An Upper-Bound on the Ergodic Capacity of Rayleigh-Fading MIMO Channels using Majorization Theory

Antonio Alisson Pessoa Guimarães and Charles Casimiro Cavalcante

Abstract— **In this paper, we investigate the ergodic capacity of multiple-input multiple-output (MIMO) wireless communication systems over spatially uncorrelated Rayleigh-fading channels, assuming that the channel state information (CSI) is unknown at the transmitter and perfectly known at the receiver. Applying some results of majorization theory, we provide an analytical closed-form upper-bound to the ergodic capacity at any signal-to-noise ratio (SNR). In addition, we also derive an approximation to the ergodic capacity in high-SNR regimes. Finally, we present numerical results that confirm our theoretical analysis.**

Keywords— **Ergodic Capacity, MIMO systems, Rayleigh-fading, Majorization Theory.**

I. INTRODUCTION

Recently multiple-input multiple-output (MIMO) wireless communication systems have received significant attention among the research community due to the potential to provide large spectral efficiency in the presence of multi-path fading improvement over single-input single-output (SISO) systems. Then, since the works by Telatar [1] and Foschini [2], who provide that the ergodic capacity increases linearly with the minimum number of antennas at the transmitter or receiver over a Rayleigh-fading MIMO channel, the analysis on the ergodic capacity has been investigated in various fading models and settings to understanding the fundamental limits of the channel. In particular, to obtain an analytical closed-form to the ergodic capacity, especially in some cases, is still a great challenge due to difficulty in manipulating the non-Gaussian joint channel statistics. Thus, to overcome this problematic, we resort to the study of bounding techniques, which intention is to propose lower and upper bounds as close as possible to the empirical ergodic capacity, obtained through Monte Carlo methods.

Rayleigh distribution is a fading model, which is frequently used to model the short-term of mobile-radio signals [3]. In other words, the envelope of the received complex low-pass signal can be modeled as a random variable with a Rayleigh distribution for non-light-of-sight (NLoS) propagation. Several works, operating in Rayleigh-fading MIMO systems, have been published about closed-form analytical expressions for upper-bounds to the ergodic capacity. In [4], a lower-bound for independent and identically distributed (i.i.d.) flat-fading channels was derived, while [5] has been analyzed the frequency-selective fading case. In [6], tight upper and lower bounds on the ergodic capacity for spatially correlated channels were provided. The spatial double-sided correlation with keyhole has been examined in [7]. Newly, tight bounds for spatially correlated Rician MIMO channels were proposed by [8], [9] at any signal-to-noise ratio (SNR) and for any number of receive and transmit antennas. Moreover, these references devote a significant part to the study of the Rayleigh-fading channels as a particular case.

In this paper, we focus on the ergodic capacity of spatially uncorrelated Rayleigh-fading MIMO channels. Specifically, we derive a closed-form analytical upper-bound assuming that the channel state information (CSI) is available only at the receiver. Part of the motivation for this work stems from the results of [10] and [11], which have obtained capacity bounds in Nakagami-m fading MIMO channels using majorization theory [12], [13]. This mathematical tool have allowed to investigate the ergodic capacity through the distribution of the diagonal elements of the Wishart matrix. Furthermore, the analysis in this work distinguishes from previous results on Rayleigh-fading MIMO channels due the use of majorization theory applied to i.i.d. Kronecker channel model.

The remaining of this paper is organized as follows: Section II introduces the Rayleigh-fading MIMO channel model and includes the definition of ergodic capacity. Section III presents briefly the Gamma random variable. In Section IV, we introduce some basic notions and results of majorization theory. We derive an upper-bound, at any SNR value, and an approximation, in high-SNR regime, to the ergodic capacity in Section V. The theoretical and the simulation results are discussed in Section VI. Finally, we conclude the paper in Section VII.

Throughout this article, matrices and vectors will be represented by bold uppercase and lowercase letters, respectively. We use $\mathbf{I}$ or $\mathbf{I}_p$ for the identity matrix of dimension $p \times p$ and $\mathbb{C}^{m\times n}$ indicates the $m \times n$ complex vector space. The superscripts $(\cdot)^H$ denotes Hermitian transpose, while the subscript $(\cdot)_i$ is the i-th element of a vector, and $(\cdot)_{ij}$ is the (i, j)-th entry of a matrix. The operator $\prec$ denotes the majorization relation, $\mathbb{E}\{\cdot\}$ represents the statistical expectation, and $\det(\cdot)$ stands for the determinant of a square matrix. Finally, the vectors $\mathbf{d}(\cdot)$ and $\boldsymbol{\lambda}(\cdot)$ denote the main diagonal elements and eigenvalues of a Hermitian matrix, respectively.

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II. System Model and Ergodic Capacity

We focus our study on single-user MIMO communications over flat-fading wireless channels with n_T transmit antennas, and n_R receive antennas. The input-output relationship is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{n_R \times 1}$ and $\mathbf{x} \in \mathbb{C}^{n_T \times 1}$ are the received and transmitted signal vectors, respectively, while $\mathbf{n} \in \mathbb{C}^{n_R \times 1}$ is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = N_0\mathbf{I}$. We assume that the transmitted signal vector satisfies the power constraint $\mathbb{E}\left\{\mathbf{x}^H\mathbf{x}\right\} \leq P_T$. In addition, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the MIMO channel matrix, which elements h_{ij} represent the complex fading parameter between the j-th transmit and i-th receive antenna. The channel gain is considered to undergo Rayleigh-fading [14], without spatial correlation occurring at both ends of the MIMO link. Furthermore, the entries h_{ij} are i.i.d. random variables with zero mean and unit variance. In other terms, the complex entry h_{ij} can be expressed as

$$h_{ij} = h_{ij}^I + j h_{ij}^Q, \tag{2}$$

where the inphase and quadratic components (denoted by h_{ij}^I and h_{ij}^Q, respectively) follow a real-valued Gaussian random variance with zero mean and identical variance, i.e.,

$$\sigma^2_{h_{ij}^I} = \sigma^2_{h_{ij}^Q} = \sigma^2. \tag{3}$$

Since, the element h_{ij} is a complex Gaussian process with zero mean and unit variance, the envelope

$$|h_{ij}| = \sqrt{[h_{ij}^I]^2 + [h_{ij}^Q]^2} \tag{4}$$

has Rayleigh distribution, which will be denoted by

$$|h_{ij}| \sim \text{Rayleigh } (\sigma^2). \tag{5}$$

Moreover, the probability density function is given by [15]

$$p_{|h_{ij}|}(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)u(r), \tag{6}$$

where $u(r)$ represents the unit step function. In our case, we have that $\sigma^2 = 1/2$.

In the sequel, we consider that the receiver has perfect channel state information (CSI), and an equal-power allocation across the transmit antennas. In this situation, the capacity ergodic can be expressed as [8]

$$\overline{\mathcal{C}} = \mathbb{E}\left\{\log_2\left[\det\left(\mathbf{I} + \frac{\rho}{n_T}\mathbf{\Gamma}\right)\right]\right\}, \tag{7}$$

where $\rho \triangleq \frac{P_T}{N_0}$ is the received signal-to-noise (SNR) ratio and $\mathbf{\Gamma}$ is the Wishart matrix, which is defined as

$$\mathbf{\Gamma} = \begin{cases} \mathbf{H}\mathbf{H}^H, & n_R \leq n_T \\ \mathbf{H}^H\mathbf{H}, & n_R > n_T. \end{cases} \tag{8}$$

For convenience, we define the constants $r = \min\{n_R,\ n_T\}$ and $t = \max\{n_R,\ n_T\}$. Thus, $\mathbf{\Gamma}$ is always a square matrix of order $r \times r$. Furthermore, we assume that the number of receive antennas does not exceed the number of transmit antennas. However, using the identity $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, ensures that all results can be extended to the case $n_R > n_T$.

Now, using the singular value decomposition (SVD) on the matrix $\mathbf{\Gamma}$, the ergodic capacity in Eq. (7) can be represented by

$$\overline{\mathcal{C}} = \mathbb{E}\left\{\sum_{i=1}^{r}\log_2\left(1 + \frac{\rho}{n_T}\right)\lambda_i\right\}, \tag{9}$$

where λ_i denotes the i-th eigenvalue of the Wishart matrix $\mathbf{\Gamma}$.

III. The Gamma Distribution

In this section, we provide a brief discussion on the Gamma distribution. In particular, we discuss the sum of independent Gamma variables, and the possibility of obtaining a Gamma random variable from a Rayleigh distribution.

Definition 1 ([15]): A random variable X follows a Gamma distribution with parameters $\alpha > 0$ and $\beta > 0$, denoted by $X \sim \gamma(\alpha, \beta)$, if the p.d.f. of X is given by [15]

$$p_X(x) = \frac{x^{\alpha-1}\exp(-x/\beta)}{\beta^\alpha\Gamma(\alpha)}u(x), \tag{10}$$

where $\Gamma(\cdot)$ denotes the gamma function. The constants α and β are known as the shape and scale parameters of the distribution.

Now, we describe the possibility of generating a Gamma distribution from a Rayleigh random variable. This can be accomplished using the following transformation.

Lemma 1: If $X \sim$ Rayleigh (σ^2), then the random variable $Y = kX^2$ has a Gamma distribution with parameters $\alpha = 1$ and $\beta = 2k\sigma^2$, i.e., $Y \sim \gamma(1,\ 2k\sigma^2)$.

Finally, we complete this section with the sum of independent Gamma variables with different shape parameters but having the same scale parameter.

Lemma 2: Let $\{X_i\}_{i=1}^m$ be a set of m independent Gamma random variables such as $X_m \sim \gamma(\alpha_m, \beta)$, then the p.d.f. of $Y = \sum_{i=1}^m X_i$ has a Gamma distribution. Specifically,

$$Y \sim \gamma\left(\sum_{i=1}^m \alpha_i,\ \beta\right). \tag{11}$$

IV. Majorization Theory

Majorization theory is a topic of much interest in various areas of the mathematics and recently has been applied in problems of wireless communications systems [16]–[19]. The definition of majorization relationship allows to compare two vectors by reordering its coordinates in decreasing order. In other words, let $\mathbf{x}$ be a vector of $\mathbb{R}^{n\times 1}$ and $[\mathbf{x}] = \left(x_{[1]},\ x_{[2]}, \cdots,\ x_{[n]}\right)$ another vector of $\mathbb{R}^{n\times 1}$ consisting of the coordinates of $\mathbf{x}$, but putting them in decreasing order, that is, $x_{[1]} \geq x_{[2]} \geq \cdots \geq x_{[n]}$. We say that $\mathbf{y}$ majorizes $\mathbf{x}$ or $\mathbf{x}$ is majorized by $\mathbf{y}$, and writes $\mathbf{x} \prec \mathbf{y}$, if the following conditions are met:

$$\sum_{i=1}^k x_{[i]} \leq \sum_{i=1}^k y_{[i]}, \quad 1 \leq k \leq n-1, \tag{12a}$$

$$\sum_{i=1}^n x_{[i]} = \sum_{i=1}^n y_{[i]}. \tag{12b}$$

Intuitively, the vector $\mathbf{y}$ majorizes the vector $\mathbf{x}$ if the coordinates of $\mathbf{y}$ are more "dispersed" or "spread out" than the coordinates of $\mathbf{x}$ [12]. However, treating with more accuracy, the "majorization inequality" is a partial order relation on vectors of real numbers. Then, to understand this definition, we consider the vectors $\mathbf{x} = (3;\ 3;\ 3;\ 3;\ 3)$, $\mathbf{y} = (5;\ 4;\ 3;\ 2;\ 1)$ and $\mathbf{z} = (7;\ 5;\ 2;\ 0.8;\ 0.2)$ of $\mathbb{R}^{5\times 1}$. Note that, we have the following majorization relationships: $\mathbf{x} \prec \mathbf{y} \prec \mathbf{z}$. The behavior of these coordinates are illustrated in Fig. 1.

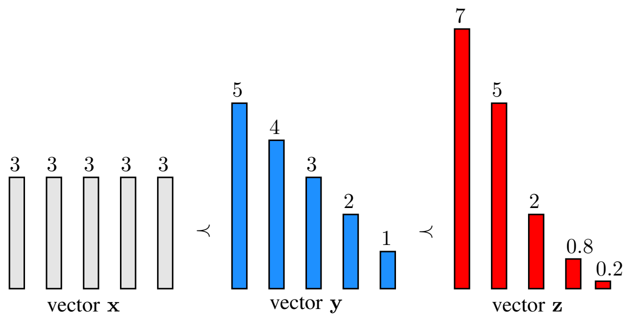

Fig. 1. Geometric interpretation of the majorization relationship.

There is an extensive list of properties involving the majorization theory, which can be found in the classical references [12], [13]. However, for this work, we highlight the following:

Lemma 3 (Schur's inequality [12, 9.B.1]): If $\mathbf{A} \in \mathbb{C}^{n\times n}$ is an Hermitian matrix, then

$$\mathbf{d}\,(\mathbf{A}) \prec \boldsymbol{\lambda}(\mathbf{A}). \tag{13}$$

Next, defines a real-valued function $\phi(\cdot)$, which applies to the previously majorized, changes the partial order relation.

Definition 2 ([12, 3.A.1]): A real-valued function $\phi(\cdot)$ on $\mathbb{R}^{n\times 1}$ is said to be Schur-concave if

$$\mathbf{x} \prec \mathbf{y} \Rightarrow \phi(\mathbf{x}) \geq \phi(\mathbf{y}). \tag{14}$$

The following result is an important case, for our purpose, of Schur-concave function.

Lemma 4 ([12, 3.C.1]): Let the real-valued function $\phi(\cdot)$ on $\mathbb{R}^{n\times 1}$. If $g : \mathbb{R} \to \mathbb{R}$ is concave, then $\phi(\cdot)$ defined by

$$\phi(\mathbf{x}) = \sum_{i=1}^{n} g(x_i) \tag{15}$$

is Schur-concave.

Finally, we present an application of *Lemma 4*, which will be used in Section V.

Example 1 ([11, Appendix II]): The real-valued function $\phi(\cdot)$ on $\mathbb{R}^{n\times 1}$, defined by $\phi(\mathbf{x}) = \sum_{i=1}^{n} \log_2(1 + \alpha x_i)$, with $\alpha > 0$, is a Schur-concave function.

V. Upper-Bound on Ergodic Capacity

Making use of the sum of independent Gamma random variables, in the case that all Gamma distributions have the same scale parameter but different shape parameters, and majorization theory, we will obtain, in this section, an analytical closed-form upper-bound to the ergodic capacity on spatially uncorrelated Rayleigh MIMO channels in terms of the Meijer G-function [20, Eq. (9.301)]. The main contribution of this paper is presented in the following theorem.

Theorem 1: The ergodic capacity of spatially uncorrelated Rayleigh MIMO channels is upper bounded by

$$\overline{\mathcal{C}} \leq \frac{r}{\Gamma(t)\ln 2} G_{3,2}^{1,3}\left(\frac{\rho}{n_T}\bigg|_{1,0}^{1-t,1,1}\right), \tag{16}$$

where ρ is the SNR, and $G_{p,q}^{m,n}(\cdot\,|\,\cdot)$ is the Meijer G-function [20, Eq. (9.301)].

Proof: We start defining the following vectors in $\mathbb{R}^{r\times 1}$

$$\mathbf{d}(\boldsymbol{\Gamma}) \triangleq (d_1,\ d_2,\ \cdots,\ d_r) \tag{17a}$$

$$\boldsymbol{\lambda}(\boldsymbol{\Gamma}) \triangleq (\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_r) \tag{17b}$$

where d_i corresponds to the i-th diagonal element of the matrix $\boldsymbol{\Gamma}$, and λ_i represents the respective eigenvalue. Now, let be the real-valued function $\phi(\cdot)$ on $\mathbb{R}^{r\times 1}$ defined by

$$\phi(\mathbf{x}) = \sum_{i=1}^{r} \log_2\left(1 + \frac{\rho}{n_T} x_i\right). \tag{18}$$

Since the vector $\boldsymbol{\lambda}(\boldsymbol{\Gamma})$ majorizes $\mathbf{d}(\boldsymbol{\Gamma})$, i.e., $\mathbf{d}(\boldsymbol{\Gamma}) \prec \boldsymbol{\lambda}(\boldsymbol{\Gamma})$ (see Schur's inequality) and $\phi(\cdot)$ is a Schur-concave function (from Example 1), we have

$$\phi(\,\boldsymbol{\lambda}\,(\boldsymbol{\Gamma})\,) \leq \phi(\,\mathbf{d}(\boldsymbol{\Gamma})\,)\,. \tag{19}$$

Applying the expectation operator $\mathbb{E}\{\cdot\}$ in the inequality (19) and observing that the ergodic capacity presented in Eq. (9) is equal to $\mathbb{E}\{\phi(\,\boldsymbol{\lambda}\,(\boldsymbol{\Gamma})\,)\}$, we obtain an upper-bound to the ergodic capacity as shown below:

$$\overline{\mathcal{C}} \leq \mathbb{E}\{\phi(\,\mathbf{d}(\boldsymbol{\Gamma})\,)\}\,. \tag{20}$$

We now express the expectation in Eq. (20) in the integral form, that is,

$$\begin{aligned}\mathbb{E}\{\phi(\,\mathbf{d}(\boldsymbol{\Gamma})\,)\} &= \mathbb{E}\left\{\sum_{i=1}^{r}\log_2\left(1+\frac{\rho}{n_T}d_i\right)\right\}\\ &= \sum_{i=1}^{r}\mathbb{E}\left\{\log_2\left(1+\frac{\rho}{n_T}d_i\right)\right\}\\ &= \frac{1}{\ln 2}\sum_{i=1}^{r}\int_0^{\infty}\ln\left(1+\frac{\rho}{n_T}\varepsilon\right)p_i(\varepsilon)d\varepsilon,\end{aligned} \tag{21}$$

where $p_i(\cdot)$ is the p.d.f. of the random variable d_i. Noting that

$$d_i = \sum_{j=1}^{t} |h_{ij}|^2, \qquad i = 1,\ 2,\ \cdots,\ r, \tag{22}$$

and from Lemmas 1 and 2, we have that d_i is given by the sum of i.i.d. Gamma random variables. Specifically, $d_i \sim \gamma(t,1)$ or

$$p_i(\varepsilon) = \frac{\varepsilon^{t-1}\exp(-\varepsilon)}{\Gamma(t)}u(\varepsilon), \tag{23}$$

for all $i = 1,\ 2,\ \cdots,\ r$. Thus, the upper-bound in Eq. (21) becomes

$$\mathbb{E}\{\phi(\,\mathbf{d}(\boldsymbol{\Gamma})\,)\} = \frac{r}{\Gamma(t)\ln 2}\int_0^{\infty}\ln\left(1+\frac{\rho}{n_T}\varepsilon\right)\varepsilon^{t-1}\exp(-\varepsilon)\,d\varepsilon. \tag{24}$$

On the other hand, using the fact that the logarithmic function can be expressed in terms of the Meijer G-function, we have [11]

$$\ln\left(1+\frac{\rho}{n_T}\varepsilon\right) = G_{2,2}^{1,2}\left(\frac{\rho}{n_T}\varepsilon\bigg|_{1,0}^{1,1}\right). \tag{25}$$

Now, substituting the Meijer G-function in Eq. (25) into to Eq. (21) and applying the following result, called Laplace transform [20, 7.813-1], [11],

$$\int_0^\infty x^{-\rho} \exp(-\beta x)\, G_{p,q}^{m,n}\left(\alpha x \middle|_{b_1,b_2,\cdots,b_q}^{a_1,a_2,\cdots,a_p}\right) dx = \beta^{\rho-1} G_{p+1,q}^{m,n+1}\left(\frac{\alpha}{\beta}\middle|_{b_1,b_2,\cdots,b_q}^{\rho,a_1,a_2,\cdots,a_p}\right), \quad (26)$$

we conclude, after some algebra, that the upper-bound is given by

$$\mathbb{E}\{\phi(\mathbf{d}(\boldsymbol{\Gamma}))\} = \frac{r}{\Gamma(t)\ln 2} G_{3,2}^{1,3}\left(\frac{\rho}{n_T}\middle|_{1,0}^{1-t,1,1}\right). \quad (27)$$

This completes the proof. ■

Though the upper-bound obtained can be expressed in a closed-form analytical expression and can be evaluated very efficiently using standard softwares like Maple® and Mathematica®, was observed in some cases (see Section VI) that our result did not presents a good approximation to the ergodic capacity in high-SNR regimes compared with a previously published upper-bound. However, to minimize this loss, we provide from Eq. (24) a tightest capacity upper-bound.

Corollary 1: In high-SNR regimes, the ergodic capacity upper-bound $\mathbb{E}\{\phi(\mathbf{d}(\boldsymbol{\Gamma}))\}$ for spatially uncorrelated Rayleigh MIMO channels can be approximated as

$$\mathbb{E}\{\phi(\mathbf{d}(\boldsymbol{\Gamma}))\} \approx \frac{r}{\ln 2}\left(\ln\left(\frac{\rho}{n_T}\right) + \psi(t)\right), \quad (28)$$

where $\psi(\cdot)$ is the Euler Digamma function [20, 8.365-4].

Proof: In high-SNR regimes (large ρ), the function $\ln(1+\rho/n_T)$ can be approximated by $\ln(\rho/n_T)$. In turn, from the Eq. (24), we define

$$\mathcal{C}_{\text{app}} \triangleq \frac{r}{\Gamma(t)\ln 2}\int_0^\infty \ln\left(\frac{\rho}{n_T}\varepsilon\right)\varepsilon^{t-1}\exp(-\varepsilon)\, d\varepsilon. \quad (29)$$

Note that $\mathcal{C}_{\text{app}} \approx \mathbb{E}\{\phi(\mathbf{d}(\boldsymbol{\Gamma}))\}$ and

$$\mathcal{C}_{\text{app}} = \frac{r}{\Gamma(t)\ln 2}\left[\ln\left(\frac{\rho}{n_T}\right)\int_0^\infty \varepsilon^{t-1}\exp(-\varepsilon)\, d\varepsilon + \int_0^\infty \ln(\varepsilon)\varepsilon^{t-1}\exp(-\varepsilon)\, d\varepsilon\right]. \quad (30)$$

Now, using the integral identities [20, 8.312-2]

$$\int_0^\infty x^{\nu-1}\exp(-x)\, dx = \Gamma(\nu) \quad (31)$$

and [20, 4.352-1]

$$\int_0^\infty \ln(x) x^{\nu-1}\exp(x)\, dx = \Gamma(\nu)\psi(\nu), \quad (32)$$

the approximation to the ergodic capacity $\mathcal{C}_{\text{app}}$ can alternatively be expressed as

$$\mathcal{C}_{\text{app}} = \frac{r}{\ln 2}\left(\ln\left(\frac{\rho}{n_T}\right) + \psi(t)\right). \quad (33)$$

■

VI. Numerical Results

To illustrate the theory described in this paper, we evaluate the capacity upper-bound to the ergodic capacity in several MIMO configurations. Moreover, we compare our result with a previously published upper-bound [9]. This reference investigates the i.i.d. Rayleigh-fading channels as a particular case of Rice-fading channels.

The ergodic capacity for 1×1, 1×2 and 1×4 systems for various values of SNR is showed in Fig. 2. It is clear that the our upper-bounds are much tighter than proposed by Jin *et al.* [9].

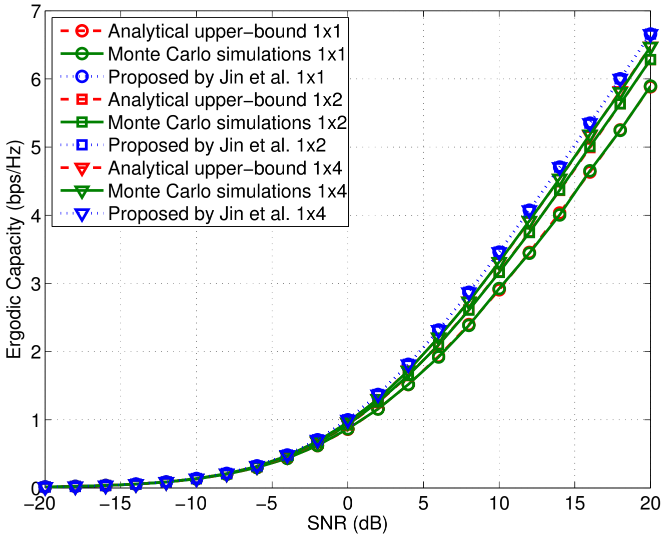

Fig. 2. Comparison of the empirical ergodic capacity and analytical upper-bound for 1×1, 1×2 and 1×4 uncorrelated Rayleigh-fading channels.

Fig. 3 and Fig. 4 depict that the analytical expression presented in Eq. (16), for low-SNR regimes, is as tight as proposed by [9]. In high-SNR regimes, for 2×2 and 2×4, we can consider that the upper-bounds are equally tight.

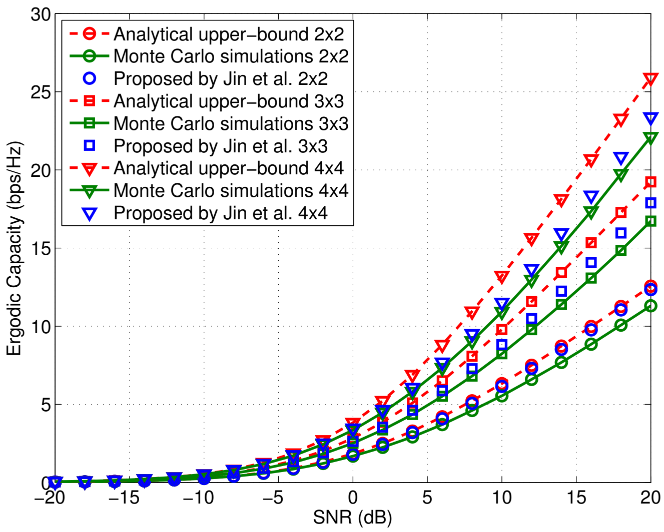

Fig. 3. Comparison of the empirical ergodic capacity and analytical upper-bound for 2×2, 3×3 and 4×4 uncorrelated Rayleigh-fading channels.

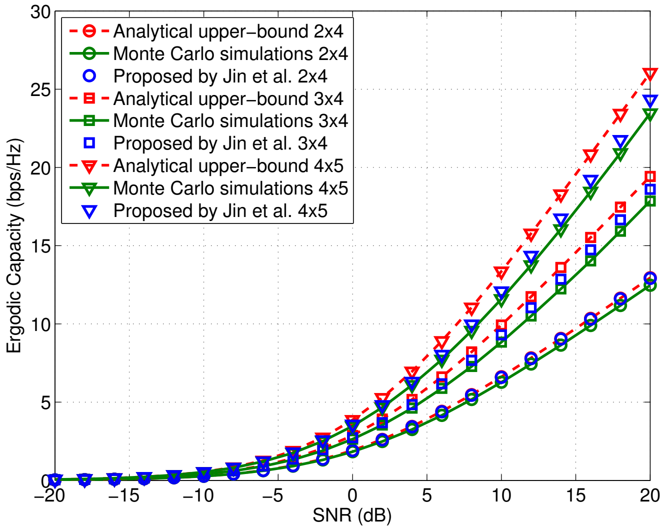

Fig. 4. Comparison of the empirical ergodic capacity and analytical upper-bound for 2×4, 3×4 and 4×5 uncorrelated Rayleigh-fading channels.

Finally, Fig. 5 plots the high-SNR approximations of the upper-bound. From the graphs, it is apparent that, for each SNR value, our propose (from Corollary 1) produces results very close to the empirical ergodic capacity.

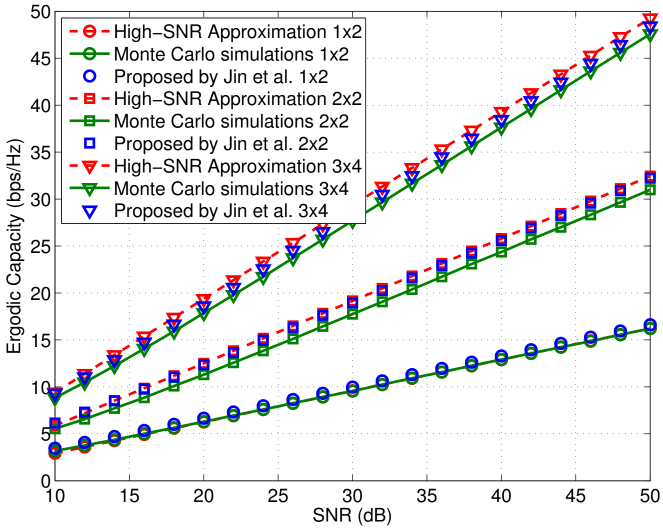

Fig. 5. Comparison of the empirical ergodic capacity and analytical high-SNR upper-bound approximation for 1×2, 2×2 and 3×4 uncorrelated Rayleigh-fading channels.

VII. Conclusions

In this paper, we present a new analytical upper-bound on the ergodic capacity for uncorrelated Rayleigh-fading MIMO channels assuming that transmitter has not knowledge of the CSI. We have also derived an analytical approximation for the ergodic capacity from upper-bound on high-SNR regimes. Based on some results on majorization theory, we demonstrated that our bound, in some cases, it is tighter than previously known upper-bound. In other ones, the result is equally tight, mainly for low-SNR regimes. Furthermore, for high-SNRs, the simulation results showed that the upper-bound approximation is very close to the ergodic capacity. In the future, we will investigate the case for Rice channels with correlated fading components.

Acknowledgments

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