A Modulation Code-Based Blind Receiver for Memoryless Multiuser Volterra Channels

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***Abstract*— This paper proposes a blind receiver for memoryless multiuser Volterra communication channels based on the use of Modulation Codes (constrained codes) and Second Order Statistics of the received signals. Modulation codes allow to model the transmitted signals as Markov chains that are used to introduce temporal correlation and to ensure the orthogonality of transmitted signal products for several time delays, inducing a Parallel Factor (PARAFAC) decomposition of a tensor composed with spatio-temporal covariance matrices.**

***Index Terms*— Multiuser Volterra channel, Modulation Code, Markov Chain, Second Order Statistics, PARAFAC decomposition.**

I. Introduction

A blind identification technique is proposed in this paper with the goal of constructing a receiver for memoryless multiuser Volterra communication channels. The proposed technique exploits the use of Modulation Codes (constrained codes) and Second Order Statistics (SOS) of the received signals. The considered channel is modeled as a memoryless Multiple-Input-Multiple-Output (MIMO) Volterra filter (multiuser Volterra model). This kind of nonlinear models has important applications in the field of telecommunications, e.g. to model uplink channels in Radio Over Fiber multiuser communication systems [1], [2].

There are few works dealing with the problem of blind channel identification or source separation in the context of multiuser nonlinear communication channels. Reference [3] proposes a blind Zero Forcing technique for multiuser Code Division Multiple Access (CDMA) systems with nonlinear channels and [4] develops a blind source separation algorithm for memoryless Volterra channels in ultra-wide-band systems.

State-dependent modulation codes (constrained codes) [5] allow to model the transmitted signals as Discrete Time Markov Chains (DTMC). They are used to introduce temporal correlation and to ensure the orthogonality of transmitted signal products for several time delays, inducing a Parallel Factor (PARAFAC) decomposition [6], [7] of a third order tensor formed from spatio-temporal covariance matrices. The redundancy provided by the codes introduces temporal correlation in a controlled way, so that the transmitted signals verify some statistical constraints associated with the channel nonlinearities.

Some coding schemes verifying these constraints are designed for 4- and 8-PSK (Phase Shift Keying) signals. A two-step Alternating Least Squares (ALS) algorithm [6], [7] is used to blindly estimate the channel.

The paper is organized as follows. Section II introduces the channel model used in this work. In Section III, some constraints are established to get a PARAFAC decomposition of the considered tensor. In Section IV, these constraints are rewritten in terms of the transition probability matrix (TPM) of the Markov chain. Section V designs TPM's verifying the constraints for 4- and 8-PSK input signals and Section VI presents the blind channel estimation algorithm. In Section VII, we evaluate the performance of the proposed algorithm by means of simulations and some conclusions are drawn in Section VIII.

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II. Channel Model

The sampled baseband equivalent model of the nonlinear communication channel is assumed to be expressed as:

$$y_r(n) = \sum_{k=0}^{\bar{K}} \sum_{t_1=1}^{T} \cdots \sum_{t_{k+1}=t_k}^{T} \underbrace{\sum_{t_{k+2}=1}^{T} \cdots \sum_{t_{2k+1}=t_{2k}}^{T}}_{t_{k+2},\ldots,t_{2k+1} \neq t_1,\ldots,t_{k+1}} h_{2k+1}^{(r)}(t_1,\ldots,t_{2k+1}) \prod_{i=1}^{k+1} s_{t_i}(n) \prod_{i=k+2}^{2k+1} s_{t_i}{}^*(n) + v_r(n), \quad (1)$$

where $y_r(n)$ $(1 \leq r \leq R)$ is the signal received by antenna r at the time instant n, R is the number of receive antennas, $(2\bar{K}+1)$ is the nonlinearity order of the model, $s_t(n)$ $(1 \leq t \leq T)$ is the stationary PSK modulated signal transmitted by the t^{th} user at the time instant n, T is the number of users, $h_{2k+1}^{(r)}(t_1,\ldots,t_{2k+1})$ are the coefficients of the r^{th} sub-channel and $v_r(n)$ $(1 \leq r \leq R)$ is the Additive White Gaussian Noise (AWGN). It is assumed that the transmitted signals $s_t(n)$ are independent from each other and that the noise components $v_r(n)$ are zero mean, independent from each other and from the transmitted signals $s_t(n)$.

A matrix writing of (1) is given by:

$$\mathbf{y}(n) = \mathbf{H}\mathbf{w}(n) + \mathbf{v}(n), \tag{2}$$

where $\mathbf{y}(n) = [y_1(n) \ldots y_R(n)]^T \in \mathbb{C}^{R\times 1}$, $\mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_R]^T \in \mathbb{C}^{R\times Q}$ is the channel matrix, with $\mathbf{h}_r = [h_{r,1}\, h_{r,2} \ldots h_{r,Q}]^T \in \mathbb{C}^{Q\times 1}$ containing the Volterra system parameters $h_{2k+1}^{(r)}(t_1,\ldots,t_{2k+1})$ associated with the r^{th} sub-channel, $\mathbf{v}(n) = [v_1(n) \ldots v_R(n)]^T \in \mathbb{C}^{R\times 1}$ and $\mathbf{w}(n) = [w_1(n) \ldots w_Q(n)]^T \in \mathbb{C}^{Q\times 1}$ is the nonlinear input vector containing all the nonlinear combinations of $s_t(n)$ present in (1), Q being the dimension of the vector $\mathbf{w}(n)$.

The nonlinear combinations corresponding to $t_i = t_j$, for all $i \in \{1,...,k+1\}$ and $j \in \{k+2,...,2k+1\}$, are absent in (1) due to the fact that, for constant modulus signals, the term $|s_{t_i}(n)|^2$ is a multiplicative constant that can be absorbed by the channel coefficients. As a consequence, some nonlinear terms degenerate in terms of smaller order. Besides, the even-order terms are also absent in (1) as they generate distortions producing spectral components lying outside of the channel bandwidth, which can be eliminated by bandpass filters. The configuration of the nonlinear terms including only-odd power terms with $k+1$ non-conjugated terms and k conjugated terms, allows to represent baseband equivalent nonlinear distortions [8].

III. PARAFAC Decomposition of a Tensor of Covariance Matrices

The proposed receiver relies on the PARAFAC decomposition of a tensor composed of spatio-temporal covariances of the received signals, given by:

$$\mathbf{R}(d) = \mathbb{E}\left[\mathbf{y}(n+d)\mathbf{y}^H(n)\right] = \mathbf{H}\mathbf{C}(d)\mathbf{H}^H \in \mathbb{C}^{R\times R}, \tag{3}$$

with

$$\mathbf{C}(d) = \mathbb{E}\left[\mathbf{w}(n+d)\mathbf{w}^H(n)\right] \in \mathbb{C}^{Q\times Q}, \tag{4}$$

where $0 \le d \le D-1$ and D is the number of delays (covariance matrices) taken into account. The noise covariance matrix is not considered in $\mathbf{R}(0)$ since it can be estimated and then subtracted from $\mathbf{R}(0)$ [9].

A third-order tensor $\mathcal{R} \in \mathbb{C}^{D\times R\times R}$ can be defined from the matrices $\mathbf{R}(d)$ in such a way that the *first-mode slices* of $\mathcal{R}$, denoted by $\mathbf{R}_{(d+1)\cdot\cdot}$, have the form:

$$\mathbf{R}_{(d+1)\cdot\cdot} = \mathbf{R}(d), \quad d = 0,...,D-1, \tag{5}$$

where a *first-mode slice* of $\mathcal{R}$ is obtained by fixing the first dimension index of $\mathcal{R}$ and varying the indices of the two other modes. The scalar notation of the tensor $\mathcal{R}$ is given by:

$$r_{(d+1),r_1,r_2} = \sum_{q_1=1}^{Q}\sum_{q_2=1}^{Q} h_{r_1,q_1}h^*_{r_2,q_2}c_{q_1,q_2}(d), \tag{6}$$

where $r_{(d+1),r_1,r_2} = [\mathcal{R}]_{(d+1),r_1,r_2}$, $h_{r,q} = [\mathbf{H}]_{r,q}$ and $c_{q_1,q_2}(d) = [\mathbf{C}(d)]_{q_1,q_2}$. Note that equation (6) corresponds to the scalar writing of a Tucker2 model [10]. If the covariance matrices $\mathbf{C}(d)$ of the nonlinear input vector are diagonal for $0 \le d \le D-1$, equation (6) becomes:

$$r_{(d+1),r_1,r_2} = \sum_{q=1}^{Q} h_{r_1,q}h^*_{r_2,q}c_{q,q}(d), \tag{7}$$

which corresponds to the scalar writing of a PARAFAC model. The advantages of the PARAFAC model over the Tucker2 model are its simplicity (number of parameters) and the essential uniqueness of its factors [6], [7], assured if the Kruskal condition is verified [11]:

$$2k_{\mathbf{H}} + k_{\bar{\mathbf{C}}} \ge 2Q + 2, \tag{8}$$

where $k_{\mathbf{A}}$ is the k-rank of matrix $\mathbf{A}$, i.e. the greatest integer k such that every set of k columns of $\mathbf{A}$ is linearly independent, and the matrix $\bar{\mathbf{C}} \in \mathbb{C}^{D\times Q}$, containing the diagonal components of $\mathbf{C}(d)$ for $0 \le d \le D-1$, is defined such that $\bar{c}_{d,q} = c_{q,q}(d-1)$.

The following theorem states sufficient conditions to ensure that the matrices $\mathbf{C}(d)$ are diagonal for $0 \le d \le D-1$. It is assumed that $P_t > 2\bar{K}+1$, for $t = 1,\ldots,T$, where P_t is the number of points of the PSK constellation associated with the t^{th} user.

Theorem 1: The off-diagonal elements of $\mathbf{C}(d)$ are null if the following conditions are satisfied for $(T-1)$ users and $0 \le d \le D-1$:

(C1) $\mu_t^{(i,j)}(d) = 0$, for all $0 \le i,j \le \bar{K}+1$ with $i \ne j$;

(C2) $\varrho_t^{(i,j)}(d) = 0$, for all $0 \le i,j \le \bar{K}+1$ with i or/and $j \ne \bar{K}+1$;

where

$$\mu_t^{(i,j)}(d) \equiv \mathbb{E}\left[s_t^i(n+d)\left[s_t^j(n)\right]^*\right] \tag{9}$$

and

$$\varrho_t^{(i,j)}(d) \equiv \mathbb{E}\left[s_t^i(n+d)s_t^j(n)\right]. \tag{10}$$

Proof: The elements of $\mathbf{C}(d)$ are defined as:

$$c_{q_1,q_2}(d) = \mathbb{E}[w_{q_1}(n+d)w^*_{q_2}(n)], \tag{11}$$

where $w_{q_1}(n)$ and $w_{q_2}(n)$ can be written respectively as:

$$w_{q_1}(n) = \prod_{t=1}^{T} s_t^{\alpha_t}(n)\left[s_t^{\beta_t}(n)\right]^*, \tag{12}$$

$$w_{q_2}(n) = \prod_{t=1}^{T} s_t^{\alpha'_t}(n)\left[s_t^{\beta'_t}(n)\right]^*, \tag{13}$$

for some integers $\alpha_t, \beta_t, \alpha'_t, \beta'_t$ verifying:

$$\sum_{t=1}^{T}\alpha_t = k+1, \qquad \sum_{t=1}^{T}\beta_t = k,$$
$$\sum_{t=1}^{T}\alpha'_t = k'+1 \text{ and } \sum_{t=1}^{T}\beta'_t = k'. \tag{14}$$

with $0 \le \alpha_t, \alpha'_t \le \bar{K}+1$ and $0 \le \beta_t, \beta'_t \le \bar{K}$. Moreover, from the constraints $t_{k+2},...,t_{2k+1} \ne t_1,...,t_{k+1}$ in (1), it can be concluded that $\alpha_t = 0$ or $\beta_t = 0$; and $\alpha'_t = 0$ or

$\beta_t^{'} = 0$, for all $t = 1, ..., T$. (They all may be equal to zero.) Hence, (12) and (13) can be rewritten respectively as:

$$w_{q_1}(n) = \prod_{t=1}^{T} \dot{s}_t^{\gamma_t}(n) \quad \text{and} \quad w_{q_2}(n) = \prod_{t=1}^{T} \ddot{s}_t^{\gamma_t^{'}}(n), \tag{15}$$

where $\gamma_t = \max(\alpha_t, \beta_t)$, $\gamma_t^{'} = \max(\alpha_t^{'}, \beta_t^{'})$,

$$\dot{s}_t(n) = \begin{cases} s_t(n), & \text{if } \beta_t = 0, \\ s_t^*(n), & \text{if } \alpha_t = 0, \end{cases} \quad \ddot{s}_t(n) = \begin{cases} s_t(n), & \text{if } \beta_t^{'} = 0, \\ s_t^*(n), & \text{if } \alpha_t^{'} = 0. \end{cases} \tag{16}$$

Substituting (15) into (11), we get:

$$c_{q_1,q_2}(d) = \prod_{t=1}^{T} \mathbb{E}\left[\dot{s}_t^{\gamma_t}(n+d)\left[\ddot{s}_t^{\gamma_t^{'}}(n)\right]^*\right]. \tag{17}$$

If $q_1 \neq q_2$, there is at least two users t_1 and t_2 such that $(\alpha_{t_1}, \beta_{t_1}) \neq (\alpha_{t_1}^{'}, \beta_{t_1}^{'})$ and $(\alpha_{t_2}, \beta_{t_2}) \neq (\alpha_{t_2}^{'}, \beta_{t_2}^{'})$. So, (17) can be rewritten as:

$$c_{q_1,q_2}(d) = \prod_{\substack{t=1 \\ t \neq t_1, t_2}}^{T} \mathbb{E}\left[\dot{s}_t^{\gamma_t}(n+d)\left[\ddot{s}_t^{\gamma_t^{'}}(n)\right]^*\right]$$

$$\mathbb{E}\left[\dot{s}_{t_1}^{\gamma_{t_1}}(n+d)\left[\ddot{s}_{t_1}^{\gamma_{t_1}^{'}}(n)\right]^*\right]\mathbb{E}\left[\dot{s}_{t_2}^{\gamma_{t_2}}(n+d)\left[\ddot{s}_{t_2}^{\gamma_{t_2}^{'}}(n)\right]^*\right]. \tag{18}$$

The last two terms of (18) can be written as one of the following forms:

- $\mu_t^{(i,j)}(d)$, with $0 \leq i, j \leq \bar{K}+1$ and $i \neq j$;
- $\left[\mu_t^{(i,j)}(d)\right]^*$, with $0 \leq i, j \leq \bar{K}$ and $i \neq j$;
- $\varrho_t^{(i,j)}(d)$, with $0 \leq i \leq \bar{K}+1$, $0 \leq j \leq \bar{K}$;
- $\left[\varrho_t^{(i,j)}(d)\right]^*$, with $0 \leq i \leq \bar{K}$, $0 \leq j \leq \bar{K}+1$.

Therefore, conditions $(C1)$ and $(C2)$ are sufficient to ensure that the off-diagonal elements of $\mathbf{C}(d)$ are null.

■

IV. Transmitted Markov Signals

In this section, the orthogonality constraints of Theorem 1 are written in terms of the Transition Probability Matrix (TPM) that characterizes the DTMC of the transmitted signals. The states of the DTMC are given by the P_t PSK symbols $a_p = \{A_t \cdot e^{j2\pi(p-1)/P_t}\}$, for $p = 1, 2, ..., P_t$, where A_t is the amplitude of the signal of the t^{th} user. The state transitions are defined by a set of L_B bits, denoted by $B_n = \{b_n^{(1)}, b_n^{(2)}, ..., b_n^{(L_B)}\}$, that are uniformly distributed over the set $\{0, 1\}$ with $2^{L_B} < P_t$. In addition, it is assumed that the $b_n^{(l)}$ (l=1, ..., L_B) are mutually independent. For each of the P_t states, the set B_n of bits defines 2^{L_B} equiprobable possible transitions. Therefore, this coding imposes some restrictions on the symbol transitions. For each state, there is $\left(P_t - 2^{L_B}\right)$ not assigned transitions. The code rate is then given by $(L_B/\log_2 P_t)$. For further details about this coding scheme, see [2].

Let us denote by $\mathbf{T} = \{T_{p_1,p_2}\}$, with $p_1, p_2 \in \{1, 2, ..., P_t\}$, the TPM for a given user, T_{p_1,p_2} being the probability of the transition from the state a_{p_1} to the state a_{p_2}. Note that $\sum_{p_2=1}^{P_t} T_{p_1,p_2} = 1$ and $T_{p_1,p_2} \in \{0, 2^{-L_B}\}$. Moreover, let T_{n,p_1,p_2} be the $(p_1,p_2)^{th}$ element of $\mathbf{T}^n$. So, T_{n,p_1,p_2} represents the probability of being in the state a_{p_2} after n transitions, supposing that the current state is a_{p_1}. The following theorem reformulates the conditions of Theorem 1 in terms of the TPM associated with the DTMC of each user.

Theorem 2: If the following conditions hold:

(C3) $\sum_{p_2=1}^{P_t} T_{p_1,p_2} = 1$, for $1 \leq p_1 \leq P_t$;
(C4) $\sum_{p_1=1}^{P_t} T_{p_1,p_2} = 1$, for $1 \leq p_2 \leq P_t$;
(C5) the DTMC is irreducible and aperiodic;

and the initial state of the DTMC is chosen in an equiprobable way among the PSK symbols, then, the moments (9) and (10) can be rewritten as:

$$\mu_t^{(i,j)}(d) = \frac{1}{P_t}\left[\mathbf{a}^{\bullet j}\right]^H \mathbf{T}^d \mathbf{a}^{\bullet i}, \tag{19}$$

and

$$\varrho_t^{(i,j)}(d) = \frac{1}{P_t}\left[\mathbf{a}^{\bullet j}\right]^T \mathbf{T}^d \mathbf{a}^{\bullet i}. \tag{20}$$

where $\mathbf{a} = [a_1, a_2, ... a_{P_t}]^T$ and $\mathbf{a}^{\bullet i} = \left[a_1^i, a_2^i, ... a_{P_t}^i\right]^T$.

Proof: Condition C3 must be verified for all TPM's and Conditions C4 and C5 ensure that the stationary distribution of the DTMC is unique and equal to the uniform distribution [12], [2]. So, if the initial state is chosen in an equiprobable way among the PSK symbols, the DTMC is stationary with a uniform distribution.

Moreover, by exploiting the symmetry of PSK constellations, it can be demonstrated that the constraints of Theorem 1 are verified for $d = 0$ in the case of uniformly distributed PSK signals [2]. So, for $d \neq 0$ we get:

$$\mu_t^{(i,j)}(d) = \mathbb{E}\left[s_t^i(n+d)\left[s_t^j(n)\right]^*\right] = \sum_{p_1=1}^{P_t}\sum_{p_2=1}^{P_t}$$

$$\mathsf{p}(\alpha_n = a_{p_1})\left[a_{p_1}^j\right]^* \mathsf{p}(\alpha_{n+d} = a_{p_2}|\alpha_n = a_{p_1}) a_{p_2}^i, \tag{21}$$

where $\mathsf{p}(\alpha_n = a_{p_1})$ is the probability of being in the state a_{p_1} at the time n and $\mathsf{p}(\alpha_{n+d} = a_{p_2}|\alpha_n = a_{p_1})$ is the conditional probability of being in the state a_{p_2} at the time $(n+d)$, given the state a_{p_1} at the time n. So, we have:

$$\mu_t^{(i,j)}(d) = \sum_{p_1=1}^{P_t}\sum_{p_2=1}^{P_t}\frac{1}{P_t}\left[a_{p_1}^j\right]^* T_{d,p_1,p_2} a_{p_2}^i \tag{22}$$

$$= \frac{1}{P_t}\left[\mathbf{a}^{\bullet j}\right]^H \mathbf{T}^d \mathbf{a}^{\bullet i}. \tag{23}$$

Equation (20) can be obtained in a similar way.

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V. Design of the Transition Probability Matrices

For all (i, j) such that $1 \leq i, j \leq \bar{K}+1$, conditions C1 and C2 of Theorem 1 can be written in a matrix form respectively as:

$$\mathbf{A}^H \mathbf{T}^d \mathbf{A} = \mathbf{0}_{(\bar{K}+1),(\bar{K}+1)} \quad \text{and} \quad \mathbf{A}^T \mathbf{T}^d \mathbf{A} = \mathbf{0}_{(\bar{K}+1),(\bar{K}+1)} \tag{24}$$

where

$$\mathbf{A} = \left(\mathbf{a}_1 \ \mathbf{a}_2 \cdots \mathbf{a}_{\bar{K}+1}\right) \in \mathbb{C}^{P_t \times (\bar{K}+1)} \tag{25}$$

and $\mathbf{0}_{(\bar{K}+1),(\bar{K}+1)}$ is the zero matrix of dimension $(\bar{K}+1) \times (\bar{K}+1)$. Equation (24) can be rewritten as:

$$\left(\mathbf{A}^T \otimes \mathbf{A}^H\right) \text{vec}\left(\mathbf{T}^d\right) = \mathbf{0}_{(\bar{K}+1)^2,1} \tag{26}$$

and

$$\left(\mathbf{A}^T \otimes \mathbf{A}^T\right) \text{vec}\left(\mathbf{T}^d\right) = \mathbf{0}_{(\bar{K}+1)^2,1}, \tag{27}$$

with vec$(\cdot)$ denoting the operator that stacks the columns of the matrix argument and $\otimes$ the Kronecker product. Conditions C1 and C2 for i or j equal to zero are not considered as they are equivalent to the case $d = 0$, which is assured from conditions C3, C4 and C5.

Note that for $d = 1$, equations (26) and (27) are linear with respect to the elements of $\mathbf{T}$. Thus, for $d = 1$, conditions C1-C4 can be written as the following set of linear equations:

$$\begin{bmatrix} \Theta_1 \left(\mathbf{A}^T \otimes \mathbf{A}^H\right) \\ \Theta_2 \left(\mathbf{A}^T \otimes \mathbf{A}^T\right) \\ \Theta_3 \\ \Theta_4 \end{bmatrix} \text{vec}(\mathbf{T}) = \begin{bmatrix} \mathbf{0}_{2(\bar{K}+1)^2-(\bar{K}+2),1} \\ \mathbf{1}_{2P_t-1,1} \end{bmatrix}, \tag{28}$$

where $\mathbf{1}_{2P_t-1,1}$ is the all ones vector of dimension $(2P_t - 1)$, $\Theta_1 \in \mathbb{C}^{[(\bar{K}+1)^2-(\bar{K}+1)]\times(\bar{K}+1)^2}$ is a matrix that selects the rows of $\left(\mathbf{A}^T \otimes \mathbf{A}^H\right)$ corresponding to $\left(\mathbf{a}_i^T \otimes \mathbf{a}_j^H\right)$ with $i \neq j$ and $\Theta_2 \in \mathbb{C}^{[(\bar{K}+1)^2-1]\times(\bar{K}+1)^2}$ is a matrix that selects the rows of $\left(\mathbf{A}^T \otimes \mathbf{A}^T\right)$ corresponding to $\left(\mathbf{a}_i^T \otimes \mathbf{a}_j^T\right)$ with i or/and $j \neq \bar{K} + 1$. Moreover, the matrices $\Theta_3 \in \mathbb{R}^{P_t \times P_t^2}$ and $\Theta_4 \in \mathbb{R}^{(P_t-1)\times P_t^2}$ are given respectively by:

$$\Theta_3 = \mathbf{1}_{P_t,1}^T \otimes \mathbf{I}_{P_t} \tag{29}$$

and

$$\Theta_4 = \left[\mathbf{I}_{P_t-1}\ \mathbf{0}_{P_t-1,1}\right] \left(\mathbf{I}_{P_t} \otimes \mathbf{1}_{P_t,1}^T\right). \tag{30}$$

Thus, the desired TPM's must satisfy (28) and C5. It should be highlighted that, once chosen the values of P_t and L_B, the orthogonality constraints only depend on the matrix $\mathbf{T}$ with $T_{p_1,p_2} \in \{0, 2^{-L_B}\}$, which means that $\mathbf{T}$ can be a priori designed. In the sequel, TPM's verifying such constraints are given for some particular cases.

A. Cubic system with 4-PSK input signal

This case corresponds to $\bar{K} = 1$, $P_t = 4$ and, hence, $L_B = 1$. Taking the fact that $s_t^2(n) \in \mathbb{R}$ into account, the constrains C1 and C2 reduce $\varrho_t^{(1,1)}(\tau) = \varrho_t^{(1,2)}(\tau) = \varrho_t^{(2,1)}(\tau) = 0$.

In this case, due to the reduced dimension of $\mathbf{T}$ (4×4), all the TPM's verifying (28) and C5, with $T_{p_1,p_2} \in \{0, 0.5\}$, can be found by an exhaustive search. They are given by:

$$\mathbf{T}_1^{(A)} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_2^{(A)} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix},$$

$$\mathbf{T}_3^{(A)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}, \mathbf{T}_4^{(A)} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

Moreover, it can be proved that these matrices verify C1-C5 for all $D \geq 2$.

B. Fifth-order system with 8-PSK input signal

In this case, due to the high dimension of $\mathbf{T}$ (8×8), it is impractical to find all the TPM's verifying (28) by an exhaustive search. However, some examples are given by:

$$\mathbf{T}_i^{(B)} = \mathbf{J}^{i-1}\mathbf{T}_1^{(B)}, \tag{31}$$

for $L_B = 1$, and

$$\mathbf{T}_i^{(C)} = \mathbf{J}^{i-1}\mathbf{T}_1^{(C)}, \tag{32}$$

for $L_B = 2$, with $i = 1, 2, ..., 8$,

$$\mathbf{J} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{33}$$

$$\mathbf{T}_1^{(B)} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{34}$$

and

$$\mathbf{T}_1^{(C)} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{35}$$

for $D \geq 2$.

VI. Estimation Algorithm

The channel estimation problem is solved by minimizing in an alternate way the two following least squares cost functions:

$$J = \left\|\hat{\mathbf{R}}_1 - \left(\bar{\mathbf{C}} \diamond \mathbf{H}_a\right) \mathbf{H}_b^T\right\|_F^2 = \left\|\hat{\mathbf{R}}_3 - \left(\mathbf{H}_b \diamond \bar{\mathbf{C}}\right) \mathbf{H}_a^T\right\|_F^2, \tag{36}$$

where $\|\cdot\|_F$ denotes the *Frobenius norm*, $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_3$ are sample estimates of the *unfolded matrices* of the tensor $\mathcal{R}$, defined as:

$$\mathbf{R}_1 \equiv \begin{bmatrix} \mathbf{R}_{1\cdot\cdot} \\ \vdots \\ \mathbf{R}_{D\cdot\cdot} \end{bmatrix}, \qquad \mathbf{R}_3 \equiv \begin{bmatrix} \mathbf{R}_{\cdot\cdot 1} \\ \vdots \\ \mathbf{R}_{\cdot\cdot R} \end{bmatrix}, \tag{37}$$

where the matrices $\mathbf{R}_{d\cdot\cdot}$, and $\mathbf{R}_{\cdot\cdot r}$ are respectively the first and third-mode slices of $\mathcal{R}$.

Thus, the it^{th} iteration of the Alternating Least Squares (ALS) algorithm can be described by the following steps:

$$\hat{\mathbf{H}}_b^{(it)} = \left[\left(\bar{\mathbf{C}} \diamond \hat{\mathbf{H}}_a^{(it-1)}\right)^{\dagger} \hat{\mathbf{R}}_1\right]^T, \ \hat{\mathbf{H}}_a^{(it)} = \left[\left(\hat{\mathbf{H}}_b^{(it)} \diamond \bar{\mathbf{C}}\right)^{\dagger} \hat{\mathbf{R}}_3\right]^T, \tag{38}$$

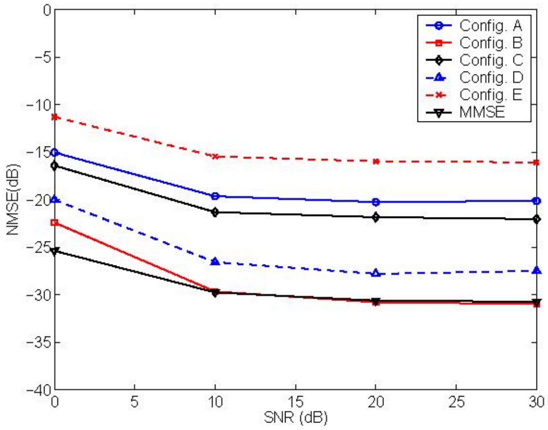

Fig. 1. NMSE of the channel parameters versus SNR.

where $\hat{\mathbf{H}}_a^{(0)}$ and $\hat{\mathbf{H}}_b^{(0)}$ are chosen as $R \times Q$ Gaussian random matrices and $(\cdot)^{\dagger}$ denotes the matrix pseudo-inverse. The algorithm iterates until the convergence of the estimated parameters is achieved. Three channel estimates can then be obtained: $\hat{\mathbf{H}}_a^{(it)}$, $(\hat{\mathbf{H}}_b^{(it)})^*$ and $0.5[\hat{\mathbf{H}}_a^{(it)} + (\hat{\mathbf{H}}_b^{(it)})^*]$, the final channel estimate being chosen as the one providing the small value of the cost function (36). The matrix $\bar{\mathbf{C}}$ is assumed to be known as it can be calculated using (19) and (20).

VII. Simulation Results

A MIMO Wiener model of an uplink channel of a Radio Over Fiber multiuser communication system [2], [1], [13] is considered for the simulations. The wireless interface is a memoryless $R \times T$ linear mixer, consisting in an uniformly spaced array of R antennas. The antennas are half-wavelength spaced and the transmitted signals are narrowband with respect to the array aperture. The E/O conversion in each antenna is modelled as a memoryless polynomial [13]. All the results in this section were obtained via Monte Carlo simulations using $N_R = 100$ independent data realizations and a data block of $N = 2000$ symbols.

The proposed channel estimation method is evaluated by means of the Normalized Mean Squared Error (NMSE) of the estimated channel parameters, defined as: $NMSE = \frac{1}{N_R}\sum_{l=1}^{N_R}\frac{\|\mathbf{H}-\hat{\mathbf{H}}_l\|_F^2}{\|\mathbf{H}\|_F^2}$, where $\hat{\mathbf{H}}_l$ represents the channel matrix estimated at the l^{th} Monte Carlo simulation. Fig. 1 shows the NMSE versus SNR for various simulation configurations described in Table I. We also show the performance of the supervised Minimum Mean Squared Error (MMSE) receiver. Note that Configuration B provides a performance very close to that of the MMSE receiver and, as expected, the performance degrades when the number of quasi-sources Q increases.

VIII. Conclusion

In this paper, a blind channel estimation method has been proposed for memoryless multiuser Volterra communication channels. Orthogonality constraints for the transmitted signals have been established to get a PARAFAC decomposition of a tensor composed of spatio-temporal covariance matrices of the received signals. Modulation codes have been used to satisfy these constraints, by designing the TPM's associated with the DTMC that characterizes each transmitted signal. A two-steps version of the ALS algorithm is used for estimating the channel. The proposed method has been applied for identifying an uplink channel in a ROF multiuser communication system, providing a performance close to the one of the supervised MMSE receiver. In future works, other kinds of diversities will be investigated, in particular for Code Division Multiple Access systems.

TABLE I

Simulation Configurations

Config.	T	R	$2\vec{K}+1$	TPM of user 1	TPM of user 2	TPM of user 3
A	2	4	3	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_1^{(A)}$	-
B	2	4	3	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_2^{(B)}$	-
C	2	4	3	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_1^{(C)}$	-
D	2	4	5	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_2^{(B)}$	-
E	3	10	3	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_2^{(B)}$	$\mathbf{T}_3^{(B)}$
MMSE	2	4	3	$\mathbf{T}_1^{(B)}$	$\mathbf{T}_2^{(B)}$	-

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