01	1.02

It is important to mention here an observation about Clarke's (1973) procedure for generation of monthly flows for intermittent rivers. Clarke's procedure is based on an adaptation of the Thomas and Fiering method in the following way: 1) from historical data,compute, for each month, the probability of the inflow being zero(p_i); 2) before the generation of any monthly discharge, generate a random number, r, uniformly distributed in the interval[0,1]; 3) then, if r happens to be less than p_i, the inflow for that month will be zero, otherwise it will be generated normally using the Thomas and Fiering method.

It is easy to see that according to that procedure the probability of the annual inflow being zero is equal to the product of p_i's i = 1,2,..12, (Πp_i). Hence, this formula tends,in many cases, to make PI too low. Nevertheless, as it was pointed out before, this can introduce a bias into the sizing process, mainly in cases of high reliability reservoirs. A procedure that avoids that bias is: 1) from historical data get the probability of the annual flow being zero and the probabilities for each month of a zero inflow(p_i); 2) before starting the generation for a given year, generate a random number U[0,1]. If this number is less than PI-Πp_i then make the annual flow equal to zero otherwise, follow as in Clarke procedure. Now, the expected value of PI is PI - Πp_i +Πp_i = PI.

C. The Sizing Procedure

The required reservoir capacity can be represented by the equation:

$$K = (\mu, \sigma^2, E_V, \alpha, M, PE, PI) \qquad [4.1]$$

To simplify the model, we used these parameters in a dimensionless form as follows:

$$K = (C_V, f_E, f_M, PE, PI) \qquad [4.2]$$

As a result, the number of parameters drop from seven in equation 4.1 to five in equation 4.2.

However, even five parameters is a big number with which to develop the graphical procedure. Therefore, supported by the results from section 'B,' we decided to compute the required capacity using the equivalent continuous p.d.f., that is, dropping the PI parameter. For specials cases of high reliability and high PI, we introduced a correction factor into the required capacity. Appendix A shows how to perform this correction. Now equation [4.3] becomes:

$$f_K = f(C_V, f_E, f_M, PE) \qquad [4.3]$$

<u>1. Range of input/output parameters</u> To define the range for input/output parameters we analyzed a collection of hydrologic studies developed by the Grupo Executivo de Irrigacao para o Desenvolvimento Agricola (GEIDA) (1970). These studies cover most of Brazil's Northeast dams existent at that time. As a result, we got a range wide enough to permit the application of the procedure to other areas around the world with similar hydrologic conditions. The range is:

f_E- 0.05, 0.10, 0.15, 0.20,...,0.40.

C_V- 0.6, 0.7, 0.8,...., 1.4.

f_M- 0.10-0.60.

f_K- 1.0, 1.5, 2.0, 2.5, 3.0, 3.5.

PE- 0-20%.

PI- 0-10%.

2. Applying the sizing procedure To make clear the application of the model let us present an example. Assume that you want to size a reservoir with the following data:

- mean annual inflow 700MCM;
- inflow coefficient of Variation 1.0;
- reservoir shape factor 16000;
- evaporation depth during the dry season 1.80m;
- probability of annual inflow being zero 0;
- probability of emptiness 10%.

The information being sought is the relationship between release and reservoir capacity.

The procedure to accomplish that is:

a) compute the dimensionless evaporation factor --

$f_E = 3\,\alpha^{1/3} E_V / \mu^{1/3} = 0.15$;

b) take in Appendix A the graphic for $C_V = 1.0$ and $f_E = 0.15$ and draw one horizontal line starting at the ordinate PE = 10%. This horizontal line crosses six f_K curves -- the first curve, from right to left, represents $f_K = 1.0$ following $f_K = 1.5$, $f_K = 2.0$, $f_K = 2.5$, $f_K = 3.0$ and $f_K = 3.5$ -- From the cross points draw a vertical line and get the f_M value in the abscissas axis. Using these points you can build the table 4.5.

As a result, you have defined the curve yield versus reservoir capacity. That curve used together with economic data provide the tool to get the reservoir size.

3. Testing the sizing procedure The objective of the sizing procedure is to provide, in a short time and with acceptable accuracy, results that can, for engineering purposes, substitute for the results from more elaborate, and as result more accurate and more time consuming, methods. As a consequence, the objective of this present test was to verify how well the probability of failure from the sizing procedure fits the corresponding values from a simulation using real data.

Table 4.5. Relationship between reservoir capacity and release for the case example.

f_K	K (MCM)	f_M	$M=f_M*\mu$ (MCM)
1.0	700	.32	224
1.5	1050	.38	266
2.0	1400	.425	298
2.5	1750	.47	329
3.0	2100	.50	350

To perform the test we obtained data from two reservoirs, Orós and Aires de Sousa, both located in Ceara, Brazil.(See table 4.5.)

Table 4.5. Characteristics of hydrologic stations used to test the sizing procedure.

Dam	River	Watershed (Km^2)	Lat. North	Long.	Period
Orós	Jaguaribe	25770	6°14'	38°55'	1927/57
Aires de Sousa	Jaibaras	1068	3°47'	40°31'	1934/67

The steps used in computing the reservoir probability of emptiness using real data are listed below.

1. The following data was collected: monthly inflows, monthly mean evaporation depth and storage versus height table.

2. The recorded data was routed through the reservoir using 3 values for reservoir capacity and five values for annual release. The probability of emptiness was computed dividing the number of years the reservoir ended empty by the total number of years simulated.

3. To wash out the initial storage we performed the routing as follows:

   a. Assuming the reservoir starting full, we computed the time for first emptiness F_{CO};

   b. Assuming the reservoir starting empty, we computed the time for first fullness F_{OC};

   c. We took the time $T_s = \min\{F_{CO}, F_{OC}\}$[1] to start other routing. The method for this is given below.

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[1] Chapter V section A.2 presents a detailed explanation on this point.

i. If $F_{CO} < F_{OC}$, we started with reservoir empty at time F_{CO};

ii. if $F_{OC} > F_{CO}$, we started with the reservoir full at time F_{OC}.

4. Results of the Test The results of the test are illustrated in figures 4.1 to 4.6 and in appendix B. Let us analyze these results. To simplify let us assign:

PE_R as the probability of emptiness attained from the route;

PE_S as the probability of emptiness attained from the sizing procedure.

Then we have:

Test 1- Aires de Sousa with $f_K = 2.0$.

$PE_R < PE_S$ for all values of f_M

The difference between results seems acceptable for practical purposes.

Test 2- Aires de Sousa with $f_K = 2.5$.

$PE_R < PE_S$ for small values of f_M

$PE_R > PE_S$ for larger values of f_M.

The difference between results seems acceptable for practical purposes.

Test 3- Aires de Sousa with $f_K = 3.0$.

$PE_R < PE_S$ for small values of f_M

$PE_R > PE_S$ for larger values of f_M.

The difference between results seems acceptable for practical purpose.

Test 4- Orós with $f_K = 2.0$.

$PE_R > PE_S$ for small values of f_M

$PE_R < PE_S$ for larger values of f_M.

The difference between results seems acceptable for practical purpose.

Test 5- Orós with $f_K = 2.5$.

Excellent fit.

Test 6- Orós with $f_K = 3.0$.

Excellent fit.

<u>Comments</u> The difference in results has two major explanations.

1. The sizing model is free from sampling errors, so the curves are smooth. On the other hand, the real data represents just one realization of the process. In this case the sample can have, as is usual, some bias -- sometimes under-estimating and sometimes overestimating the probability of emptiness.
2. As the amount of recorded data is relatively small, the observed frequency of failure takes values in a discrete field. To make it clear, assume 30 years of observations. In this case the frequency of failure, taken as a measure for the probability of failure, can take values of 0/30, 1/30, 2/30,.. ..,30/30. As a consequence, the curve obtained from simulation is formed for a series of steps. On the other hand, the curve from

the sizing procedure is smooth, thus, we can expect one curve crossing the other.

It is important keep track of the following. The sizing procedure is based on a two-season time step and it is free from sampling errors. On the other hand, the results from simulation are based on a 12-season time step (monthly), but as the availability of real data is limited, it incorporates some sampling errors.[2]

As a final conclusion one can state that the model is accurate enough for practical purposes.

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[2] Of course it is always possible using the Thomas-Fiering/Clarke model to generate synthetic samples and get rid from sampling errors. But, that model, as was shown before, introduces some bias in the process, and, futhermore, it is not a well tested model. Thus a question arises, if we use this model what are we testing, the sizing procedure or the Clarke model?

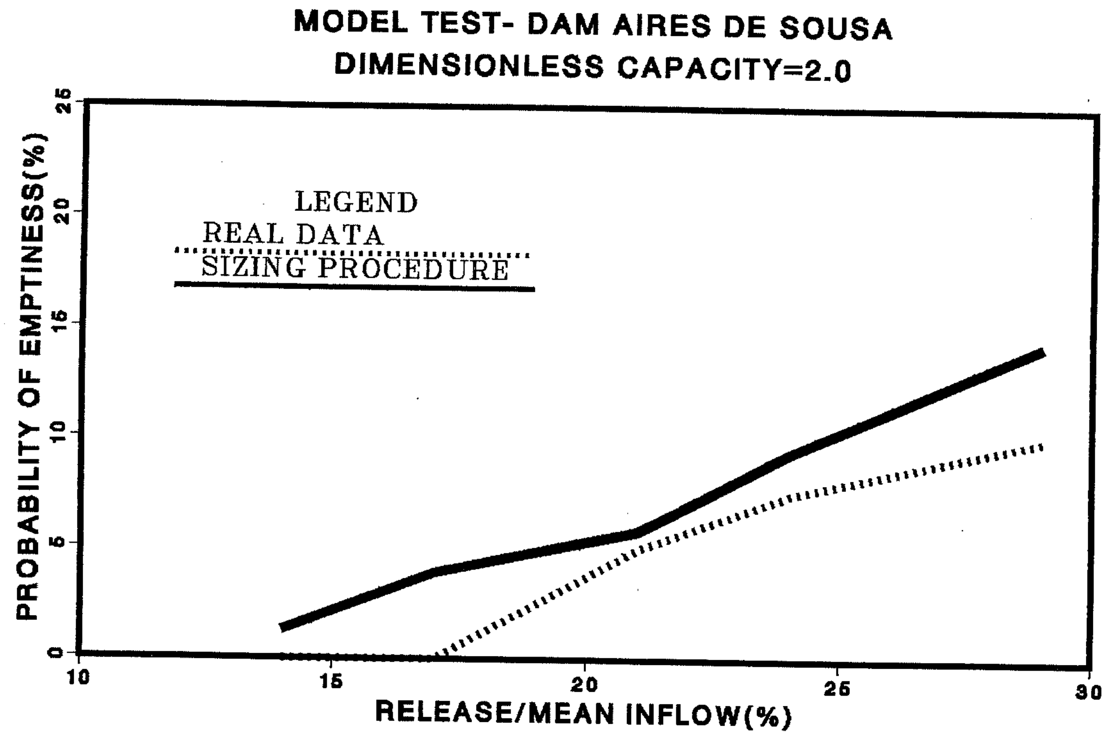

Figure 4.1. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Aires de Sousa Dam -- Ceara, Brazil -- with f_K = 2.0.

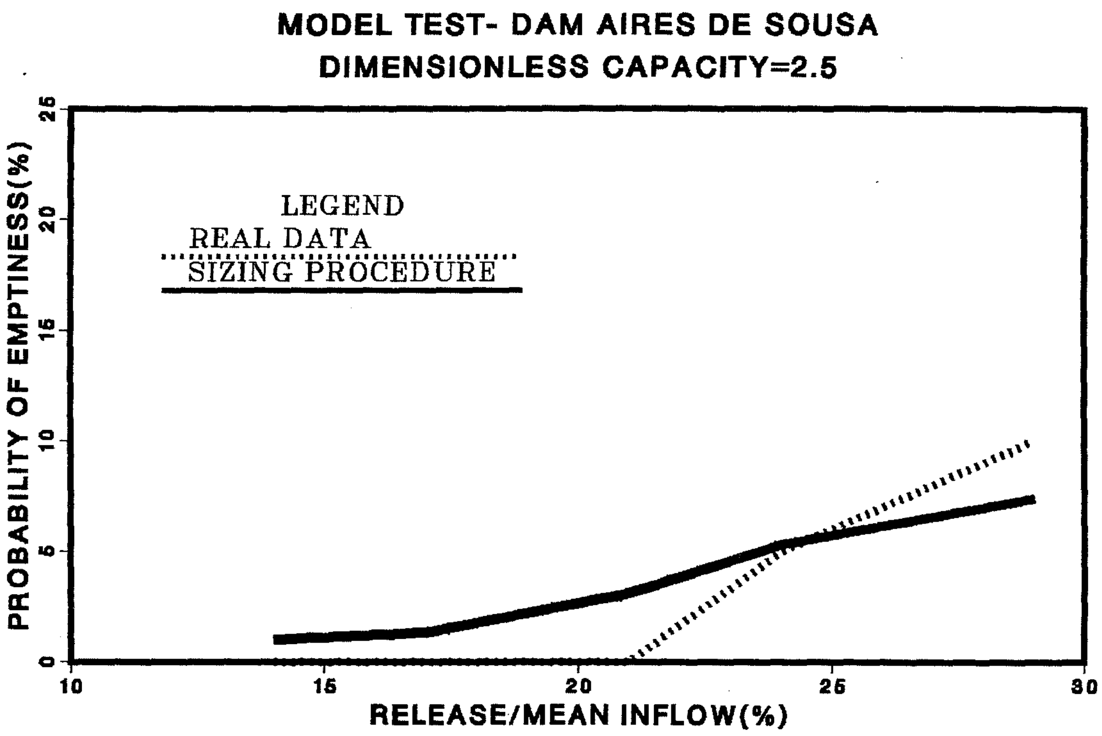

Figure 4.2. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Aires de Sousa Dam -- Ceara, Brazil -- with $f_K = 2.5$.

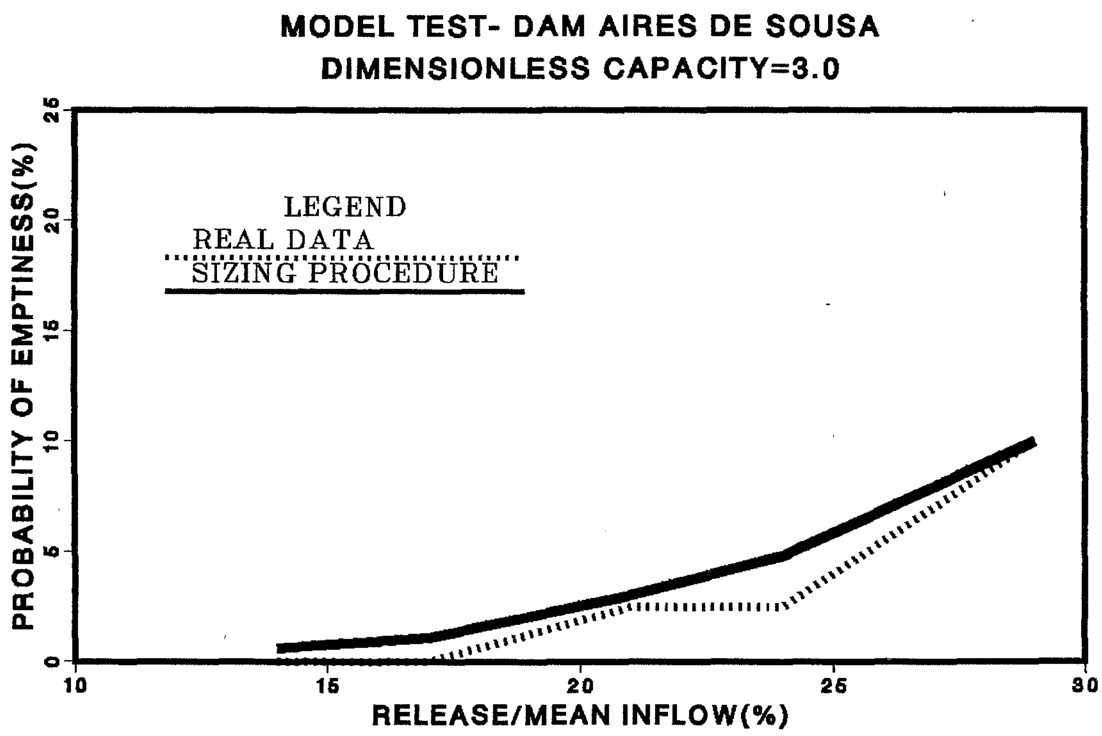

Figure 4.3. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Aires de Sousa Dam -- Ceara, Brazil -- with $f_K = 3.0$.

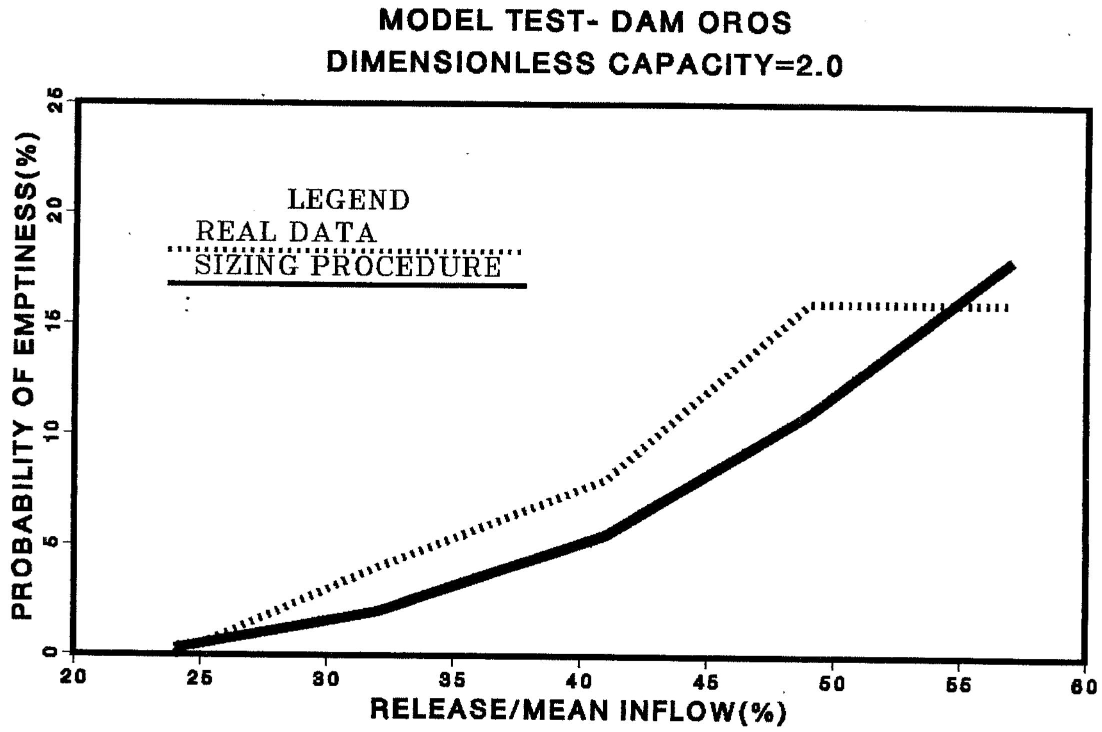

Figure 4.4. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Oros Dam -- Ceara, Brazil -- with $f_K = 2.0$.

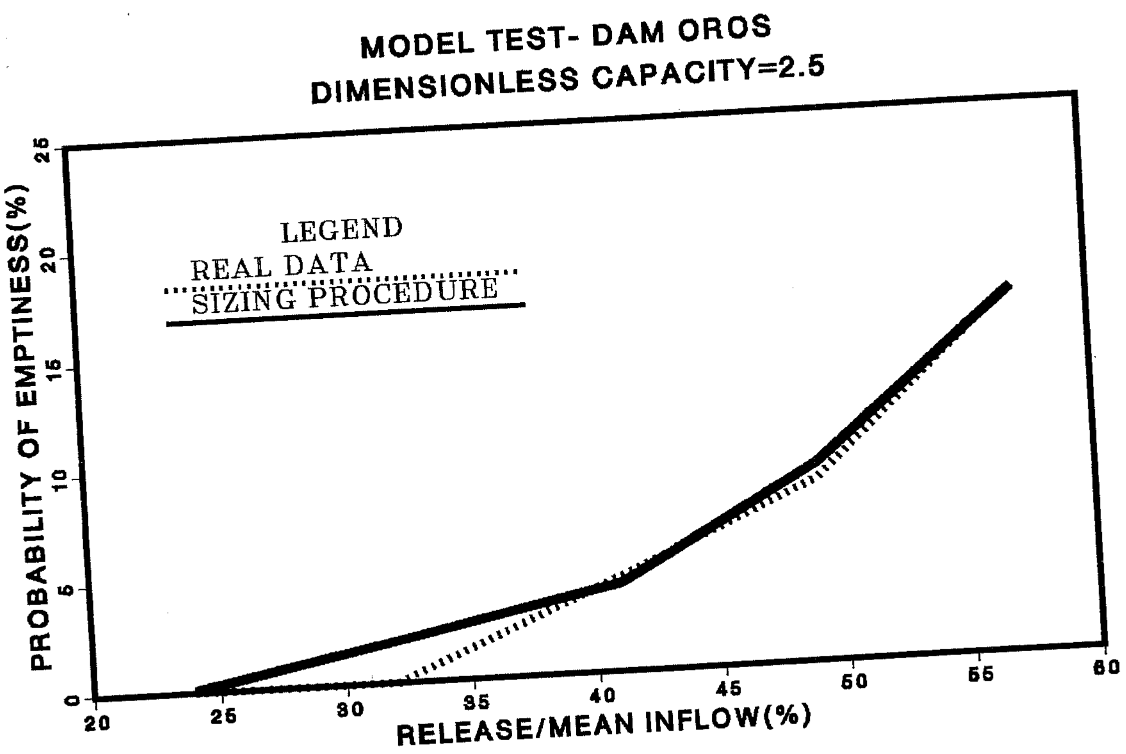

Figure 4.5. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Oros Dam -- Ceara, Brazil -- with $f_K = 2.5$.

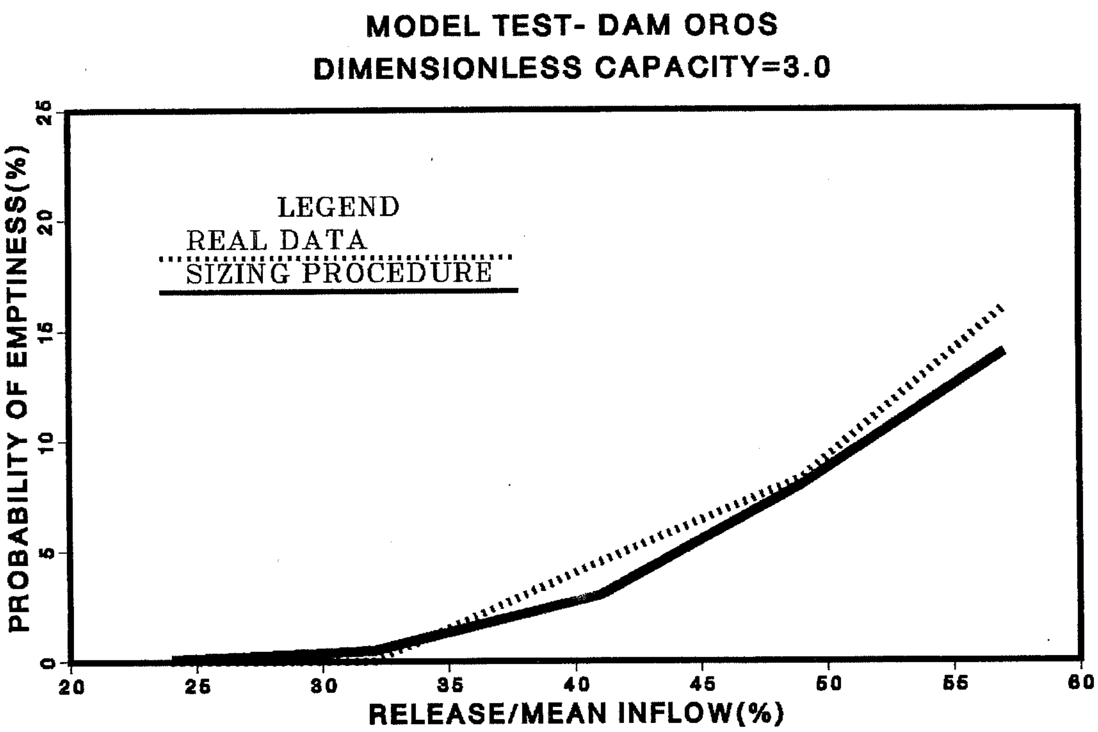

Figure 4.6. Probability of emptiness versus dimensionless release curves from real data and from the sizing procedure for Oros Dam -- Ceara, Brazil -- with $f_K = 3.0$.

CHAPTER V

ADDITIONAL STUDIES ON INITIAL STORAGE AND SEASONAL INTERMITTENCE

This chapter presents some additional studies on the effects of the initial storage and seasonal intermittence on the reservoir sizing process, and it introduces a simple procedure to wash out the effects of initial storage on computations using the Monte Carlo technique. It also introduces a random variable to measure the time during which the reservoir performance depends on the initial storage. None of these are, however, in-depth studies.

In the case of the initial storage, one reason for the studies was to support the use of steady state in computing the reservoir reliability. Another reason was to analyze the relationship between required capacity and reservoir horizon life. In the case of seasonal intermittence the reason was to justify the way the way the sizing procedure divides the year into seasons.

A. Effect of the Initial Storage

The influence of initial storage(S_0) on the reservoir sizing process has been the object of a few studies and as a result one can find in literature some procedures to handle the problem. The most common are:

i) starting with a full reservoir,

ii) starting with a half full reservoir,

iii) making initial storage equal to final storage,

iv) choosing the initial storage by chance, and

v) computing the risk of emptiness for steady state conditions.

Among these procedures, only the last one yields results which, in fact, do not depend on initial storage. The others, usually applied in simulation models or embodied in some analytical methods, can drive an unaware designer to wrong conclusions. This is particularly true for cases when the time span used to size the reservoir is short.

<u>1. Relationship between required storage capacity and reservoir horizon life</u> In the literature sometimes we can find an idea that "the longer the reservoir horizon life the larger the storage capacity required."(Langbein 1958).The support for that affirmation comes from the application of the range theory $E\{R_n\} \approx \sqrt{n}$. Notwithstanding, this is only partially true. For a finite reservoir there are situations where "the longer the horizon life the less the required capacity." The appropriateness of one these of these statements depends, mainly, on the initial storage.

In this section we will prove that the plane with the reliability versus time curves has two regions: an upper region where the first statement holds, and a lower region where the second statement is true. Let us analyze some particular cases.

Case 1. Reservoir starting empty Let us investigate the behavior of two reservoirs of sizes K_1 and K_2 with $K_1>K_2$. Assume both reservoirs yield the same amount of water, M. Under these assumptions we can write:

$$R_{a,t}(S_0=0,K=K_1) \geq R_{a,t}(S_0=0,K=K_2),$$

where,

$R_{a,t}(S_0=0,K=K_i)$ = Reservoir reliability at time t given that the initial storage is zero for the reservoir with capacity K_i.

From the Markovian chain theory we can expect the reservoir reliability to tend asymptotically to the steady state reliability. Then, we have:

$$R_{a,t}(S_0=K,K=K_i) \rightarrow R_a^*(K_i) \quad i=1,2 \qquad [5.2]$$

As $K_1>K_2$,

$$R_a^*(K_1) > R_a^*(K_2)$$

$R_a^*(K_i)$ = reservoir reliability at steady state for storage capacity K_i.

Now, we can sketch the reliability versus time curves. They look like those in Figure 5.1. From that figure it is easy to see that if you want to size a reservoir for a certain reliability and two different time spans, t_1 and t_2 with $t_1<t_2$, you get $K_1>K_2$. In conclusion, when the reservoir starts empty you have: "the longer the reservoir horizon life the smaller the storage capacity required."

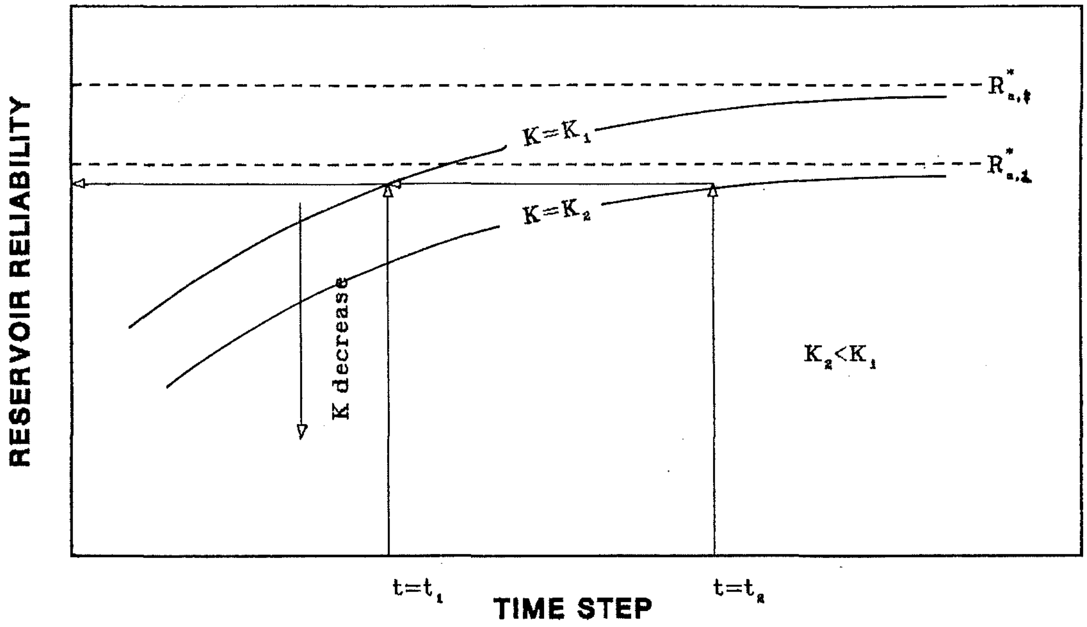

Figure 5.1. Sketch for the reliability versus time curve, for several storage capacities, when the reservoir starts empty.

To check this statement we generated 2000 traces of gamma-II independent annual inflows. Using these traces we performed the reservoir simulation for the following cases:

μ =10.0 C_V=1.0 M=10.0 S_0=0. K=10,15,20,30,40,50,∞;

μ=10.0 C_V=0.5 M=10.0 S_0=0 K=10,15,20,30,40,50,∞;

μ =10.0 C_V=1.0 M=8.0 S_0=0. K=10,15,20,30,40,50,∞.

The results, shown in figures 5.10-5.12, hold with our assertion. For example, for μ = 10.0, Cv = 1.0, M = 10.0 and R_a = 75%, for a time horizon of 29 years, the required capacity is 50 units, while for 40 years the capacity is 40 units.

<u>Case 2. Reservoir starting full</u> Using the same reasoning, for $K_1 > K_2$ we have:

$R_{a,t}(S_0=K1,K=K_1) \geq R_{a,t}(S_0=K_2,K=K_2)$

$R_{a,t}(S_0=K,K=K_i) \rightarrow R_a^*$ for large t and i=1,2

$R_{a,t1}(S_0=K_i,K=K_i) \leq R_{a,t2}(S_0=K_i,K=K_i) \quad t_1<t_2;\ i=1,2.$

In this case the reliability versus time curves look like those in Figure 5.2. Now the conclusion is: "the longer the horizon life the larger the required capacity."

<u>Case 3. Reservoir starting with a given storage</u> Now we know the relationship between the required storage capacity and the reservoir horizon life for the cases where the reservoir starts full and when it starts empty. We also know that the reservoir behavior in one case is the opposite of the behavior in the other. The logical question is: " what happens in between?"

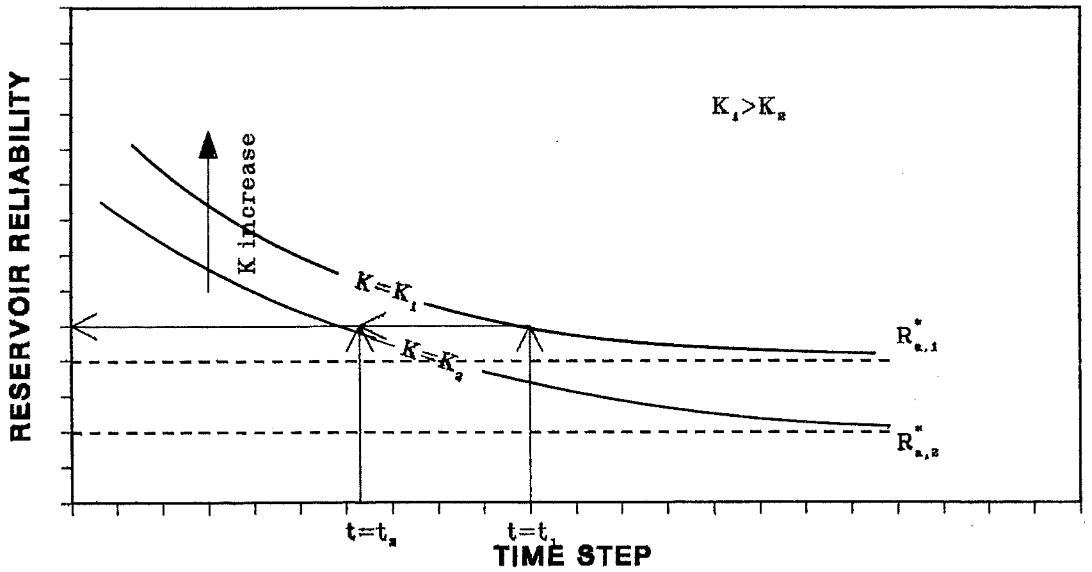

Figure 5.2. Sketch for the reliability versus time curve, for several storage capacities, when the reservoir starts full.

Then, let us try to answer this question using some experimentation and logical thinking. Let us first analyze the case where the reservoir has a fixed capacity starting at different initial storage levels. Using 2000 traces of inflows we simulated the reservoir behavior for the following values:

$C_V = 0.5 \quad \mu = 10.0 \quad M = 10.0 \quad K = 40.0$

$C_V = 1.0 \quad \mu = 10.0 \quad M = 10.0 \quad K = 40.0$

$C_V = 2.5 \quad \mu = 10.0 \quad M = 10.0 \quad K = 40.0$

$C_V = 0.5 \quad \mu = 10.0 \quad M = 10.0 \quad K = 20.0$

$C_V = 1.0 \quad \mu = 10.0 \quad M = 10.0 \quad K = 20.0$

For K = 40.0 we assumed initial storage :

0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 12.0, 15.0, 20.0 ,40.0

For K = 20.0 we assumed:

0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 12.0, 14.0, 16.0, 20.0

The results from this simulation are in figures 5.13-5.18. Looking at those figures you can observe the following: for certain values of S_0, the reliability decreases until it reaches a minimum. After this point, it starts increasing toward the steady state reliability. You can also notice that when S_0 increases, the time that the reservoir reliability takes to reach its minimum also increases. Figures 5.3 shows a sketch where you can see this behavior in amplified form.

An explanation for that behavior comes from the theory of the Markovian chain at continuous time. Assume the simultaneous model; it means that the inflow and outflow occur at the same time. In this case if $S_0 = 0$ the reliability at time zero is zero. On the other side, for any S_0 greater than zero the instantaneous reliability is one[1]. This is true because the reservoir has water to

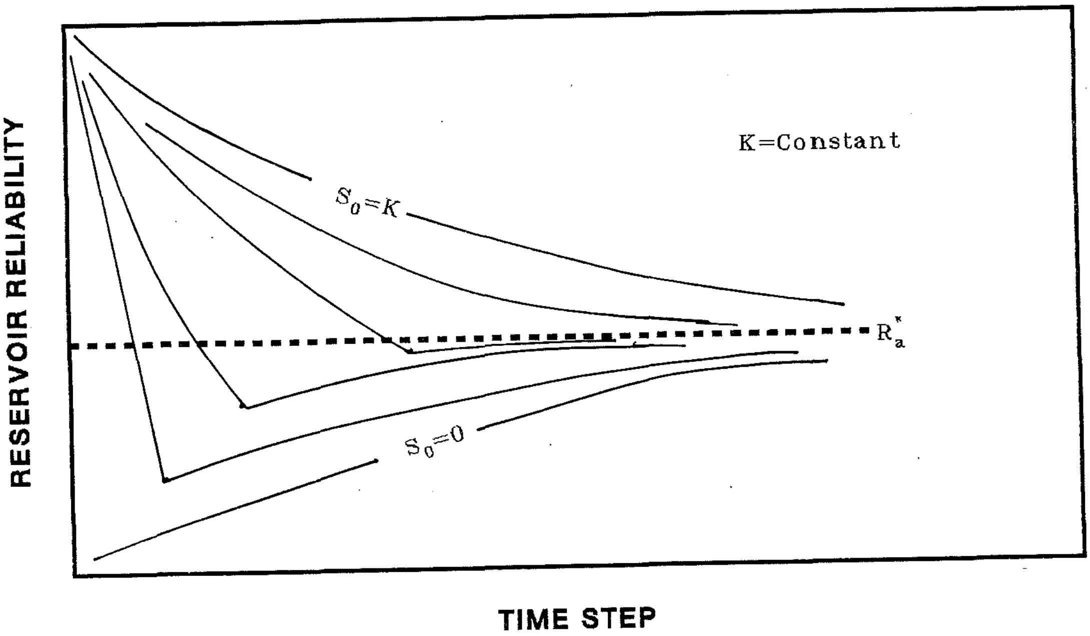

Figure 5.3 Sketch for the reliability versus time curves for a reservoir with capacity K starting at different storage.

supply the demand during a given time interval. Now assume the reservoir starting at an infinitesimal initial storage, $S_0 = e$. We already know that, in this case, the reliability at time zero is 100%. Nevertheless, "what about this reliability a finite time-interval later?" In other words, what is the difference in reliability, at time "1," between one reservoir which had at time "0," "e" units of water and

[1] An analytical prove for a similar case is in Clarke and Disney(1970).

another that was empty. Of course this difference must also be infinitesimal.In summary, for an initial storage S_0 = "e," the reliability starts with its maximum value, then, after a given time interval dT, it jumps to a value slightly less than the reliability for the case where the reservoir started empty. After this point it should go toward the steady state reliability in the same way that the curve for $S_0 = 0$ does.

Now let us analyze the behavior of reservoirs of different sizes all starting with the same volume. We simulated the following cases:

CV = 0.5 $\mu = 10.0$ M = 10.0 $S_0 = 5.0$

CV = 1.0 $\mu = 10.0$ M = 10.0 $S_0 = 5.0$

CV = 2.5 $\mu = 10.0$ M = 10.0 $S_0 = 5.0$

In all cases we made K = 10.0, 15.0, 20.0, 30.0, 40.0, 50.0 and infinite. The results are in figures 5.19 to 5.21.

Let us analyze, for example, some results from the curve for $C_V = 0.5$, $S_0 = 5$ (Figure 5.19).

For a reliability of 84%:

when the horizon life N = 29 years K = 50;

N = 40 years K = 40.

While for a reliability of 65%:

when the horizon life N = 1 year K = 10;

N = 10 years K = 15.

In brief, in the case of a reservoir starting with a given initial volume, we can expect two regions in the reliability versus time plane: an upper region, let us name it Zone I, where "the longer the reservoir horizon life the smaller the required capacity;" and a lower zone, let us call it Zone II, where " the longer the horizon life the larger the required capacity." Figure 5.4 shows a sketch where the contour of such a line is accentuated for a better view.

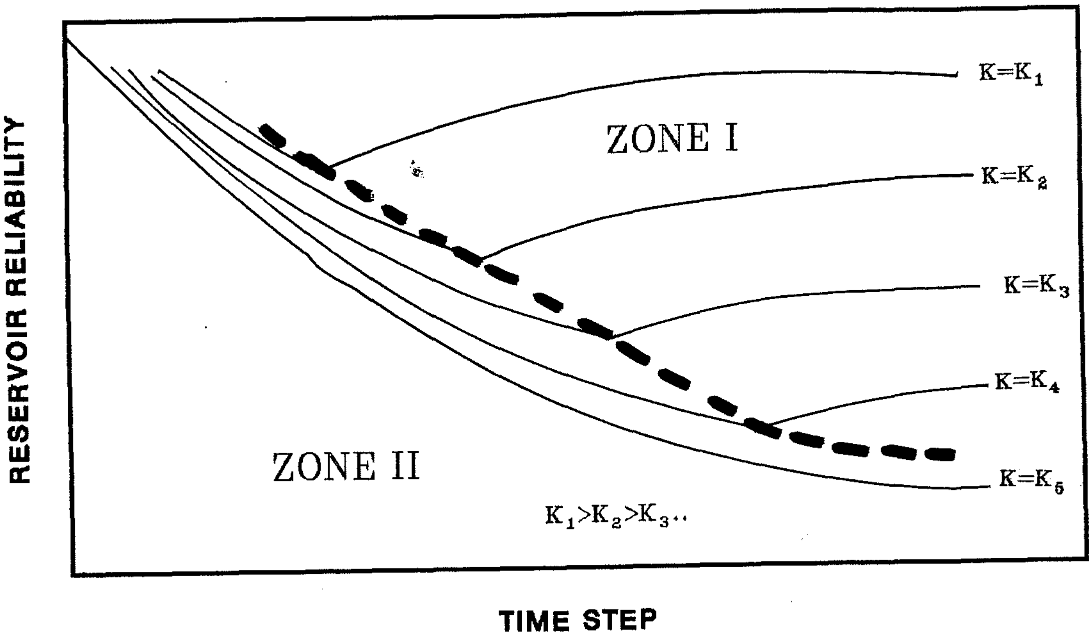

Figure 5.4 Sketch for the reliability versus time curves for reservoirs with different capacities all starting with the same volume.

In brief, we can expect the following behavior: when the reservoir starts empty, the plane has only Zone I; for low initial storage values the zone I must have a larger area than zone II; for larger initial storage values, zone II must have a larger area than zone I; finally, for fullness as the initial storage values the plane has only zone II.

The following two observations look important at this point.

a) There is no symmetry -- as assumed by Jeng (1967)-- in the way the reliability versus time curves approach the steady state. Jeng assumed that the curve for $S_0 = cK$ is symmetrical to the curve $S_0 = (1-c)K$, with c in the interval 0.-0.50.

b) Another point is with regard to the question: "What is the S_0 value that makes the reliability versus time curve more quickly approach the steady state reliability?" The answer is: " It depends on the time horizon." Let us explain with an example. Assume that you know the value of R_a^*. Thus, for a time t equal one, you can find an initial storage that yields the reliability for that time that is equal to R_a^*, solving the equation:

$$P_r\{Z_1=0/S_0=S_0^1\} = R_a^*$$

For a time t = 2 the equation is:

$$P_r\{Z_2=0/S_0=S_0^2\} = R_a^*$$

If you solve this two equations you get:

$$S_0^1 \neq S_0^2$$

This comes from the reliability versus time curves of Figure 5.3.

<u>2. Evaluation of the time of influence of the initial storage</u> Let us introduce the concept of time of influence of initial storage(T_S). Assume that you are studying a reservoir for a capacity K and a yield M, and that you have

a number of inflow traces for the site. Then, to compute T_S, proceed as follows.

a) With the reservoir starting full, compute the first passage time to empty(F_{C0}). This time represents a convergence point because any other route through the reservoir with an initial storage S_0<full will reach the empty condition at the same time, as in Figure 5.5.

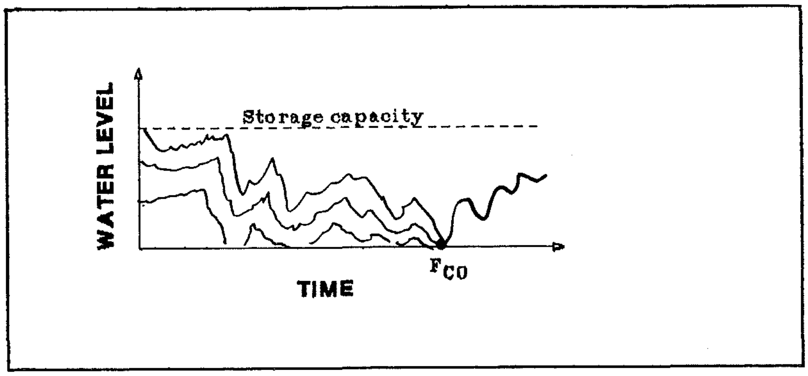

Figure 5.5. Schematic representation for the first passage time full-empty.

b) With the reservoir starting empty, compute the first passage time to full(F_{0C}). In the same way, this time represents a convergence point because any route through the reservoir with an initial storage S_0 > empty will reach the full condition at the same point, as in Figure 5.6.

c) Compute the time of influence of initial storage as $T_S = \min[F_{0C}, F_{C0}]$. As both times F_{C0} and F_{0C} represent convergence points for any initial storage, the reservoir behavior beyond the time T_S does not depend on the starting storage.

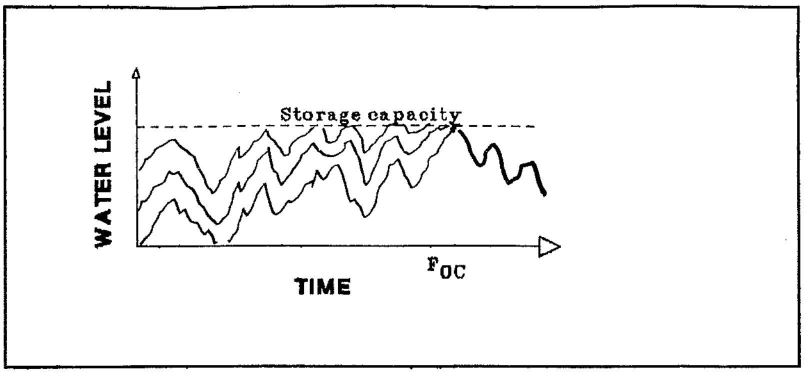

Figure 5.6. Schematic representation of the first passage time empty-full.

d) Repeat steps a-c for all other traces. Each trace must yield, likely, a different value for T_S. Thus, you can compute its mean and variance.

It is important to observe that if you get a T_S value close to the reservoir horizon life, you can draw two conclusions. First, that the storage capacity is likely oversized -- see figure 5.7, -- and second, that the value of the initial storage is important in the determination of the reservoir reliability.

<u>3. Simulation model for computing the steady state reliability</u> The most common procedure to perform a stochastic simulation is:

a) generate N_T traces of inflow, each one with duration equal to the reservoir horizon life N_L.;

b) choose an initial storage S_0 = s0 and route the N_T traces through the reservoir for several values of yield and storage capacity;

c) Using the results from route compute

$$E\{PE\ S_0=s_0,\ Life=N_L\},$$

$$Var\{PE\ S_0=s_0,\ Life=N_L\}.$$

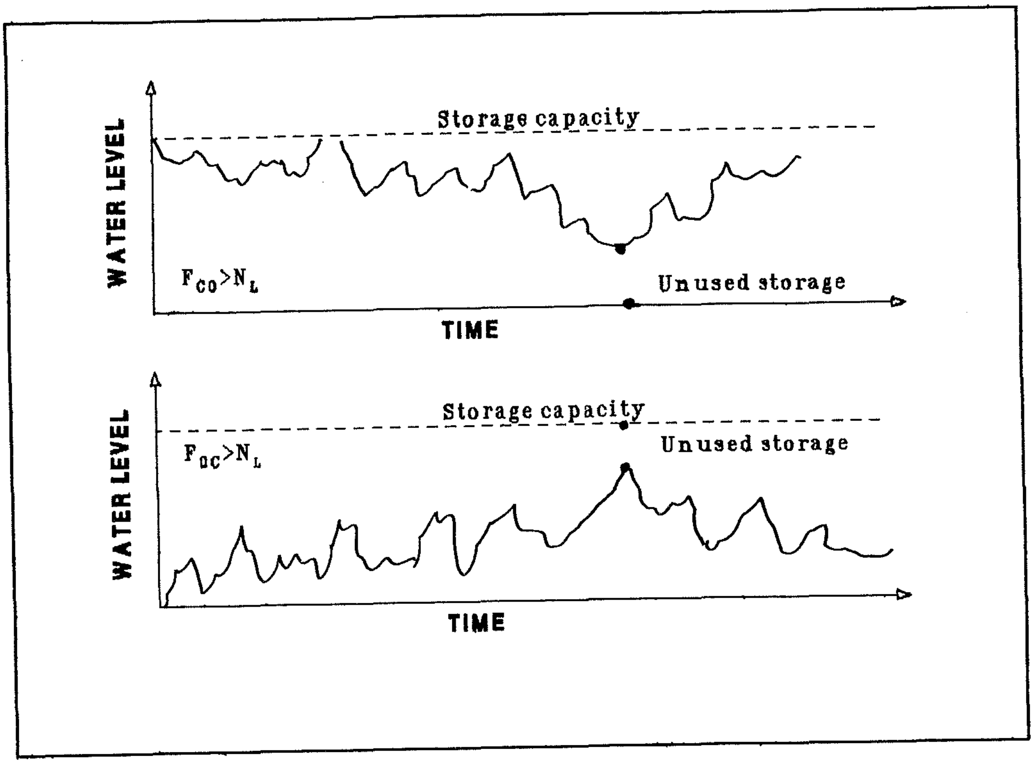

Figure 5.7. Representation of the time of influence of initial storage when the reservoir is oversized.

It is easy to see in this approach the results depend on the initial storage. To investigate the reservoir behavior at equilibrium state, we modified that approach as follows:

a) generate one trace with duration equal to $(N_T+1)N_L$;

b) perform the simulation, as described in B.1, to find the time of influence of the initial storage; (It usually gives a $T_S<N_L$, otherwise the total duration of the trace must be increased correspondingly.)

c) from the first convergence point determined in previous step, perform the simulation for the remaining years;

d) divide the total trace in N_T sub-traces, each one of length N_L. Then, starting at the first sub-trace following T_S, determine, for each of these sub-traces, the number of failures, f_i, or other parameter to measure the reservoir performance.

The difference between the first and second approaches is the starting value of the storage. In the first approach, the reservoir always starts with the same volume, so in each route the effect of initial storage is re-introduced. On the other hand, the second approach deals with the initial storage as if it were a random variable being drawn from its real probability distribution function.

B. Intermittence of Seasonal Inflows

The assumption of a two-season process with inflow concentrated in the wet season and outflow concentrated in the dry season transforms the real inflow and outflow hydrograph as in Figure 5.8. In other words, the assumption is based on transferring small amounts of input that occur in the dry season to the wet season, and a small demand of the wet season to the dry season.

Then, let us analyze how that simplification affects the sizing process.

-Transferring inflow from the dry season to the wet season has the effect of increasing the probability of spill, thus increasing the mean amount of spill losses and, as a result, decreasing the availability of water for controlled release.

-Transferring demand from the wet season to the dry season also has the effect of increasing the amount of spill losses and thus decreasing the controlled release.

In summary, that simplification results in under-valuing the release.

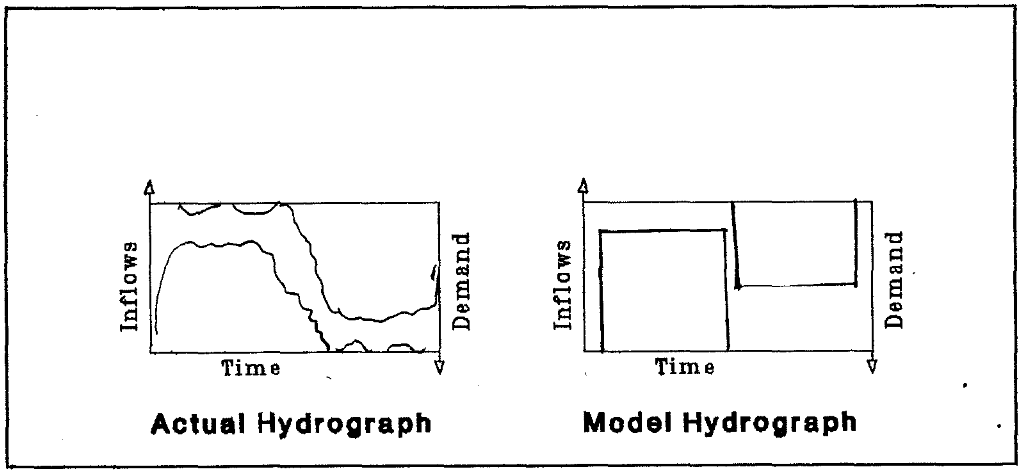

Figure 5.8. Real and transformed hydrographs.

Another point that is worth analysis here is the time step used in the sizing process. When one changes the time step, for example, from one day to one season, all trade-off occurs inside the season. That is, the net input in each season is preserved. Then, let us analyze how this trade-off affects the required reservoir capacity.

1) Assume that during a given season the net input is always positive. Under that hypothesis let us analyze the difference between using a daily or a seasonal step.

a) Assume, first, that computing using a daily basis the reservoir spills at a given time. It means that, for this period, the net input is larger than the available initial storage. As a result, in the seasonal computation, the reservoir will finish up the season in a full condition. That is, in both computations the reservoir storage at the end of the season is the same, and the gain in using a daily step is only the knowledge about the day that the reservoir started spilling. If one is not interested in that information there is no reason to use the daily step.

b) Assume that the reservoir does not spill during the period. Hence, in both approaches the final storage is equal to the starting storage plus the net input. Then, if one is not interested in knowing exactly how the water level fluctuated inside the season there is no reason to use the daily step.

2) Assume that during a given season the net input is always negative.

a) Assuming, first, that no failure occurs from the daily computation, then the final storage is equal to the initial storage plus the net input. In this case the conclusions are the same as in case 1b.

b) Assume that a failure occurs at a given time from the daily balance. It means that for this period the initial storage is not enough to supply the excess of demand, and as a result, the final storage in both approaches is equal to zero. Thus, the gain in information in using the daily step is only the knowledge about the day that the reservoir became empty.If this information is not important there is no reason to use the daily step.

3)Assume that during the season the net input is sometimes positive and sometimes negative. Let us analyze two situations:

a) A reservoir failure occurs in the seasonal computation. It means that the available storage at the beginning of the period is not enough to supply the excess of demand. A failure also occurs in the daily computation. Hence, as in case 2b, the gain in using daily step is the knowledge about the days where the failure(s) happened.

b) No failure occurs when the balance is performed using a seasonal step. It means that the net input for the whole season is positive. But, situations like those, shown in Figure 5.9, with sub-periods with of negative net input, can occur. From that figure it is easy to conclude that, in such a situation, the daily balance can detect failures that do not appear in the seasonal computation. In this case, and only in this case, lies the bias introduced in the sizing process by assuming a larger time step.

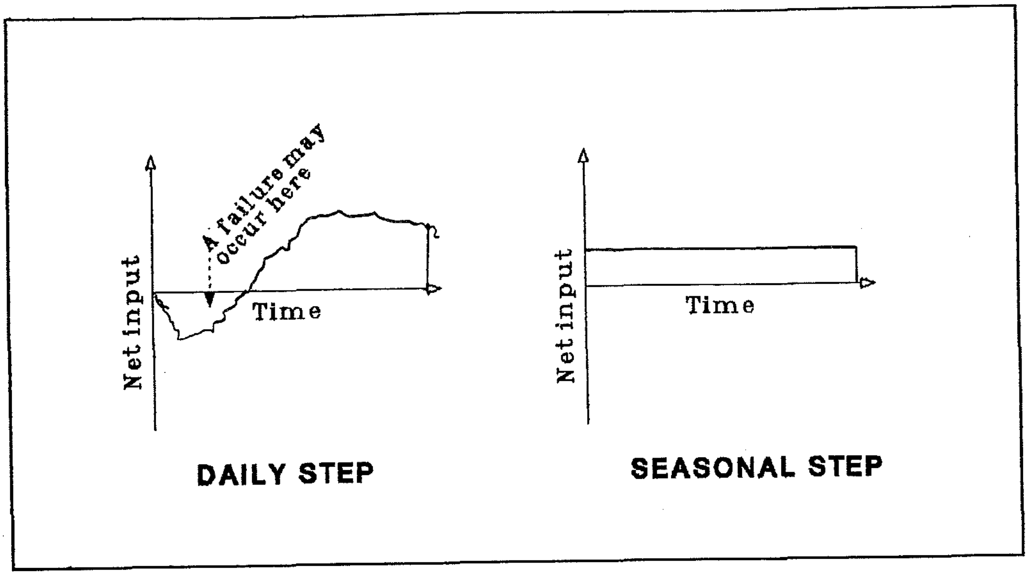

Figure 5.9. Schematic representation of a case where a failure can be detected in a daily base and not in a seasonal base.

Now let us return to our sizing procedure to see the implications of this reasoning. There are two ways of dividing the year into two seasons:

The first, is to assume all annual inflow in the wet season and all outflow in the dry season. This procedure, used by Moran's followers, is the one we used in testing the model. Nevertheless, we concluded that it tends to underestimate the release.

The second, is dividing the year in two periods of fixed duration. The first period, with net input positive, is named surplus period, and the second period, with negative net input negative, is named deficit period. Then, compute the mean and coefficient of variation of the net inputs for the surplus and deficit periods. The surplus period can be used to compute the input matrix, while the deficit period can be used to compute the release matrix. This procedure tends to overestimate the release for a given probability of failure.

Now, let us return to the situation of the study area. Of course, it is impossible, in a probabilistic sense, to find a time period where the input surpass the demand 100 percent of the time. But, in a practical sense, it is possible to find time periods where, let us say, 90 percent of the time the net input is positive or negative. In such a case the error introduced is not significant. This situation, in fact, occurs in the study area, which means that a two-season approach is not a bad assumption.

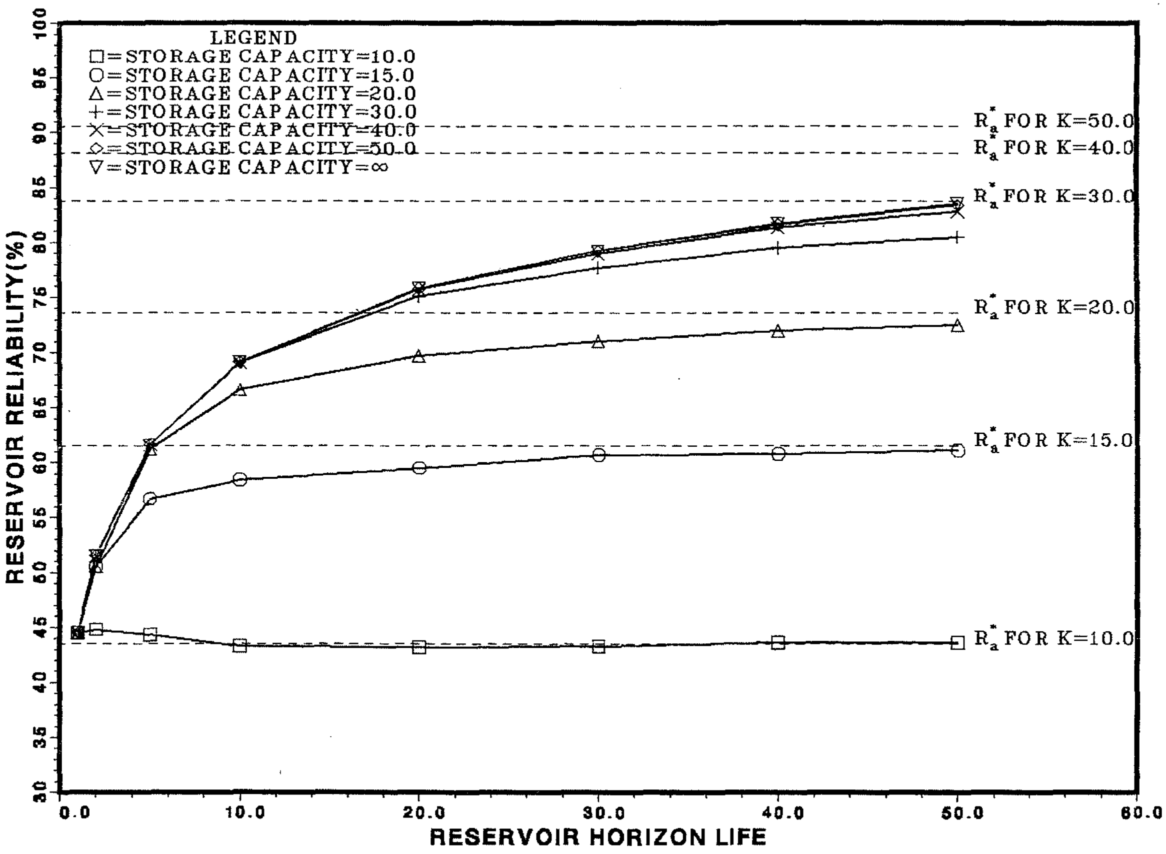

Figure 5.10. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 0.5, initial storage = 0 and capacities 10, 15, 20, 30, 40, 50 and infinite.

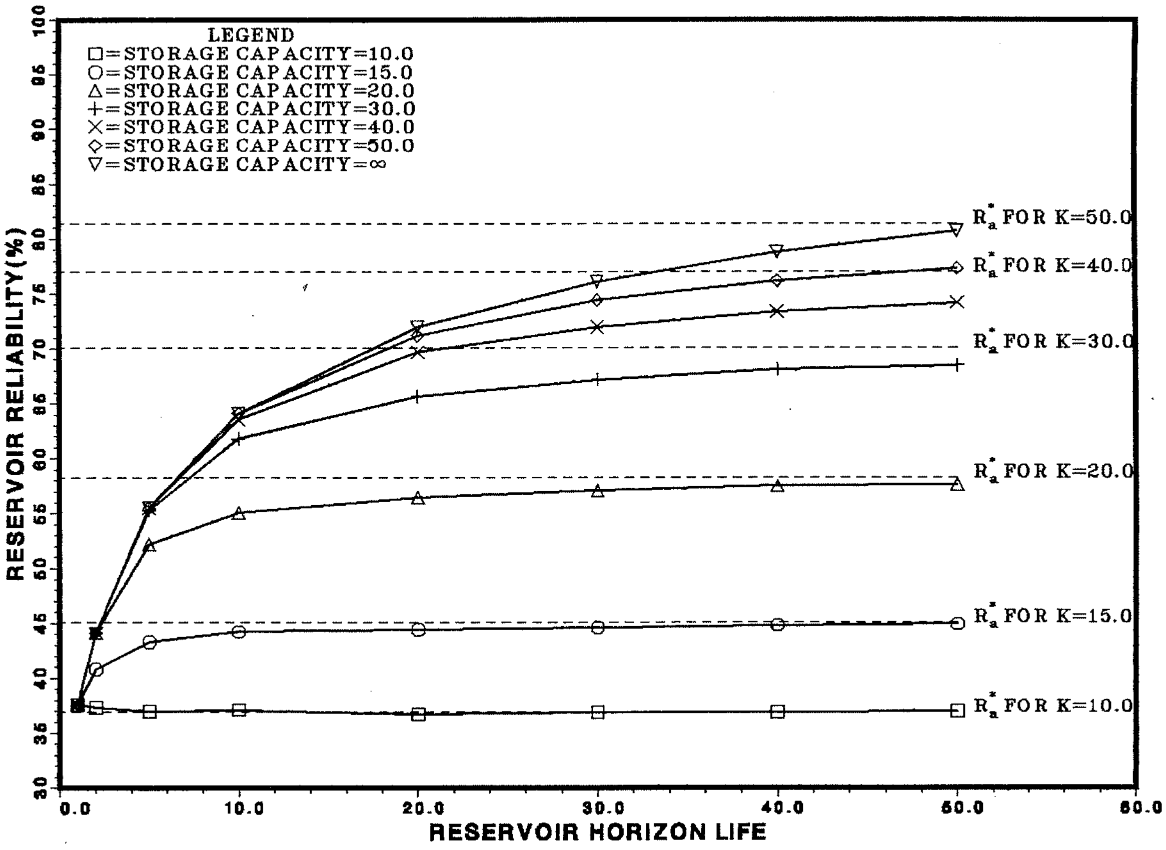

Figure 5.11. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 1.0, initial storage = 0 and capacities 10, 15, 20, 30, 40, 50 and infinite.

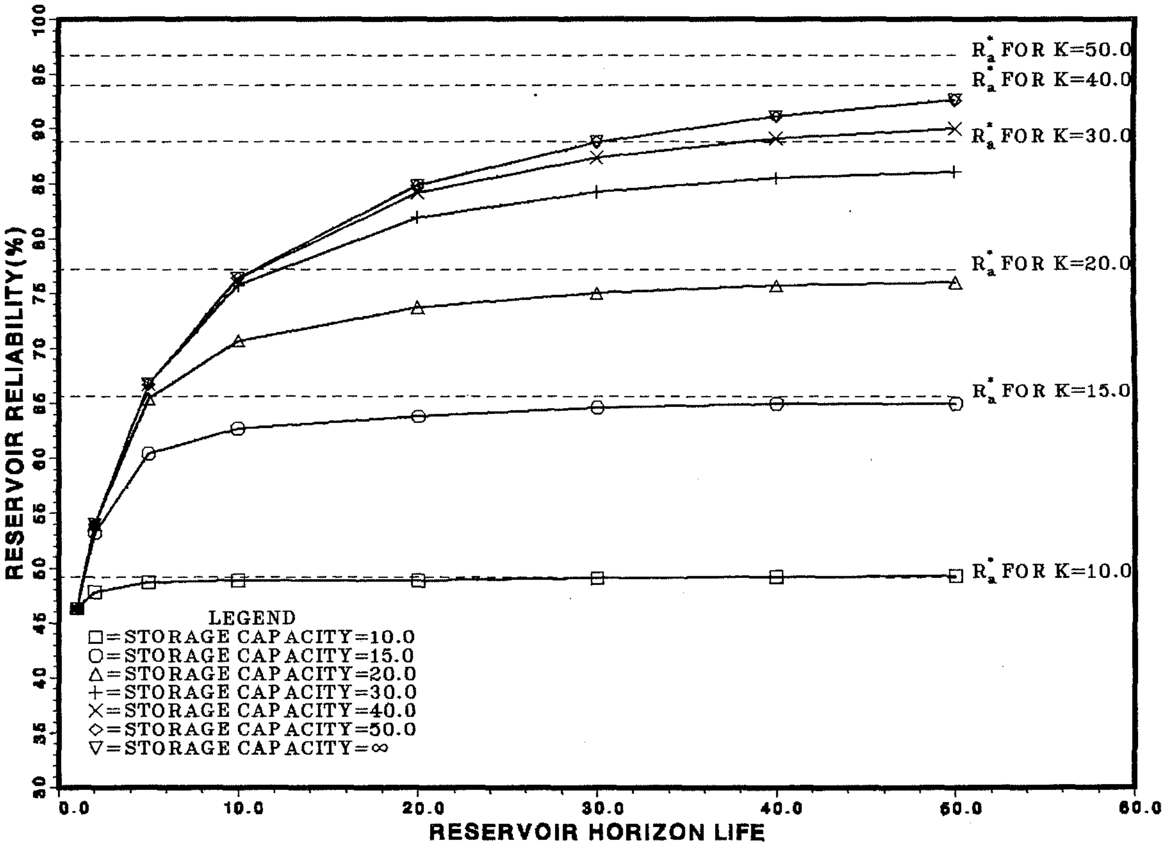

Figure 5.12. Reliability versus time curves for a reservoir with mean inflow = 10, release = 8, coefficient of variation = 1.0, initial storage = 0 and capacities 10, 15, 20, 30, 40, 50 and infinite.

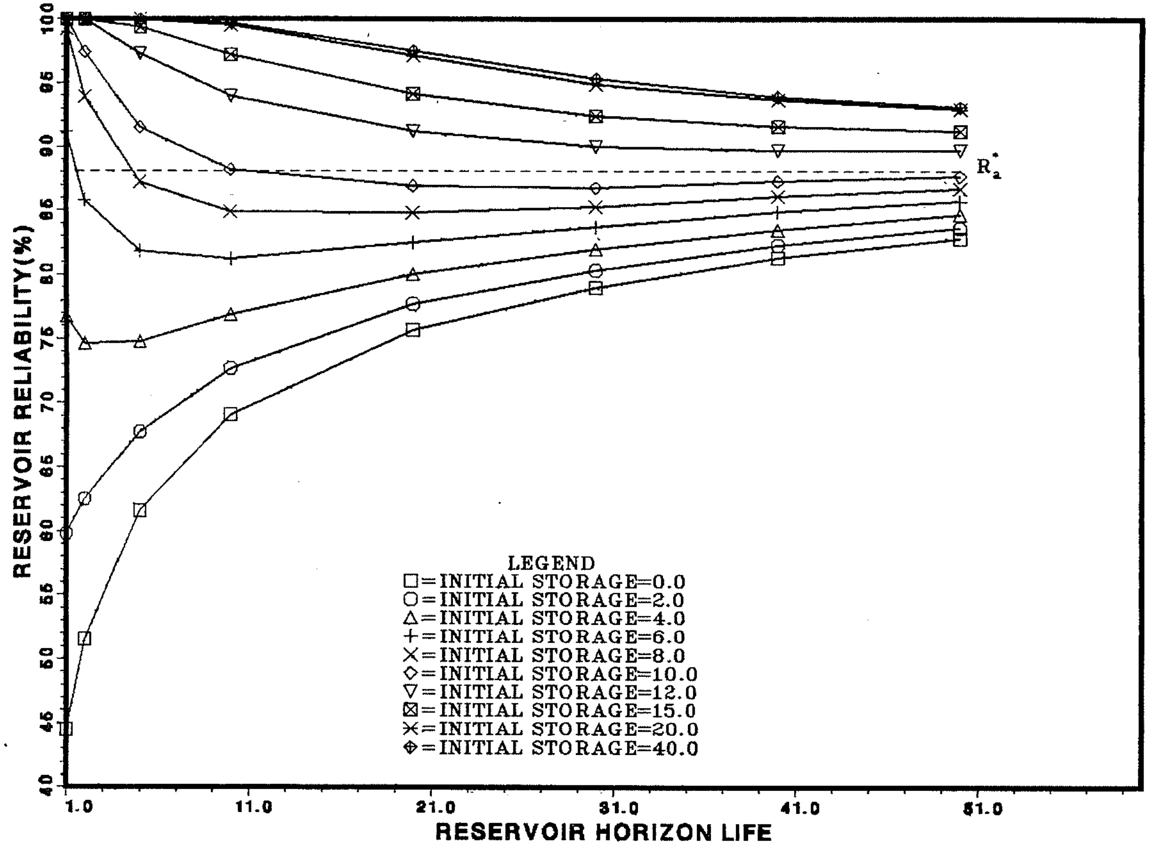

Figure 5.13. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 0.5, reservoir capacity = 40, and initial storage 0, 2, 4, 6, 8, 10, 12, 15, 20 and 40.

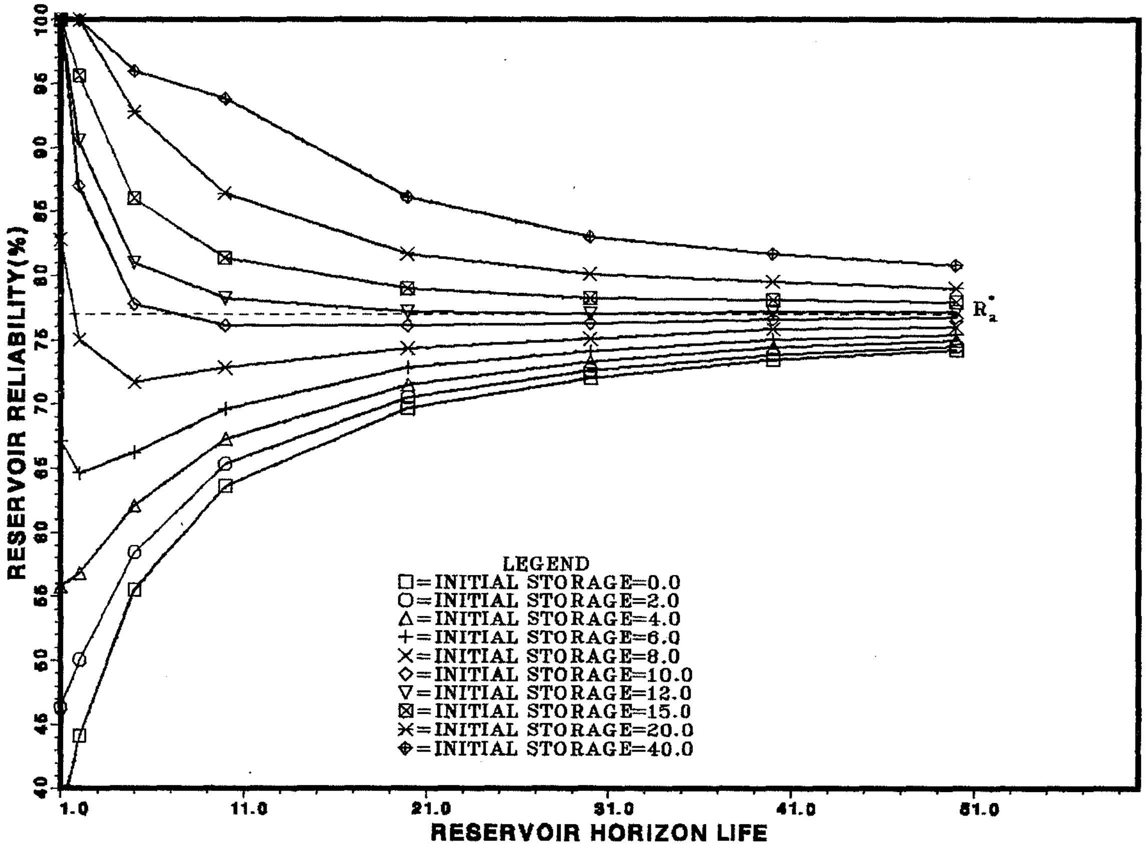

Figure 5.14. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 1.0, reservoir capacity = 40, and initial storage 0, 2, 4, 6, 8, 10, 12, 15, 20 and 40.

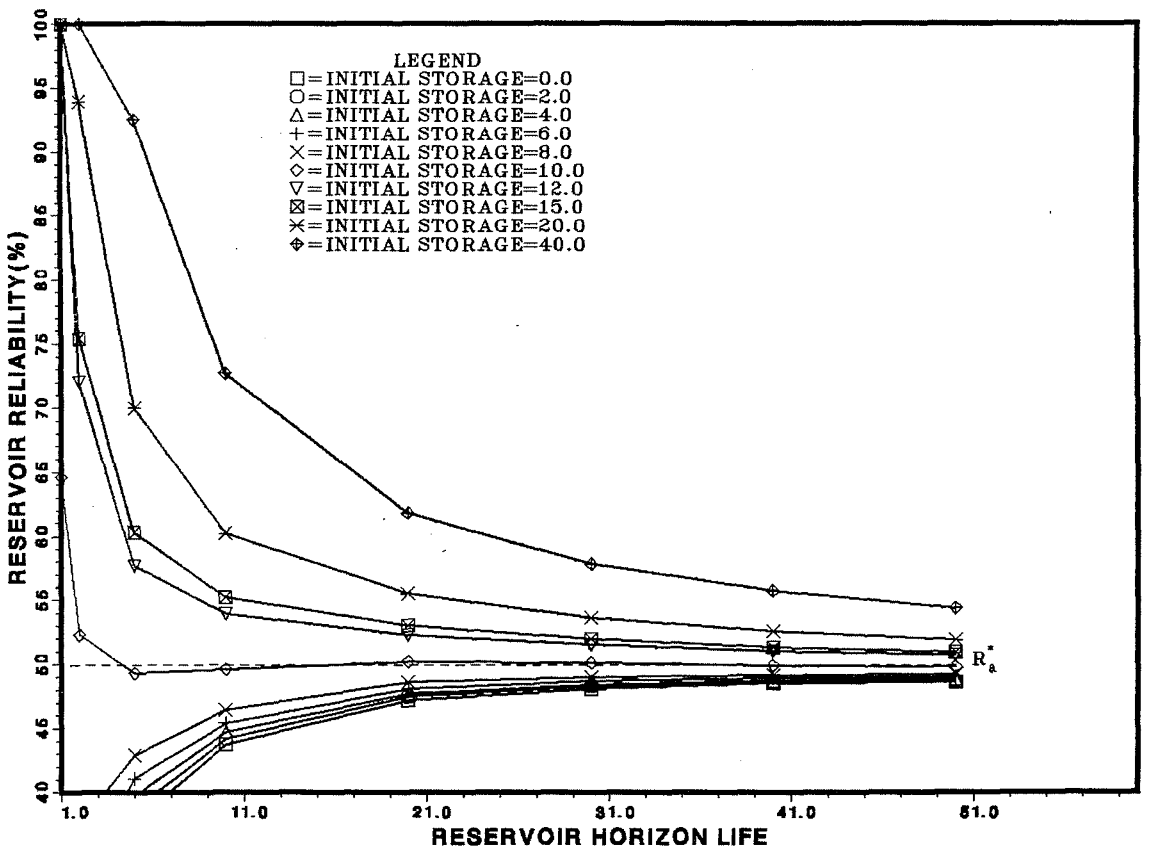

Figure 5.15. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 2.5, reservoir capacity = 40, and initial storage 0, 2, 4, 6, 8, 10, 12, 15, 20 and 40.

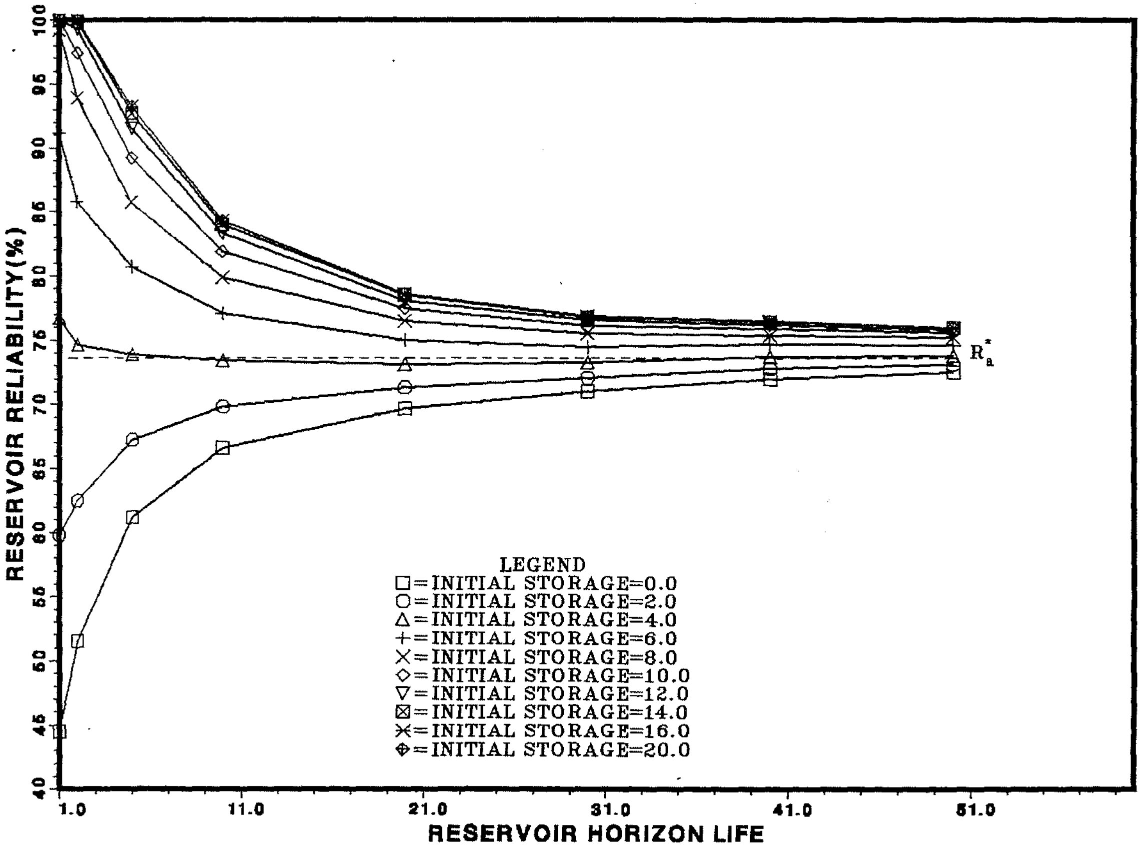

Figure 5.16. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 0.5, reservoir capacity = 20, and initial storage 0, 2, 4, 6, 8, 10, 12, 14, 16 and 20.

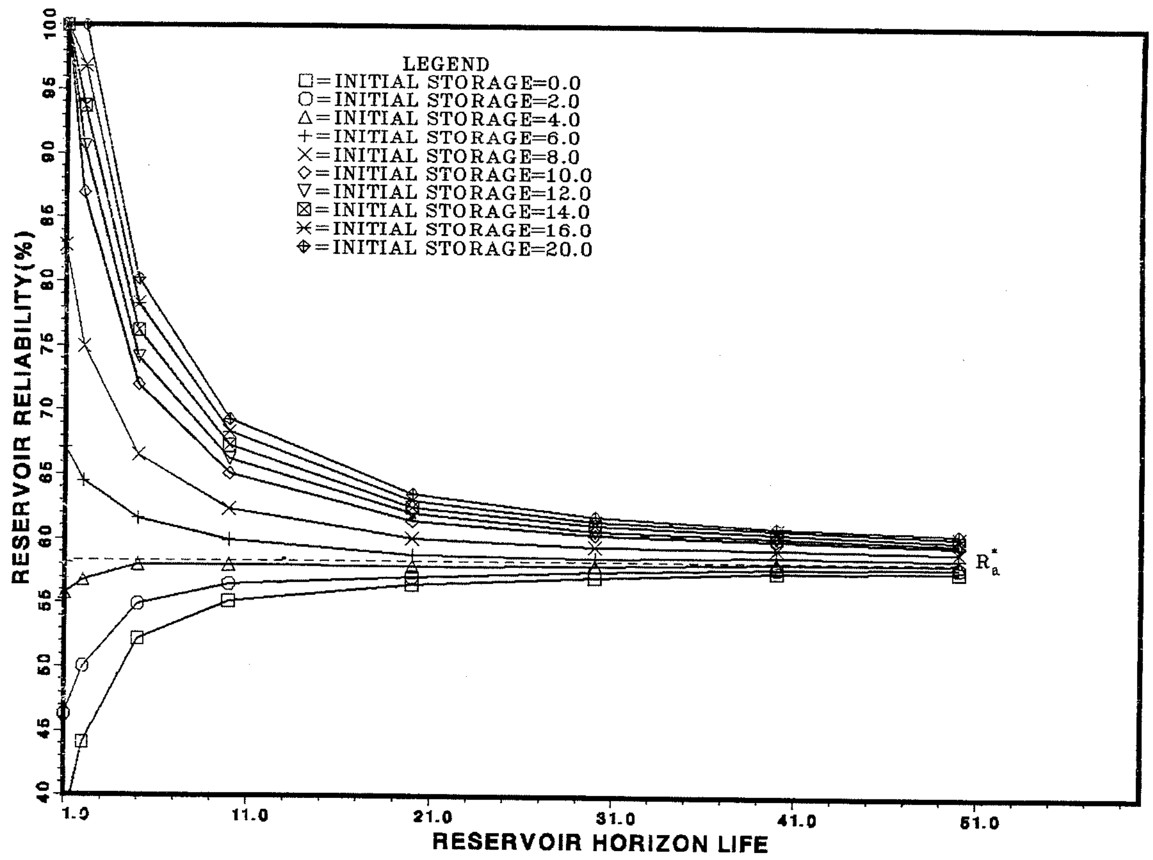

Figure 5.17. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 1.0, reservoir capacity = 20, and initial storage 0, 2, 4, 6, 8, 10, 12, 14, 16 and 20.

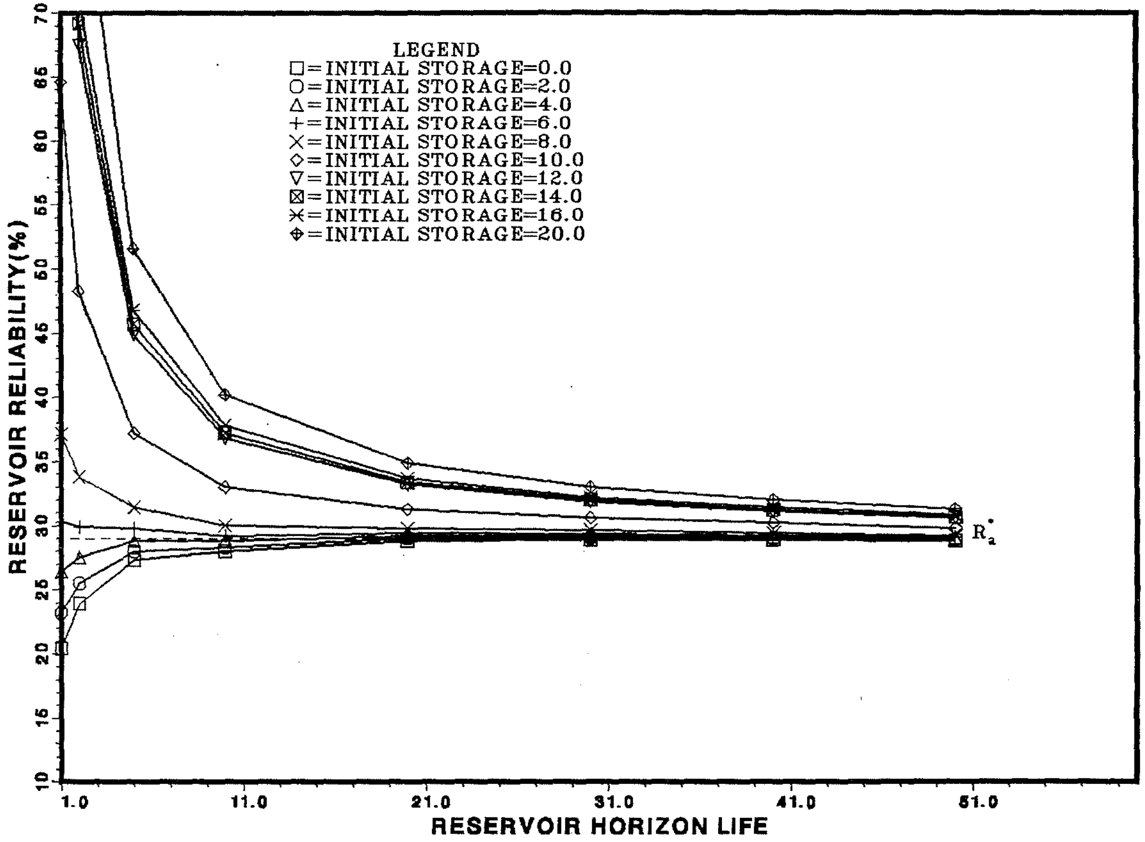

Figure 5.18. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 2.5, reservoir capacity = 20, and initial storage 0, 2, 4, 6, 8, 10, 12, 14, 16 and 20.

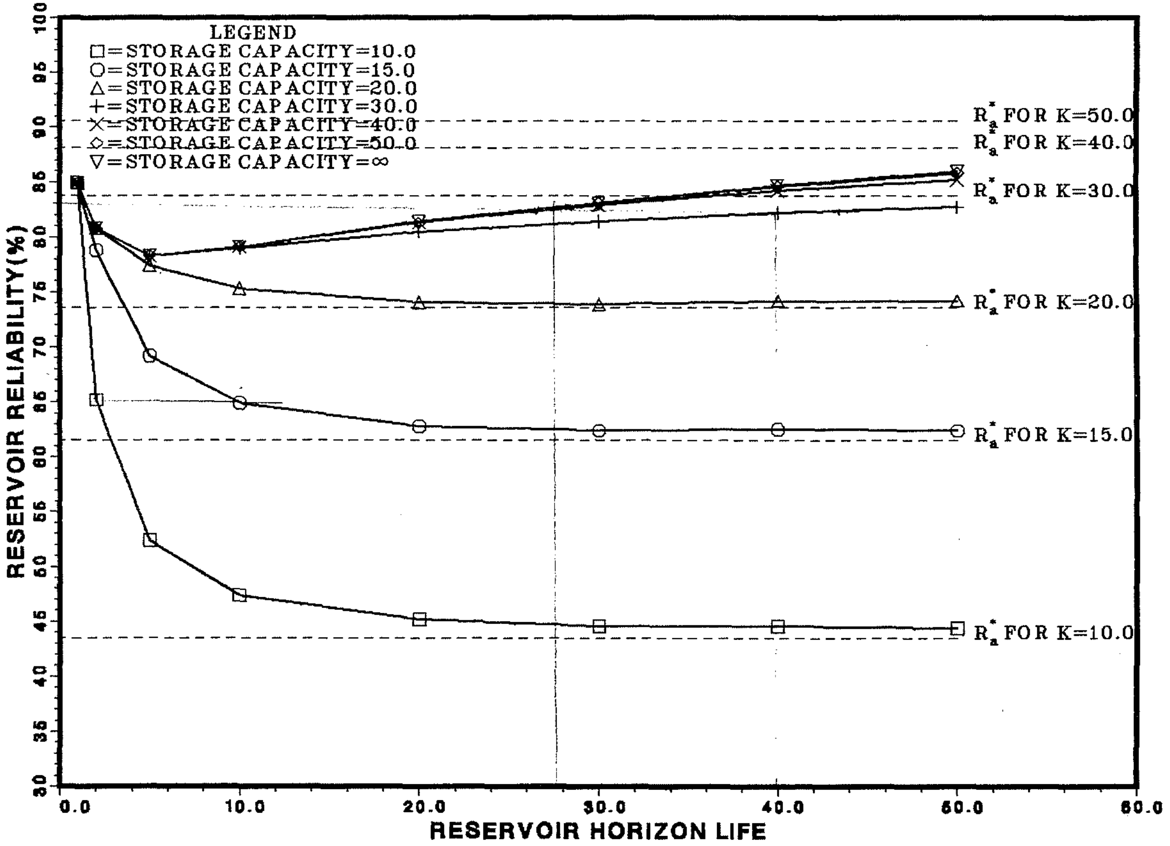

Figure 5.19. Reliability versus curves time for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 0.5, initial storage = 5 and capacities 10, 15, 20, 30, 40, 50 and infinite.

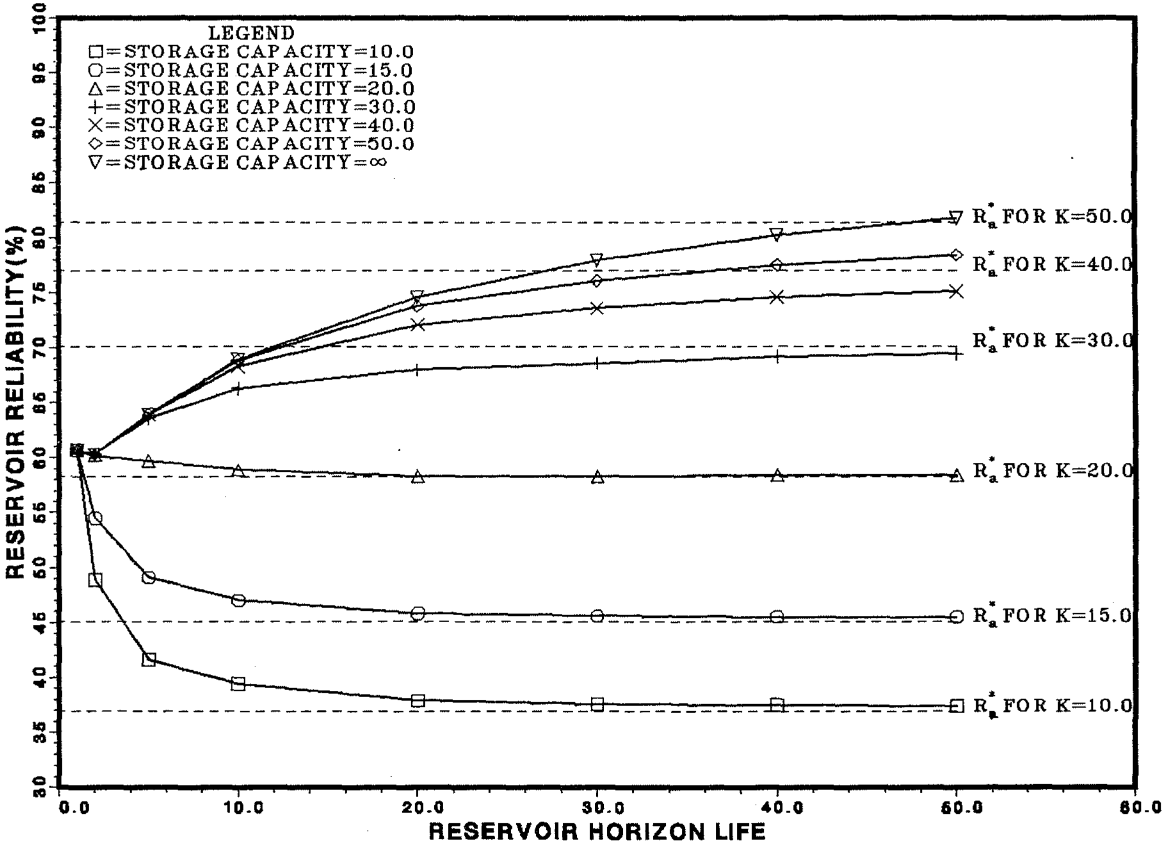

Figure 5.20. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 1.0, initial storage = 5 and capacities 10, 15, 20, 30, 40, 50 and infinite.

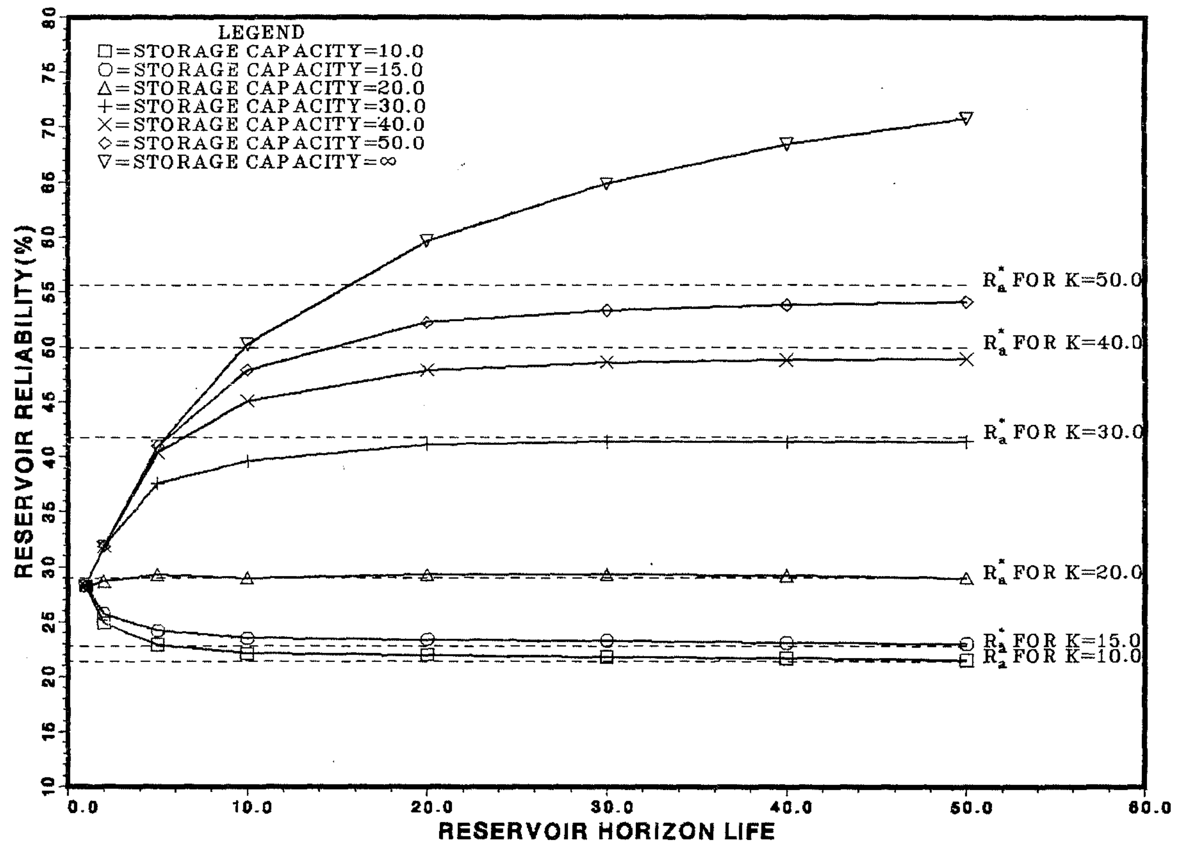

Figure 5.21. Reliability versus time curves for a reservoir with mean inflow = 10, release = 10, coefficient of variation = 2.5, initial storage = 5 and capacities 10, 15, 20, 30, 40, 50 and infinite.

CHAPTER VI

CONCLUSIONS AND FURTHER STUDIES REQUIRED

The development of a procedure for sizing reservoirs in semi-arid/arid areas on intermittent rivers was the main purpose of this dissertation. Related to that subject, the research also investigated how evaporation, annual and seasonal intermittence, and initial storage capacity affect the required storage capacity.

A. Conclusions

The main conclusions drawn from the research are the following.

1\) The sizing procedure seems to have an acceptable accuracy and to be appropriate for application in regions where: a) rivers are intermittent; b) the probability distribution function of inflows is mixed with a probability mass for inflow equal to zero; c) the evaporation losses are important, and d) the annual inflows have no, or very low, serial correlation.

2\) The transformation of the mixed probability function of inflows in a probability density function introduces a bias in the evaluation of the required capacity. This bias undersizes the reservoir capacity. The bias is greater for designs with high reliability and for higher values of the probability mass at inflow zero.

3) When generating synthetic traces of monthly inflows for intermittent rivers, it is important to preserve the probability mass for inflow equal to zero. A suggestion for how to achieve that is presented.

4) Introducing a correction factor on the reservoir capacity to take into account the evaporation losses can overestimate the controllable release. This is particularly true for high regulations and for a high evaporation rate.

5)A better way to evaluate the effect of evaporation on reservoir performance is to use the dimensionless evaporation factor $f_E=(3\,\alpha^{1/3}E_V)/(\,\mu^{1/3})$. It means that the evaporation losses are directly proportional to the evaporation rate on the site, proportional to the factor of shape of the lake to one-third power, and inversely proportional to the mean inflow to one-third power.

6) For regions where the evaporation rate is high, it is better to store water in rivers with high discharges whenever a site with a low factor of shape of the lake is available.

7) The belief that the "longer the reservoir horizon life the larger the required reservoir capacity" is only partially true. This belief likely comes from applying the range theory in the reservoir sizing process. For a finite reservoir the process works as follows: when the reservoir is assumed starting full, the prior statement holds. When

the reservoir is assumed starting empty, the statement changes to "the longer the reservoir horizon life the less the required capacity;" when the reservoir is assumed starting with a fixed initial volume, the reservoir reliability versus time plane has two different regions,--a lower region, with lower reliability, where the first assertion holds, and an upper region, with higher reliability, where the second statement prevails.

8) For a given reservoir capacity and certain values of the initial storage, the reservoir reliability reaches a minimum value after a time interval. This time interval increases when the initial storage increases.

9) The time of influence of the initial storage on the computation of reservoir reliability (T_S) can be estimated, as a random variable, by the relationship: $T_S = \min[F_{0C}, F_{C0}]$. F_{0C} and F_{C0} are the first passage time from empty to full and full to empty, respectively.

10) The ratio between the time of influence of initial storage, T_S, and the reservoir horizon life, N_L, measures the importance of the assumption of the initial storage in the computation of the reservoir reliability. It also gives indications if the reservoir is oversized. Values of that ratio close to one means that the storage capacity is oversized and the reservoir reliability is sensitive to the assumption of initial storage.

B. Further Studies Required

1) The extension of the model to include areas with a significant value of the serial correlation is possible using a bi-variate Markovian chain.

2) It is possible, using numerical procedures, to fit an equation to the data generated by the theoretical model. The equation has the advantage of making the rapid procedure more elegant. The trade-off is loosing accuracy.

3) We saw that for small initial storage values the reliability curves decrease, reach a minimum, change direction and increase toward the steady state reliability. Let us represent the time interval starting at time zero up to the time of minimum reliability by t_m and the difference between the minimum reliability and the steady state reliability by R_m. We showed, by numerical experiments , that for certain initial storage values when S_0 increases, t_m increases and R_m decreases. We can speculate without proof that this pattern is the same for all values of S_0. In other words, it means that when S_0 tends to K, t_m tends to infinity and R_m tends to zero. If that is true for any time there exists an initial storage that makes the reliability at that time equal to the steady state reliability. In our numerical experiments we detected values of t_m close to 30 years. The problem in going after this point is that R_m becomes too small, so small that is beyond the limits of accuracy of the simulation procedure. In conclusion only

further studies can prove whether or not the prior speculation is true.

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APPENDIX A

GUIDELINES TO USE OF THE SIZING PROCEDURE

A1. Organization of the Graphics

The graphics wich form the sizing procedure consist of a family of six curves each one for a dimensionless capacity f_K. It is important to keep in mind that for all graphics the curves, from right to left, correspond to $f_K = 1.0, 1.5, 2.0, 2.5, 3.0$ and 3.5 respectively.

A2. Input Data

a) primary data.

μ=mean annual inflow in volume units

C_V= Coefficient of variation of annual inflows in volume units.

PI= Mass probability of annual inflow being zero.

E_V= Mean evaporation depth during the dry season.

Volume x height table for the lake.

b) Input data computed from primary data.

α = It comes from the regression V= H^3, using the data from volume x height table.

f_E= dimensionless evaporation factor. Computed by the relationship:

$f_E=3\,\alpha^{1/3}E_V/\,\mu^{1/3}$

A3. Decisions/Output Variables

M= annual release in volume units.

K= required reservoir capacity.

PE= probability of emptiness, that is the probability of the reservoir running empty at the end of the year.

fM= Dimensionless release (M/μ).

f_K= Dimensionless capacity (K/μ).

A4. Procedures

a) Computing the relationship between storage capacity required versus annual release for a given probability of emptiness.

-Choose the graphic with the closest value of f_e and C_V.

-Draw a horizontal line from PE up to reach the f_K curve. Get the correspondent f_M in the abscissas axis. Repeat the process for all f_K curves.

-If PI>0 go to figure A1 and get the correction factor R_K for the point (PI,PE). Correct the required capacity: $f_K' = f_K * R_K$

b) Compute the relationship between annual release versus probability of emptiness for a given reservoir capacity.

-Choose the graphic with the closest value of C_V and f_E.

-Choose the curve correspondent to the f_K value. OBS-

Only curves with f_K=1.0, 1.5, 2.0, 2.5, 3.0, 3.5 are available. Some interpolations may be necessary.

-Choose a f_M and get the PE from the f_K curve.

-If PI>0 then do as follows:

Choose the f_K curve.

Choose a f_M and get PE from the f_K curve.

With PE and PI get the correction of R_K in figure A.1. Make $f_K^1=R_K*f_K$.

Repeat the procedure with the new f_K.

c) Compute the relationship between reservoir capacity and probability of emptiness for a given release.

-Choose the graphic with the closest value of C_V and f_E.

-Draw a vertical line from f_M up to the curve f_K and get the PE value. Repeat the procedure for other values of f_K.

-If PI>0 than go to figure A.1 and get R_K for the pair (PE,PI). Make $f_K^1=f_K*R_K$.

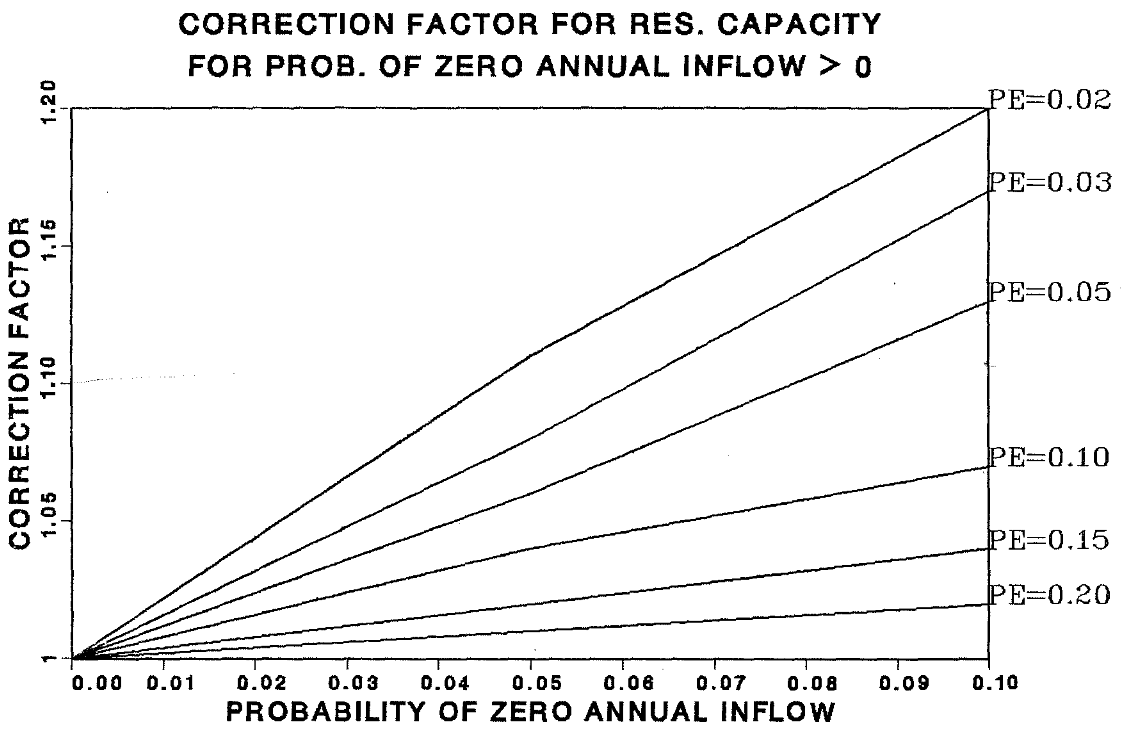
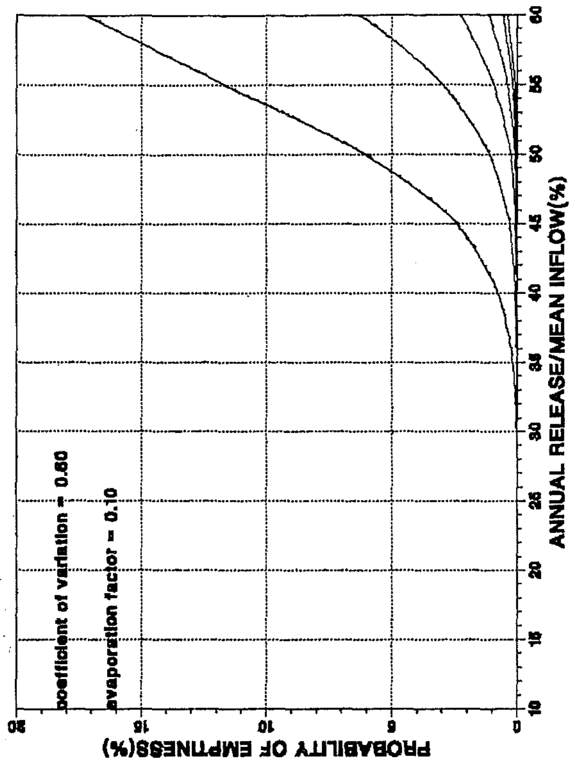


Figure A1. Factor to correct the required storage capacity when PI > 0.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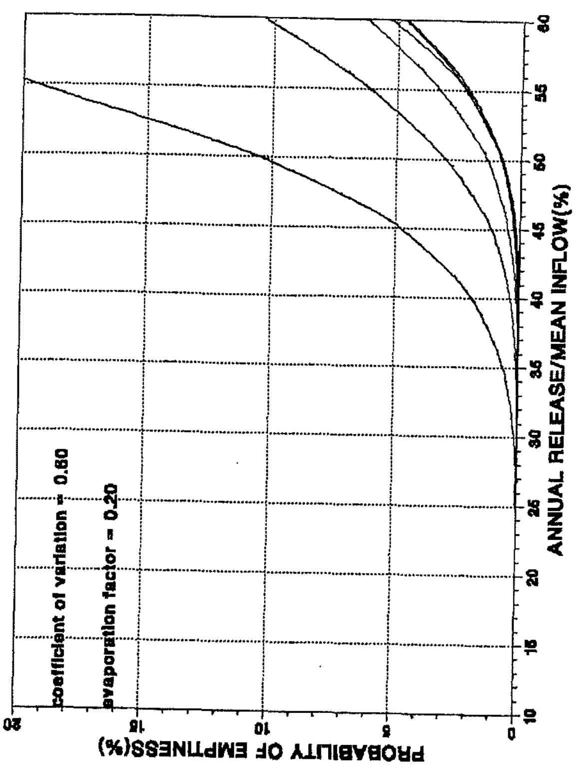
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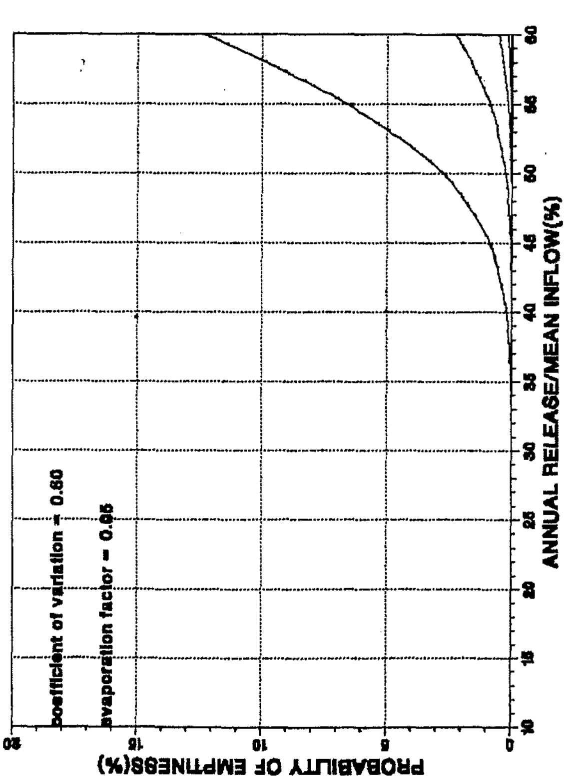
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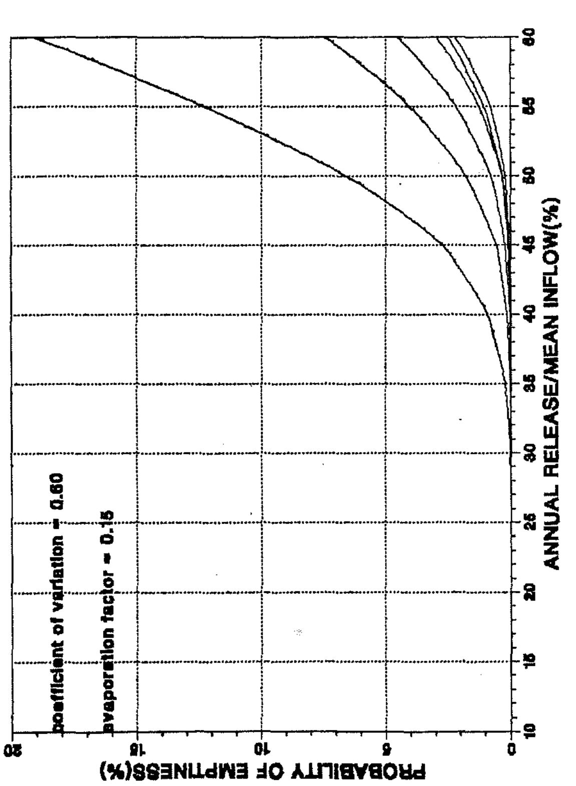
PROBABILITY OF EMPTINESS(%)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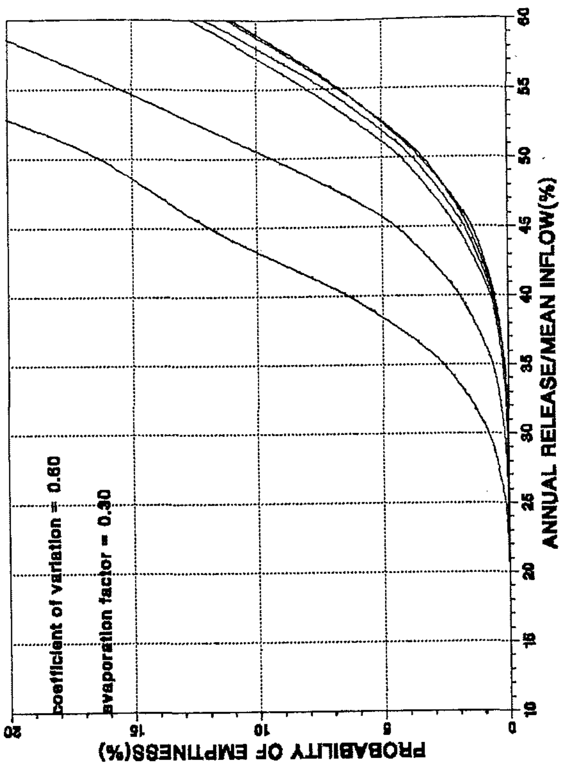
PROBABILITY OF EMPTINESS(%)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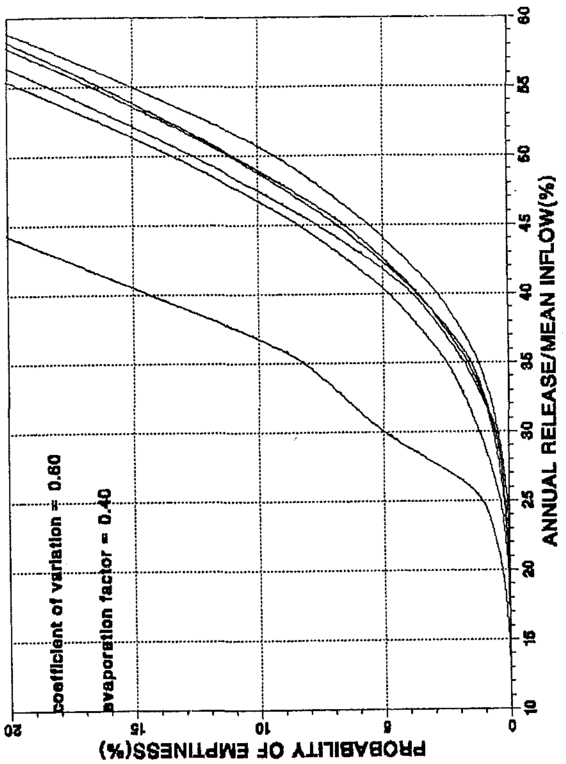
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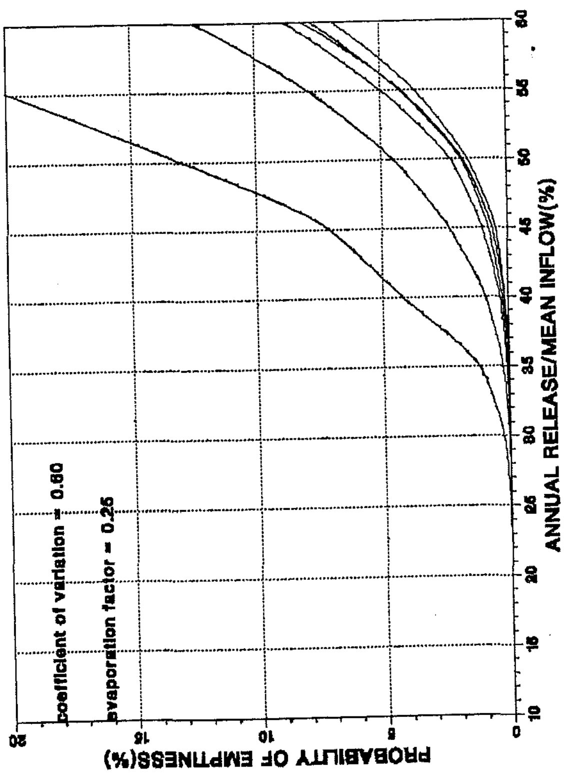
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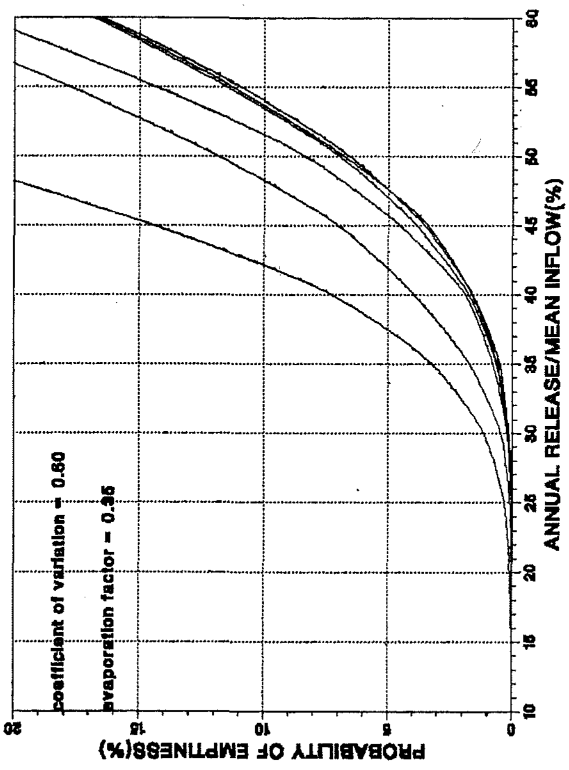
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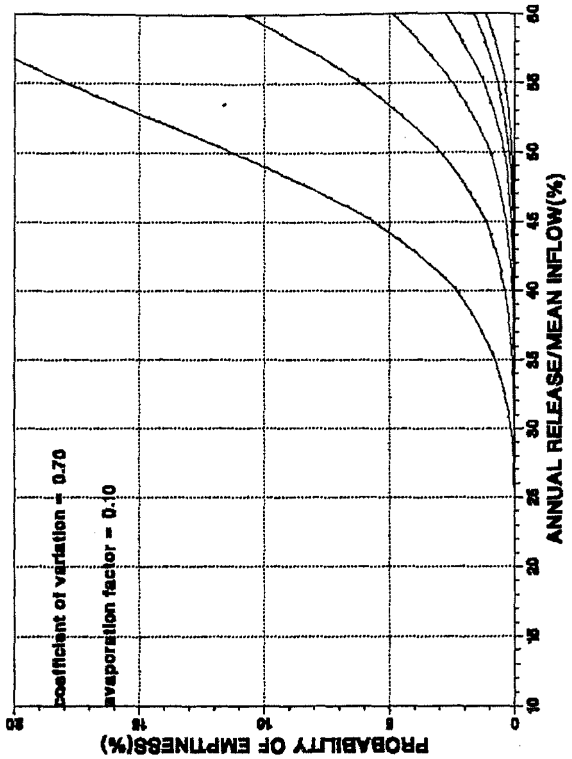
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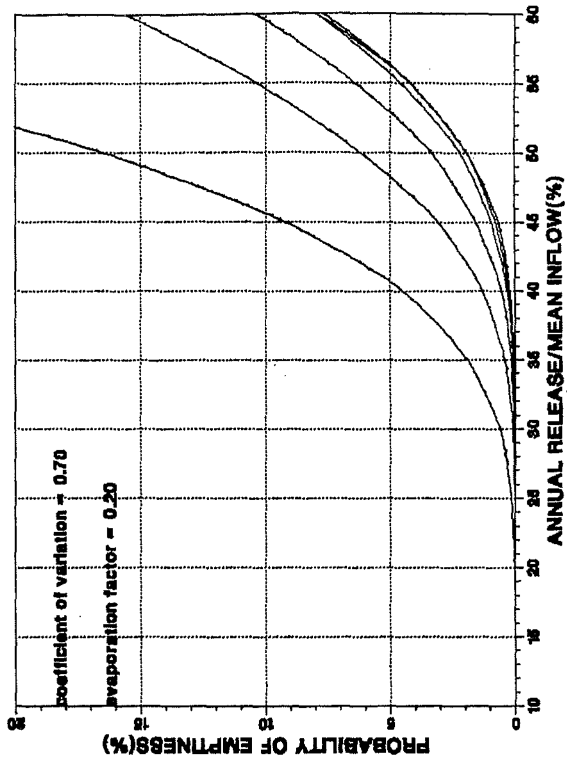
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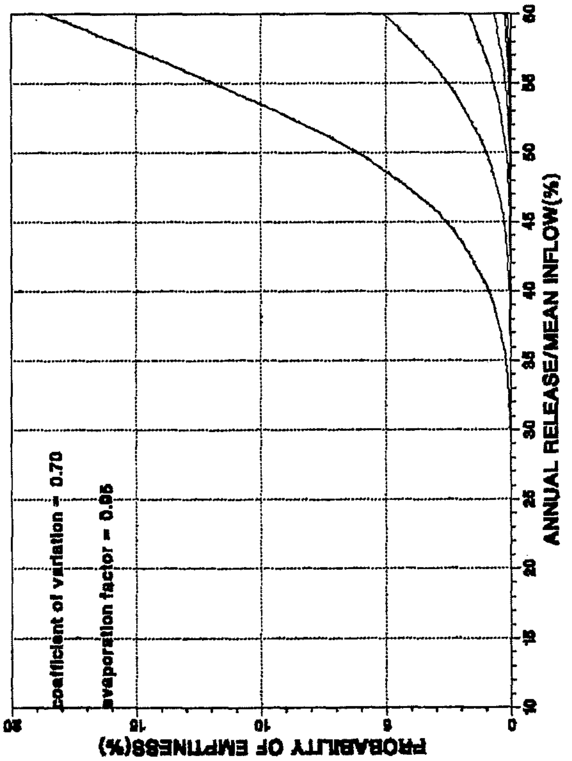
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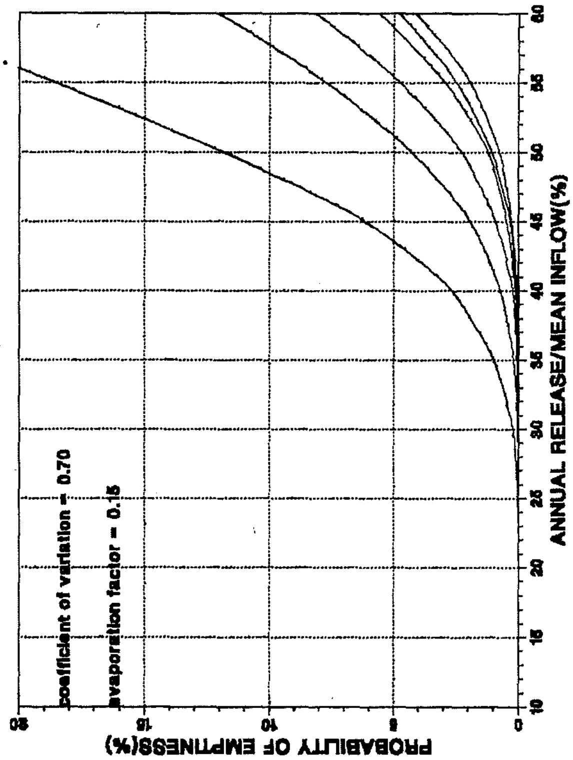
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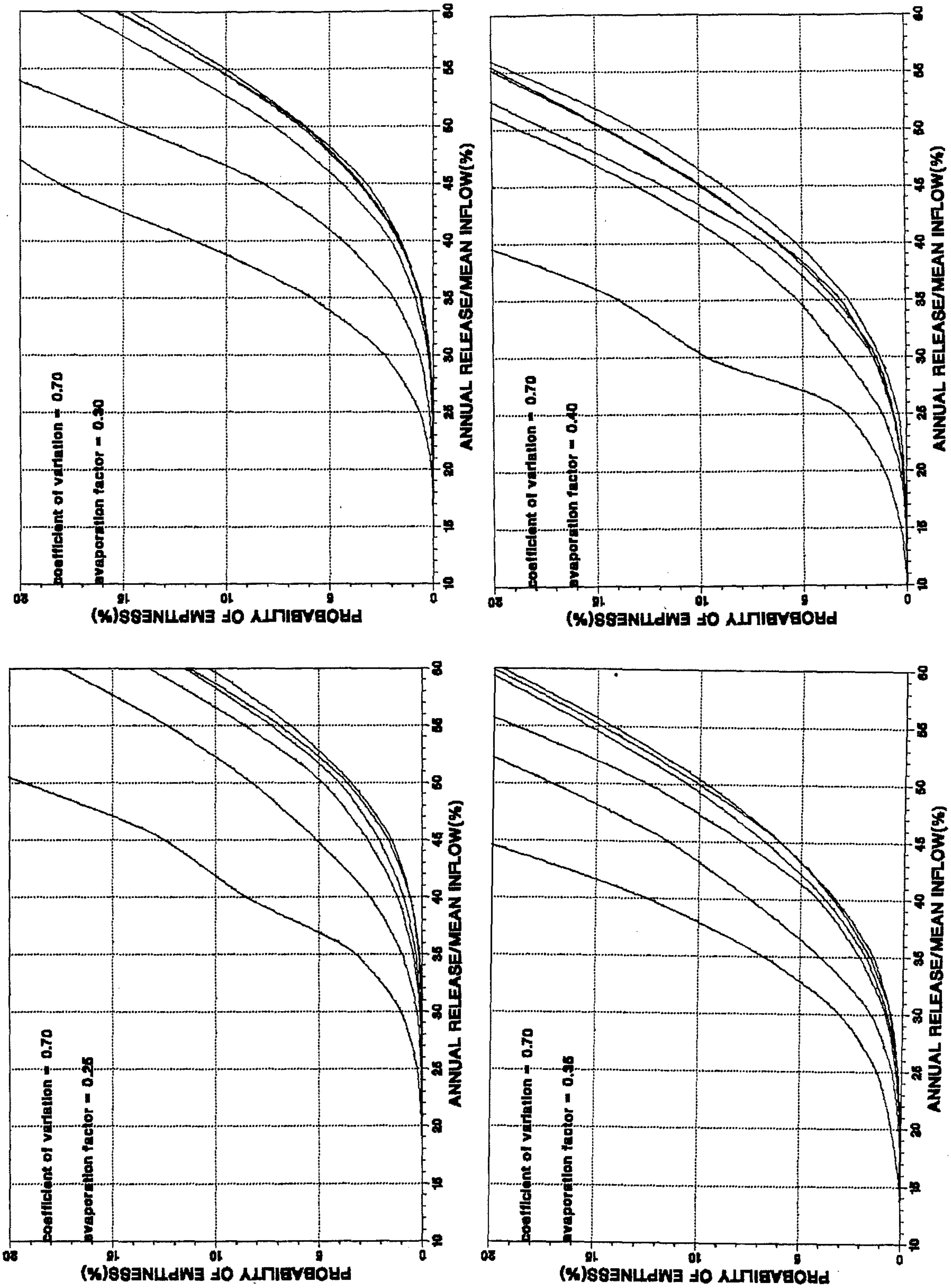

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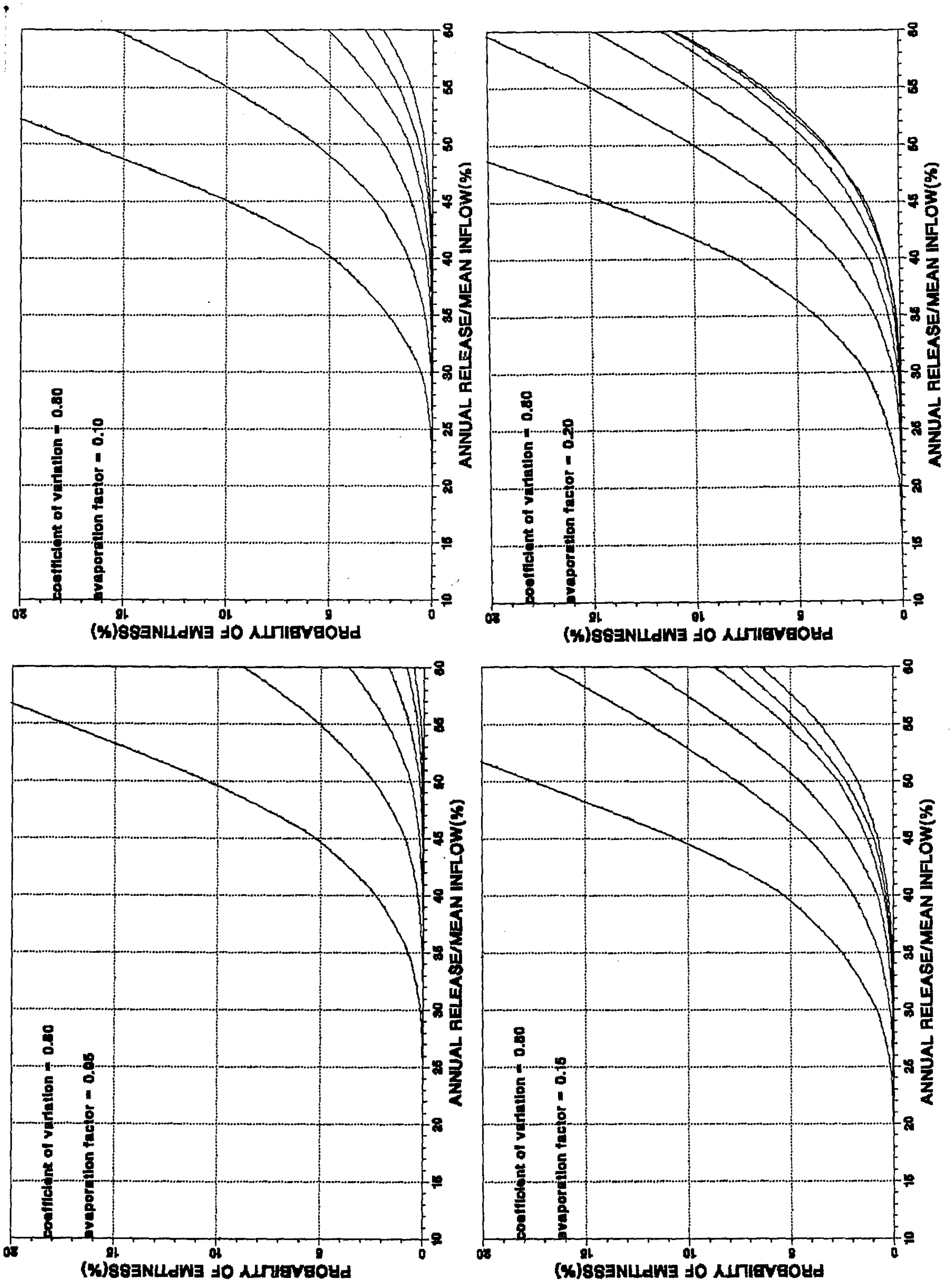
PROBABILITY OF EMPTINESS(%)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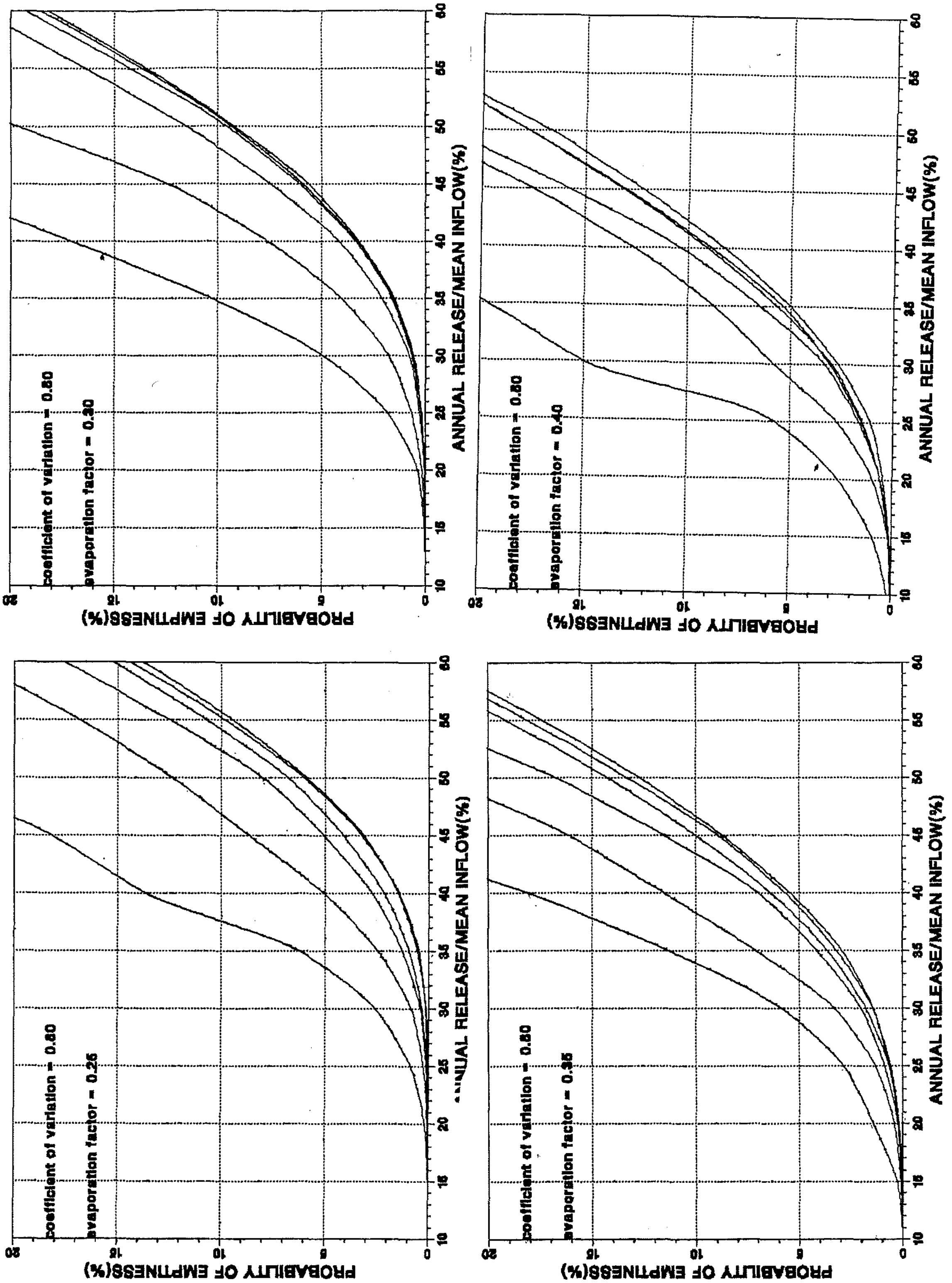
PROBABILITY OF EMPTINESS(%)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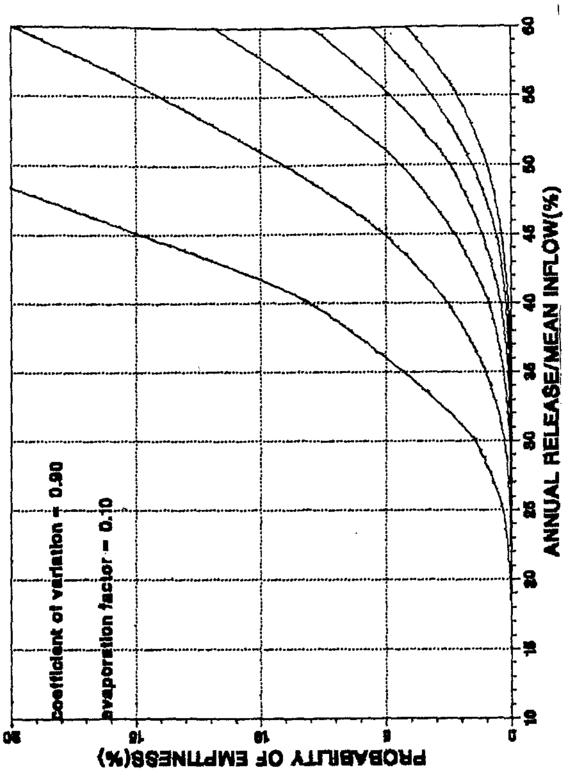
ANNUAL RELEASE/MEAN INFLOW(%)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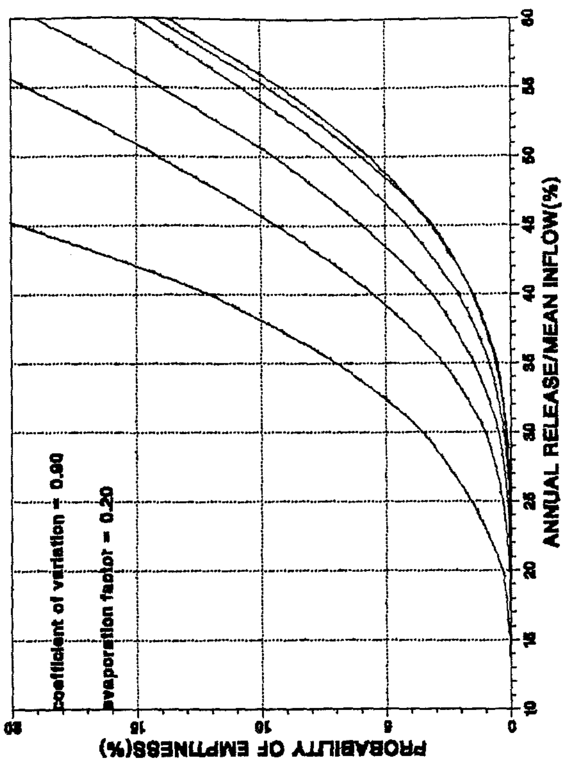
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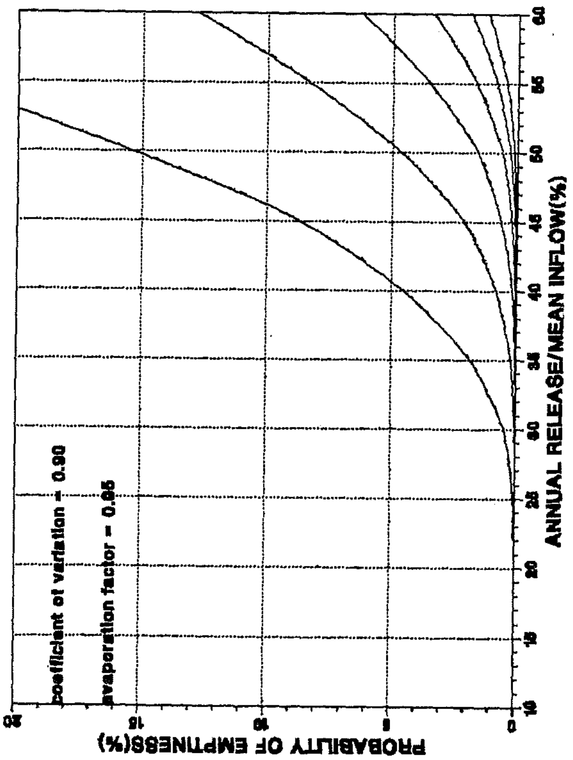
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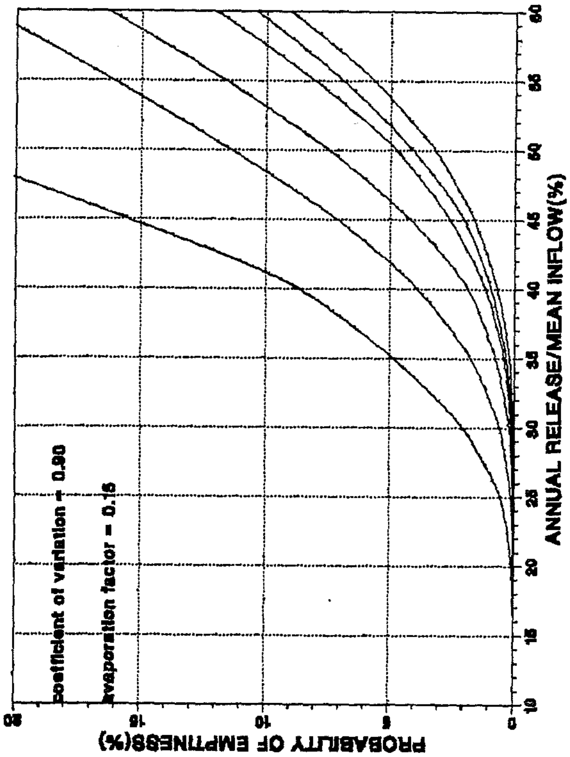
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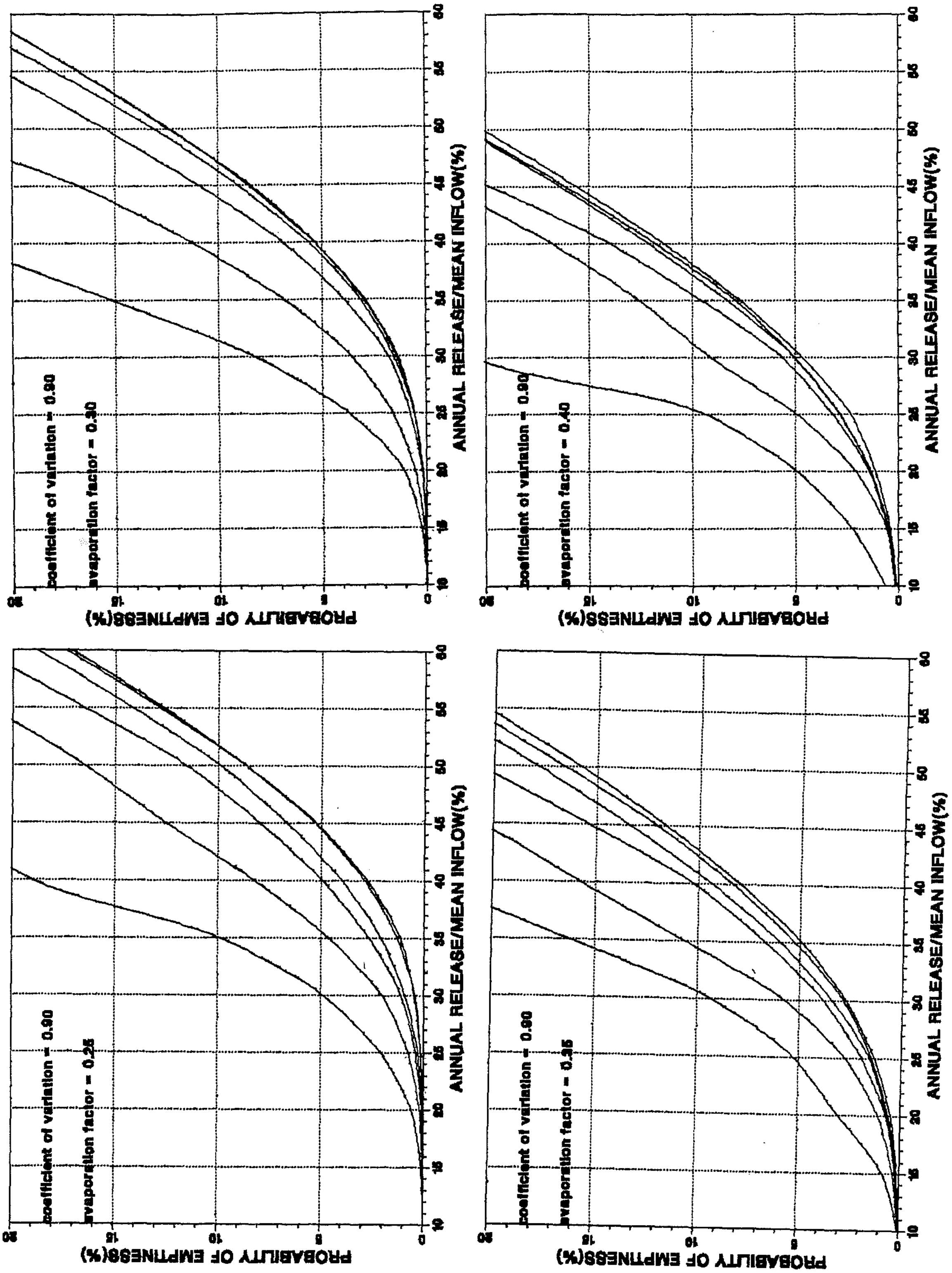
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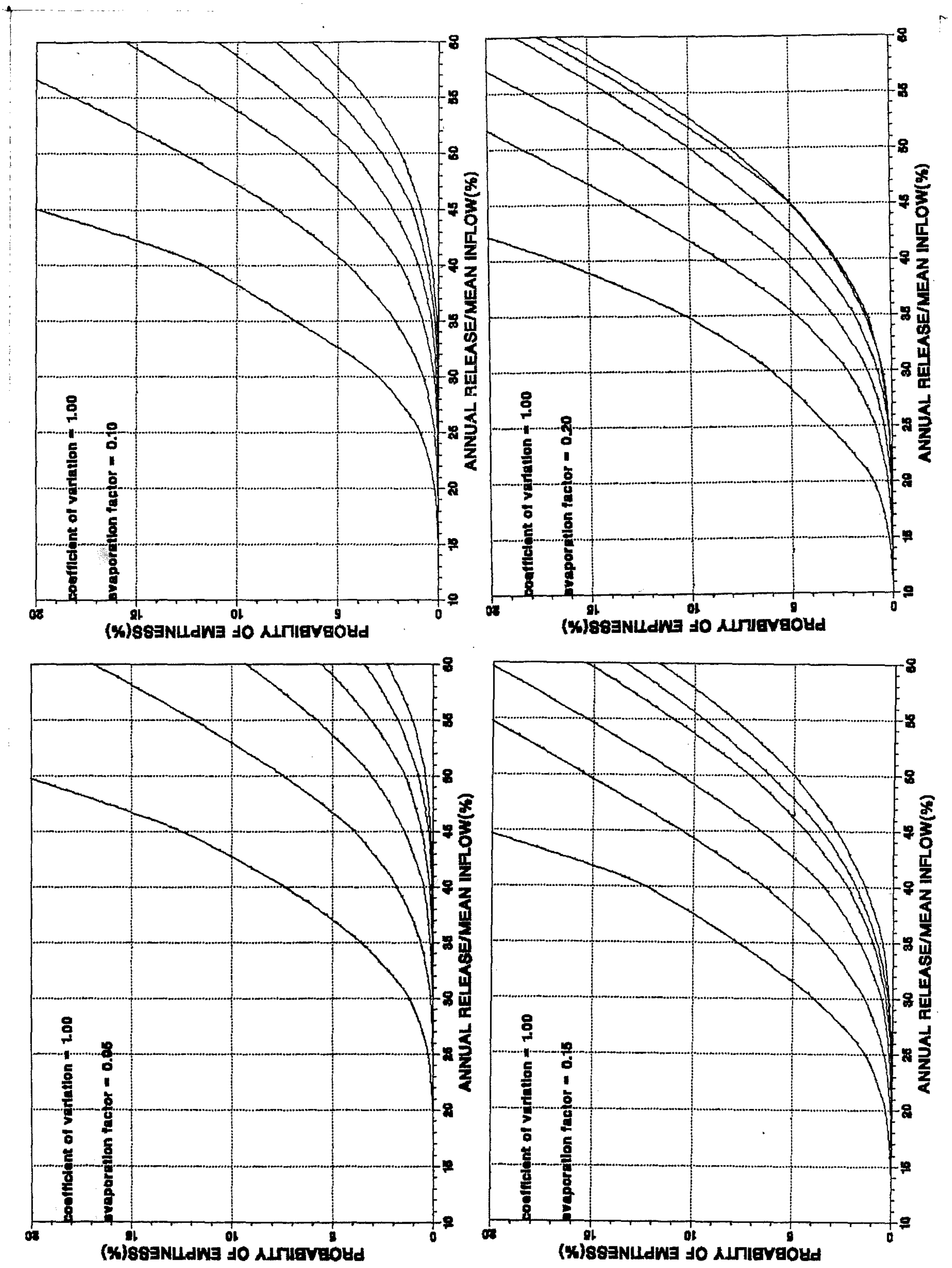
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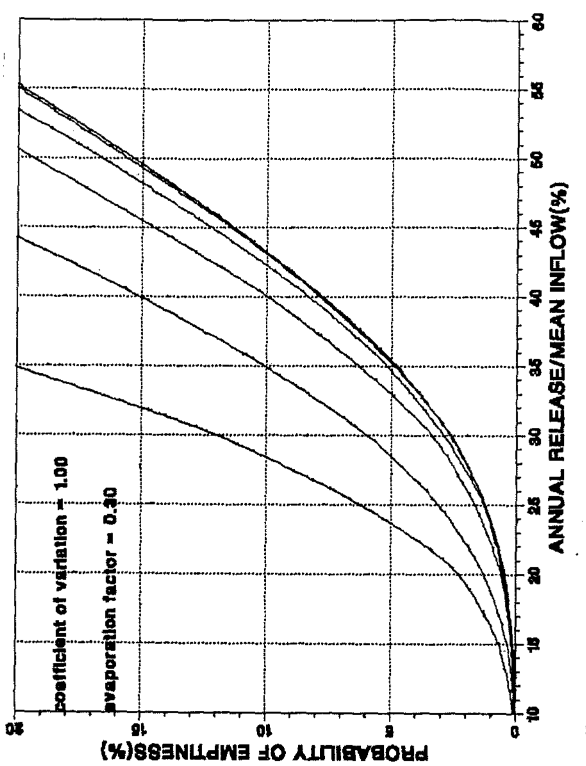
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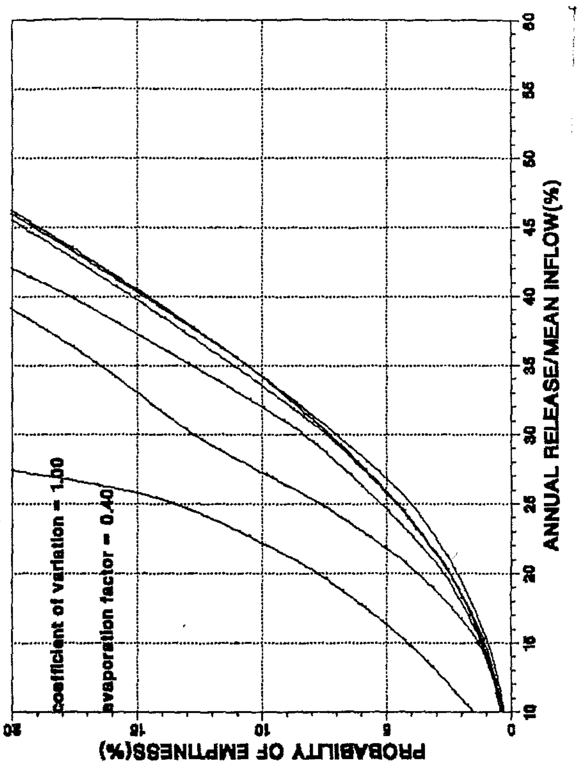
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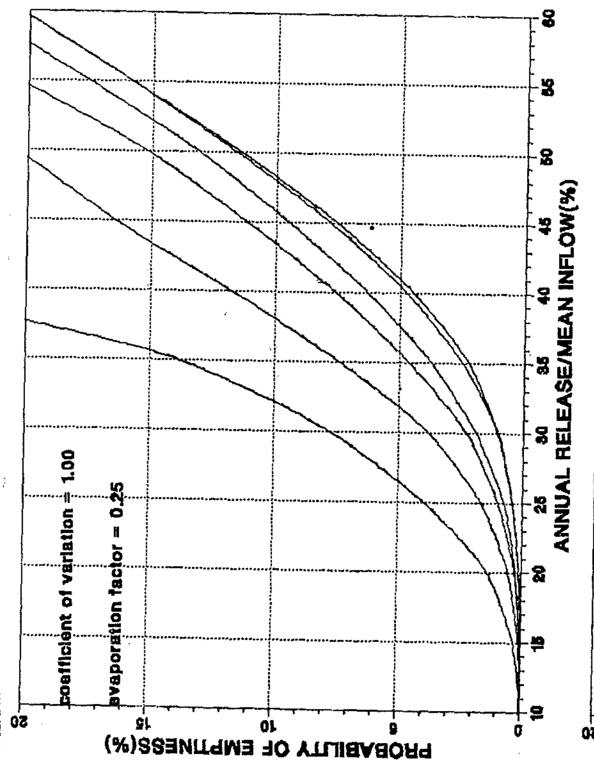
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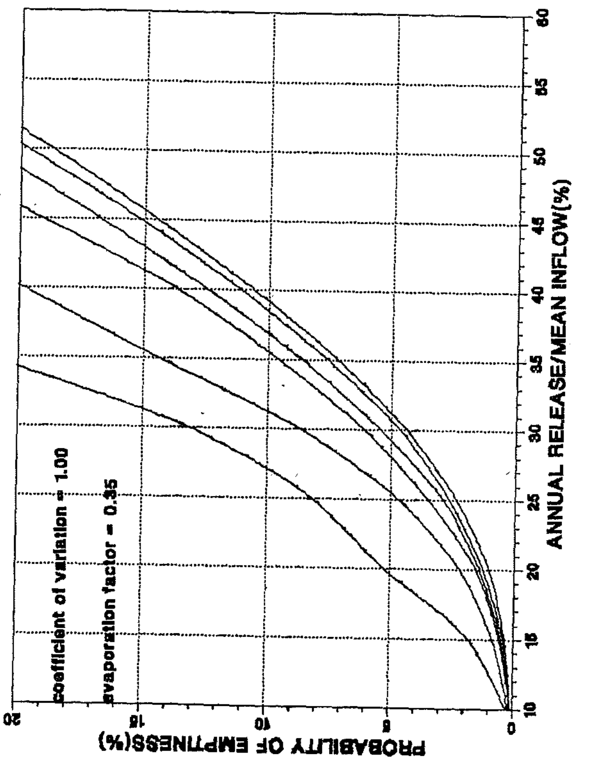
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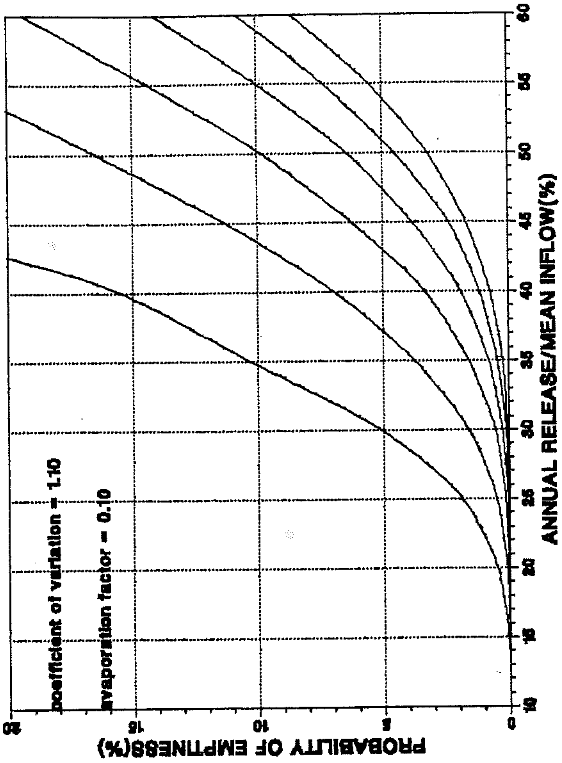
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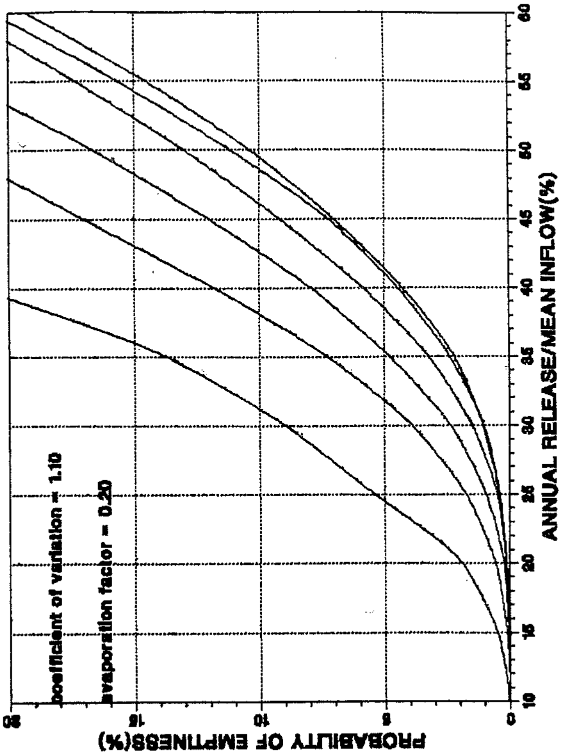
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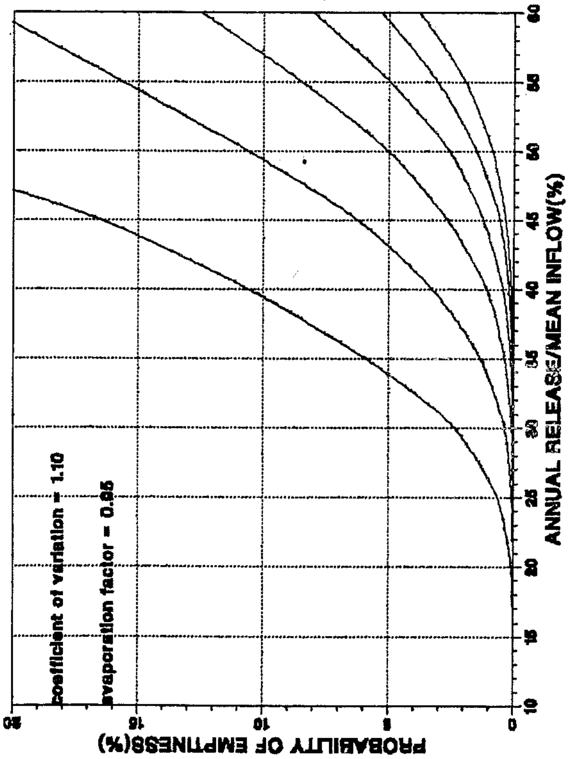
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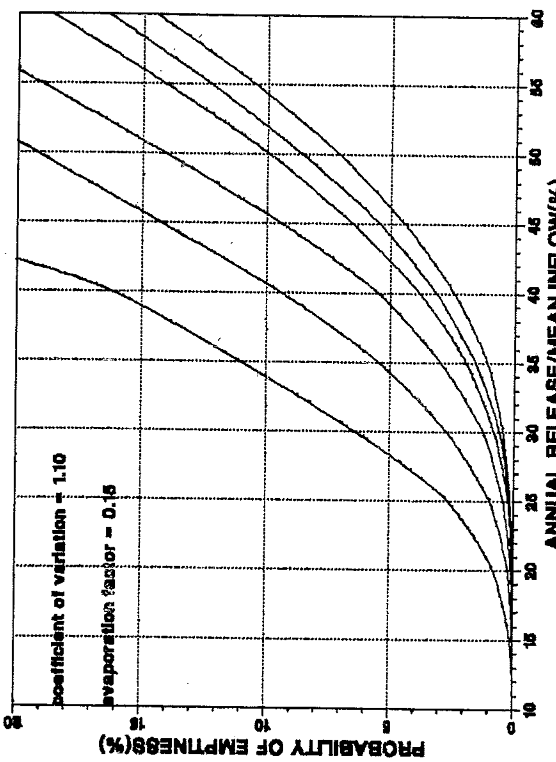
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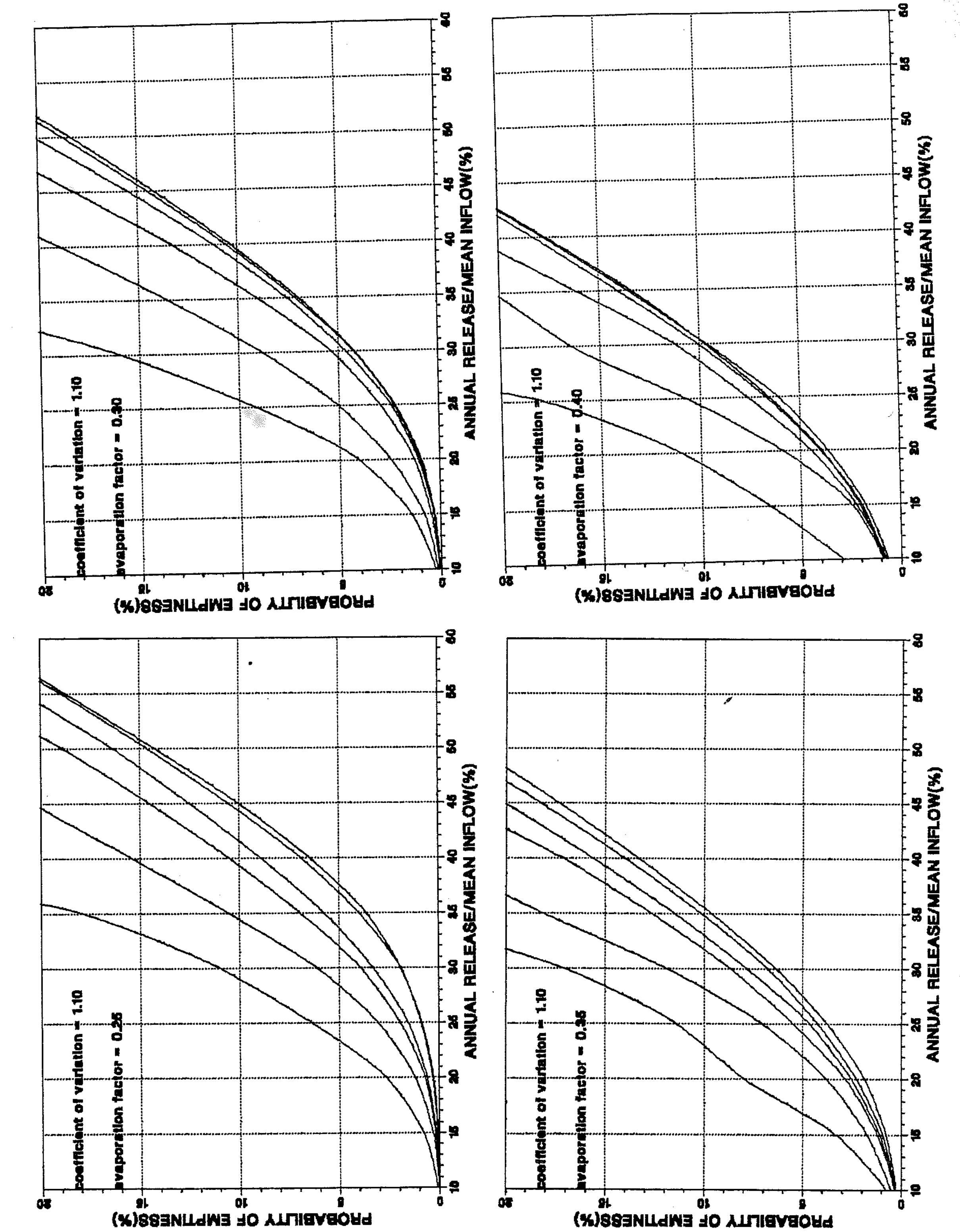
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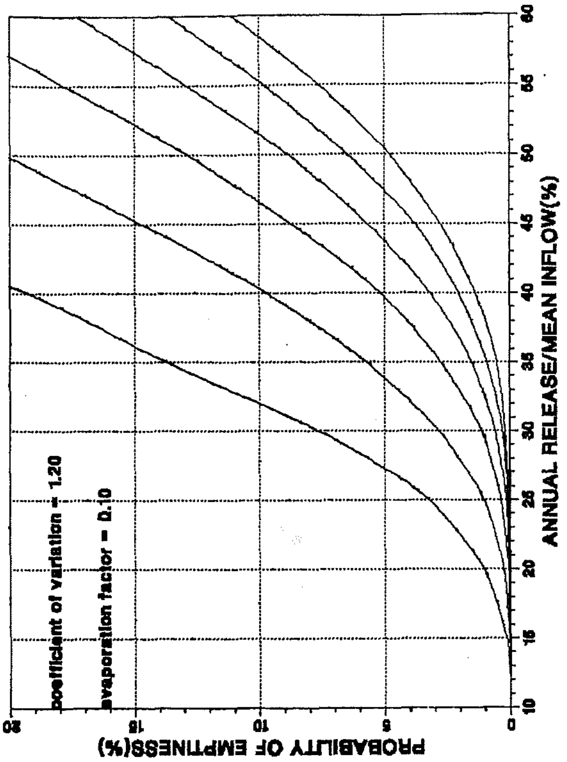
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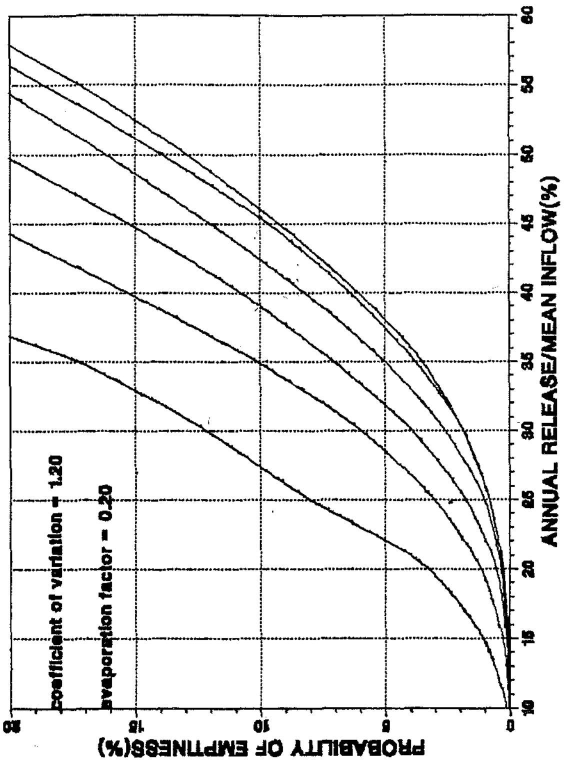
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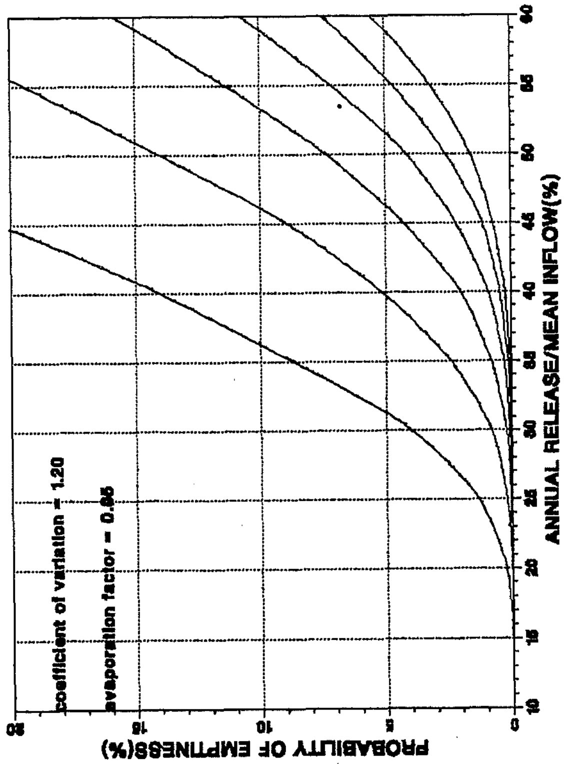
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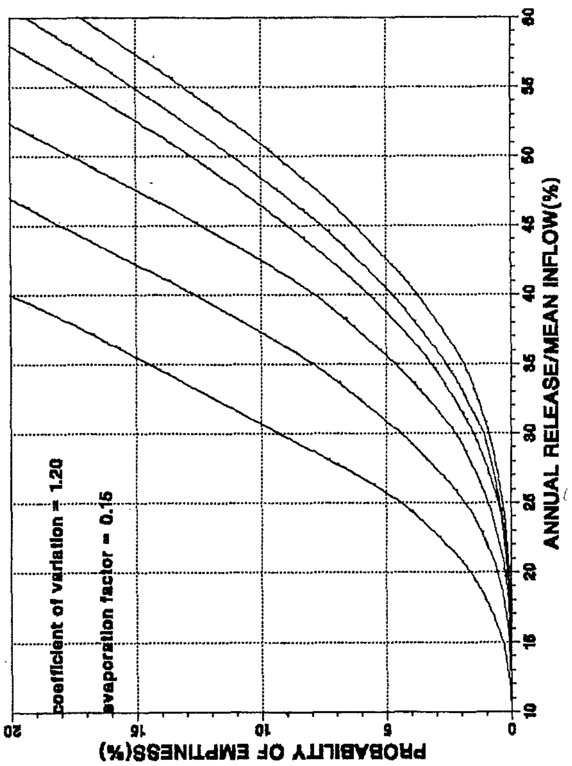
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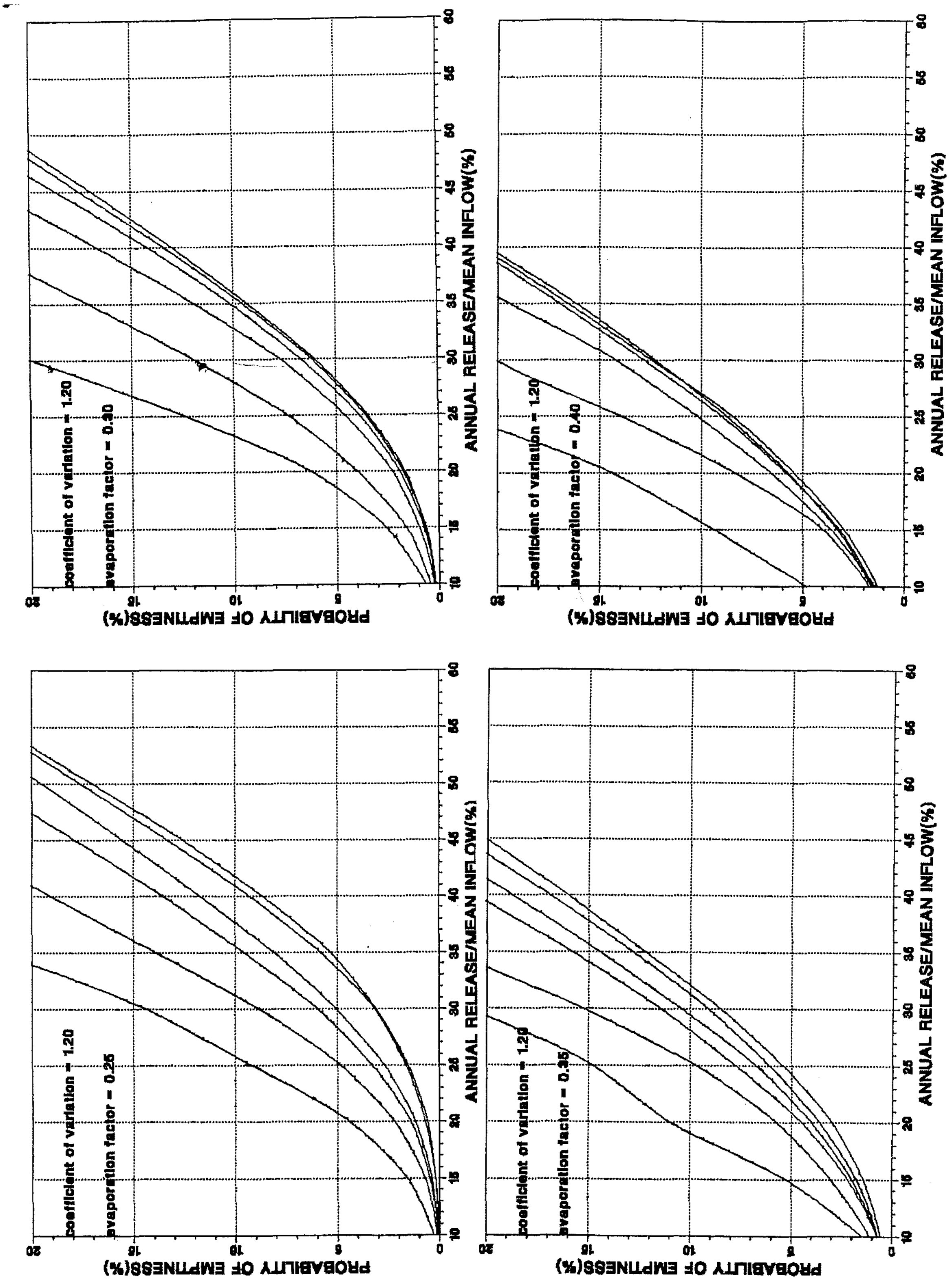
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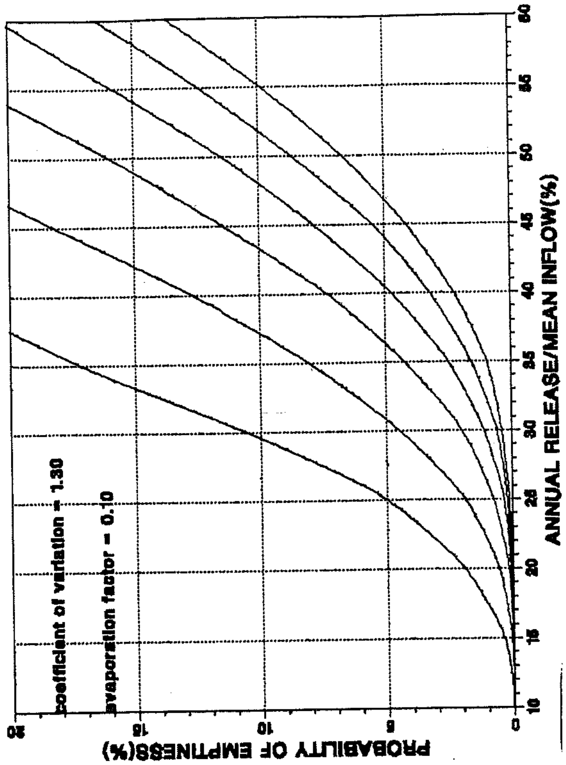
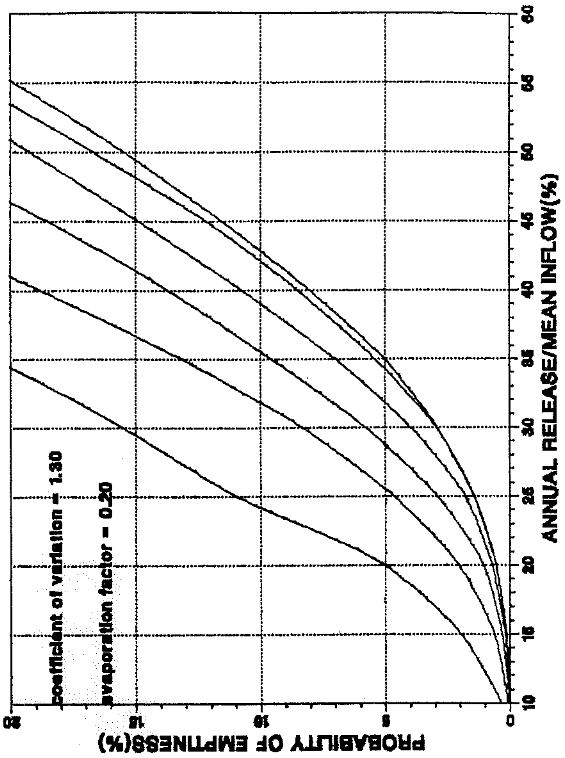
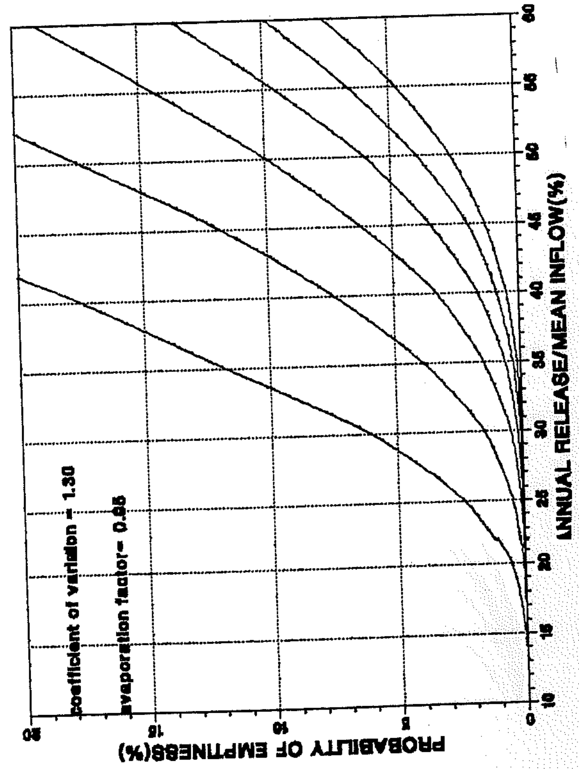
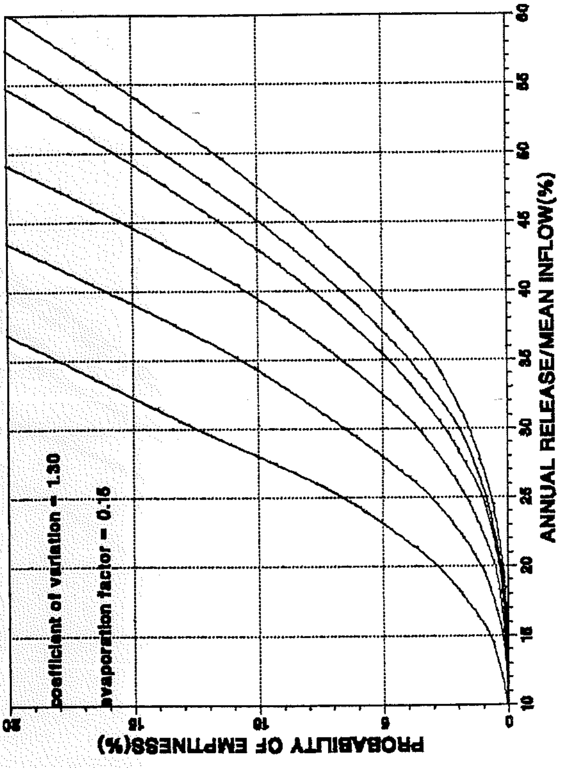
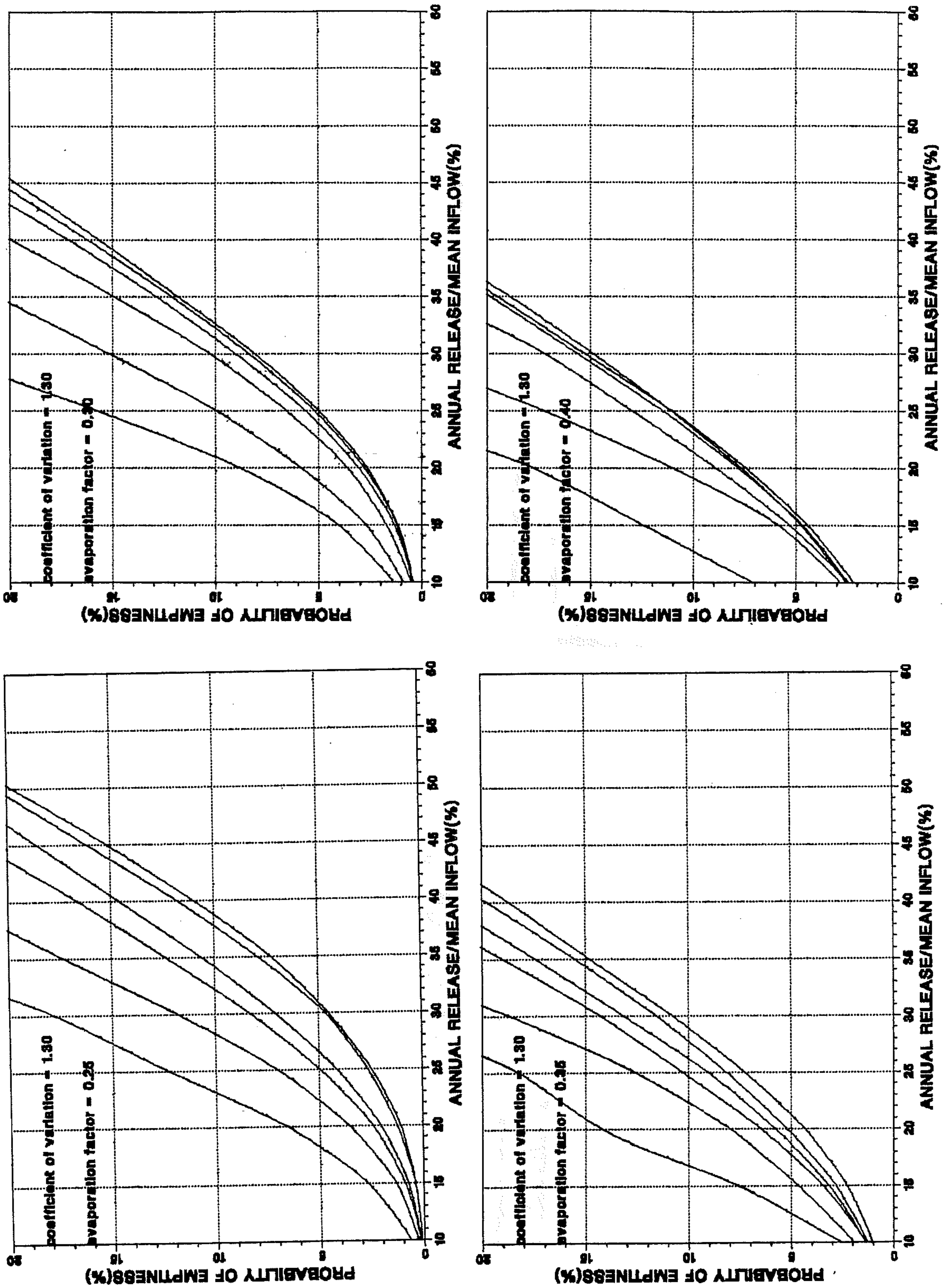
evaporation factor = 0.30
coefficient of variation = 1.20
evaporation factor = 0.40
coefficient of variation = 1.20
evaporation factor = 0.25
coefficient of variation = 1.20
evaporation factor = 0.35
PROBABILITY OF EMPTINESS(%)
ANNUAL RELEASE/MEAN INFLOW(%)

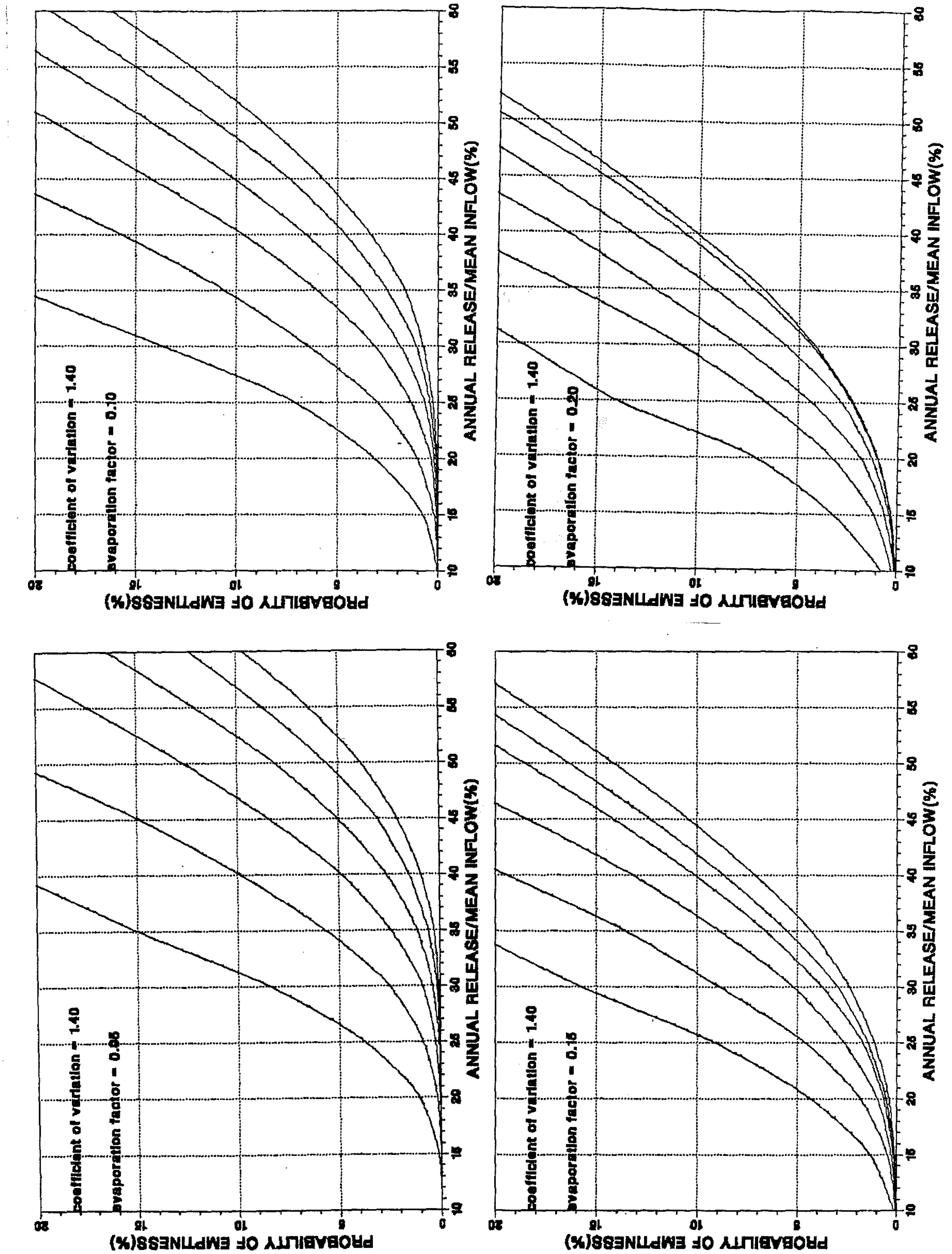

coefficient of variation = 1.30
evaporation factor = 0.10
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)

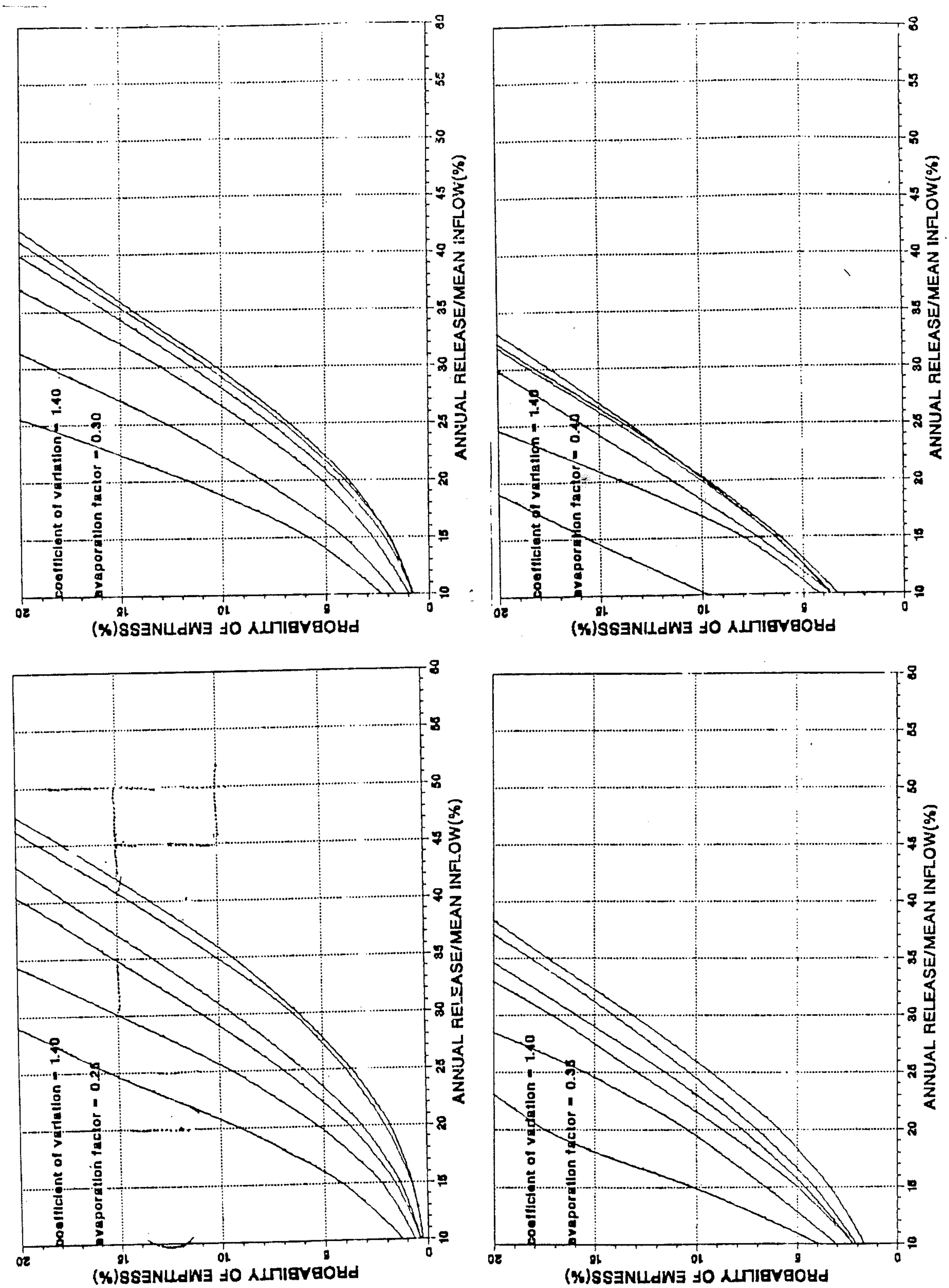

coefficient of variation = 1.30
evaporation factor = 0.20
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)


coefficient of variation = 1.30
evaporation factor = 0.05
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)


coefficient of variation = 1.30
evaporation factor = 0.15
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)


coefficient of variation = 1.30
evaporation factor = 0.30
PROBABILITY OF EMPTINESS(%)
ANNUAL RELEASE/MEAN INFLOW(%)
coefficient of variation = 1.30
evaporation factor = 0.40
PROBABILITY OF EMPTINESS(%)
ANNUAL RELEASE/MEAN INFLOW(%)
coefficient of variation = 1.30
evaporation factor = 0.25
PROBABILITY OF EMPTINESS(%)
ANNUAL RELEASE/MEAN INFLOW(%)
coefficient of variation = 1.30
evaporation factor = 0.35
PROBABILITY OF EMPTINESS(%)
ANNUAL RELEASE/MEAN INFLOW(%)


coefficient of variation = 1.40
evaporation factor = 0.10
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.20
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.05
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.15
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)


coefficient of variation = 1.40
evaporation factor = 0.30
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.40
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.25
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)
coefficient of variation = 1.40
evaporation factor = 0.35
ANNUAL RELEASE/MEAN INFLOW(%)
PROBABILITY OF EMPTINESS(%)

APPENDIX B

RESULTS FROM THE TEST OF THE SIZING PROCEDURE

TEST #1- Aires de Sousa.

DATA:

= 70MCM	C_V = 1.29
= 8830	E_V = 2.00m
PI = 0.05	f_E = 0.30

Results for f_K= 2.0

M	f_M	PE model	PE simul.
10	.14	1.3	.0
12	.17	3.9	0.0
15	.21	5.8	5.0
17	.24	9.4	7.5
20	.29	14.3	10.0

RESULTS: For f_K = 2.5

M	f_M	PE model	PE simul.
10	.14	1.0	.0
12	.17	1.3	.0
15	.21	3.1	.0
17	.24	5.3	5.0
20	.29	7.4	10.0

Results for f_K= 3.0

M	f_M	PE model	PE simul.
10	.14	0.6	.0
12	.17	1.1	.0
15	.21	3.3	2.5
17	.24	4.8	2.5
20	.29	7.4	10.0

Test#2- Oros.

DATA:

= 616MCM　　　C_V = 0.90

= 28400　　　E_V = 2.15m

PI = 0.0　　　f_E = 0.25

Results for f_K = 2.0

M	f_M	PE model	PE simul.
150	.24	0.3	.0
200	.32	2.0	4.0
250	.41	5.5	8.0
300	.49	11.0	16.0
350	.57	18.0	16.0

Results for f_K = 2.5

M	f_M	PE model	PE simul.
150	.24	.1	.0
200	.32	1.9	.0
250	.41	4.0	4.3
300	.49	9.0	8.3
350	.57	16.5	16.6

Results for f_K = 3.0

M	f_M	PE model	PE simul.
150	.24	<.1	.0
200	.32	0.5	.0
250	.41	3.0	4.5
300	.49	8.0	8.3
350	.57	4.0	16.0