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THE USE OF ORTHONORMAL BASES IN EQUALIZATION STRUCTURES

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ABSTRACT

In this work we propose the use of an ARMA equalizer structured on generalized orthonormal bases for communication purposes. This equalizer structure presents a tapped line of all-pass and low-pass filters. Such a structure is inherently stable since all its poles are within the unit circle. We also discuss a method for bases parameterizing (poles choice) based on the channel characteristics. The proposed structure performances are compared with conventional FIR equalizer ones. The results are evaluated in terms of MSE and overall numerical complexity. Traditional trained algorithms are employed for filter weight adapting. Our simulations show that the proposed structure leads to enhanced performances offering an alternative reduced complexity solution for communication channel equalization problem.

1. INTRODUCTION

The fact that IIR filters could be useful in adaptive signal processing has been widely explored in the last years. It is expected that these adaptive filters improve the performance of their FIR counterparts in many areas, as for example communication channel equalization.

The ARMA (Autoregressive Moving Average) structure is adapted for IIR filtering, for the fact that its dynamics can be represented by zeros and poles. Therefore, the aspects of stability and convergence of the ARMA structure can be controlled from convenient criteria. The ARMA filters can be implemented in several forms: blocks of coupled or decoupled filters, in cascade, etc.

The so-called orthonormal filters such as Laguerre and Kautz filters form an other class of ARMA filters. Contrarily to the conventional IIR filters, the orthonormal filters don't have stability problems since theirs poles are inherently stables and can be defined by the user according to a priori or posteriori knowledge of the channel.

Traditionally, the orthonormal bases are employed in the context of system representation and identification problems. The main motivation for using orthonormal bases in this context is the possibility to provide parsimonious representation of systems by a strategy of fixing poles near where the poles of the underlying dynamics are believed to lie. This strategy leads to an improved accuracy representation of the systems using a reduced number of coefficients in the filter as compared to conventional FIR filters.

The equalizer complexity becomes an important issue for its implementation in real time applications, principally in the future high capacity communications systems. That's why a considerable amount of work is dedicated to the numerical complexity reduction of such structures. For this sake, there are two principal ways for decreasing the computational cost of the equalizer sub-systems. We can work in the algorithm complexity bringing it more efficient or we can act in the equalizer structure in manner to obtain a reduction in the number of coefficients to adapt. It's well known that there is a direct relationship between the number of coefficients (filter order) and the algorithm computational cost. For instance, conventional LMS algorithm requires $\mathcal{O}(N)$ computations per sample while RLS need $\mathcal{O}(N^2)$ per sample and DMI requires $\mathcal{O}(N^3)$ operations, where N is the filter order.

In this work, we propose the use of ARMA filters based on the orthonormal functions in order to achieve a reduced complexity equalizer. The reduction in the computational cost is possible by decreasing the number of needed coefficients to obtain a certain mean square error. There are several important reasons to consider orthonormal basis functions instead of the usual FIR implementations. First, the use of Laguerre or Kauts models (or more generally, orthonormal basis functions) to describe the dynamical behavior of a wide class of systems has been studied extensively in many works on system identification and control [3], [4], [5], [6]. Second, the orthonormality property of such a models offers many benefits in estimation problems, including better numerical conditioning of the data. Third, one of the primary motivations in using all-pass basis functions for adaptive filtering is the fact that it requires fewer parameters to model systems with long impulse responses. In echo cancellation applications, for example, a long FIR filter may be necessary to model the echo path and adaptive IIR techniques have been proposed as possible alternatives (e.g.,[1], [2]). These techniques are nevertheless known to face stability problems due to the arbitrary pole locations during filter operation.

In the following sections we describe how the orthonormal bases can be employed for the equalization problem. We put in evidence, through simulation results, the improved performances of the proposed structure, face to minimum and non-minimum phase channels, when compared to conventional strategies. The rest of the paper is organized as follow: in section II we introduce the orthonormal bases. In section III we explain how the orthonormal bases can be used for equalization. In section IV we describe a simple method for base choice. The results are compared to conventional structures in section V. Finally, we come with the conclusions in section VI.

2. GENERALIZED ORTHONORMAL BASES

The orthonormal bases is widely used in system representation problems. It is possible to show that every Hilbert space has an orthonormal bases. This fact is important because it allows an unique representation of any element of the space as an orthonormal series expansion in terms of the elements of the Basis:

$$x = \sum_{i=1}^{\infty} c_i B_i \qquad (1)$$

where c_i is a Fourrier coefficient determined as $c_i = \langle x, B_i \rangle$. It is clear that the representation in (1) is not useful in practical problems where only a finite number of terms can be handled. The solution is then to approximate x by a trucked series like:

$$\hat{x} = \sum_{i=1}^{N} c_i B_i \qquad (2)$$

where $\hat{x}$ is an approximation of x.

Obviously the error decreases as the representation order N increases. For a same representation order, the error can vary according to the chosen orthonormal basis, i.e. an orthonormal basis with similar dynamics provides better approximation than other basis with different to x dynamics. So, the basis choice plays a fundamental role in system representation.

Consider the Hilbert space ℓ^2 formed by the sequences $f(n)$ with finite energy. The most common orthonormal basis on ℓ^2 are the well known FIR basis that corresponds to the choice:

$$B_i = z^{-i}$$

The use of FIR model structures to represent systems with long (possibly infinite) impulse responses has the disadvantage that the number of terms in the series expansion necessary to provide an acceptable approximation of the system is high and this may lead to poor accuracy in the estimated model.

Another well known bases are the Laguerre and Kautz bases. The first one is well adapted for first order system representation while the second one is indicated for resonant systems. For systems with several resonant dynamics, more general orthonormal bases allowing the incorporation of prior information about several modes would be more desiderable. Examples of such more general basis are the orthonormal basis generated by Inner Functions introduced by Heuberger, Van den Hof and co-workers or the generalized orthonormal basis with fixed poles studied by Ninness and co-workers in [4]. The Kautz, Laguerre and FIR model structures are all special cases of these methods.In this work, we consider the use of generalized orthonormal basis with fixed poles, henceforth called simply generalized orthonormal basis or GOB. The generalized orthonormal basis functions are built as follows:

$$B_i(z) = z^{-d} \frac{\sqrt{1 - p_i p_i^*}}{1 - p_i z^{-1}} \prod_{j=1}^{i-1} \frac{z^{-1} - p_j^*}{1 - p_j z^{-1}} \qquad (3)$$

where $d = 0$ or $d = 1$. Each $p_{k,k=1,\ldots,i}$ $|p_k| < 1$, is a basis function pole. The factor z^{-d} doesn't influence the base orthonormality. Its incorporation permits the strictly causal system representation.

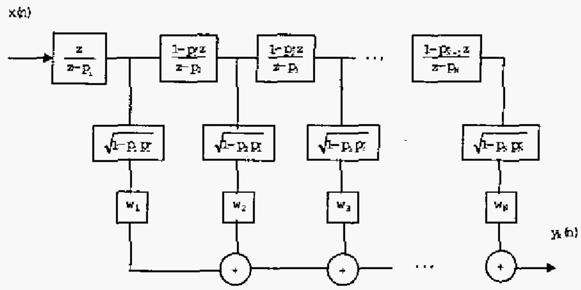

Fig. 1. Structure of an N order GOF.

3. THE UTILITY OF GOB FOR EQUALIZATION

In this paper we propose the use of equalization filters based on generalized orthonormal bases. The motivation of our proposition is to achieve an improved performance as compared to traditional FIR structures using lower order filters. This is possible due to GOB characteristics, which are well adapted for system identification/representation. In fact, the main GOB virtue is its flexibility for the pole choice. In the equalization context, the problem can be stated as the identification of the channel transfer function inverse (Zero Forcing approach) or the identification of a transfer function that minimizes some criterion (e.g. Minimum Square Error criterion). The GOB based filter, hereafter called GOF (Generalized Orthonormal Filter), structure is illustrated in figure 1.

To obtain an efficient representation of a dynamical system using orthonormal expansion it is important that the basis functions are calibrated to the underlying system characteristics. This gives increased rate of convergence of these expansions and hence accurate model with few parameters. So, the basis choice is a fundamental point if we pretend achieve an increased performance with GOF. Actually, a bad choice of the pole set that characterizes the GOB can leads to worst results when compared to FIR implementations. That's why an adaptive algorithm should be employed to select the pole set that parameterizes the GOF. The stochastic gradient algorithms form, with no doubt, one of the most popular class of adaptive algorithm due, principally, to their simplicity. However, the convergence of these algorithms is influenced by the existence of local minimums. This problem is observed even when just one pole is adapted, e.g. for Laguere basis selection. Some authors have proposed other algorithms to accomplish this task, i.e. to select a pole set which optimizes the filter performances according to a certain criterion. The methods proposed in [7], [8], present a common point: they consider that all poles are real. This consideration simplify an algorithm construction, but it is not adequate for our case, since the presence of pairs of complex conjugate poles may be allowed in order to compensate the complex conjugate zeros of the communication channel.

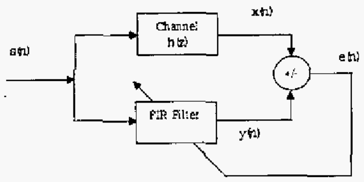

Fig. 2. Channel identification.

4. THE BASIS CHOICE

In this paper we propose a simple method for the generalized orthonormal function pole selection. We divide the task in two parts. In the first one, we proceed with the channel identification. In the second one, the poles are calculated according to the channel characteristics. In our work we consider that the channel $h(z)$ is modeled as an all zero transfer function (MA model):

$$h(z) = \prod_{i=1}^{L}(z_i - z)$$

where L is the channel order.

Consider the chain formed by the transmitted symbols $s(n)$, the channel, the modified symbols $x(n)$, the equalizer and the recuperated symbols $y(n)$.

To recuperate the transmitted information, the equalizer must compensate the distortions produced by the channel. To do that, the channel zeros should be compensated by the equalizer poles. In our approach, the poles are determined according to the channel zeros. More precisely, we identify the channel in the first part of our method. The identification procedure is accomplished using an ordinary FIR filter as shown in fig.(2). A training sequence is employed for the filter weight adjustments. After the training period the channel coefficients are estimated. We use this information to determine the channel order and to calculate the channel zeros. The DMI (Direct Matrix Inversion) algorithm is used for the channel identification.

In the second part, the zeros calculated are used to determine each pole of the GOB. There are two possible scenarios according to channel zero nature: minimum phase channel and non-minimum phase channel. For each scenario we use a different strategy described as follows.

4.1. Minimum Phase Channel

When all the channel zeros are within the unit circle, our algorithm acts in a very simple way: for each zero z_i we place a pole p_i where $p_i = z_i$. In this case, the filter is chosen to be equal to the channel order. However, if we want, the filter order can be increased putting more poles in the origin, i.e. the poles $p_i = 0 \; \forall i > L$, where L is the channel order. We expect with this method to compensate each channel zero with a GOF pole. In a ideal case (perfectly channel identification), if we choose $N = L$, we observe that the combined transfer function channel + filter will be $h_{res}(z) = \alpha_1 P_1(z) + \alpha_2 P_2(z) + \cdots + \alpha_N P_N(z)$, where $P_i(z)$ is an $N-1$ degree polynomial and $\alpha_i = w_i\sqrt{1 - p_i p_i^*}$. If the Zero Forcing criterion is applied, $h_{res}(z) = 1$.

4.2. Non-Minimum Phase Channel

For the non-minimum phase channel, the previous procedure must be rethinked. Indeed, for stability reasons the zeros outside the circle cannot be used as described before to select the GOF poles. Then, we divide the channel zeros in two categories: the zeros within the unit circle and the zeros outside the circle. For the zeros within the unit circle we place a correspondent pole in the same position. For the zeros outside the circle, we must use another approach. For each non-minimum phase zero we place a pole in the symmetrical point within the unit circle, i.e. $p_i = 1/z_i^*$. This pair of zero/pole forms an all-pass filter. The all-pass filter created by the channel zero and the GOF pole will be responsible for a phase distortion. Then, we must use a phase correction network to overcome the all-pass filter effects. Such a procedure requires the realization of inverse all-pass systems. It is well known that an inverse transfer function of an all-pass filter can be approximated by a FIR filter. Let us discuss the problem of the FIR approximation of poles which is necessary for inverse all-pass modeling. Consider the first order transfer function:

$$H(z) = \frac{z}{z - p_i} \text{ with } |p_i| \geq 1$$

under the specific condition of the pole outside the unit circle, $H(z)$ can be described by the following series expansion:

$$H(z) = -\sum_{n=1}^{\infty} p_i^{-n} z^n \tag{4}$$

which converges since $|p_i| > 1$. Thus the system can be interpreted as causal and non-stable or non-causal and stable. In the second case we get the impulse response:

$$h(n) = -p_i^n{}_{;n=1,2,3,\ldots}$$

which can be approximated by an N order causal FIR filter by time truncation and shift. This causal impulse response can be written as:

$$H_{FIR}(z) = \sum_{n=1}^{N} p_i^{-n} z^n = H(z)[z^{-N} - p_i^{-N}] \tag{5}$$

the second term describes N zeros in the z-plane, equidistantly spaced on a circle with the radius $|p_i|$. One of these zeros (located at the position of the approximated pole) is canceled by the term $H(z)$. This form of approximation is based on a rectangular windowing of the true impulse response. Of course, this method can be applied for all-pass filter inversion.

Now, let us continue with the pole choice issue. Until this point, we proposed the placement of a GOF pole in the same position of each channel minimum phase zero and a GOF pole in the symmetrical position for each channel non-minimum phase zero. We have discussed about the use of FIR filters to correct the phase distortion produced by the all-pass filters formed by the pairs of non-minimum phase zeros and their corresponding GOF poles, but how can we combine the phase correction using FIR filters and the GOF pole choice? This can be accomplished in a very simple way. First of all, we must remember that a FIR is a particular case of a GOF when all poles are placed in the origin. Then, the GOF structure can put into practice the channel zero compensation and the phase distortion correction tasks. To do that, we must just place extra poles in the origin to implement a FIR. The all-pass inversion is accomplished using the GOF with a pertinent decision delay. The following example illustrates our method. Consider a channel with zeros $= \{z_1\, z_2 \ldots z_i\, z_{i+1} \ldots z_L\}$, where $|z_k| < 1_{,k=1\ldots i}$ and $|z_k| > 1_{k=i+1\ldots L}$. The GOF order is chosen to be $N > L$. The GOF poles are chosen to be $p_j = z_j$ for $j = 1 \ldots i$ and $p_j = 1/z_j^*$ for $j = i+1 \ldots l$. For $j = L+1 \ldots N$ $p_j = 0$. The number of extra poles in the origin (i.e. the correction phase FIR order) will increase as the number of all-pass filters to inverse increases. In our simulations, the decision delay is chosen to match the central FIR tap, that is, the delay $\delta = (N-L)/2$[1]. A more detailed description of this method is available in [9].

5. PERFORMANCE EVALUATION

In this section the GOF performances are evaluated and compared with the conventional FIR structures. Both approaches are compared under two aspects: mean square error and computational cost. For the filter weight adjustments we employed two traditional trained algorithms: LMS and Direct Matrix Inversion. In each case, we use the same parameters (adapting step, training sequence, input sequence, etc.) for the two approaches.

Initially, we consider a noiseless channel. In these conditions the channel identification can be realized without problems. In a second moment, the simulations are realized in a channel with Additive White Gaussian Noise. In this case, the channel identification will not be perfect. As consequence, the channel zeros estimation will be less accurate. The channel zero estimation accuracy influences the performances of the GOF equalizer.

In our simulations, we use the BPSK modulation and we consider that the transmitted data are identically distributed random variable with zero mean and unit variance, following the BPSK modulation alphabet. The channel identification is accomplished using the DMI algorithm. A training sequence of 64 symbols is employed in this task.

5.1. Noiseless Channel

In this subsection, we compare the FIR and GOF performances in different channels. The first one is a minimum phase channel, the second one is a non-minimum phase channel and the two others have zeros on the unit circle. The zeros of each channel are presented in Tab.(1).

In fig (3) and (4), we show the simulation results for channel 1 and 2 respectively. These simulations show us the square error evolution when a LMS algorithm is employed for filter weight adjustment. In fig(3) the order of both filters is $N = 8$. In fig(4) the order is $N = 7$. We observe that,

[1] The optimal delay determination is out of scope of this work.

Channel id.	Channel zeros
1	-0.63 -0.95 0 0.2 0.2 0.3 0.5 0.95
2	-0.7 -0.3 0.8+0.8i 0.8-0.8i 0.9
3	0.5 0.7 1 1.1
4	-1i 1i -0.86+0.5i 0.86-0.5i

Table 1. Channel zeros.

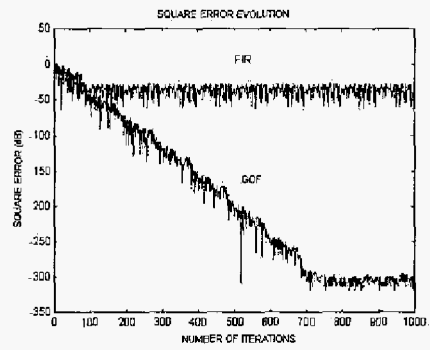

Fig. 3. FIR and GOF performances. Channel 1.

in the same conditions, the GOF performances are superior to the FIR performances in both channels. Indeed, The FIR reaches the GOF performances when the FIR order is increased to $N = 90$ for the channel 1 and $N = 150$ for the channel 2.

Now, we compare the computational cost of the two filters. For a given performance in terms of EQM we compare the filter order required to reach the desired EQM. We also compare the number of floating point operations needed for each case. For the GOF filter, we consider the computational cost for the channel identification plus the computational cost for the channel equalization. The LMS and DMI algorithms are used. In the LMS algorithm, the number of iterations (N-ite) that leads the desired performance is also illustrated. The results are showed in Tab(2).

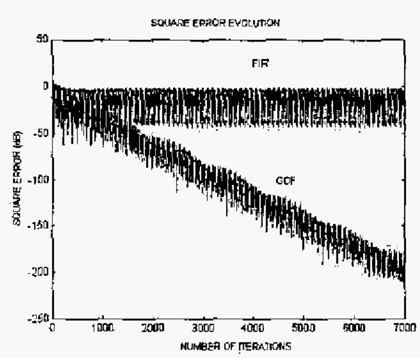

Fig. 4. FIR and GOF performances. Channel 2.

Channel 3 Desired EQM = -26dB				
Filter	Order	DMI Flops	LMS N-ite	LMS Flops
GOF	6	532	20	336
FIR	20	8000	350	7000

Channel 4 Desired EQM = -20dB				
Filter	Order	DMI Flops	LMS N-ite	LMS Flops
GOF	4	160	30	120
FIR	60	216000	200	12000

Table 2. Computational cost comparative.

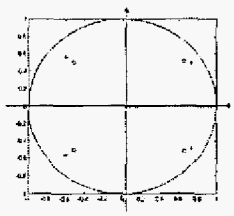

Fig. 5. Channel zero estimation. SNR= 3dB.

5.2. Noisy Channel

It is well known that the noise influences the second order statistics based algorithms. So, this is the case of the DMI algorithm used in the first stage, the channel identification, of our method. Actually, the noise brings more difficult the correct channel identification and the zeros estimation. In the fig (5), we show the noise effect over the zero estimation. The simulation was realized using a SNR of 3dB. The circles represent the channel zeros and the ⊗ represent the zero estimation. As the zero estimation plays a fundamental role in the GOF results, the overall filter performances decreases in the presence of noise. Nevertheless, the GOF performance remains greater than the FIR one. This is illustrated in fig(6). In this simulation, we compare the FIR and GOF performances in terms of MSE as function of the SNR. The filter order $N = 8$ and the channel 1 is employed in the simulation.

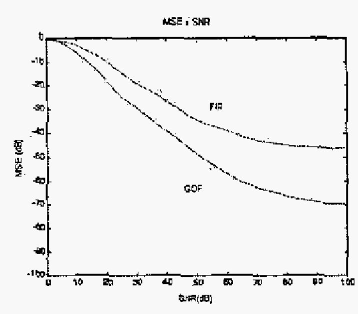

Fig. 6. MSE evolution as function of the SNR.

6. CONCLUSIONS

We have proposed a new equalizer filter structure based on generalized orthonormal bases. We also have presented an experimental method for basis parameterizing. The simulation results show us that the GOF structure presents better performances in terms of EQM as compared to FIR structures with the same order. For the same performance, the GOF structure presents a reduced computational cost due to the fewer coefficient number as compared to the traditional FIR solutions. Nevertheless, our results were obtained in stationary channels. In this condition, the basis parameterizing must be accomplished once during the equalization process. If the channel changes during the equalization, the poles of bases should be updated. According to our method, the pole updating requires a new channel identification. That is why the proposed method is suitable for packet data transmission systems, where a training sequence is generally employed for equalization purposes.

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