Joint Bit Loading and Subcarrier Matching in Two-Hop Cooperative Systems

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***Abstract*— Relaying and cooperative communications are key technologies present in 4G (4th Generation) and probably in 5G (5th Generation) networks. Efficient RRA (Radio Resource Allocation) applied in relay networks has the potential to fully exploit the performance gains in terms of spatial diversity, coverage and spectral/energy efficiency. Previous works in the literature have studied RRA for relay networks considering the unrealistic assumption of continuous mapping between SNR (Signal-to-Noise Ratio) and transmit data rate. In fact, the mapping between channel quality state and transmit data rates in real systems is discrete and depends on the employed MCSs (Modulation and Coding Schemes). In this work we reformulate the total data rate maximization problem with this new assumption, show an approach to obtain the optimal solution and propose low-complexity quasi-optimal solutions to the considered scenario.**

***Keywords*— Subcarrier matching, power allocation, cooperative communications, combinatorial optimization.**

I. INTRODUCTION

Relaying technology is an important driver of heterogeneous networks that is part of 4G (4th Generation) standard and has been considered as a relevant component of 5G (5th Generation). Relaying technology is able to provide spatial diversity gains and coverage extension, improving spectral efficiency, reducing interference and achieving energy efficiency [1].

The relay architecture and the potential cooperation among network nodes allow for new degrees of freedom when allocating system resources especially when OFDMA (Orthogonal Frequency Division Multiple Access) is assumed. Therefore, the potential gains of relaying technology can only be fully exploited if efficient RRA (Radio Resource Allocation) is performed. RRA algorithms are responsible for managing the scarce radio resources such as power, time slots, spatial channels and frequency chunks [2].

Transmit power and subcarriers are the main system radio resources to be optimized by RRA in relay networks. In [3] the authors consider the transmit power optimization in single-channel relay networks in order to minimize the outage probability. Subcarrier and power allocation optimization was studied in [4]. In that article, the authors present the concept of *subcarrier pairing* or *matching* that consists in associating a subcarrier of one hop with a subcarrier of the next hop. In this case, the data transmitted on a subcarrier in the previous hop is forwarded to the corresponding paired subcarrier of the next hop. A proper matching among subcarriers can lead to important performance gains.

In [5] the need for subcarrier matching is well motivated and the authors prove that assuming equal power allocation, the optimal subcarrier matching for data rate maximization consists in sorting the subcarriers of the two hops according to the channel state. Subcarrier matching based on channel gain sorting was also proposed in parallel by [6] and was evaluated only through computational simulations. Majorization theory was used by [7] in order to prove that channel gain sorting is optimal for subcarrier matching with equal power allocation when the objective to be optimized is the total data rate or the sum of the SNR (Signal-to-Noise Ratio). Closed form expressions for average end-to-end SNR were provided in [8] assuming channel gain sorting subcarrier matching.

The joint subcarrier matching and power allocation was first optimally solved in [9] assuming a unique total power constraint for source and relay nodes, where it was shown that subcarrier matching and power allocation can be decoupled into two subproblems without loss of optimality for the total data rate maximization. The subcarrier matching is optimally solved by sorting the subcarriers according to the channel gains while the power allocation is solved by water filling. The work [10] consider the same problem as [9] but with the realistic approach of separate or individual total power constraint per node. In fact, the assumption of separate total power constraint per node turns the analysis of the problem more difficult. The work [11] proves that the channel gain sorting is an optimal strategy of subcarrier matching even when the number of hops is higher than two.

All previous works have considered that the mapping between SNR and transmit data rate is done by a continuous log-shaped function such as the Shannon capacity expression. Furthermore, some of the proofs and algorithms are strongly dependent of that continuous mapping. However, in practical systems the set of achievable transmit data rates is discrete and finite. For each combination of modulation schemes and channel coding rates, different data rates can be transmitted according to the SNR. The work [12] has considered a discrete mapping between SNR and transmit data rate for a different problem involving energy efficiency. The modelling of discrete MCSs (Modulation and Coding Schemes) has an important impact on the structure of the problem studied in the previous works: the problem becomes completely combinatorial. Therefore, the power allocation subproblem cannot be solved by waterfilling-like algorithms or convex optimization methods.

As far we know, the total data rate maximization problem in a two hop scenario with discrete MCSs has not been studied

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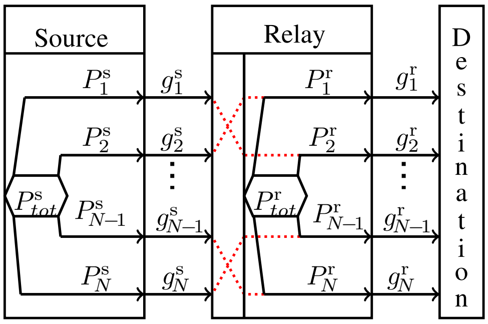


Fig. 1. System model of a two-hop system.

in the literature. Our main contributions in this article are: Problem formulation assuming a discrete mapping between SNR and transmit data rate; Optimal solution of the studied problem by converting the original problem that is nonlinear integer in an ILP (Integer Linear Problem); Proposal of efficient and low-complexity suboptimal solutions for the studied problem.

The remainder of the article is organized as follows. In section II we present the system model and main assumptions. In section III we formulate the studied problem in optimization form and in section IV we show an approach to obtain the optimal solution. Low-complexity and efficient solutions to the studied problem are provided in section V whereas performance results are presented in section VI. The main conclusions and perspectives are presented in section VII.

II. System Modeling

The considered scenario consists in a two-hop cooperative system containing a source, a relay and a destination as illustrated in Figure 1. The system has a total of N OFDM (Orthogonal Frequency Division Multiplexing) subcarriers that are commonly used in the first hop (Source-Relay) and in the second hop (Relay-Destination) multiplexed in time, i.e., in the first time slot the N OFDM subcarriers are used to transmit data from source to relay and in the next time slot the N subcarriers are reused to transmit data from relay to the destination. We assume that the subcarriers experience Rayleigh fading and that the system nodes have perfect CSI (Channel State Information).

The total available power at the source and relay nodes that should be distributed among subcarriers are $P^{\text{s}}_{\text{tot}}$ and $P^{\text{r}}_{\text{tot}}$, respectively. The variables P^{s}_n and P^{r}_n $\forall n \in \{1, \dots, N\}$ are the allocated power to subcarrier n in the first and second hops, respectively. We define $g^{\text{s}}_n \equiv \frac{|h^{\text{s}}_n|^2}{\sigma^2}$ and $g^{\text{r}}_q \equiv \frac{|h^{\text{r}}_q|^2}{\sigma^2}$ as the normalized channel gains for the subcarriers n and $q \in \mathcal{N}$ of hops 1 and 2, respectively; h^{s}_n and h^{r}_q as the channel frequency response for the subcarriers n and $q \in \mathcal{N}$ of hops 1 and 2, respectively; and σ^2 as the average noise power.

The bits transmitted on each subcarrier of the first hop should be retransmitted in a corresponding (paired) subcarrier of the second hop. We define as subcarrier matching the system functionality responsible for associating the subcarriers of the first hop with the ones of the second hop. As an example, we can see in Figure 1 that subcarriers 1, 2, $N-1$ and N of the first hop are paired with the subcarriers 2, 1, N and $N-1$ of the second hop. Furthermore, we assume that the DF (Decode and Forward) protocol is applied in the relay node.

Differently of the majority of the works in the literature, we consider here that the mapping between channel quality state, e.g., SNR, and the transmit data rate is represented by a discrete mapping. We consider a generic monotonic increasing function $f(\cdot)$ that maps the achieved SNR to a specific discrete transmit data rate. According to this, the transmit data rate r_m is achieved when the SNR is higher than or equal to SNR_m and lower than SNR_{m+1}, where $SNR_i \, \forall i$ can be obtained from link level curves and $SNR_m < SNR_{m+1}$. The function $f(\cdot)$ models the use of discrete MCSs in practical systems. In this article, the MCS m is a corresponding modulation and channel coding scheme that leads to the transmit data rate of r_m. We assume a total of M MCS levels.

III. Problem Formulation

Before formulating the joint subcarrier matching and bit loading optimization problem that maximizes the spectral efficiency of a two-hop relaying system, we present the definition of other important variables of our model. $y^{\text{s}}_{n,m}$ is a binary variable that assumes the value 1 if the m^{th} MCS level is used in subcarrier n of the first hop, and assumes the value 0 otherwise. Furthermore, $y^{\text{r}}_{n,q,m}$ is a binary variable which assumes value 1 if the m^{th} MCS level is used in subcarrier q of the second hop that is matched to subcarrier n of the first hop, and assumes the value 0 otherwise. Both variables $y^{\text{s}}_{n,m}$ and $y^{\text{r}}_{n,q,m}$ are the optimization variables of our problem since they are responsible for the subcarrier matching and bit loading in the hops 1 and 2.

The minimum power needed for the subcarrier n of first hop to achieve the MCS level m is given by $P^{\text{s}}_{n,m}$. Analogously, $P^{\text{r}}_{q,m}$ is the minimum power needed to subcarrier q of second hop achieve the MCS level m. These transmit power values can be previously calculated based on the knowledge of channel frequency response, noise power and link adaptation function $f(\cdot)$. Finally, $\mathcal{N} = \{1, \dots, N\}$ is the set of all subcarriers and $\mathcal{M} = \{1, \dots, M\}$ is the set of all MCS levels.

The optimization problem that aims to maximize the end-to-end data rate in two-hop system is formulated as follows:

$$\max_{\mathbf{y}^{\text{s}}, \mathbf{y}^{\text{r}}} \sum_{n=1}^{N} \min \left\{ \sum_{m=1}^{M} y^{\text{s}}_{n,m} \, r_m, \sum_{q=1}^{N} \sum_{m=1}^{M} y^{\text{r}}_{n,q,m} \, r_m \right\} \tag{1a}$$

subject to :

$$\sum_{m=1}^{M} y^{\text{s}}_{n,m} = 1 \text{ and } \sum_{q=1}^{N} \sum_{m=1}^{M} y^{\text{r}}_{n,q,m} = 1, \; \forall n \in \mathcal{N}, \tag{1b}$$

$$\sum_{n=1}^{N} \sum_{m=1}^{M} y^{\text{r}}_{n,q,m} = 1, \; \forall q \in \mathcal{N}, \tag{1c}$$

$$\sum_{n=1}^{N} \sum_{m=1}^{M} y^{\text{s}}_{n,m} \, P^{\text{s}}_{n,m} \leq P^{\text{s}}_{\text{tot}}, \tag{1d}$$

$$\sum_{n=1}^{N} \sum_{q=1}^{N} \sum_{m=1}^{M} y^{\text{r}}_{n,q,m} \, P^{\text{r}}_{n,q,m} \leq P^{\text{r}}_{\text{tot}}, \tag{1e}$$

where $\mathbf{y}^{\text{s}}$ and $\mathbf{y}^{\text{r}}$ are vectors that collect the variables $y^{\text{s}}_{n,m}$ and $y^{\text{r}}_{n,q,m}$, respectively. The objective function (1a) represents the end-to-end data rate and is nonlinear due to the use of $\min\{\cdot,\cdot\}$ operator, that is a consequence of the DF protocol employed in the relay. The constraints (1b) and (1c) ensure that all subcarriers will achieve some MCS level and the subcarrier pairing occurs in a one-to-one relation. The equations (1d) and (1e) represent power constraints in the source and relay, respectively.

IV. Optimal Solution

The problem presented in 1 belongs to the class of nonlinear integer optimization problems that are in general hard to be solved optimally. A direct approach to optimally solve this problem is to employ the exhaustive search or brute force procedure considering that problem 1 can be split into SM (Subcarrier Matching) and BL (Bit Loading) subproblems. Considering exhaustive search, the total number of possible solutions for SM and BL are $N!$ and M^{2N}, respectively. Therefore, the exhaustive search procedure should enumerate $N!M^{2N}$ solutions to find the optimum one.

Due to the high computational complexity of obtaining the optimal solution by exhaustive search, we propose here a reformulation of the original problem in order to obtain an ILP. In order to do that we use a strategy to linearize the objective function at the cost of adding new auxiliary optimization variables and constraints. Specifically, the maximization of $\sum_{\forall i}\min\{A_i, B_i\}$ is equivalent to maximize $\sum_{\forall i} x_i$ subject to $x_i \leq A_i \;\; \forall i$ and $x_i \leq B_i \;\; \forall i$. Based on this we get the problem (1) in ILP form. This class of problem can be optimally solved by algorithms based on BB (Branch and Bound) method that is capable of drastically reducing the search space. Although BB-based solutions are more efficient than exhaustive search procedures, its worst-case computational complexity is exponential in terms of the number of subcarriers and MCS schemes: $\mathcal{O}\left(2^{MN^2}\right)$. Accordingly, efficient and low-complexity solutions are required for this problem.

V. Low Complexity Solution(s)

In order to obtain a low-complexity solution for the studied problem we split it into two subproblems: SM and BL. For each subproblem we proposed different heuristics that will be evaluated later in this article.

A. SM subproblem

According to the mapping between SNR and data rate shown in section II, we have that $r_m = f(SNR_m)$. Therefore, we can rewrite the equation (1a) as following:

$$r^{\text{s,r}}_n = \min\left\{f(P^{\text{s}}_{n,m}\, g^{\text{s}}_n), f(P^{\text{r}}_{n,q,m}\, g^{\text{r}}_q)\right\}, \forall n \in \mathcal{N}. \quad (2)$$

If we fixed BL-scheme in (2), i.e., $P^{\text{s}}_{n,m}$ and $P^{\text{r}}_{n,q,m}$ are constant, the data rate $r^{\text{s,r}}_n$ is a function on the variables g^{s}_n and g^{r}_q. Since that the function $f(\cdot)$ is monotonic increasing and the data rate $r^{\text{s,r}}_n$ is limited by the hop of lowest rate, we can conclude that if the gain channels g^{s}_n and g^{r}_q are very unbalanced ($g^{\text{s}}_n \gg g^{\text{r}}_q$ or $g^{\text{s}}_n \ll g^{\text{r}}_q$) then the value of $r^{\text{s,r}}_n$ will be low. Based in this reasoning, we propose two SM configurations:

- **Channel gain sorted SM**: The subcarriers of the first and second hops are sorted in the descending order according to the normalized channel, g^{s}_n and g^{r}_n $\forall n \in \mathcal{N}$, respectively. Assume that after the sorting procedure we have $g^{\text{s}}_1 \geq g^{\text{s}}_2 \geq \ldots \geq g^{\text{s}}_N$ and $g^{\text{r}}_1 \geq g^{\text{r}}_2 \geq \ldots \geq g^{\text{r}}_N$. Therefore, the subcarrier pairing is given by $(g^{\text{s}}_1, g^{\text{r}}_1), (g^{\text{s}}_2, g^{\text{r}}_2), \ldots, (g^{\text{s}}_N, g^{\text{r}}_N)$;
- **Random SM**: In this case the subcarriers of the first hop are paired with the ones of the second hop at random.

B. BL subproblem

Once the SM is done, i.e., the subcarriers of the first and second hops are matched, the bit loading should be applied. Assume, without loss of generality, that the subcarriers indices of the first and second hops are rearranged after applying SM solution so that the subcarriers pairs are $(g^{\text{s}}_1, g^{\text{r}}_1), (g^{\text{s}}_2, g^{\text{r}}_2), \ldots, (g^{\text{s}}_N, g^{\text{r}}_N)$. In the following, we present two bit loading solutions.

1) HH (Hughes Hartogs)-based solution: In multi-channel (subcarriers) point-to-point connections, the optimal power allocation when the mapping between SNR and transmit data rate is discrete is the well known HH solution [13]. The main idea of HH algorithm is to iteratively increase the transmit data rate (or MCS level) of the subcarriers that need less transmit power. Here, we propose applying the HH solution in both links source-relay and relay-destination separately. Note that, according to this solution, the achieved data rates (or MCS levels) of two paired subcarriers may differ. In this case the actual end-to-end transmit data rate of the paired subcarriers is given by equation 1a.

2) MPRP (Maximization of the Product of the Residual Powers): Our second proposal is the iterative MPRP solution shown in Algorithm 1. The MPRP solution is based on two premises:

- Same MCS level for paired subcarriers: As the end-to-end transmit data rate through paired subcarriers is limited by the worst link, i.e., source-relay or relay-destination, any difference of MCS level for paired subcarriers would be waste of transmit power;
- Efficient use of transmit power: The iterative update of MCS levels employed on each paired subcarriers should be based on how efficient is the use of the transmit power at the source and relay.

Before describing the algorithm, some important variables should be explained. The variables $P^{\text{s}}_{m,n}$ and $P^{\text{r}}_{m,n}$ consist in the transmit power needed to achieve the MCS level m at subcarrier n in the first and second hops, respectively. These variables are calculated in the lines 1 where SNR_m, g^{s}_n and g^{r}_n were defined in section II. The variables $\Delta P^{\text{s}}_{m,n}$ and $\Delta P^{\text{r}}_{m,n}$ calculated between lines 2 and 4 are the additional power needed to update the MCS level from $m-1$ to m of subcarrier n at the first and second hops, respectively. The binary variable $\beta^{\text{s,r}}_n$ is used along the algorithm to evaluate whether the MCS level of the paired subcarriers n of the first and second hops can be updated without violating the total available transmit

Algorithm 1 MPRP

1: $P^{\mathrm{s}}_{m,n} = SNR_m/g^{\mathrm{s}}_n$ and $P^{\mathrm{r}}_{m,n} = SNR_m/g^{\mathrm{r}}_n, \forall m \in \mathcal{M}$ and $\forall n \in \mathcal{N}$
2: $\Delta P^{\mathrm{s}}_{m-1,n} = P^{\mathrm{s}}_{m,n} - P^{\mathrm{s}}_{m-1,n}, \forall m \in \{M, ..., 2\}$ and $\forall n \in \mathcal{N}$
3: $\Delta P^{\mathrm{r}}_{m-1,n} = P^{\mathrm{r}}_{m,n} - P^{\mathrm{r}}_{m-1,n}, \forall m \in \{M, ..., 2\}$ and $\forall n \in \mathcal{N}$
4: $\Delta P^{\mathrm{s}}_{M,n} = P^{\mathrm{s}}_{\mathrm{tot}}$ and $\Delta P^{\mathrm{r}}_{M,n} = P^{\mathrm{r}}_{\mathrm{tot}}, \forall n \in \mathcal{N}$
5: $\epsilon^{\mathrm{s}}_n = P^{\mathrm{s}}_{\mathrm{tot}} - \Delta P^{\mathrm{s}}_{1,n}$ and $\epsilon^{\mathrm{r}}_n = P^{\mathrm{r}}_{\mathrm{tot}} - \Delta P^{\mathrm{r}}_{1,n}, \forall n \in \mathcal{N}$
6: **for** $n \leftarrow 1$ **to** N **do**
7:     **if** $\epsilon^{\mathrm{s}}_n \geq 0$ **and** $\epsilon^{\mathrm{r}}_n \geq 0$ **then** $\beta^{\mathrm{s,r}}_n = 1$
8:     **else** $\beta^{\mathrm{s,r}}_n = 0$
9:     **end if**
10: **end for**
11: **while** $\sum_{\forall j \in \mathcal{N}} \beta^{\mathrm{s,r}}_j > 0$ **do**
12:     $f_n = (\epsilon^{\mathrm{s}}_n \cdot \epsilon^{\mathrm{r}}_n \cdot \beta^{\mathrm{s,r}}_n), \forall n \in \mathcal{N}$
13:     $i = \mathbf{argmax}(f_1, \ldots, f_n)$
14:     $\theta^{\mathrm{s,r}}_i = \theta^{\mathrm{s,r}}_i + 1$
15:     $aux^{\mathrm{s}} = \epsilon^{\mathrm{s}}_i \quad aux^{\mathrm{r}} = \epsilon^{\mathrm{r}}_i$
16:     $t_n = \theta^{\mathrm{s,r}}_n + 1, \forall n \in \mathcal{N}$
17:     $\epsilon^{\mathrm{s}}_n = aux^{\mathrm{s}} - \Delta P^{\mathrm{s}}_{t_n,n}$ and $\epsilon^{\mathrm{r}}_n = aux^{\mathrm{r}} - \Delta P^{\mathrm{r}}_{t_n,n}, \forall n \in \mathcal{N}$
18:     **for** $n \leftarrow 1$ **to** N **do**
19:         **if** $\epsilon^{\mathrm{s}}_n \geq 0$ **and** $\epsilon^{\mathrm{r}}_n \geq 0$ **then** $\beta^{\mathrm{s,r}}_n = 1$
20:         **else** $\beta^{\mathrm{s,r}}_n = 0$
21:         **end if**
22:     **end for**
23: **end while**

power at the source and relay nodes. It assumes the value 1 when the total power constraints are not violated and 0, otherwise. The variable $\theta^{\mathrm{s,r}}_n$ controls the current MCS level assigned to the paired subcarriers n of the first and second hops.

In the main loop of the MPRP algorithm between lines 11 and 23, the MCS levels of each paired subcarriers are iteratively updated. In line 12, the utility of selecting the n^{th} subcarrier of the first and second hops to have the MCS level updated, f_n, is quantified. Our utility function consists in the product of ϵ^{s}_n, ϵ^{r}_n and $\beta^{\mathrm{s,r}}_n$. The variables ϵ^{s}_n and ϵ^{r}_n consist in the remaining or residual power at the source and relay nodes, respectively, if the subcarrier n of the first and second hops have their MCS level increased by a unit. In line 13 the paired subcarriers with maximum utility is selected. Therefore, the selected paired subcarriers to have the MCS level updated are the ones that most efficiently use the available transmit power at the source and relay nodes. In line 14 the current MCS level of the n^{th} subcarriers of the first and second hops, $\theta^{\mathrm{s,r}}_n$, are updated. Between lines 17 and 22 the variables ϵ^{s}_n, ϵ^{r}_n and $\beta^{\mathrm{s,r}}_n$ are updated according to the previous decision.

VI. Performance Evaluation

A. Simulation parameters

The performance of the proposed solutions in this article were evaluated by means of Monte Carlo simulations. Basically, we consider an OFDM system with $N = 64$ subcarriers, each one experiencing Rayleigh-distributed fading. The channel gain in each subcarrier of hops 1 and 2 is a exponential random variable with mean $\overline{g}^{\mathrm{s}} = N\,\overline{SNR}^{\mathrm{s}}/P^{\mathrm{s}}_{\mathrm{tot}}$ and $\overline{g}^{\mathrm{r}} = N\,\overline{SNR}^{\mathrm{r}}/P^{\mathrm{r}}_{\mathrm{tot}}$, respectively. The source and relay nodes have each one an available transmit power of $P^{s}_{\mathrm{tot}} = P^{r}_{\mathrm{tot}} = 1W$. According to the model presented in section II, we assume that there are 12 possible MCS levels $m \in \{1, 2, ..., 12\}$ and the data rate achieved in each MCS level is $r_m \in \{0, 1, ..., 11\}$ bits and the minimal SNR needed to transmit r_m bits is given by $SNR_m = 2^{r_m/0.9} - 1$, i.e., the mapping between SNR and data rate corresponds to the discretization of 90% of the Shannon capacity. For each simulated point, we considered 4,000 independent Monte Carlo repetitions.

The algorithms to be evaluated are the combinations of SM solutions (channel gain sorted and random) and BL solutions (HH-based BL and MPRP). Additionally, we also simulated the optimal solution obtained according to section IV. In order to obtain the optimal solution we used the IBM ILOG CPLEX software that provides an ILP solver [14].

The performance is measured in terms of the achieved spectral efficiency of the presented solutions since this is the objective of the proposed optimization problem, i.e., problem 1. The system load is emulated by varying the averaged SNR experienced at the first and second hops.

B. Simulation results

In Figure 2 we present the relative performance loss of each proposed solution compared with the optimal solution when the average SNR of the first and second hops are the same. Firstly, we can see that the performance of all solutions are improved (relative to the optimal one) as the average SNR of the two hops is increased. This is expected since the gains of adaptive algorithms such as subcarrier assignment and power allocation are mainly noticeable in low-SNR regime, where the need for intelligence in resource allocation is less significant since any choice for resource allocation are good enough.

When the performance of the SM strategies are concerned, we can observe that the channel gain sorted solution achieves the best performance independently of the considered BL solution. This is a consequence of the end-to-end transmit data rate expression that is limited by the worst hop. Basically, when the channel gains of the links in both hops are (directly) sorted, the potential of the subcarriers can be better exploited and less radio resources are wasted. This is not the case when the subcarriers are randomly paired. The best combined SM and BL solution is the channel gain sorted with the MPRP solution (or shortly, sorted + MPRP). We can see that the relative performance loss of this solution to the optimal one is not greater than $3{\cdot}10^{-2}$% for the simulated SNR range (almost optimal). The MPRP algorithm boosts the performance of the considered system since it assures that the same MCS is used for paired subcarriers in both hops, and also allocates power to the paired subcarriers that can use it more efficiently. These premises are not present in the HH-based BL solution.

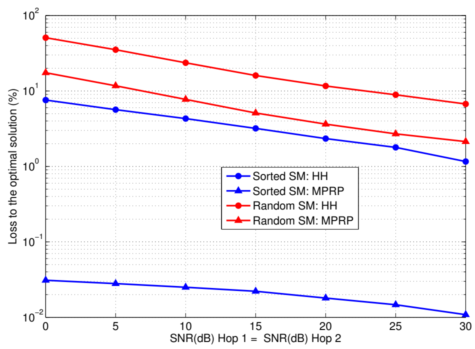
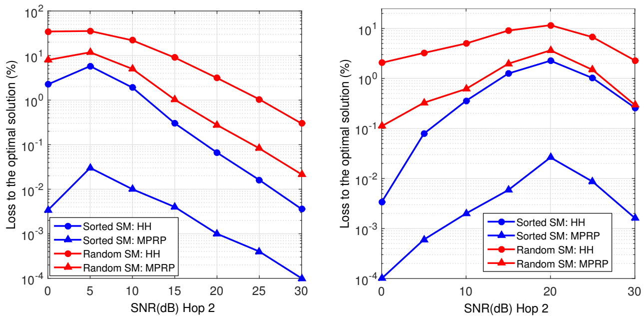


Fig. 2. Relative performance loss of the proposed solution to the optimal one versus the average SNR of hops 1 and 2.



(a) (b)

Fig. 3. (a) and (b) are relative performance loss of the proposed solution to the optimal one versus the average SNR of hop 2. In (a) the average SNR of hop 1 was fixed at 5 dB and (b) was fixed at 20 dB.

In Figures 3(a) and 3(b) we present the relative performance loss to the optimal solution of the proposed solutions when the average SNR of the first hop is fixed in 5dB and 20 dB, respectively, and the average SNR of the second hop is varied. We can see in these figures that the relative performance among the presented algorithms is the same as presented in Figure 2. The sorted + MPRP once again is able to perform almost optimally with a maximum performance loss of $3{\cdot}10^{-2}\%$.

Another aspect to highlight in Figures 3(a) and 3(b) is that the worst performance, i.e., maximum relative performance loss to the optimal solution, occurs in general when the average SNR of both hops are the same. In order to understand this result, we should firstly see that the maximum transmit data rate is achieved when the SNR of both hops are the same, since the end-to-end transmit data rate is limited by the worst link. In this case, the adaptive resource allocation strategies have maximum "degrees of freedom" to improve the system performance. As the hops becomes unbalanced, the system performance is dominated by the worst hop independently of the resource allocation decisions taken in the other hop. In this case (as shown in Figures 3(a) and 3(b)), different adaptive solutions tend to present similar performance. Therefore, the performance of the algorithms shown in Figure 2 can be considered as the worst-case scenario.

The computational complexity of the sorted SM + MPRP solution is dominated by the operation in the line 13. As the main loop iterates at most NM times and to find the maximum we need $N-1$ comparisons, this solution has a worst-case computational complexity equal to $\mathcal{O}(N^2M)$ (polynomial). As shown in previous figures, the sorted + MPRP solution presents an almost-optimal performance and reduced the computational complexity when compared with the optimal solution that is exponential: $\mathcal{O}\left(2^{MN^2}\right)$. Therefore, the proposed sorted SM + MPRP solution is able to achieve a good performance-complexity trade-off.

VII. Conclusions

In this article we revisit the problem of maximizing the total data rate in relay networks by adding a practical and appealing aspect in modern networks: discrete MCS (Modulation and Coding Scheme) levels. This aspect has been ignored in previous studies. The impact of introducing this feature changes the class of the formulated optimization problem: the problems formulated in the literature were continuous whereas our new formulated problem is nonlinear integer.

After some algebraic manipulations and introduction of new variables and constraints we were able to convert the nonlinear integer optimization problem in an ILP (Integer Linear Problem). This problem can be solved by solvers based on BB (Branch and Bound) technique. Due to the high computational complexity, we structured the original problem in SM (Subcarrier Matching) and BL (Bit Loading) subproblems. Then, we proposed some heuristic strategies for each subproblem. Through performance results, we showed that the sorted SM + MPRP (Maximization of the Product of the Residual Powers) solution performed almost optimally with a relative loss to the optimal solution no greater than 0.03%. This good performance is obtained with a drastically lower computational complexity compared to the optimal solution.

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