XXXVII SIMPÓSIO BRASILEIRO DE TELECOMUNICAÇÕES E PROCESSAMENTO DE SINAIS - SBrT2019, 29/09/2019–02/10/2019, PETRÓPOLIS, RJ

Tensor-Based Joint Downlink and Uplink Channel Estimation in MU-MIMO Communication Systems

Paulo R. B. Gomes and André L. F. de Almeida

***Abstract*—In this paper, the problem of joint downlink (DL) and uplink (UL) channel estimation is adressed in a multiuser multiple-input multiple-output (MIMO) wireless communication system. We consider that the known training sequence sent by a multiple-antenna base-station (BS) is received by single-antenna mobile stations (MSs) and then reported back to the BS over multiple adjacent subcarriers. For this scenario, the signal received at the BS is modeled as a third-order tensor that follows a parallel factors (PARAFAC) decomposition. By exploiting the tensor structure of the received signal, we propose two semi-blind receivers for the joint estimation of the DL and UL channels at the BS. The first one is an iterative estimator based on the alternating least squares (ALS) principle, while the second one is a closed-form estimator based on the least squares Khatri-Rao factorization (LS-KRF) algorithm. The idea of the proposed receivers is to concentrate most of the processing burden for channel estimation at the BS, thus avoiding additional processing with high computational cost at the power-limited MSs side. Moreover, they allow multiple MSs to share the same feedback channel and UL-DL channel reciprocity assumption can be relaxed. Simulation results show that the proposed receivers achieve performance close to their equivalent minimum mean square error (MMSE)-based channel estimator, with the advantage of avoiding additional processing for channel estimation in each MS as well as dedicated feedback channels.**

***Keywords*—Wireless communication systems, channel estimation, alternating least squares, least squares Khatri-Rao factorization, PARAFAC decomposition.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) system is being applied to many wireless standards because it can increase the capacity and reliability in wireless communication systems [1]. In recent years, great attention has been given to multiuser MIMO systems (MU-MIMO) [2], where a base station (BS) equipped with multiple antennas simultaneously serves a set of single-antenna mobile stations (MSs). In contrast to the point-to-point MIMO systems, the MU-MIMO system is generally more tolerant to the propagation environment [1], [3]. Futhermore, it is commercially attractive because expensive equipment is only needed in the BS while the MSs can be cheap power-limited single-antenna devices [1].

The performance of MU-MIMO systems strongly depends on the efficient measurement of the channel state information (CSI) by the BS. This is possible from a training phase in which the BS transmits known training signals to the MSs. According to [4], the BS can learn the CSI from limited feedback in frequency division duplexing (FDD) [5] or assuming channel reciprocity in time division duplexing (TDD) [3]. However, in practice, the downlink (DL) channel estimated by the uplink (UL) channel considering channel reciprocity may not be accurate [6]. Futhermore, the channel acquisition becomes a challenge if the BS has to transmit long DL training sequences, and the MS has to report back its large channel matrix estimates. The amount of overhead in the DL channel estimation due to the long training sequences and in the UL channel due to the large matrices to be reported can severely decrease the spectral efficiency of the system, calling for novel solutions that enable us to reduce the unnecessary overhead.

In this paper, we obtain the relationship between DL and UL signals using a tensor formalism. We consider a novel training-based scheme for joint DL and UL channel estimation in MU-MIMO wireless communication systems. By assuming that a known training sequence sent by multiple-antenna BS is received by single-antenna MSs and then reported back to the BS by means of multiple adjacent subcarriers, the signal received at the BS is modeled as a third-order parallel factors (PARAFAC) decomposition. Motivated by the multidimensional structure of the received signal, we develop two tensor-based semi-blind receivers which offer greater practical appeal and less systemic limitations comparared to classical approaches for channel estimation (i.e., when the DL and UL channels are estimated separately at the MSs and BS, for instance, through least-squares (LS) or minimum mean square error (MMSE) estimators). In contrast to such classical approaches, the proposed tensor-based semi-blind receivers concentrate most of the processing burden for channel estimation at the BS side, thus avoiding additional processing with high computational cost at the power-limited MSs side. Moreover, they allow multiple MSs to share the same feedback channel while UL-DL channel reciprocity assumption can be relaxed.

Notation: Scalars are represented as non-bold lower-case letters (a), column vectors as lower-case boldface letters $(\boldsymbol{a})$, matrices as upper-case boldface letters $(\boldsymbol{A})$, and tensors as calligraphic upper-case letters $(\boldsymbol{\mathcal{A}})$. The superscripts $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^{*}$, $\{\cdot\}^{\mathrm{H}}$ and $\{\cdot\}^{\dagger}$ stand for transpose, conjugate, conjugate transpose and pseudo-inverse operations, respectively. The operator $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix or tensor. The operator $D_i(\boldsymbol{A})$ forms a diagonal matrix from the i-th row of $\boldsymbol{A}$. $\mathrm{vec}(\boldsymbol{A})$ converts $\boldsymbol{A} \in \mathbb{C}^{I_1 \times R}$ to a column vector $\boldsymbol{a} \in \mathbb{C}^{I_1 R}$ by stacking its columns on top of each other. As its inverse operation, $\mathrm{unvec}_{I_1 \times R}(\boldsymbol{a})$ reshapes the column vector $\boldsymbol{a} \in \mathbb{C}^{I_1 R}$ into a matrix $\boldsymbol{A} \in \mathbb{C}^{I_1 \times R}$. The symbols $\circ$, $\otimes$ and $\diamond$ represent the outer product, Kronecker product and Khatri-Rao product (i.e., the column-wise Kronecker product), respectively.

Paulo R. B. Gomes and André L. F. de Almeida are with the GTEL-Wireless Telecom Research Group, Federal University of Ceará, Fortaleza-CE, Brazil, e-mails: {paulo,andre}@gtel.ufc.br. This work was partially supported by FUNCAP.

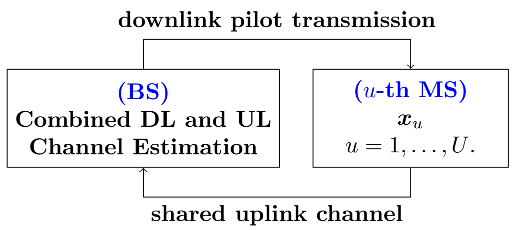

Fig. 1. System model representation. A known training sequence is sent by the BS and received by each MS. Then, each MS reports its received signal $\boldsymbol{x}_u$ ($u = 1, \ldots, U$) back to BS without performing any channel estimation processing.

Due to the space limitation the definitions, operations involving tensors and details about the PARAFAC decomposition are referenced for [7]. Throughout this paper, we shall make use of the following property:

$$\boldsymbol{a} \otimes \boldsymbol{b} = \text{vec}\left(\boldsymbol{b} \circ \boldsymbol{a}\right). \tag{1}$$

II. System and Channel Models

We consider a MU-MIMO wireless communication system consisting of a BS and U MSs. The BS is equiped with a uniform linear array (ULA) of N elements while each MS is a single-antenna device. The training sequence sent by the BS is received by each MS and only reported back to the BS, i.e., no processing to the DL channel estimation is done by the MSs. Each MS only applies a linear combining on the received signal before sending it back to the BS. The details of such an operation will be given in the next section. We assume that the MSs share the same uplink feedback channel. The BS then receives the sum of U co-channel signals. Moreover, the DL and UL channels are possibly different, which means channel reciprocity may not hold. Figure 1 illustrates this scenario.

The DL and UL channels of each MS are assumed block-fading narrowband channels and time-invariant during the training stage. Both are modeled as a combination of L_u ($u = 1, \ldots, U$) paths per user, each one characterized by an angle of arrival (AoA) $\theta_{u,l} \in [0, 2\pi]$ in the uplink, an angle of departure (AoD) $\phi_{u,l} \in [0, 2\pi]$ in the downlink and a complex fading coefficient $\alpha_{l,u}$ for the l-th path of the u-th MS.[1] The DL or UL channel of the the u-th MS, denoted by $\boldsymbol{h}_u^{(dl,ul)} \in \mathbb{C}^N$, can be expressed as[2]

$$\boldsymbol{h}_u^{(dl,ul)} = \sum_{l=1}^{L_u} \alpha_{u,l}^{(dl,ul)} \boldsymbol{a}_{BS}(\theta_{u,l}), \tag{2}$$

where $\alpha_{u,l}$ is the fading coefficient associated with the l-th path of the u-th MS and $\boldsymbol{a}_{BS}(\theta_{u,l}) \in \mathbb{C}^N$ denotes the BS antenna array response associated with the l-th path of the u-th MS. Since ULA is assumed, we have $\boldsymbol{a}_{BS}(\theta_{u,l}) = \left[1, e^{-j(2\pi/\lambda)d\sin(\theta_{u,l})}, \ldots, e^{-j(2\pi/\lambda)(N-1)d\sin(\theta_{u,l})}\right]^{\mathrm{T}}$.

Let $\boldsymbol{S} \in \mathbb{C}^{T \times N}$ be a known training sequence sent by the BS. The signal $\boldsymbol{x}_u \in \mathbb{C}^T$ received by the u-th MS is given by

$$\boldsymbol{x}_u = \boldsymbol{S}\boldsymbol{h}_u^{(dl)} + \boldsymbol{v}_u^{(dl)}, \tag{3}$$

where $\boldsymbol{v}_u^{(dl)} \in \mathbb{C}^T$ denotes the additive white Gaussian noise term at the u-th MS.

The signal $\boldsymbol{x}_u$ of each MS is then reported back to the BS, i.e., no additional processing for channel estimation is performed at the MSs side. The matrix $\boldsymbol{Y} \in \mathbb{C}^{N \times T}$ that represents the sum of U co-channel signals received at the BS can be written as

$$\boldsymbol{Y} = \sum_{u=1}^{U} \boldsymbol{h}_u^{(ul)} \boldsymbol{x}_u^{\mathrm{T}} + \boldsymbol{V}^{(ul)} = \boldsymbol{H}^{(ul)} \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{V}, \tag{4}$$

where $\boldsymbol{X} = \left[\boldsymbol{S}\boldsymbol{h}_1^{(dl)}, \ldots, \boldsymbol{S}\boldsymbol{h}_U^{(dl)}\right] \in \mathbb{C}^{T \times U}$ is given by

$$\boldsymbol{X} = \boldsymbol{S}\boldsymbol{H}^{(dl)}, \tag{5}$$

and collects the signals from the U MSs, $\boldsymbol{H}^{(ul)} = \left[\boldsymbol{h}_1^{(ul)}, \ldots, \boldsymbol{h}_U^{(ul)}\right] \in \mathbb{C}^{N \times U}$ and $\boldsymbol{H}^{(dl)} = \left[\boldsymbol{h}_1^{(dl)}, \ldots, \boldsymbol{h}_U^{(dl)}\right] \in \mathbb{C}^{N \times U}$ are the UL and DL channel matrices, respectively. The overall noise at the BS is given by

$$\boldsymbol{V} = \sum_{u=1}^{U} \boldsymbol{h}_u^{(ul)} \boldsymbol{v}_u^{(dl)\mathrm{T}} + \boldsymbol{V}^{(ul)} \in \mathbb{C}^{N \times T}, \tag{6}$$

which takes into account the contribuitions of the additive white Gaussian noise at the BS and MSs.

III. Proposed Tensor-Based Semi-Blind Receivers

In this section, we initially propose a novel multicarrier-based training scheme that concentrates computational burden associated with channel estimation at the BS. Then, we propose two tensor-based semi-blind receivers for joint DL and UL multiuser channel estimation.

A. Novel multicarrier-based training scheme

Let us consider that the signal $\boldsymbol{x}_u$, $u = 1, \ldots, U$, received at the u-th MS is spread in the frequency-domain across K adjacent subcarriers over which the channel is considered to be constant, i.e. affected by the same fading coefficient $\alpha_{l,u}$. The u-th MS loads the received pilots into the k-th subcarrier using the weight factor $f_{k,u}$. Then, the coded multicarrier uplink pilots are fed back to the BS by the multiple MSs. We can write the received signal at the BS as

$$\begin{aligned}\boldsymbol{Y}_k &= \sum_{u=1}^{U} f_{k,u} \boldsymbol{h}_u^{(ul)} \boldsymbol{x}_u^{\mathrm{T}} + \boldsymbol{V}_k^{(ul)} \\ &= \boldsymbol{H}^{(ul)} D_k\left(\boldsymbol{F}\right) \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{V}_k \in \mathbb{C}^{N \times T}, \quad k = 1, \ldots, K\end{aligned} \tag{7}$$

where $f_{k,u}$ is the (k,u)-th entry of the so called spreading matrix $\boldsymbol{F} = [\boldsymbol{f}_1, \ldots, \boldsymbol{f}_U] \in \mathbb{C}^{K \times U}$ assumed here as a DFT matrix. The u-th column of $\boldsymbol{F}$ contains the set of spreading coefficients, $f_{1,u}, \ldots, f_{K,u}$, used by the u-th MS.

According to [7], the received closed-loop signal $\boldsymbol{Y}_k$ denotes the k-th frontal slice of the PARAFAC tensor $\boldsymbol{\mathcal{Y}} \in \mathbb{C}^{N \times T \times K}$

[1] Since only the fading coefficients vary for the different operating frequencies in the DL and UL, we have assumed that $\theta_{u,l} = \phi_{u,l}$ for simplicity of representation.

[2] This model is a particular case of the general geometric channel model $\boldsymbol{H}_u = \sum_{l=1}^{L_u} \alpha_{u,l} \boldsymbol{a}_{BS}(\theta_{u,l}) \boldsymbol{a}_{MS}^{\mathrm{H}}(\phi_{u,l})$ when single-antenna devices are considered.

that admits the following factorizations in terms of its unfolding (or flattening) and factor matrices:

$$[\mathcal{Y}]_{(1)} = \boldsymbol{H}^{(ul)} (\boldsymbol{F} \diamond \boldsymbol{X})^{\mathrm{T}} \in \mathbb{C}^{N \times TK}, \tag{8}$$

$$[\mathcal{Y}]_{(2)} = \boldsymbol{X} \left(\boldsymbol{F} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}} \in \mathbb{C}^{T \times NK}, \tag{9}$$

$$[\mathcal{Y}]_{(3)} = \boldsymbol{F} \left(\boldsymbol{X} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}} \in \mathbb{C}^{K \times NT}. \tag{10}$$

Our aim is to jointly estimate the channel matrices $\boldsymbol{H}^{(dl)}$ and $\boldsymbol{H}^{(ul)}$ at the BS from the received signal tensor $\mathcal{Y}$ in (7). For this purpose, we formulate in the following two PARAFAC-based semi-blind receivers based on the alternating least squares (ALS) and least squares Khatri-Rao factorization (LS-KRF) algorithms, respectively.

B. ALS-PARAFAC semi-blind receiver

From $\mathcal{Y}$ described in (7), estimates of $\boldsymbol{H}^{(ul)}$, $\boldsymbol{X}$ and $\boldsymbol{F}$ can be obtained by solving the following quadratic optimization problem:

$$\min_{\hat{\boldsymbol{H}}^{(ul)}, \hat{\boldsymbol{X}}, \hat{\boldsymbol{F}}} \left\| \mathcal{Y} - \sum_{u=1}^{U} \hat{\boldsymbol{h}}_u^{(ul)} \circ \hat{\boldsymbol{x}}_u \circ \hat{\boldsymbol{f}}_u \right\|_{\mathrm{F}}^2 . \tag{11}$$

This problem can be solved by means of the ALS algorithm [8]. It consists of estimating in an alternating way the factor matrices from the unfolding matrices $[\mathcal{Y}]_{(n)}$, $n = 1, \ldots, 3$ by solving the following three linear LS problems:

$$\underset{\boldsymbol{H}^{(ul)}}{\operatorname{argmin}} \left\| [\mathcal{Y}]_{(1)} - \boldsymbol{H}^{(ul)} (\boldsymbol{F} \diamond \boldsymbol{X})^{\mathrm{T}} \right\|_{\mathrm{F}}^2, \tag{12}$$

$$\underset{\boldsymbol{X}}{\operatorname{argmin}} \left\| [\mathcal{Y}]_{(2)} - \boldsymbol{X} \left(\boldsymbol{F} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}} \right\|_{\mathrm{F}}^2, \tag{13}$$

$$\underset{\boldsymbol{F}}{\operatorname{argmin}} \left\| [\mathcal{Y}]_{(3)} - \boldsymbol{F} \left(\boldsymbol{X} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}} \right\|_{\mathrm{F}}^2 . \tag{14}$$

The analytic solutions of which are given by $\hat{\boldsymbol{H}}^{(ul)} = [\mathcal{Y}]_{(1)} \left[(\boldsymbol{F} \diamond \boldsymbol{X})^{\mathrm{T}}\right]^{\dagger}$, $\hat{\boldsymbol{X}} = [\mathcal{Y}]_{(2)} \left[\left(\boldsymbol{F} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}}\right]^{\dagger}$ and $\hat{\boldsymbol{F}} = [\mathcal{Y}]_{(3)} \left[\left(\boldsymbol{X} \diamond \boldsymbol{H}^{(ul)}\right)^{\mathrm{T}}\right]^{\dagger}$, respectively.

Each interation of the ALS-PARAFAC receiver has three LS updating steps. At each step, one factor matrix is updated while the remaining factor matrices are assumed fixed to their values obtained in the previous steps. This procedure is repeated until the convergence of the algorithm. Denoting by

$$\varepsilon^{(i)} = \left\| [\mathcal{Y}]_{(1)} - [\hat{\mathcal{Y}}]_{(1)} \right\|_{\mathrm{F}}^2 \tag{15}$$

the residual error between the received signal tensor and the reconstructed signal tensor at the i-th iteration, defined as $[\hat{\mathcal{Y}}]_{(1)} = \hat{\boldsymbol{H}}^{(ul)} \left(\hat{\boldsymbol{F}} \diamond \hat{\boldsymbol{X}}\right)^{\mathrm{T}}$, the convergence at the i-th iteration is declared when $|\varepsilon^{(i)} - \varepsilon^{(i-1)}| \leqslant 10^{-6}$.

Remark: By assuming a coordinated scenario in which the frequency spreading matrix $\boldsymbol{F}$ is known at the BS, only column scaling ambiguity in the estimated factor matrices $\hat{\boldsymbol{H}}^{(ul)}$ and $\hat{\boldsymbol{X}}$ exists. It can be eliminated with a simple normalization procedure by assuming knowledge of the first row of $\boldsymbol{H}^{(ul)}$. In practice, the knowledge of the first row of $\boldsymbol{H}^{(ul)}$ can be obtained using a simple supervised procedure in which a known pilot symbol is sent to the BS by each MS [9]. We assume that before transmission, each MS send a known pilot sequence to estimate its link between the first receive antenna at the BS. This "pre-phase" is essential for the receiver to remove the scaling ambiguity in the estimated matrices. Note that, after the convergence of the ALS-PARAFAC receiver, an estimate for the DL channel matrix $\hat{\boldsymbol{H}}^{(dl)}$ can be obtained from the output matrix $\hat{\boldsymbol{X}} = \boldsymbol{S}\hat{\boldsymbol{H}}^{(dl)}$ through a classical training-based LS or MMSE channel estimator. The proposed ALS-PARAFAC semi-blind receiver for joint estimation of the DL and UL channels is summarized in **Algorithm 1**.

Algorithm 1: Proposed ALS-PARAFAC Semi-Blind Receiver for Joint DL and UL Channel Estimation

1. *Set* $i = 0$;
Randomly initialize $\hat{\boldsymbol{X}}_{(i=0)}$;
2. $i = i + 1$;
3. *Using* $[\mathcal{Y}]_{(1)}$, *find an LS estimate of* $\hat{\boldsymbol{H}}_{(i)}^{(ul)}$:
$$\hat{\boldsymbol{H}}_{(i)}^{(ul)} = [\mathcal{Y}]_{(1)} \left[\left(\boldsymbol{F} \diamond \hat{\boldsymbol{X}}_{(i-1)}\right)^{\mathrm{T}}\right]^{\dagger};$$
4. *Using* $[\mathcal{Y}]_{(2)}$, *find an LS estimate of* $\hat{\boldsymbol{X}}_{(i)}$:
$$\hat{\boldsymbol{X}}_{(i)} = [\mathcal{Y}]_{(2)} \left[\left(\boldsymbol{F} \diamond \hat{\boldsymbol{H}}_{(i)}^{(ul)}\right)^{\mathrm{T}}\right]^{\dagger};$$
5. *Repeat steps 2-4 until convergence.*
6. *From* $\hat{\boldsymbol{X}}$ *obtain an LS or MMSE estimate for* $\hat{\boldsymbol{H}}^{(ul)}$.

C. LS-KRF semi-blind receiver

The proposed tensor-based LS-KRF semi-blind receiver is a closed-form solution that operates on the received UL signal after frequency-domain combining. The idea is to filter the received signal tensor by exploiting the knowledge of the frequency spreading matrix $\boldsymbol{F}$ at the BS, and then solve a set of rank-1 approximation problems.

By assuming $K \geq U$, and multiplying both sides of $[\mathcal{Y}]_{(3)}^{\mathrm{T}}$ in (10) by the pseudo-inverse of $\boldsymbol{F}^{\mathrm{T}}$ from the right hand side, we obtain

$$\begin{aligned} \boldsymbol{X} \diamond \boldsymbol{H}^{(ul)} &= [\mathcal{Y}]_{(3)}^{\mathrm{T}} \left(\boldsymbol{F}^{\mathrm{T}}\right)^{\dagger} \\ &= \left[\boldsymbol{x}_1 \otimes \boldsymbol{h}_1^{(ul)}, \ldots, \boldsymbol{x}_U \otimes \boldsymbol{h}_U^{(ul)}\right] \in \mathbb{C}^{NT \times U}. \end{aligned} \tag{16}$$

In accordance with property in (1), the u-th column of (16) can be rewritten as

$$\boldsymbol{x}_u \otimes \boldsymbol{h}_u^{(ul)} = \operatorname{vec}\left(\boldsymbol{h}_u^{(ul)} \circ \boldsymbol{x}_u\right), \tag{17}$$

that denotes the vectorization operation of the rank-one matrix $\boldsymbol{W}_u = \boldsymbol{h}_u^{(ul)} \circ \boldsymbol{x}_u \in \mathbb{C}^{N \times T}$. Being $\boldsymbol{U}_u \boldsymbol{\Sigma}_u \boldsymbol{V}_u^{\mathrm{H}}$ the singular value decomposition (SVD) of $\boldsymbol{W}_u$, estimates for $\boldsymbol{h}_u^{(ul)}$ and $\boldsymbol{x}_u$, $u = 1, \ldots, U$, can be obtained by truncating this SVD to a rank-one approximation as follows [10]:

$$\hat{\boldsymbol{h}}_u^{(ul)} = \sqrt{\sigma_1} \boldsymbol{u}_1 \quad \text{and} \quad \hat{\boldsymbol{x}}_u = \sqrt{\sigma_1} \boldsymbol{v}_1^*, \tag{18}$$

where $\boldsymbol{u}_1 \in \mathbb{C}^{N \times 1}$ and $\boldsymbol{v}_1 \in \mathbb{C}^{T \times 1}$ are the first left and right singular vectors of $\boldsymbol{U}_u$ and $\boldsymbol{V}_u$, respectively, and σ_1 is the largest singular value. Final estimates for the matrices $\boldsymbol{X}$ and $\boldsymbol{H}^{(ul)}$ are obtained by repeating this process for $u = 1, \ldots, U$. The proposed LS-KRF semi-blind receiver is summarized in **Algorithm 2**.

Algorithm 2: Proposed LS-KRF Semi-Blind Receiver for Joint DL and UL Channel Estimation

for $u = 1, \ldots, U$

1. *Apply the* $\text{unvec}_{N\times T}$ *operator in the u-th column of (16) and obtain the rank-one matrix* $\boldsymbol{W}_u \in \mathbb{C}^{N\times T}$;
2. *Compute the SVD of* $\boldsymbol{W}_u = \boldsymbol{U}_u \boldsymbol{\Sigma}_u \boldsymbol{V}_u^{\mathrm{H}}$, *then obtain the estimates for the u-th column of the matrices* $\hat{\boldsymbol{H}}^{(ul)}$ *and* $\hat{\boldsymbol{X}}$ *as follows*:
   $\hat{\boldsymbol{h}}_u^{(ul)} = \sqrt{\sigma_1}\boldsymbol{u}_1$ *and* $\hat{\boldsymbol{x}}_u = \sqrt{\sigma_1}\boldsymbol{v}_1^*$,
   where $\boldsymbol{u}_1 \in \mathbb{C}^{N\times 1}$ *and* $\boldsymbol{v}_1 \in \mathbb{C}^{T\times 1}$ *are the first left and right singular vectors of* $\boldsymbol{U}_u$ *and* $\boldsymbol{V}_u$, *respectively, and* σ_1 *is the largest singular value.*

end

3. *From* $\hat{\boldsymbol{X}}$, *obtain an LS or MMSE estimate for* $\hat{\boldsymbol{H}}^{(ul)}$.

IV. Identifiability Issues

In this section, we examine the identifiability issues under which the downlink $\boldsymbol{H}^{(dl)}$ and uplink $\boldsymbol{H}^{(ul)}$ channel matrices can be jointly and uniquely recovered using the proposed tensor-based ALS-PARAFAC and LS-KRF semi-blind receivers.

A. ALS-PARAFAC semi-blind receiver

According to [11], the PARAFAC decomposition of $\boldsymbol{\mathcal{Y}}$ is essentially unique if the following Kruskal's condition, that is based in the Kruskal-rank concept (please see [11] for more details), is satisfied:

$$\kappa_{\boldsymbol{H}^{(ul)}} + \kappa_{\boldsymbol{X}} + \kappa_{\boldsymbol{F}} \geq 2U + 2. \quad (19)$$

We assume the following: (i) the antenna array response $\boldsymbol{a}_{BS}(\theta_{u,l})$ has a Vandermonde structure; (ii) the multipaths have different AoAs, AoDs and fading coefficients and (iii) the factor matrix $\boldsymbol{S}$ is randomly generated and follow a uniform distribution, while $\boldsymbol{F}$ is a DFT matrix. Under the assumptions (i) and (ii), the UL channel matrix $\boldsymbol{H}^{(ul)}$ has full rank with probability one, i.e., $\kappa_{\boldsymbol{H}^{(ul)}} = \min(N, U)$. Assumption (iii) implies that the matrices $\boldsymbol{S}$ and $\boldsymbol{F}$ have full rank, and consequently $\kappa_{\boldsymbol{X}} = \min(T, U)$ and $\kappa_{\boldsymbol{F}} = \min(K, U)$. From this analysis, the Kruskal's condition (19) can be equivalently written as follows:

$$\min(N, U) + \min(T, U) + \min(K, U) \geq 2U + 2. \quad (20)$$

Since the assumption that the number of antennas at the BS is greater than the number of MSs is reasonable in MU-MIMO scenarios, the identifiability condition (20) simplifies to

$$\min(T, U) + \min(K, U) \geq U + 2. \quad (21)$$

From condition (21), we can analyze the following scenarios:

1) Considering $T \geq U$, only $K = 2$ subcarriers are required to estimate the DL and UL channels of U MSs. Otherwise stated, the ALS-PARAFAC receiver requires a reduced number of frequency resources (subcarriers).

2) Considering $K \geq U$, a training sequence of length $T = 2$ pilots is enough to estimate the DL and UL channels of U MSs. Due to the small size of the training sequence, the ALS-PARAFAC receiver has a reduced training overhead.

B. LS-KRF semi-blind receiver

The LS-KRF receiver requires that the necessary and sufficient uniqueness condition $K \geq U$ be satisfied. Note that this condition represents a particular case of (21), which indicates that the application of the LS-KRF receiver requires a more restricted scenario compared to the ALS-PARAFAC receiver, since the number of used frequency resources (subcarriers) increases with the number of active MSs. On the other hand, the LS-KRF receiver is a closed-form solution that allows parallel (user-wise) channel estimation and symbol detection, in contrast to the ALS-PARAFAC one, where all users are processed jointly.

V. Computational complexity

A. ALS-PARAFAC semi-blind receiver

We approximate the computational complexity of the proposed ALS-PARAFAC receiver, in terms of flops, considering only the cost associated with the SVD used to calculate the matrix pseudo-inverses in the LS solutions (12) and (13). Since $N > U$, $N > T$ and $K \geqslant U$ are acceptable assumptions, each iteration of the ALS-PARAFAC requires approximately $\mathcal{O}(U^2KT + U^2KN)$ flops.

B. LS-KRF semi-blind receiver

The computational complexity of the LS-KRF receiver can be approximated as the cost to obtain the product $\boldsymbol{X} \diamond \boldsymbol{H}^{(ul)}$ from (10) added with the cost to calculate U SVD-based rank-one approximations to estimate $\hat{\boldsymbol{H}}^{(ul)}$ and $\hat{\boldsymbol{X}}$ from the previous Khatri-Rao product. Therefore, the LS-KRF receiver requires approximately $\mathcal{O}(U^2K + UNT)$ flops.

VI. Simulation Results

In this section, we present some simulation results to evaluate the performance of the proposed tensor-based ALS-PARAFAC and LS-KRF semi-blind receivers. We evaluate the performance in DL and UL channel estimations in terms of the normalized mean square error (NMSE) of the estimated matrices compared to the actual channel matrices. We consider a ULA with $N = 32$ antennas at the BS. The channel matrices are generated in accordance with (2). The results are averaged over 2000 independent Monte Carlo runs. At each run, we assume a cluster with L_u multipath components between the u-th MS and the BS, in which L_u is set randomly between one and five for each MS. Equal AoAs and AoDs ($\theta_{u,l} = \phi_{u,l}$) to the l-th path of the u-th MS are generated in the interval $\theta_{u,l} = [\theta_u \pm \frac{\Delta\theta_u}{2}]$ for $l = 1, \ldots, L_u$, while the nominal angles θ_u for $u = 1, \ldots, U$ are randomly distributed in the interval $[0, 2\pi]$ and the angular spread $\Delta\theta_u$ are in the interval $[0, \frac{\pi}{12}]$. The complex gains $\alpha_{u,l}^{(dl,ul)}$ follow a complex-valued Gaussian distribution with zero-mean and unit variance. We also assume that the known training sequence $\boldsymbol{S}$ is BPSK modulated.

In Fig. 2, the performance is evaluated as a function of the number K of subcarriers. The parameters T and U are fixed to $T = 16$ and $U = 4$, respectively. The SNR in the DL and UL are set equal to 10 dB. This experiment shows that the accuracy in the DL channel estimate is not affected by the number of subcarriers used. In contrast, when

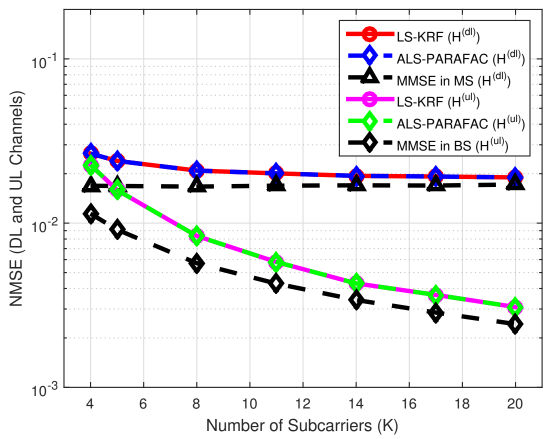

Fig. 2. NMSE (DL and UL Channels) vs. number of subcarriers (K) for $N = 32$, $T = 16$, $U = 4$ and SNR = 10 dB.

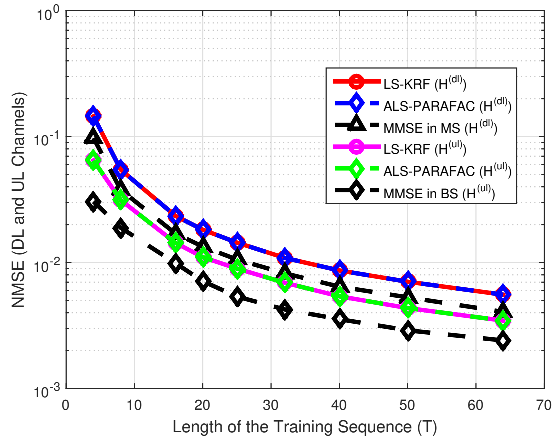

Fig. 3. NMSE (DL and UL Channels) vs. length of the training sequence (T) for $N = 32$, $K = 4$, $U = 4$ and SNR = 10 dB.

the number of subcarriers increases, the proposed receivers get better performance in the UL channel estimation. We also observe a strong proximity between the proposed methods and the MMSE-based approach (in which each MS estimates its DL channel individually).

In Fig. 3, we plot the NMSE as a function of the length T of the training sequence. We consider $K = 4$, $U = 4$ and the same SNR as the previous experiment. We can see that the performance of all methods improves when T increases. In the important case of $T < N$ we can notice that the proposed receivers present a performance very close to the MMSE estimator. For a massive scenario, this result implies in a satisfactory performance of the proposed methods even when a reduced length training sequence is used. In this case, the proposed tensor-based semi-blind receivers achieve a substancial training overhead reduction.

From these simulation results, we can notice that the proposed receivers present a performance close to the competing MMSE estimator. However, the latter requires independent estimation at the MSs and BS sides. Futhermore, assumptions such as channel reciprocity and limited feedback control signaling for each MS must be guaranteed when the MMSE channel estimator is used. In contrast, the proposed receivers concentrate the processing burden for joint DL and UL channel estimation at the BS avoiding, such previously mentioned, practical limitations as well as high complexity processing for channel estimation at each power-limited MS. Despite the same performance between the proposed tensor-based receivers, the LS-KRF receiver is more limited due to its more restrictive identifiability condition.

VII. Conclusion

We have adressed the joint DL and UL channel estimation problem for MU-MIMO wireless communication systems. We first propose a novel multicarrier-based training scheme able to concentrate the processing burden for channel estimation at the BS. Then, we have proposed the iterative ALS-PARAFAC and closed-form LS-KRF tensor-based receivers for joint DL and UL channel estimation. The proposed methods provide accurate DL and UL estimates and dispenses assumptions such as channel reciprocity and dedicated channels, which are common in classical TDD and FDD systems, respectively. Additionally, the proposed receivers achieve channel estimation accuracy close to their equivalent MMSE channel estimator.

VIII. Acknowledgment

This work has been supported by Ericsson Research, Technical Cooperation contract UFC.47.

Referências

[1] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: benefits and challenges," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 742-758, Oct. 2014.

[2] D. Gesbert, M. Kountouris, R. W. Heath. Jr., C. Chae, and T. Salzer, "From single user to multiuser communications: shifting the MIMO paradigm," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 36-46, Sep. 2007.

[3] T. L. Marzeta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Transactions on Wireless Communication*, vol. 9, no. 11, pp. 3590-3600, Nov. 2010.

[4] J. Choi, D. J. Love, and P. Bidigare, "Downlink training techniques for FDD massive MIMO systems: open-loop and closed-loop training with memory," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 802-814, Oct. 2014.

[5] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao and M. Andrews, "An overview of limited feedbak in wireless communication systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 26, no. 8, pp. 1341-1365, Oct. 2008.

[6] J. Guey and L. D. Larsson, "Modeling and evaluation of MIMO systems exploiting channel reciprocity in TDD systems," *in Proc. IEEE Veh. Technol. Conf.*, Los Angeles, USA, Sep. 2004, pp. 4265-4269.

[7] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *Society for Industrial and Applied Mathematics*, vol. 51, no. 3, pp. 455-500, Aug. 2009.

[8] A. Smilde, R. Bro and P. Geladi, "Multi-way analysis: applications in the chemical sciences," John Wiley and Sons, Chichester, U.K., 2004.

[9] C. A. R. Fernandes, G. Favier and J. C. M. Mota, "PARAFAC-based channel estimation and data recovery in nonlinear MIMO spread spectrum communication systems," *Signal Processing*, vol. 91, pp. 311-322, 2011.

[10] F. Roemer and M. Haardt, "Tensor-based channel estimation and iterative refinements for two-way relaying with multiple antennas and spatial reuse," *IEEE Transactions on Signal Processing*, vol. 58, no. 11, pp. 5720-5735, Nov. 2010.

[11] J. B. Kruskal, "Three-way arrays: rank and uniqueness of trilinear decompositions, with application to arithmetic complexity and statistics," *Society for Industrial and Applied Mathematics*, vol. 51, no. 3, pp. 455-500, Aug. 2009.