ROBUST PILOT-AIDED CHANNEL ESTIMATOR FOR TIME-VARYING OFDM CHANNELS

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ABSTRACT

This paper deals with time-varying channel estimation in *orthogonal frequency division multiplexing* (OFDM) systems by means of pilot-aided strategies. A robust estimator which does not depend on the channel statistics is proposed. The derivation is based on a constrained filtering approach and particular cases are analyzed. Simulation results are provided in order to verify the better performance of the proposal when compared to estimators reported in the literature.

1. INTRODUCTION

To achieve high data rates in *orthogonal frequency division multiplexing* (OFDM) [1] it is mandatory to employ multilevel modulation with nonconstant amplitude, such as 16-QAM. For efficient coherent demodulation, it is necessary an accurate channel estimation method capable to track the variations of the fading channel. Furthermore, the performance of many diversity decoding techniques depends heavily on good channel estimates, specially when the channel is time-varying in nature. In the works [2, 3], it is derived a *minimum mean-square error* (MMSE) channel estimator based on pilot symbols using Wiener-type filters. The disadvantage of the optimum design of these filters is the required knowledge of the channel statistics, i.e. time and frequency channel correlations, which are usually unknown at the receiver and their estimation has a high computational burden.

Based on the use of a comb pilot pattern arrangement, subspace projection and low-pass filtering, in [4] proposes an algorithm which does not depends on the channel statistics. In [5] the implementation of the subspace projection is performed by means of a factorization based on a QR matrix decomposition leading to a better performance in terms of mean square error (MSE). Further, the noise in the projection components can be attenuated via Wiener filtering and a subspace tracking (ST) algorithm can perform the signal subspace estimation.

In this paper we propose a robust OFDM channel estimator which has a twofold objective: its does not requires the knowledge about the channel statistics (temporal and frequency correlations) and has a higher performance in terms of MSE than the ones reported in the literature.

The rest of the paper is organized as follows. Section 2 describes the OFDM system and the channel model we have taken into consideration. The derivation and analysis of both MMSE and robust estimators are shown in Sections 3 and 4, respectively. Section 5 is devoted to present the simulation results and, finally, Section 6 states our conclusions and perspectives.

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2. SYSTEM MODEL

In our model, the received signal $x[m,k]$ and transmitted symbol $a[m,k]$ at the m-th OFDM symbol and k-th subcarrier, for $k = 0, \dots, N_s - 1$, are related as

$$x[m,k] = H[m,k]a[m,k] + u[m,k] + w[m,k],$$

where $H[m,k]$ is the subcarrier complex attenuation, and $u[m,k]$ and $w[m,k]$ are the *inter-carrier interference* (ICI) and noise components, respectively. We consider that the symbols $a[m,k]$ are uncorrelated for different m's and k's. The noise $w[m,k]$ is supposed i.i.d. and independent of the remaining signals. Supposing that $a[m,k]$ at the pilot positions are selected from a PSK constellation, the LS estimate of $H[m,k]$ can be found simply back-rotating $x[m,k]$, which results in

$$\tilde{H}[m,k] = x[m,k]a^*[m,k] = H[m,k] + z[m,k]$$

where $z[m,k] = (u[m,k] + w[m,k])a^*[m,k]$. Due to the uncorrelated assumption of $a[m,k]$, we can write

$$\mathbb{E}\{z^*[m_1,k_1]z[m_2,k_2]\} = 0, \quad \text{for } m_1 \neq m_2 \text{ or } k_1 \neq k_2,$$
$$\mathbb{E}\{H^*[m_1,k_1]z[m_2,k_2]\} = 0, \quad \text{for any } m_1, m_2, k_1, k_2,$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. This simplifies significantly the design of a second-order estimator for $H[m,k]$ (see [2]). We consider the pilot subcarriers are arranged in *grid*. The pilot subcarriers are allocated at positions $m = nM_t$ and $k = lM_f$, for $n \in \mathbb{N}$ and $l = 0, \dots, N_p - 1$, where N_p is the number of pilot subcarriers per OFDM symbol. Let $\tilde{\mathbf{H}}[n]$, $\mathbf{H}[n]$ and $\mathbf{z}[n]$ be column vectors containing respectively $\tilde{H}[nM_t, lM_f]$, $H[nM_t, lM_f]$ and $z[nM_t, lM_f]$, for $l = 0, \dots, N_p - 1$. Then we can state

$$\mathbf{R}_{\tilde{H}} = \mathbb{E}\{\tilde{\mathbf{H}}[n]\tilde{\mathbf{H}}^H[n]\} = \mathbf{R}_H + \rho\mathbf{I},$$

where $\mathbf{R}_H = \mathbb{E}\{\mathbf{H}[n]\mathbf{H}^H[n]\}$ and ρ is the variance of $z[nM_t, lM_f]$. We also consider a cyclic prefix with length N_{cp} in order to avoid inter-symbol interference.

The considered channel model is the WSS-US one with a constant number of paths. In this case, the base-band impulse response is given by

$$h[m,l] = \sum_{i=0}^{K-1} \gamma_i[m]g_i[l],$$

where $\gamma_i[m]$ is the complex amplitude of the i-th path, and $g_i[l] = g(lT_s - \tau_i)$, where T_s is the sample period and $g(\tau)$ is the shaping filter impulse response that satisfies the Nyquist criterion. The WSS-US assumption imposes the restriction

$$\mathbb{E}\{\gamma_{i'}^*[m']\gamma_i[m'+m]\} = \begin{cases} \rho_i r_i[m], & \text{if } i' = i, \\ 0, & \text{if } i' \neq i, \end{cases}$$

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where ρ_i and $r_i[m]$ denote the mean power and normalized correlation of the i-th path, respectively. In the frequency domain, we have [4]

$$H[n,k] = \sum_{i=0}^{K-1} \overline{\gamma}_i[n] \exp(-j2\pi k \Delta f \tau_i),$$

where $\Delta f = 1/T_s N_c$, and we denoted

$$\overline{\gamma}_i[n] = \frac{1}{N_c} \sum_{l=0}^{N_c-1} \gamma_i[(n-1)N_s + N_{\text{cp}} + l].$$

From the expansion of $\overline{\gamma}_i[n]$ in the expression above, a straightforward computation shows us

$$\begin{aligned}\mathbb{E}\{\overline{\gamma}_i^*[n']\overline{\gamma}_i[n'+n]\} &= \frac{\rho_i}{N_c^2} \sum_{i_1,i_2=0}^{N_c-1} r_i[nN_s + i_2 - i_1] \\ &= \kappa_i \rho_i \overline{r}_i[n],\end{aligned}$$

where $\overline{r}_i[n]$ is the normalized correlation of $\overline{\gamma}_i[n]$, and we have defined the normalization factor

$$\kappa_i = \mathbb{E}|\overline{\gamma}_i[n]|^2/\rho_i = \frac{1}{N_c^2} \sum_{i_1,i_2=0}^{N_c-1} r_i[i_2 - i_1].$$

Observe that this factor satisfies $0 \leq \kappa_i \leq 1$ and can be interpreted as the power loss ratio of the i-th path. With the above statements, we can write

$$\begin{aligned}r_H[n,k] &= \mathbb{E}\{H^*[n',k']H[n'+n,k'+k]\} \\ &= \sum_{i=0}^{K-1} \kappa_i \rho_i \overline{r}_i[n] \exp(-j2\pi k \Delta f \tau_i).\end{aligned}$$

Just when all paths have the same correlation function $r_t[n]$, the separability property [3] is valid:

$$r_H[n,k] = \kappa \sigma_h^2 \overline{r}_t[n] r_f[k],$$

where we define the normalized frequency correlation

$$r_f[k] = \sum_{i=0}^{K-1} \frac{\rho_i}{\sigma_h^2} \exp(-j2\pi k \Delta f \tau_i), \qquad \sigma_h^2 = \sum_{i=0}^{K-1} \rho_i,$$

and observe that all paths have the same factor κ. The channel power σ_h^2 is attenuated by the factor κ, such that the sub-carrier power is given as $\sigma_H^2 = \kappa \sigma_h^2$. The Fourier transforms of $r_t[n]$ and $\overline{r}_t[n]$, which we denote by $p_t[n]$ and $\overline{p}_t[n]$, respectively, are related according to [3]

$$\kappa \overline{p}_t(\nu) = \frac{1}{N_s} p_t\Big(\frac{\nu}{N_s}\Big) m_t\Big(\frac{\nu}{N_s}\Big),$$

where

$$m_t(\nu) = \frac{\text{sinc}^2(N_c \nu)}{\text{sinc}^2(\nu)}.$$

The function $m_t(\nu)$ is even and strictly decreasing in $[0, 1/2]$ with maximum $m_t(0) = 1$. Such property justifies the appearance of the attenuation factor $0 \leq \kappa \leq 1$, and shows how the power spectral density of $H[n,k]$ is attenuated. In addition, the factor κ can be alternatively expressed as

$$\kappa = \int_{-1/2}^{1/2} p_t(\nu) m_t(\nu) d\nu.$$

The remaining power $\sigma_h^2 - \sigma_H^2 = (\kappa - 1)\sigma_h^2$ appears as the ICI power.

In the sequel, we will be interested in the correlation $\overline{\overline{r}}_t[n] = \overline{r}_t[nM_t]$, whose Fourier transform is related to $\overline{p}_t(\nu)$ according to

$$\kappa \overline{\overline{p}}_t(\nu) = \kappa \frac{1}{N_s} \overline{p}_t\Big(\frac{\nu}{N_s}\Big).$$

3. MMSE OFDM CHANNEL ESTIMATOR

3.1. IIR Estimator

The Wiener filter coefficients $c[m,l;n',k]$ are chosen such that the estimative

$$\hat{H}[n,k] = \sum_{m=-\infty}^{\infty} \sum_{l=(k-k'-N_c)/M_f+1}^{(k-k')/M_f} c[m,l;n',k] \cdot \tilde{H}[n-n'-mM_t, k-k'-lM_f] \quad (1)$$

minimizes the mean square error

$$\mathbb{E}|\hat{H}[n,k] - H[n,k]|^2. \quad (2)$$

The indices $n' = n \mod M_t$ and $k' = k \mod M_f$ are inserted in order to simplifies calculations. Minimizing the error in Eq. (2) leads the orthogonality principle

$$\mathbb{E}\{(\hat{H}[n,k] - H[n,k])\tilde{H}^*[n-n'-mM_t, k-k'-lM_f]\} = 0. \quad (3)$$

Inserting the expression for $\hat{H}[n,k]$ in Eq. (3), we can write

$$\sum_{m_1,l_1} \kappa \overline{r}_t[(m-m_1)M_t] r_f[(l-l_1)M_f] c[m_1,l_1;n',k] - \kappa \overline{r}_t[n'+mM_t] r_f[k'+lM_f] + \rho c[m,l;n',k] = 0. \quad (4)$$

We denote

$$\begin{aligned}\mathbf{r}_f[k] &= (r_f[k], r_f[k-M_f], \ldots, r_f[k-N_c+M_f])^T \\ \mathbf{R}_f &= (\mathbf{r}_f[0], \mathbf{r}_f[M_f], \ldots, \mathbf{r}_f[N_c-M_f]),\end{aligned}$$

and

$$\mathbf{c}_2[m;n',k] = \begin{pmatrix} c[m,(k-k')/M_f;n',k] \\ c[m,(k-k')/M_f-1;n',k] \\ \vdots \\ c[m,(k-k'-N_c)/M_f+1;n',k] \end{pmatrix}.$$

The subindex '2' in $\mathbf{c}_2[m;n',k]$ indicates that the elements in the vector are ordered along l. Hence, Eq. (4) can be vectorized according to (where $*$ denotes the convolution operator)

$$(\rho \mathbf{I} + \kappa \mathbf{R}_f \overline{\overline{r}}_t[m]) * \mathbf{c}_2[m;n',k] = \kappa \overline{r}_t[n'+mM_t] \mathbf{r}_f[k].$$

Applying a Fourier transform to the equation above, we have

$$(\rho \mathbf{I} + \kappa \mathbf{R}_f \overline{\overline{p}}_t(\nu)) \mathbf{c}_2(\nu;n',k) = \kappa \overline{\overline{p}}_t(\nu) \phi(\nu;n') \mathbf{r}_f[k],$$

where we denoted $\phi(\nu;n') = \exp(j2\pi \frac{n'}{M_t}\nu)$. If we define

$$\begin{aligned}\tilde{\mathbf{R}}_f &= (\mathbf{r}_f[0], \ldots, \mathbf{r}_f[N_c-1]) \\ \mathbf{C}(\nu;n') &= (\mathbf{c}_2(\nu;n',0), \ldots, \mathbf{c}_2(\nu;n',N_c-1)),\end{aligned}$$

we can write

$$(\rho \mathbf{I} + \kappa \mathbf{R}_f \overline{\overline{p}}_t(\nu)) \mathbf{C}(\nu;n') = \kappa \overline{\overline{p}}_t(\nu) \phi(\nu;n') \tilde{\mathbf{R}}_f. \quad (5)$$

Let the eigendecomposition of $\mathbf{R}_f$ be given as

$$\mathbf{R}_f = \mathbf{U}\mathbf{D}\mathbf{U}^H, \quad (6)$$

where $\mathbf{U}$ is a unitary matrix, and $\mathbf{D}$ is the diagonal matrix containing the eigenvalues d_l. We denote the elements of the pseudo-inverse $\mathbf{D}^\dagger$

of $\mathbf{D}$ according to

$$d_l^\dagger = \begin{cases} 1/d_l, & \text{if } d_l \neq 0, \\ 0, & \text{if } d_l = 0, \end{cases}$$

From Eq. (6), the inverse of the matrix multiplying the term $\mathbf{C}(\nu; n')$ in Eq. (5) can be easily found. Let $\mathbf{\Phi}(\nu)$ be a diagonal matrix whose entries are

$$\Phi_l(\nu) = \frac{\kappa d_l \overline{\overline{p}}_t(\nu)}{\rho + \kappa d_l \overline{\overline{p}}_t(\nu)}.$$

Then, we finally have

$$\begin{aligned} \mathbf{C}(\nu; n') &= \mathbf{U}\mathbf{\Phi}(\nu)\phi(\nu; n')\mathbf{D}^\dagger \mathbf{U}^H \tilde{\mathbf{R}}_f \\ &= \mathbf{U}\mathbf{\Phi}(\nu)\phi(\nu; n')\mathbf{U}^H \mathbf{R}_f^\dagger \tilde{\mathbf{R}}_f. \end{aligned} \quad (7)$$

3.2. FIR Estimator

In Eq. (1), the index m is taken over the integers. This leads to a impractical implementation of the resulting Wiener filter. An alternative is to restrict the index m to the values $m = -M, \ldots, M$. Following analogous steps as above, Eq. (7) ends up been expressed as

$$\mathbf{C}(\nu; n') = \mathbf{U}\mathbf{\Phi}^*(\nu; n')\mathbf{U}^H \mathbf{R}_f^\dagger \tilde{\mathbf{R}}_f,$$

where $\mathbf{\Phi}(\nu; n')$ is a diagonal matrix whose l-th element is the frequency domain response a filter of length $2M+1$ whose coefficients are

$$\mathbf{c}[l; n'] = \left(\frac{\rho}{\kappa d_l}\mathbf{I} + \mathbf{R}_t\right)^{-1} \mathbf{r}_t[n'], \quad (8)$$

for $d_l \neq 0$, and zero for $d_l = 0$, where we denoted

$$\begin{aligned} \mathbf{r}_t[n'] &= (\overline{r}_t[n' + M \cdot M_t], \ldots, \overline{r}_t[n' - M \cdot M_t])^T \\ \mathbf{R}_t &= (\mathbf{r}_t[-M \cdot M_t], \ldots, \mathbf{r}_t[M \cdot M_t]). \end{aligned}$$

Here a substantial difference is that we cannot separate $\mathbf{\Phi}(\nu; n')$, while in the IIR case firstly we estimate the pilot subcarriers, and after realize a sinc interpolation in time. If the correlations $\overline{r}_t[mM_t]$ are nearly null for $|m| > M$, we can apply an approximation to the IIR case. Firstly, we estimate the pilot subcarriers using the FIR filter $\mathbf{c}[l; 0]$ of length $2M + 1$ as above, and apply a sinc interpolation in time. This is the approach we employ in the robust estimator.

4. ROBUST OFDM CHANNEL ESTIMATOR

For an optimum channel estimation, the channel correlations must be known. In practice, the estimation of such correlations are intractable, since it is computationally demand. Additionally the channel statistics may change with time. A suboptimum solution is to make an arbitrary choice for the correlations $\overline{\overline{r}}_t[n]$ and $r_f[n]$, whose MSE is near to the optimum case. This estimator is robust in the sense of not depending on the channel statistics.

In what follows, the estimation will be just considered over the pilot subcarriers. Then we are interested in the filtering structure

$$\mathbf{C}(\nu) = \mathbf{C}(\nu; 0) = \mathbf{U}\mathbf{\Phi}(\nu)\mathbf{U}^H.$$

In order to simplify notation, we will use $c[m, l; k]$ in the place of $c[m, l; 0, kM_t]$, for $k = 0, \ldots, N_p - 1$.

4.1. Derivation of the Robust Estimator

Let $\overline{\text{MSE}}$ be the mean square error averaged over pilot subcarriers, i.e.,

$$\overline{\text{MSE}} = \frac{1}{N_p} \sum_{k=0}^{N_p-1} \mathbb{E}|\hat{H}[n, kM_f] - H[n, kM_f]|^2.$$

If the correlations of the entries $\widetilde{\mathbf{H}}[n]$ are

$$\mathbf{R}_{\widetilde{H}}[n] = \mathbb{E}\{\widetilde{\mathbf{H}}[n + n']\widetilde{\mathbf{H}}^H[n']\} = \kappa\overline{\overline{p}}_t[n]\mathbf{R}_f + \rho\mathbf{I},$$

then the $\overline{\text{MSE}}$ corresponding to the filter of coefficients $\widetilde{c}[m, l; k]$ is (see [2])

$$\overline{\text{MSE}} = \frac{1}{N_p}\int_{-1/2}^{1/2} \kappa\overline{\overline{p}}_t(\nu)\,\text{tr}\{(\widetilde{\mathbf{C}}(\nu) - \mathbf{I})^H \mathbf{R}_f(\widetilde{\mathbf{C}}(\nu) - \mathbf{I})\}d\nu + \frac{1}{N_p}\int_{-1/2}^{1/2} \rho\,\text{tr}\{\widetilde{\mathbf{C}}^H(\nu)\widetilde{\mathbf{C}}(\nu)\}d\nu, \quad (9)$$

where $\widetilde{\mathbf{C}}(\nu)$ is the frequency domain response of the estimator filter.

Initially, we assume that $\widetilde{\mathbf{C}}(\nu)$ is given in the form

$$\widetilde{\mathbf{C}}(\nu) = \mathbf{U}\widetilde{\mathbf{\Phi}}(\nu)\mathbf{U}^H, \quad (10)$$

where $\mathbf{U}$ is the unitary matrix found in Eq. (6). The elements of the diagonal matrix $\widetilde{\mathbf{\Phi}}(\nu)$ are choosen according to

$$\widetilde{\Phi}_l(\nu) = \frac{\kappa d_l \overline{\overline{p}}_{tr}(\nu; l)}{\rho + \kappa d_l \overline{\overline{p}}_{tr}(\nu; l)}. \quad (11)$$

where the arbitrary terms $\overline{\overline{p}}_{tr}(\nu; l)$ satisfy the constraints

$$\int_{-1/2}^{1/2} p_{tr}(\nu; l)m_t(\nu)d\nu = \kappa, \qquad \int_{-1/2}^{1/2} p_{tr}(\nu; l)d\nu = 1. \quad (12)$$

From the above statements, Eq. (9) simplifies to

$$\overline{\text{MSE}} = \overline{\text{MSE}}'' + \overline{\text{MSE}}',$$

where

$$\overline{\text{MSE}}'' = \frac{1}{N_p}\sum_{l=0}^{N_p} \kappa d_l \int_{-1/2}^{1/2} \{\overline{\overline{p}}_t(\nu) - \overline{\overline{p}}_{tr}(\nu; l)\}|\widetilde{\Phi}_l(\nu) - 1|^2 d\nu$$

$$\overline{\text{MSE}}' = \frac{1}{N_p}\sum_{l=0}^{N_p-1} \rho \int_{-1/2}^{1/2} \widetilde{\Phi}_l(\nu)d\nu.$$

The term $\overline{\text{MSE}}'$ does not depend on the cannel statistics, and $\overline{\text{MSE}}''$ is interpreted as a "residual" $\overline{\text{MSE}}$. The performance of the estimator does not depend on the channel statistics when $\overline{\text{MSE}}''$ is null.

We will find functions $\overline{\overline{p}}_{tr}(\nu; l)$ under the constraints in Eq. (12) that maximizes $\overline{\text{MSE}}'$. Then, we will show that $\overline{\text{MSE}}''$ is nulled for these $\overline{\text{MSE}}'$ we found. This problem is formulated as

$$\text{maximize:} \quad \int_{-1/2}^{1/2} \frac{d_l y(\nu)m_t(\nu)}{\rho' + d_l y(\nu)m_t(\nu)}d\nu, \quad (13)$$

$$\text{such that:} \quad \int_{-1/2}^{1/2} y(\nu)m_t(\nu)d\nu = \kappa, \quad (14)$$

$$\int_{-1/2}^{1/2} y(\nu)d\nu = 1, \quad (15)$$

where $\rho' = N_s M_t \rho$. Using Lagrange multipliers, this optimization problems leads to

$$-\frac{d_l m_t(\nu)}{[\rho' + d_l y(\nu)m_t(\nu)]^2} + \lambda_1 m_t(\nu) + \lambda_2 = 0, \quad (16)$$

where λ_1 and λ_2 are selected in order to $y(\nu)$ satisfies the above constraints.

Inserting $\widetilde{\Phi}(\nu)$ given in Eq. (11), the expression for $\overline{\text{MSE}}''$ can be rewritten as

$$\overline{\text{MSE}}'' = \frac{1}{N_p}\sum_{l=0}^{N_p}\rho'^2\int_{-1/2}^{1/2}[p_t(\nu)-p_{tr}(\nu;l)]\cdot \frac{d_l m_t(\nu)}{[\rho'+d_l p_{tr}(\nu;l)m_t(\nu)]^2}d\nu. \quad (17)$$

Hence, the result found in Eq. (16) implies

$$\overline{\text{MSE}}'' = \frac{1}{N_p}\sum_{l=0}^{N_p}\rho'^2\int_{-1/2}^{1/2}[p_t(\nu)-p_{tr}(\nu;l)]\cdot (\lambda_1 m_t(\nu)+\lambda_2)d\nu = 0. \quad (18)$$

We have obtained

$$\overline{\text{MSE}} = \overline{\text{MSE}}',$$

and then the filter performance does not depend on the channel statistics.

We can discard or insert more constraints in the problem. If we discard one of the constraints given in Eqs. (14)-(15), the term $\overline{\text{MSE}}''$ continues equal to zero, however the performance worsen, since the resulting $\overline{\text{MSE}}'$ increases. If the maximum Doppler frequency ν_d is supposed known, depending on the constraint we discard, we have the following cases:

$\lambda_1 = 0$: In this case, Eq. (16) results in

$$\kappa_l\overline{\overline{p}}_{tr}(\nu;l) = \left[\frac{1}{I_1}\left(1+\frac{\rho'}{d_l}I_2\right)m_t^{1/2}\left(\frac{\nu}{N_sM_t}\right)-\frac{\rho'}{d_l}\right], \quad (19)$$

where

$$I_1 = \int_{-\nu_d}^{\nu_d}\frac{1}{\sqrt{m_t(\nu)}}d\nu \qquad I_2 = \int_{-\nu_d}^{\nu_d}\frac{1}{m_t(\nu)}d\nu,$$

and κ_l is inserted for making equal to unit the power of $\overline{\overline{p}}_{tr}(\nu;l)$ in the interval $2\nu_d$. We can also found

$$\overline{\text{MSE}}' = 2\nu_d\rho'\frac{K}{N_p} - \frac{1}{N_p}\sum_{l=0}^{K-1}\frac{I_1^2\rho'^2}{I_2\rho'+d_l},$$

where K is the number of eigenvalues d_l different of zero.

$\lambda_2 = 0$: In this case, we have

$$\overline{\overline{p}}_{tr}(\nu;l) = \frac{1}{2(N_sM_t\nu_d)}, \quad (20)$$

which results in

$$\overline{\text{MSE}}' = 2\nu_d\rho'\frac{K}{N_p} - \frac{1}{N_p}\sum_{l=0}^{K-1}\frac{(2\nu_d)^2\rho'^2}{(2\nu_d)\rho'+\kappa d_l}.$$

The expression for the $\overline{\text{MSE}}'$ found in these cases only differ on the summations, which result in an $\overline{\text{MSE}}'$ lower than $2\nu_d\rho'\frac{K}{N_p}$.

The estimators found above are robust in the sense that their performances, expressed as $\overline{\text{MSE}}'$, does not depend on the time channel correlations $\overline{\overline{r}}_t[n]$. It is required we know $\mathbf{R}_f$ and ρ. For eliminating the dependence on $\mathbf{R}_f$, we rewrite Eq. (10) as

$$\widetilde{\mathbf{C}}(\nu) = \mathbf{F}\widetilde{\mathbf{\Phi}}(\nu)\mathbf{F}^H,$$

where $\mathbf{F}$ is the normalized Fourier matrix, and $\widetilde{\Phi}_l(\nu) = \widetilde{\Phi}(\nu)$ is given according to Eq. (11), with $\overline{\overline{p}}_{tr}(\nu;l) = \overline{\overline{p}}_{tr}(\nu)$ and

$$d_l = \begin{cases} N_p/L, & \text{for } 0 \le l \le L-1,\\ 0, & \text{for } L \le l \le N_p-1,\end{cases}$$

where L is the channel length. Then, Eq. (9) is reduced to

$$\overline{\text{MSE}} = \frac{1}{N_p}\int_{-1/2}^{1/2}\kappa\overline{\overline{p}}_t(\nu)|\widetilde{\Phi}(\nu)-1|^2\cdot \text{tr}(\mathbf{F}^H\mathbf{R}_f\mathbf{F}\cdot\mathbf{I}')d\nu + \frac{L}{N_p}\int_{-1/2}^{1/2}\rho|\widetilde{\Phi}(\nu)|^2d\nu,$$

where $\mathbf{I}' = \text{diag}\{(\mathbf{1}_L, \mathbf{0}_{N_p-L})\}$. We know that

$$H[n,k] = \sum_{l=0}^{L-1}\bar{h}[n,l]\omega_{N_c}^{kl},$$

where $h[m,l] = \sum_{i=0}^{K-1}\bar{\gamma}_i[m]g_i[l]$ and $\omega_{N_c}^{kl} = \exp(-j2\pi\frac{kl}{N_c})$. Taking in consideration the pilot positions kM_f, we will obtain

$$H[n,kM_f] = \sum_{l=0}^{L-1}\bar{h}[n,l]\omega_{N_p}^{kl},$$

or in the vectorized form

$$\mathbf{H}_p[n] = \mathbf{W}_p\bar{\mathbf{h}}^0[n],$$

where $\mathbf{W}_p$ is the Fourier matrix of dimension $N_p \times N_p$, and $\bar{\mathbf{h}}^0[n] = (\bar{h}[n,0],\ldots,\bar{h}[n,L-1],0,\ldots,0)^T$, and $\bar{h}[m,l] = \sum_{i=0}^{K-1}\bar{\gamma}_i[m]g_i[l]$. Hence, we have

$$\kappa\mathbf{F}^H\mathbf{R}_f\mathbf{F} = N_p\,\mathbb{E}\{\bar{\mathbf{h}}^0[n](\bar{\mathbf{h}}^0[n])^H\}.$$

Since $\mathbb{E}\{(\bar{\mathbf{h}}^0[n])^H\bar{\mathbf{h}}^0[n]\} = \kappa\sigma_h^2$, it follows that

$$\text{tr}(\mathbf{F}^H\mathbf{R}_f\mathbf{F}\cdot\mathbf{I}') = N_p.$$

Therefore, we have

$$\overline{\text{MSE}} = \overline{\text{MSE}}'' + \overline{\text{MSE}}',$$

where

$$\overline{\text{MSE}}'' = \kappa\int_{-1/2}^{1/2}\{\overline{\overline{p}}_t(\nu)-\overline{\overline{p}}_{tr}(\nu)\}|\widetilde{\Phi}(\nu)-1|^2d\nu$$

$$\overline{\text{MSE}}' = \frac{L}{N_p}\int_{-1/2}^{1/2}\rho|\widetilde{\Phi}(\nu)|^2d\nu.$$

The analysis for the choice of $\overline{\overline{p}}_{tr}(\nu)$ is analogous.

The robust estimator derived above depends on ρ e L, since $\widetilde{\Phi}(\nu) = \widetilde{\Phi}_l(\nu)$ in Eq. (11) is given in terms of these parameters.

Since in practice the filters have finite impulse response length, we use Eq. (8), which is rewritten bellow for $n' = 0$:

$$\mathbf{c}[l] = \left(\frac{\rho}{\kappa d_l}\mathbf{I}+\mathbf{R}_t\right)^{-1}\mathbf{r}_t, \quad (21)$$

where the indices of $\mathbf{c}[l;0]$ and $\mathbf{r}_t[0]$ was omitted. The correlations found in Eqs.(19) and (20) can be inserted in Eq. (21) given above. This turn out in a practical implementation of a FIR robust filter.

We eliminate the dependence on ρ in $\overline{\overline{p}}_{tr}(\nu;l)$, given in Eq. (19), by making $\rho' = 0$, such that we obtain

$$\overline{\overline{p}}_{tr}(\nu) = \kappa_l^{-1}I_1^{-1}m_t^{1/2}(\nu/N_sM_t). \quad (22)$$

Observe that $\overline{\overline{p}}_{tr}(\nu)$ given in Eq. (20) does not depende on ρ. The dependence on ρ given in Eq. (21) can be eliminated if we substitute $\rho/\kappa d_l$ for a small constant δ, such that the inverse there exist in this equation. We have

$$\mathbf{c}[l] = (\delta\mathbf{I}+\mathbf{R}_t)^{-1}\mathbf{r}_t. \quad (23)$$

5. SIMULATION RESULTS

The simulations consider an OFDM system with 800 kHz of total bandwidth, each OFDM symbol is composed by $N_c = 128$ subcarriers and a cyclic prefix of length $N_{\text{cp}} = 15$. The pilot symbols are arranged in $N_p = 16$ subcarriers equally spaced, with each one chosen from a 4-PSK uniformly distributed constellation. The data symbols are chosen from a 16-QAM constellation. The channel model is the TDL model with Jakes spectrum. The channel and symbol power are equal to $\sigma_h^2 = 1$ and $\sigma_a^2 = 1$, respectively. And the number of multipaths is assumed fixed with $K = K_{\max} = 4$.

Fig. 1 shows the filtering strategies we developed and analyzed in this paper. The algorithms are described in what follows.

Initially, we consider a "semi-robust" case where the matrix $\mathbf{U}$, the number of multipaths K, and the parameters κ, ρ and d_l are perfectly known. The algorithms taken into account are the following. U-P: The LS estimate is projected into the signal subspace [4]. U-Wiener: The coefficients from the temporal section are selected according to Eq. (8), where we suppose known the channel correlations and parameters κ, ρ and d_l. U-ERPCa: The parameters κ and ρ are also known, and the coefficients are selected from Eq. (8), with the channel correlations given according to Eq. (19). U-ERPCb: The same as U-ERPCa, with the channel correlations given in Eq. (20). U-FA: The estimate is given by the QR decomposition based algorithm found in [5].

For the totally robust case, we consider the matrix $\mathbf{F}$ in the place of $\mathbf{U}$, and we assume that the channel length L is known. We have the following filtering cases. F-P: The LS estimate is projected into the signal subspace [4]. F-ERa: The coefficients from the temporal section are selected according to Eq. (23), which does not require we know the parameters κ, ρ and d_l. The channel correlations are given according to Eq. (22). F-ERb: The same as F-ERa, with the channel correlations given in Eq. (20). F-FA: The estimate is given by the QR decomposition based algorithm found in [5].

As expected, the robust algorithms outperforms the semi-robust algorithms. With exception of F-FA, which estimates adaptively the matrix $\mathbf{U}$, convergence is meaningless since the others estimators are given in a closed-form solution. In both cases, more the correlations are estimated or inserted, better results are obtained. A more detailed comparison, with broader set of parameters, can be found in [6].

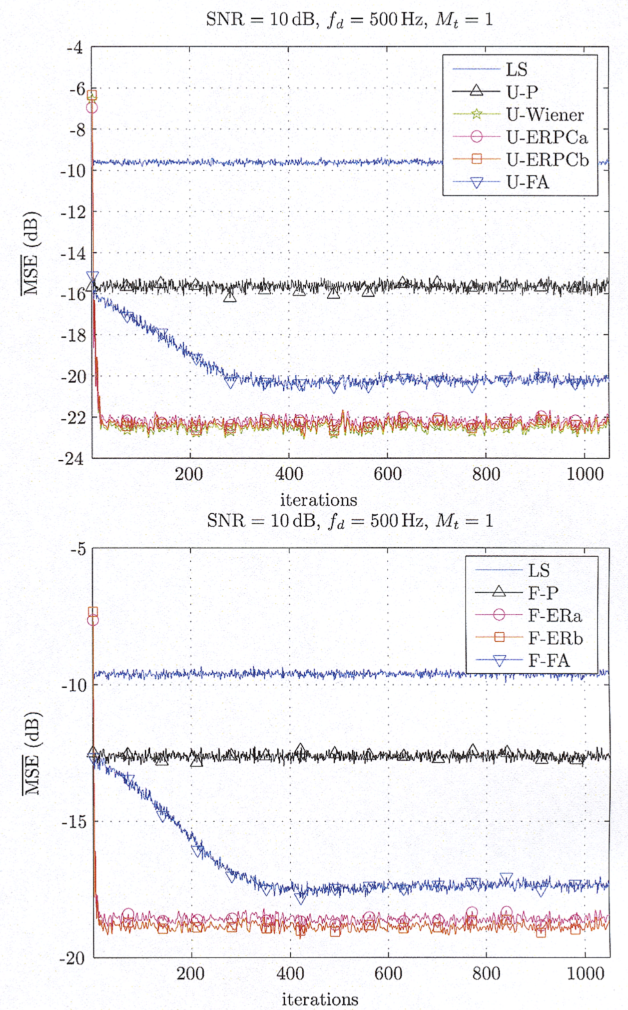

Fig. 1. Partially and totally robust cases for 200 Monte Carlo simulations.

6. CONCLUSIONS

In this paper a robust channel estimator is derived for OFDM systems in a time-varying scenario. The aim of the estimator is to be independent from the channel statistics, i.e. time and frequency correlations, in order to avoid computational effort, which is usually high, for estimation of such statistics. Despite the derivation of the criteria take into account particular constraints the simulations show a superior performance of the proposed estimators when compared to existing methods reported in the literature confirming a great capability of tracking for time-varying channels. A straightforward continuation of this work is the optimization regarding the λ_1 and λ_2 for the derivation of a more general estimator.

7. REFERENCES

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