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A CONSTRAINED VERSION OF FITTING PDF ALGORITHM FOR BLIND SOURCE SEPARATION

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ABSTRACT

An adaptive multiuser constrained version of a fitting probability density function (pdf) algorithm (FPA) for source separation is proposed. The novel algorithm is derived from the multiuser kurtosis (MUK) procedure proposed in the literature. In fact, the proposed approach cay be viewed as a generalization of MUK one. The resulting algorithm is evaluated through computational simulations in the context of a space division multiple access (SDMA) system. The obtained results show that a better performance is attained w.r.t. the unconstrained version.

1. INTRODUCTION

The problem of blind source separation (BSS) has been largely investigated in the last 10 years due to its wide applicability in several fields [1]. In fact, the search of unsupervised techniques has begun in the middle of 70's with Sato [2] and later in 80 with Godard [3], but in those works only single-input single-output (SISO) systems were considered.

Multiple-input multiple-output (MIMO) systems have received a lot of attention and interest during the last decade with the several multiple access systems that emerged at that time. Through BSS techniques, several strategies have shown promising results in the context of multiuser detection. To cite a few references, [4–6] reveal the potential of equalization strategies to source separation.

In a previous work [7] we have proposed an algorithm for source separation based on probability density function (pdf) estimation, called fitting pdf (FP) algorithm, that uses the explicit decorrelation procedure proposed in [6].

Recently, the algorithm based on the kurtosis minimization, called multiuser kurtosis (MUK) algorithm, have been shown as a plausible alternative to the task of source separation due to its globally convergence ability, assured by a constraint over the global system response [8, 9].

In the present paper, we propose the use of the orthogonality constraint criterion [8, 9], jointly with FP criterion, instead of the explicit decorrelation procedure in order to provide separation of the sources.

The paper is organized as follows. In Section 2 is devoted to our assumptions about the system model as well as some recalls about the FP algorithm. On Section 3 the MUK algorithm is recalled. The proposed constrained FP algorithm is the posed in Section 4. Simulation results are presented in Section 5 and, finally, our conclusions are stated in Section 6.

2. SYSTEM MODEL AND FITTING PDF CRITERION

2.1. System Model

We consider an space division multiple access (SDMA) scenario with several users transmitting simultaneously. By the use of a linear antenna array in the receiver, we can write the discrete time model for the received signal of the K users as

$$\mathbf{x}(n) = \sum_{k=1}^{K} \sqrt{P_k} \cdot a_k(n) \cdot \mathbf{f}\left(\theta_k\right) + \mathbf{v}(n), \qquad (1)$$

where P_k is the transmission power, a_k is the symbol sequence, $\mathbf{v}(n)$ is a vector of spatial AWGN samples, θ_k is the direction of arrival (DOA) of the k-th user and $\mathbf{f}\left(\theta_k\right) = \left[\begin{array}{ccc} f_1\left(\theta_k\right) & \cdots & f_M\left(\theta_k\right) \end{array}\right]^T$ is the array response vector where the array phase response at the m-th antenna is modeled as

$$f_m\left(\theta_k\right) = \exp\left(\frac{2 \cdot d \cdot \pi \cdot (m-1) \cdot \sin\left(\theta_k\right)}{\lambda}\right), \qquad (2)$$

where d is the inter-element spacing in the array and λ is the carrier wavelength. We assume in this paper that $d = \frac{\lambda}{2}$. The channel for all users is then denoted by the matrix $\mathbf{F} = \left[\ \mathbf{f}(\theta_1) \ \mid \ \cdots \ \mid \ \mathbf{f}(\theta_K)\ \right]$.

In order to detect/separate multiple users, adaptive spatial filters are provided to each desired user as shown in Figure 1. The output for the k-th user, provided by the k-th beamformer, denoted by $\mathbf{w}_k$, is given at time instant n by

$$y_k(n) = \mathbf{w}_k^H(n)\mathbf{x}(n), \tag{3}$$

where the superscript ^{H} stands for Hermitian transposition. Moreover, the vector of beamformers outputs is denoted by $\mathbf{y}(n) = \left[\ y_1(n) \ \cdots \ y_K(n)\ \right]^T$.

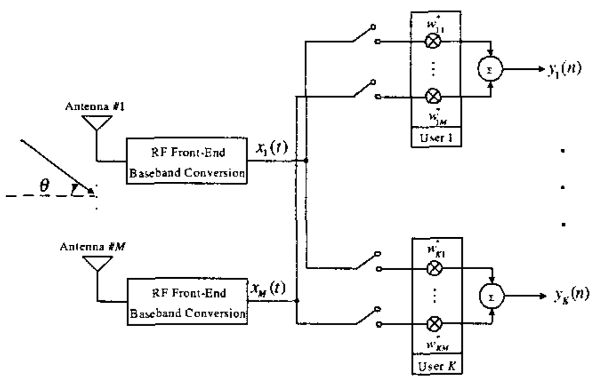

Fig. 1. Antenna array architecture for multiuser detection.

2.2. Recalls on FP Criterion

The FP criterion is based on the pdf estimation of an ideally equalized signal. If we consider a linear system we can easily show the pdf of the ideally equalized signal is a mixture of Gaussians centered at the transmitted constellation symbols [10, 11].

Then, we can construct the criterion in order to minimize the "*distance*" between the desired pdf (ideally equalized one) and the one provided by a parametric model that fits the system order and pdf features. Thus, the well known *Kullback-Leibler divergence* (KLD) [12] is used to minimize the divergence between both pdfs. The criterion may be written as [11]

$$J_{\text{FP}}(\mathbf{w}) = D_{p_{Y,\text{ideal}}(y)\|\Phi(y)}, \tag{4}$$

where $D_{\circ\|\bullet}$ is the KLD between the pdfs, $p_{Y,\text{ideal}}$ is the pdf of the ideally equalized signal and $\Phi(y)$ is the parametric model given by

$$\Phi(y) = \frac{1}{\sqrt{2\pi\sigma_r^2}} \sum_{i=1}^{S} \exp\left(-\frac{\left|\mathbf{w}^H(n)\mathbf{x}(n) - a_i\right|^2}{2\sigma_r^2}\right), \tag{5}$$

where σ_r^2 is the variance of each Gaussian in the model, S is the number of symbols in the transmitted constellation and a_i is the i-th symbol from the constellation. It worths to mention that minimize Equation (4) corresponds to maximize the log-likelihood function [12] and also to find the entropy of y if $\Phi\left(y, \sigma_r^2\right) = p_{Y,\text{ideal}}(y)$ [7, 10, 13].

A stochastic version for filter adaptation of the k-th user is given by

$$\begin{aligned} \nabla J_{\text{FP}}\left(\mathbf{w}_k(n)\right) &= \frac{\sum_{i=1}^{S} \exp\left(-\frac{|y_k(n)-a_i|^2}{2\sigma_r^2}\right)\left(y_k(n) - a_i\right)}{\sigma_r^2 \cdot \sum_{i=1}^{S} \exp\left(-\frac{|y_k(n)-a_i|^2}{2\sigma_r^2}\right)} \mathbf{x}^* \\ \mathbf{w}_k(n+1) &= \mathbf{w}_k(n) - \mu \nabla J_{\text{FP}}\left(\mathbf{w}_k\right), \end{aligned} \tag{6}$$

and this adaptive algorithm, which uses the FP criterion, is called ***Fitting pdf Algorithm*** **(FPA)**.

In [7], the multiuser version of the FPA has been proposed. It is based on the explicit decorrelation procedure reported in [6], where a penalization term is inserted to force the beamformers outputs to be uncorrelated. Then, the ***MU-FPA*** (multiuser FPA) is the adaptive (stochastic) version of the criterion for the k-th user, given by:

$$J_{\text{MU-FP}}\left(\mathbf{w}_k\right) = J_{\text{FP}}\left(\mathbf{w}_k\right) + \gamma \sum_{i=1}^{K} \sum_{\substack{j=1 \\ j \neq 1}}^{K} \left|r_{ij}\right|^2, \tag{7}$$

where γ is a regularization parameter and r_{ij} is the correlation term between the i-th and j-th beamformers outputs.

The influence of the γ parameter on the convergence was investigated in [14], and an adaptive procedure is also proposed in order to improve the convergence rate and the steady state error. In this paper we propose an alternative solution based on the necessary conditions for blind signal recovering as stated in [8, 9] and described in the sequel.

3. THE MUK ALGORITHM

A multiuser algorithm based on the maximization of the kurtosis for blind signals recovering is proposed in [8, 9].

The criterion is based on the Shalvi-Weinstein one [15] for SISO systems. Such criterion states that if the received power (after equalization) is assured to be equal to the transmitted one, it is sufficient to equalize one higher-order moment to achieve equalization, except by a phase rotation. It is normally chosen the unnormalized kurtosis, also known as forth-order cumulant [9].

Also, in order to provide all signals recovering (no lost users) some additional conditions are required [8, 9]. If $a(n)$ and $y(n)$ are considered to be, respectively, the input and output sequences, the following conditions must hold:

C1. $a_l(n)$ is i.i.d. and zero mean $(l = 1, \ldots, K)$;

C2. $a_l(n)$ and $a_q(n)$ are statistically independent for $l \neq q$ and have the same pdf;

C3. $|\kappa[y_l(n)]| = |\kappa_a| \quad (l = 1, \ldots, K)$;

C4. $E\left\{|y_l(n)|^2\right\} = \sigma_a^2 \quad (l = 1, \ldots, K)$;

C5. $E\{y_l(n)y_q^*(n)\} = 0, \quad l \neq q$.

where $a_l(n)$ is the transmitted sequence by the l-th source, $E\{\cdot\}$ stands for expectation, κ_a is the kurtosis and σ_a^2 is the variance of transmitted sequence, and $\kappa[\cdot]$ is the kurtosis operator. Condition C5 corresponds to the explicit decorrelation procedure described in [6].

As proved in [8, 9], those conditions are sufficient for recovering the signal, and the MUK criterion is able to provide source sepparation. MUK is a constrained criterion that maximizes the kurtosis of the signals subject to the constraint of normalized global response, it means,

$$\left\{ \begin{array}{c} \max\limits_{\mathbf{G}} J_{\text{MUK}}(\mathbf{G}) = \sum\limits_{j=1}^{K} |\kappa[y_j]| \\ \text{subject to: } \mathbf{G}^H\mathbf{G} = \mathbf{I} \end{array} \right. , \tag{8}$$

where $\mathbf{G} = \mathbf{F}^T\mathbf{W}$ is the global system response, $\mathbf{I}$ is the identity matrix and $\mathbf{W} = \left[\begin{array}{c|c|c} \mathbf{w}_1 & \cdots & \mathbf{w}_K \end{array} \right]$.

In fact, the source separation task is divided into two parts: the equalization step, that maximizes the kurtosis, and the separation one, that performs the decorrelation of the beamformers outputs. For each task, we denote the beamformers by $\mathbf{W}^e$ for equalization part, and $\mathbf{W}$ for the later one. The constraint step corresponds to a Gram-Schimdt orthogonalization of matrix $\mathbf{W}^e$ [9]. Therefore, those two parts can be, respectively, written in their adaptive versions by [8, 9]:

$$\mathbf{W}^e(n+1) = \mathbf{W}(n) + \mu\,\text{sign}(\kappa_a)\,\mathbf{x}^*(n)\mathcal{Z}(n), \tag{9}$$

where $\mathcal{Z}(n) = \left[\, |y_1(n)|^2 y_1(n) \quad \cdots \quad |y_K(n)|^2 y_K(n)\,\right]$ and Equation (9) corresponds to the equalization step. For the orthogonalization one we then have, for the j-th user

$$\mathbf{w}_j(n+1) = \frac{\mathbf{w}_j^e(n+1) - \sum\limits_{l=1}^{j-1}\left(\mathbf{w}_l^H(n+1)\mathbf{w}_j^e(n+1)\right)\mathbf{w}_l(n+1)}{\left\|\mathbf{w}_j^e(n+1) - \sum\limits_{l=1}^{j-1}\left(\mathbf{w}_l^H(n+1)\mathbf{w}_j^e(n+1)\right)\mathbf{w}_l(n+1)\right\|}. \tag{10}$$

The MUK algorithm is summarized in Table 1 [9].

The strategy of processing, in a "decoupled" way, of the MUK procedure is a very interesting issue because it allows to perform the task of source separation easier, through the use of an equalization strategy constrained to a orthogonal global system response. In fact, this feature is the main point for our approach that is described in the next section.

Table 1. MUK algorithm.

1. Initialize $\mathbf{W}(0)$
2. for $n > 0$
3. Obtain $\mathbf{W}^e(n+1)$ from Equation (9)
4. Obtain $\mathbf{w}_1(n+1) = \frac{\mathbf{w}_1^e(n+1)}{\|\mathbf{w}_1^e(n+1)\|}$
5. for $j = 2 : K$
6. Compute $\mathbf{w}_j(n+1)$ from Equation (10)
7. Go to 5
8. Go to 2

4. MULTIUSER CONSTRAINED FP APPROACH

As we have mentioned before, the use of such approach of "decoupled" processing makes possible the use of equalization techniques, in order to obtain source separation constraining the global system response to be orthogonal. The key point is that the considered equalization criterion has to respect the necessary and sufficient conditions for blind signal recovering described in Section 3.

We have noticed that, the FP criterion can be viewed as a general case of the Shalvi-Weinstein one. This is due to the fact that, to equalize the pdfs of signals corresponds to equalize *all* higher-order moments (and cumulants) of the signals, and matching all cumulants comprises the kurtosis one, as stated by Shalvi-Weinstein and MUK criteria. Indeed, we take profit from the parametric form of pdf estimation provided by the FP criterion to cope with the high computational load required to match all cumulant between the input and output of the system.

It indicates that we can replace the pdf estimation procedure instead the kurtosis maximization, in the MUK criterion, and the set of conditions is still valid as necessary and sufficient conditions for blind source separation. With this approach, we replace the processing with a penalization term that is performed by the MU-FPA, see Equation 7, by a constrained procedure aiming to improve the performance.

Moreover, the criterion is then given by

$$\left\{ \begin{array}{c} \min\limits_{\mathbf{W}} J_{\text{FP}}(\mathbf{W}) = \sum\limits_{j=1}^{K} D_{p_{Y,\text{ideal}}(y)\|\Phi(y_j)} \\ \text{subject to: } \mathbf{G}^H\mathbf{G} = \mathbf{I} \end{array} \right. , \tag{11}$$

that is pretty similar to Equation (8) except for the equalization part. and the fact that we stress only the filter dependent term for minimization $(\mathbf{W})$ instead of the global response one $(\mathbf{G})$ what gives the same solution.

Thus, the new adaptive algorithm is obtained by replacing the step 3 in Table 1 by the following computation:

$$\mathbf{W}^e(n+1) = \mathbf{W}(n) - \mu \nabla J_{\mathrm{FP}}\left(\mathbf{W}(n)\right), \quad (12)$$

where $\nabla J_{\mathrm{FP}}\left(\mathbf{W}(n)\right)$ is given in Equation (6). The resulting algorithm is then called ***Multiuser Constrained FPA (MU-CFPA)***.

In terms of computational complexity, the FPA is a LMS-like algorithm and the orthogonalization procedure slightly increases this issue.

5. SIMULATION RESULTS

A SDMA system with 4 users is used as the evaluation scenario of the algorithm. In the receiver we use a linear antenna array with 8 elements. The DOAs of each user are indicated in Table 2. In each direction 2000 QPSK symbols are transmitted over 100 Monte Carlo trials. In this environment we have assumed that the power control is achieved to assure the same transmission power to all users, and the signal-to-noise ratio (SNR) equals 30 dB.

Table 2. SDMA system configuration.

User	DOA (degrees)
1	1°
2	-52°
3	29°
4	76°

In order to evaluate the performance of the proposed algorithm we use the constant modulus error (CME) defined for the k-th user as follows:

$$\mathrm{CME}_k(n) = \left(|y_k|^2 - R\right)^2. \quad (13)$$

The constant R is related to the power of the transmitted constellation. In our case we will assume a normalized power, it means, $R = 1$.

Figure 2 shows the evolution of the CME for both versions of the FPA, the MU-FPA (with explicit decorrelation) and the MU-CFPA. The simulation parameters for the algorithms are: $\mu = 10^{-3}$, $\gamma = 10^{-3}$, $\sigma_r^2 = 0.1$ and initializations are performed as $\mathbf{W}(0) = \mathbf{I}$. For the MU-FPA, the forgetting factor used in the correlations matrices estimation is set to $\alpha = 0.96$.

As one can easily note, the MU-CFPA outperforms the MU-FPA in terms of steady state error, where the difference of performance between the two algorithms is about 10 dB. This is due to the drop of the decorrelation term in Equation (7) that has a strong trade-off between steady state error and number of lost users [14]. In the case of MU-CFPA,

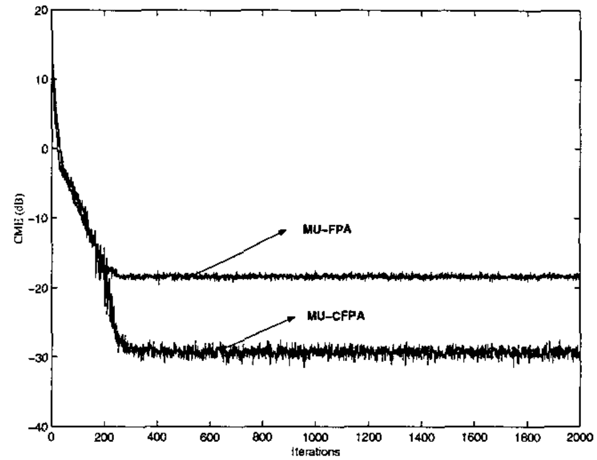

Fig. 2. Constant modulus error evolution.

this term is not taken into account and the decorrelation procedure is done by means of the orthogonalization procedure provided by Equation (10) that improves the steady state performance.

Another important feature of the MU-CFPA is that phase rotation is not observed in the output signals, except rotations of $c \cdot \frac{2\pi}{S}$ where c is a constant [7, 11]. By the way, those rotations cannot be detected by any blind algorithm and differential modulations become necessary [16].

6. CONCLUSIONS

We have proposed a new blind constrained algorithm based on the pdf estimation of the received signals for source (users) separation. The proposed algorithm is based on the MUK procedure proposed in [8] substituting the stage of kurtosis maximization by the maximization of the log-likelihood of a parametric function that estimates the statistical distribution of the ideally equalized data on the output and preserving the ortoghonalization constraint over the global system response.

The proposed algorithm can be viewed as a general form of the MUK since kurtosis is also equalized and maximized when the pdfs are matched.

Simulation study has shown a great improvement of the MU-CFPA over its past version, the MU-FPA. Due to the drop of the explicit decorrelation term, problems with steady state error enhancement are avoided.

Immediate perspectives to this work are related to prove that the FP criterion may be viewed as a generalization of the Shalvi-Weinstein one. So that global convergence can be assured as in the MUK case. Another research front is the study of the influence of other higher-order moments (cumulants) on the final performance of adaptive constrained algorithms, revisiting the Shalvi-Weinstein and Benvenist-Goursat-Rouget theorems [15, 17].

Also, an adaptive version to update the σ_r^2 parameter is under investigation. Further, the use of such strategy in a CDMA and/or space-time models context is envisioned.

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