FILTERED DELAY-SUBSPACE APPROACH FOR PILOT ASSISTED CHANNEL ESTIMATION IN OFDM SYSTEMS

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ABSTRACT

In this paper we propose a pilot-based OFDM channel estimator based on the combination of low-pass filtering and delay-subspace projection. The proposed estimator, which we abbreviate ST-LP, is robust in the sense it does not require prior statistical knowledge of the channel. The only assumptions are the *least-square* (LS) estimates have limited spectrum and the channel follows the *tapped delay line* (TDL) model, which are commonly taken in practice. Since it is desirable slow delay variations operability acceptance, the delay-subspace is tracked by a *subspace tracking* (ST) algorithm. The ST-LP estimator can be implemented by two filtering structures, which provide a trade-off between accuracy and complexity. Simulation results confirm the superior performance of the ST-LP estimator when compared to methods already reported in the literature.

1. INTRODUCTION

To achieve high data rates in *orthogonal frequency division multicarrier modulation* (OFDM) [1] it is mandatory to employ multilevel modulation with nonconstant amplitude, such as 16-QAM. For efficient coherent demodulation, it is necessary an accurate channel estimation method capable to track the variations of the fading channel. Furthermore, the performance of many diversity decoding techniques depends heavily on good channel estimates. In the works [2, 3], it is derived a *minimum mean-square error* (MMSE) channel estimator based on pilot symbols using Wiener-type filters. The disadvantage of the optimum design of these filters is the required knowledge of the channel statistics, which are usually unknown at the receiver. This problem is avoided in the estimator we propose.

With the use of a comb pilot pattern arrangement [4, 5], we propose a channel estimator based on the application of subspace projection and low-pass filtering. Observing that the pilot subcarrier *least-square* (LS) estimates have limited spectrum, we then apply a low-pass filter that results in a significant decreasing of noise level. In the pilot subcarrier spectrum region that is not affected by the filter, part of the noise is discarded by projection onto the delay-subspace. From the parametric channel model, we have derived the delay-subspace [6]. The slow delay-subspace variations are tracked by the subspace tracking algorithm presented in [7]. The proposed estimator is robust in the sense it is required a limited Doppler spectrum and delays of slow variations and these assumptions are commonly observed in practice. As we will present in the paper, the estimator can be implemented by means of two forms. The first one gives more accurate estimates and presents higher computational complexity. The later has inferior performance and lower computational complexity. As verified by computer simulation, the proposed estimator gives much more accurate estimates w.r.t. to the LS channel estimator.

The rest of the paper is organized as follows. In Section 2, we describe the OFDM system and the channel model we have taken into consideration. Section 3 exposes the development of the proposed estimators. In Section 4, some computer simulation results and the effective performance of the proposed estimator are shown. The paper ends in Section 5 with our conclusions and perspectives.

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2. CONSIDERED MODELS

2.1. OFDM System Model

Fig. 1 shows the base-band model of an OFDM system.

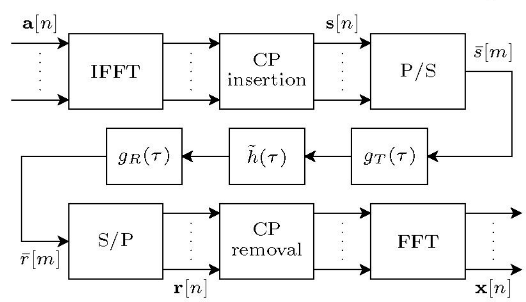

Fig. 1. OFDM System

Let $\mathbf{a}[n] = (a[n,0],\ldots,a[n,N_c-1])^T$ be the vector containing the frequency domain symbols allocated at the N_c subcarriers of the nth OFDM symbol. The transmitted OFDM symbol is constituted of the time domain version of $\mathbf{a}[n]$ (obtained by a normalized IDFT) added of its cyclic prefix. Let $\mathcal{F}$ be the Fourier matrix of (k,l)-entry given by $\exp(j2\pi kl/N_c)$. The addition of the cyclic prefix is provided by the application of the matrix $\mathbf{\Theta} = \left(\begin{smallmatrix} 0 & \mathrm{I}_{N_{\mathrm{cp}}} \\ & \mathrm{I}_{N_c} \end{smallmatrix}\right)$, that copies to the top of the resulting vector the last N_{cp} elements of a vector of length N_c. Hence, we have the following OFDM symbol of length $N = N_{\text{cp}} + N_c$:

$$\mathbf{s}[n] = \tfrac{1}{\sqrt{N_c}} \mathbf{\Theta} \mathcal{F}^H \mathbf{a}[n], \qquad (1)$$

of elements $s[n,k]$. Each vector $\mathbf{s}[n]$ is serialized, resulting in the transmitted signal $\overline{s}[m] = s[\lfloor m/N \rfloor, (m)_N]$, where $\lfloor\cdot\rfloor$ and $(\cdot)_N$ are the floor and module operator, respectively.

Let $h[m,l]$ be the discrete time base-band channel impulse response. In order to avoid interference between different OFDM symbols, we assume that the maximum channel delay L satisfies $L \leq N_{\text{cp}} + 1$. With a white and circularly symmetric additive Gaussian noise $\eta[m]$ at the receiver, the received signal is given by

$$\overline{r}[m] = \sum_{l=0}^{L-1} h[m,l]\overline{s}[m-l] + \eta[m]. \qquad (2)$$

We obtain the received OFDM symbol by serial-to-parallel conversion of $\overline{r}[m]$, which gives the vector $\mathbf{r}[n] = (\overline{r}[nN_c], \ldots, \overline{r}[nN_c + N_c - 1])^T$. The received OFDM symbol is demodulated by the removal of the cyclic prefix and the application of a normalized DFT. Further, we assume that $\mathbf{\Theta}^{-1} = (\mathbf{0}_{N_c \times N_{\text{cp}}}, \mathbf{I}_{N_c})$ is the matrix responsible by discarding the cyclic prefix. Thus, the vector containing the signal at the subcarriers is given by

$$\mathbf{x}[n] = \tfrac{1}{\sqrt{N_c}} \mathcal{F} \mathbf{\Theta}^{-1} \mathbf{r}[n]. \qquad (3)$$

The signal at the subcarriers can be written as

$$\mathbf{x}[n] = \text{diag}(\mathbf{a}[n])\mathbf{H}[n] + \mathbf{u}[n] + \mathbf{z}[n], \qquad (4)$$

where $\mathbf{H}[n]$ is the channel response at the frequency domain, $\mathbf{u}[n]$ is the *intercarrier interference* (ICI), $\mathbf{z}[n] = \frac{1}{\sqrt{N_c}} \mathcal{F} \boldsymbol{\eta}[n]$ is the noise at the frequency domain and diag$(\cdot)$ gives a diagonal matrix formed with the elements of the argument in the diagonal.

Defining the vector $\mathbf{h}[m] = (h[m,0], \ldots, h[m,L-1])^T$ and $\mathbf{h}^0[m]$ a version of $\mathbf{h}[m]$ trailed by $N_c - L$ zeros, after some calculations, we can write

$$\mathbf{H}[n] = \mathcal{F}\left(\frac{1}{N_c} \sum_{m=nN+N_{\text{cp}}}^{(n+1)N-1} \mathbf{h}^0[m] \right), \qquad (5)$$

i.e., the $N_c \times 1$ vector $\mathbf{H}[n]$ is the frequency domain version of the time average over N_c consecutive channel realizations. If the channel do not vary in time, it means $\mathbf{h}^0 = \mathbf{h}^0[m]$ for all m, then we have $\mathbf{h}[n] = \mathcal{F}\mathbf{h}^0$, and in this case the ICI is totaly supressed, i.e., $\mathbf{u}[n] = \mathbf{0}$.

2.2. Channel Model

We consider here the *tapped delay line* (TDL) channel model [8]. The base-band representation of the time varying channel impulse response is given by

$$\widetilde{h}(t,\tau) = \sum_{k=0}^{K-1} \gamma_k(t)\delta(\tau - \tau_k), \qquad (6)$$

where $\gamma_k(t)$ and τ_k are the time-varying complex amplitude and the delay of the kth path, respectively. In $\widetilde{h}(t,\tau)$ we have not included the modulate filter $g_T(\tau)$ at the transmitter and the matched filter $g_R(\tau)$ at the receiver. The delay τ_k is considered almost constant, varying slowly with time. The complex amplitude $\gamma_k(t)$ is a *wide sense stationary* (WSS) process w.r.t. time. For different k's, the realizations of $\gamma_k(t)$ are independent. We assume that each $\gamma_k(t)$ has the same normalized correlation function, according to

$$r_{\gamma_k}(\Delta t) = \mathbb{E}\{\gamma_k(t+\Delta t)\gamma_k^*(t)\} = \rho(\tau_k) r_t(\Delta t). \qquad (7)$$

Considering the channel obeys the Jakes' model [9], we have

$$r_t(\Delta t) = J_0(2\pi f_d \Delta t), \qquad (8)$$

$$\rho(\tau) = \tfrac{1}{\tau_{\max}} \exp(-\tau/\tau_{\max}), \qquad (9)$$

where J_0 is the zero-order Bessel function of the first kind, and $\tau_{\max}$ is the maximum delay spread of the channel. The normalized spectral density of $\gamma_k(t)$ is limited to the maximum Doppler frequency f_d and is given by

$$P_J(f) = \begin{cases} \frac{1}{\pi f_d} \frac{1}{\sqrt{1-(f/f_d)^2}}, & \text{for } |f| < f_d, \\ 0, & \text{otherwise.} \end{cases} \qquad (10)$$

The overall base-band channel impulse response is given by $h(t,\tau) = g(\tau) * \widetilde{h}(t,\tau)$, where $g(\tau) = g_T(\tau) * g_R(\tau)$ is the composite impulse response of the modulate and matched filters. These filters are designed to obey the Nyquist criterion. Generally $g(\tau)$ is a raised-cosine with roll-off α. Adopting the sampling period T, we have

$$h[m,l] = h(mT, lT) = \sum_{k=0}^{K-1} \gamma_k(mT) g(lT - \tau_k - \tau_0'), \qquad (11)$$

where τ_0' was inserted in order to make $h[m,l]$ causal. Since $g(\tau) \approx 0$ for sufficiently large τ, we can consider $h[m,l] \approx 0$ for l different of $\{0, \ldots, L-1\}$. We can write

$$\mathbf{h}[m] = \mathbf{G}_{\boldsymbol{\tau}} \boldsymbol{\gamma}[m], \qquad (12)$$

where $\mathbf{h}[m]$ has lth element equal to $h[m,l]$, the matrix $\mathbf{G}_{\boldsymbol{\tau}}$ has $g(lT - \tau_k - \tau_0')$ as its (l,k)th entry, and $\gamma_k(mT)$ is the kth element of $\boldsymbol{\gamma}[m]$. $\mathbf{h}[m]$, $\mathbf{G}_{\boldsymbol{\tau}}$ and $\boldsymbol{\gamma}[m]$ have dimensions $L \times 1$, $L \times K$ and $K \times 1$, respectively. Let $\mathbf{G}_{\boldsymbol{\tau}}^0$ be the matrix $\mathbf{G}_{\boldsymbol{\tau}}$ trailed by $N_c - L$ null rows such that we can write $\mathbf{h}^0[m] = \mathbf{G}_{\boldsymbol{\tau}}^0 \boldsymbol{\gamma}[m]$. From Eq. (5), we then have

$$\mathbf{H}[n] = \mathcal{F}\mathbf{G}_{\boldsymbol{\tau}}^0 \left(\frac{1}{N_c} \sum_{m=nN+N_{\text{cp}}}^{(n+1)N-1} \boldsymbol{\gamma}[m] \right) = \mathbf{W}_{\boldsymbol{\tau}} \mathbf{\Gamma} \boldsymbol{\alpha}[n], \qquad (13)$$

where $\mathbf{W}_{\boldsymbol{\tau}} = \mathcal{F}\mathbf{G}_{\boldsymbol{\tau}}^0$, $\mathbf{\Gamma} = \text{diag}\big(\sqrt{\rho(\tau_0)}, \ldots, \sqrt{\rho(\tau_{K-1})}\big)$, and $\boldsymbol{\alpha}[m] = \mathbf{\Gamma}^{-1}\big(\frac{1}{N_c}\sum_{m=nN+N_{\text{cp}}}^{(n+1)N-1} \boldsymbol{\gamma}[m]\big)$. We have inserted $\mathbf{\Gamma}$ in order that $\mathbb{E}\{\boldsymbol{\alpha}[n]\boldsymbol{\alpha}^H[n]\} = \mathbf{1}$. Since $g(\tau)$ satisfies the Nyquist criterion, the matrix $\mathbf{W}_{\boldsymbol{\tau}}$ of dimension $N_c \times K$ has elements given approximately by (such error is due to the truncation error)

$$(\mathbf{W}_{\boldsymbol{\tau}})_{k_1 k_2} \approx \exp\left(-j \frac{2\pi}{N_c} \frac{k_1(\tau_{k_2} + \tau_0')}{T} \right). \qquad (14)$$

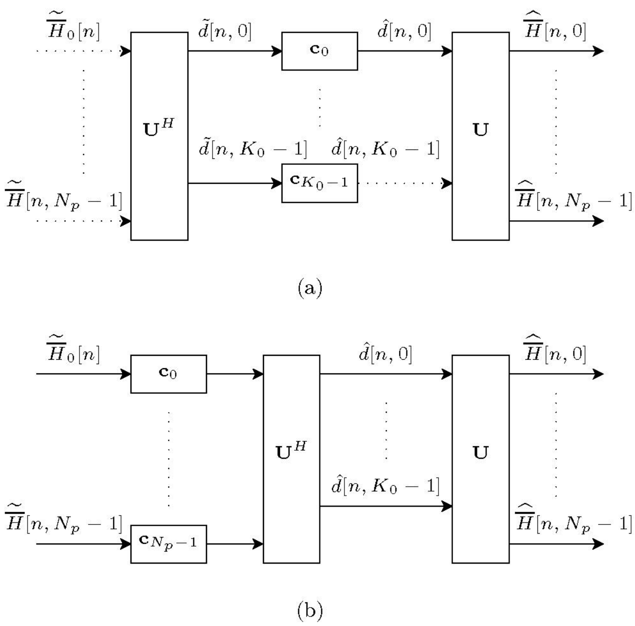

Fig. 2. Filtering structure of (a) Form I and (b) Form II.

3. PROPOSED ESTIMATORS

Our system adopt a comb pilot pattern [4, 5], where the N_p pilot symbols are allocated at equally spaced positions $k = lN_c/N_p$, with $l = 0,\dots,N_p-1$. In order to facilitate the receiver design, the pilot symbols assume values $a[n, lN_p/N_p] = p[n,l] \in \{-1,1\}$. The others symbols $a[n,k]$ can be taken from any constellation type.

In what follows, the bar over matrix or vector indicates we are just considering, respectively, the rows or elements of positions $k = lN_c/N_p$, for $l = 0,\dots,N_p-1$. We have

$$\overline{\mathbf{x}}[n] = \mathrm{diag}(\overline{\mathbf{a}}[n])\overline{\mathbf{H}}[n] + \overline{\mathbf{u}}[n] + \overline{\mathbf{z}}[n]. \qquad (15)$$

Since $\overline{\mathbf{a}}[n] = \mathbf{p}[n] = (p[n,0],\dots,p[n,N_p-1])^T$, the *least square* (LS) estimative of $\overline{\mathbf{h}}[n]$ is simply given by

$$\begin{aligned}\widetilde{\overline{\mathbf{H}}}[n] &= \mathrm{diag}(\mathbf{p}[n])\overline{\mathbf{x}}[n] \\ &= \overline{\mathbf{H}}[n] + \mathrm{diag}(\mathbf{p}[n])(\overline{\mathbf{u}}[n] + \overline{\mathbf{z}}[n]).\end{aligned} \qquad (16)$$

The LS estimator gives poor results, since the ICI plus noise term is still significant.

Its is possible to obtain more accurate estimatives of the channel by filtering the LS estimate. The filtering structure able to make full use of both time and frequency correlation of the channel is depicted in Fig. 2-(a). Such filtering structure will be called Form I. The choice in the MMSE sense of optimal matrix $\mathbf{U}$ and filter coefficients $\mathbf{c}_l$ were derived in [2]. Since this choice requires knowledge of the channel statistics, it is also derived a suboptimum filter where $\mathbf{U}$ consists of a Fourier matrix $\mathcal{F}$ and the $K_0 = L$ (channel order) filters $\mathbf{c}_l$ are sinc low-pass filters.

In this paper we explore the form of Eq. (13) that is not considered explicitly in [2]. Let the SVD of $\mathbf{B} = \overline{\mathbf{W}}_{\boldsymbol{\tau}}\boldsymbol{\Gamma}$ be given by $\mathbf{B} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H$, such that we can write

$$\overline{\mathbf{H}}[n] = \mathbf{U}\mathbf{d}[n], \qquad \mathbf{d}[n] = \boldsymbol{\Lambda}\mathbf{V}^H\boldsymbol{\alpha}[n]. \qquad (17)$$

With $\rho = \mathrm{rank}(\mathbf{B}) \le \min(N_p, K)$, we see that $\mathbf{d}[n]$ has dimension $\rho \times 1$. This fact will be explored in a tentative of computational effort decreasing. Alternatively, the orthogonal matrix $\mathbf{U}$ can be found by calculating the EVD of

$$\begin{aligned}\mathbb{E}\{\overline{\mathbf{H}}[n]\overline{\mathbf{H}}^H[n]\} &= \mathbf{B}\,\mathbb{E}\{\boldsymbol{\alpha}[n]\boldsymbol{\alpha}^H[n]\}\mathbf{B}^H \\ &= \mathbf{U}\boldsymbol{\Lambda}^2\mathbf{U}^H.\end{aligned} \qquad (18)$$

Above we have used the fact that the elements of $\boldsymbol{\alpha}[n]$ are independent. Since we just have access to the LS estimate $\widetilde{\overline{\mathbf{H}}}[n]$, we calculate the EVD of the sample correlation matrix $\mathbf{R}_H[n] = \sum_{i=1}^{n} \widetilde{\overline{\mathbf{H}}}[i]\widetilde{\overline{\mathbf{H}}}^H[i]$ and build the matrix $\mathbf{U}[n]$ with columns constituted of the eigenvectors of the ρ largest eigenvalues of $\mathbf{R}_H[n]$. In what follows, we assume $\rho = K \le N_c$, although it is possible to have $\mathrm{rank}(\mathbf{B}) < \rho$.

We have a better estimate of $\overline{\mathbf{H}}[n]$ given by the projection of $\widetilde{\overline{\mathbf{H}}}[n]$ onto the subspace signal spanned by $\mathbf{U}[n]$, as follows:

$$\tilde{\mathbf{d}}[n] = \mathbf{U}^H[n]\widetilde{\overline{\mathbf{H}}}[n], \qquad \widehat{\overline{\mathbf{H}}}[n] = \mathbf{U}[n]\tilde{\mathbf{d}}[n]. \qquad (19)$$

The direct EVD calculation of $\mathbf{R}_H[n]$ requires a large amount of computational burden. To avoid this, we make use of the *subspace tracking* (ST) algorithm [7] summarized in Table 1. Although it is possible to estimate the rank of $\mathbf{B}$ adaptively, for simplicity we assume that we can find at the receiver side $\rho = K$ that satisfies $\rho \ge \mathrm{rank}(\mathbf{B})$.

Table 1. ST Algorithm

Initialization:
ρ; $\mathbf{U}[0] = \binom{\mathbf{I}_\rho}{0}$; $\boldsymbol{\Theta}[0] = \mathbf{I}_\rho$; $\mathbf{A}[0] = \mathbf{0}$;
For each n:
$\mathbf{Z}[n] = \mathbf{U}^H[n-1]\widetilde{\overline{\mathbf{H}}}[n]$
$\mathbf{A}[n] = \mathbf{A}[n-1]\boldsymbol{\Theta}[n-1] + \widetilde{\overline{\mathbf{H}}}[n]\mathbf{Z}^H[n]$
$\mathbf{A}[n] = \mathbf{U}[n]\mathbf{R}[n]$ (QR factorization)
$\boldsymbol{\Theta}[n] = \mathbf{U}[n-1]\mathbf{U}[n]$

In [6] it is proposed the *subspace amplitude tracking* (SAT) algorithm, whose adaptation equations are

$$\mathbf{e}[n] = \widetilde{\overline{\mathbf{H}}}[n] - \mathbf{U}[n]\hat{\mathbf{d}}[n-1] \qquad (20)$$

$$\hat{\mathbf{d}}[n] = \hat{\mathbf{d}}[n-1] + \mu\mathbf{U}^H[n]\mathbf{e}[n] \qquad (21)$$

$$\widehat{\overline{\mathbf{H}}}[n] = \mathbf{U}[n]\hat{\mathbf{d}}[n],$$

where the forgetting factor μ satisfies $0 < \mu < 2$. After the substitution of Eq. (20) in Eq. (21), we have

$$\hat{\mathbf{d}}[n+1] = \mu\hat{\mathbf{d}}[n] + (1-\mu)\hat{\mathbf{d}}[n-1], \qquad (22)$$

whose transfer function is $c(z) = \mu/(1+(\mu-1)z^{-1})$. Observe that $c(z)$ is a low-pass filter. The SAT algorithm has the same filtering structure of Form I, with the exception that now $\mathbf{U}$ is substituted by its time varying version $\mathbf{U}[n]$ and the filters $\mathbf{c}_l$ have all the same transfer function $c(z)$.

Due to variability of $\mathbf{U}[n]$, the estimate $\widehat{\widehat{\mathbf{H}}}[n]$ can present some degraded performance.

If the filters employed in Fig. 2-(a) are all the same, we can write

$$\widehat{\widehat{\mathbf{H}}}[n] = \mathbf{U}\Big(\sum_{i=0}^{\infty} c_i \mathbf{U}^H \widetilde{\widetilde{\mathbf{H}}}[n]\Big) = \mathbf{U}\mathbf{U}^H \Big(\sum_{i=0}^{\infty} c_i \widetilde{\widetilde{\mathbf{H}}}[n]\Big), \quad (23)$$

where the c_i's are the filter coefficients. Defining $\widetilde{\widetilde{\mathbf{H}}}_f[n] = \sum_{i=0}^{\infty} c_i \widetilde{\widetilde{\mathbf{H}}}[n]$ and using $\mathbf{U}[n]$ instead of $\mathbf{U}$, we have

$$\widehat{\widehat{\mathbf{H}}}[n] = \mathbf{U}[n]\mathbf{U}^H[n]\widetilde{\widetilde{\mathbf{H}}}_f[n]. \quad (24)$$

The Eq. (24) defines the filtering structure shown in Fig. 2-(b), where the LS estimate $\widetilde{\widetilde{\mathbf{H}}}_f[n]$ is first filtered and after projected onto the subspace of $\mathbf{U}[n]$. This filtering structure will be called Form II. Compared to the Form I with time-varying $\mathbf{U}[n]$, the Form II presents more accurate estimates. This fact is justified by the time variability of $\mathbf{U}[n]$. On the other hand, the Form I has the advantage of lower computational complexity, since the filters are applied to $\widetilde{\mathbf{d}}[n]$, whose dimension ρ is smaller than dimension N_p of $\widetilde{\widetilde{\mathbf{H}}}[n]$.

The channel estimators we propose are based on Form I or II with the estimate $\mathbf{U}[n]$ in the place of $\mathbf{U}$, and $\widetilde{\mathbf{d}}[n]$ been filtered by low-pass filters. These estimators will be called ST-LP that stands for *subspace tracking low-pass filters*. Since the complex amplitudes $\gamma_k(t)$ have spectrum limited to the maximum Doppler frequency f_d, we can conclude that $\mathbf{d}(\omega) = \sum_{n=-\infty}^{\infty} \mathbf{d}[n]e^{-jn\omega}$ is limited to $[-2\pi f_d NT, 2\pi f_d NT]$. So the elimination of noise over the rest of the spectrum of $\widetilde{\mathbf{d}}[n]$ by the application of the discrete time filter $\mathrm{sinc}(2f_d NTn)$ will result in better estimates of $\mathbf{H}[n]$.

The Form I of the ST-LP estimator presents a little loss of performance if compared to its respective Form II. As seen, it is due to the variability of $\mathbf{U}[n]$. In fact, this loss can be slightly alleviated with the use of low-pass filters of short impulse response. We can design these filters by any method. In order to avoid phase delay complications, we adopted here the simpler one that consists of a sinc filter truncation and posterior application of a Chebyshev window.

4. SIMULATION RESULTS

The OFDM system we have simulated employs a total bandwidth of 800 kHz, with each OFDM symbol been constituted of $N_c = 128$ subcarriers and cyclic prefix of length $N_{\mathrm{cp}} = 15$. The pilot symbols were allocated at $N_p = 16$ equally spaced subcarriers, each one assuming the values $\{-1, 1\}$ equally likely. The data symbols were taken from a 16-QAM constellation. The channel realizations were simulated according to the Jake's model given by Eqs. (8)-(9). The shaping filter $g(\tau)$ was implemented by a sinc instead of a raised-cosine response. We used $\rho = 4$ delays. At each simulation run, the delays τ_k were independently and uniformly selected in the interval $[0, TN_{\mathrm{cp}} - T_g]$, where the time guard $T_g = 4T$ was empirically chosen in order that the channel length satisfies $L \le N_{\mathrm{cp}} + 1$. In the SAT algorithm, we adopted $\mu = 0.6$. The low-pass filters was designed by selecting 41 values of $\mathrm{sinc}(2f_d NTn)$ and the application of a Chebyshev window with Fourier transform sidelobe magnitude 20 dB below the mainlobe magnitude (see [10]). Obviously it is possible to have a more efficient implementation of these filters.

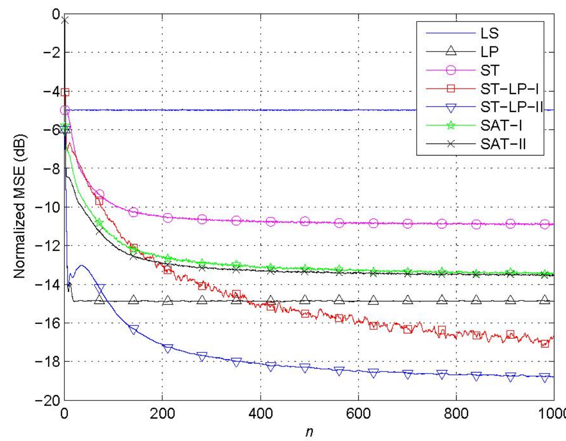

Fig. 3. Normalized MSE curves for a OFDM system operating over SNR = 5 dB for $f_d = 200$ Hz.

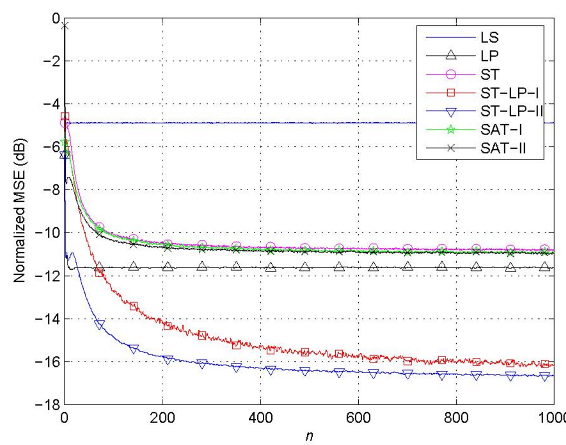

Fig. 4. Normalized MSE curves for a OFDM system operating over SNR = 5 dB for $f_d = 500$ Hz.

Firstly, we investigate the normalized MSE between the channel and its estimate over the pilot subcarriers. For SNR = 5dB, different curves of normalized MSE *vs.* number of processed symbols are shown in Figs. 3 and 4 for maximum Doppler frequency f_d of 200 and 500 Hz, respectively. The MSE was estimated by averaging over 10^4 OFDM symbols. The delay-subspace projection (ST label) and the low-pass filtering (LP label) performance are also considered separately. As expected, both techniques provide accurate estimates, with better results obtained by the low-pass filtering. The Form II type filtering structure given by the combination of low pass-filtering and delay-subspace projection outperform considerably the LS estimator by up to 14 dB and 11 dB for $f_d = 200$ and 500 Hz, respectively. The

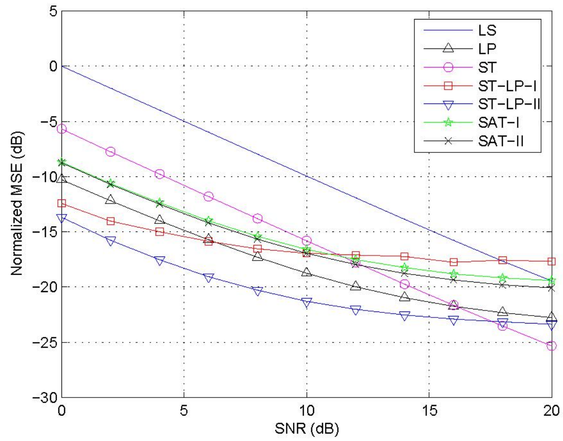

Fig. 5. Normalized MSE *vs.* SNR for different estimators.

less complex Form I presents inferior performance if compared to Form II. For effects of comparison, we see that the Form II of the SAT estimator slightly outperform its respective Form I implementation.

Fig. 5 shows the MSE pilot subcarriers estimate steady-state value *versus* SNR for different estimators operating at $f_d = 200\,\text{Hz}$. These curves were obtained by averaging over $2 \cdot 10^3$ OFDM symbols. We can observe the linear performance of the ST estimator. Theoretically, the low-pass filtering based estimators should have the same linear decreasing behavior. The observed saturation in such estimators is due to filtering imperfections. Since in the SAT estimator the filtering naturally attenuates all the spectrum region, its normalized MSE curve tends to saturate faster. We can eliminate such saturation effects by better designing of the low-pass filter. There is also the influence of the ICI, whose level does not change since the Doppler frequency is the same. The real ICI contribution to the loss of performance still needs further investigation.

5. CONCLUSIONS AND PERSPECTIVES

Based on low-pass-filtering and delay-subspace projection, we developed an efficient pilot based channel estimator for OFDM systems. The delay-subspace estimation is implemented by a subspace tracking algorithm capable to track slow delay variations. The estimator is robust in the sense it does not require knowledge of channel statistics. We can implement the estimator by two filtering structure forms. The first form provides better estimates and requires a higher computational complexity. The second has inferior performance but presents lower computational complexity. As shown by simulations, the proposed estimator, if implemented in Form I, outperforms significantly the LS estimator and presents almost constant MSE performance for a wide range of SNR values.

Improvements on the design of the low-pass filter is a natural continuation of this work. An adequate method that allows to have a good trade-off between performance and impulse response length is required since the filters should be as shortest as possible. Another direction we aim to go further is the investigation of the loss of performance due to the variation of the matrix $\mathbf{U}[n]$. We expect to find suitable constraints of such matrix in order to diminish the loss of performance. Finally, a deeper understanding of the contribution of the ICI in the performance of the LP-based estimators is also a research line we will pursuit.

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