Application of Fuzzy Set Theory on the Saint Venant Equations to Study Flood Wave Propagation in Natural Rivers

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Abstract: The knowledge of the propagation of flood waves, in natural channels, through Saint Venant's equations, has been object of studies on the part of engineers and scientists, along years. With the progress of the digital computers, the mathematical models become a great option, in the analysis of this class problem. In this context, the uses of numerical models, in the treatment of the hydrodynamic models, has allowed more select studies of the behavior of the flood routing to be accomplished. This work treats of the study of a flood wave, through a simplified methodology, for solution of the equations of the hydrodynamic that allows the use of a numerical model with explicit solution. The influence of hydraulic parameters, inherent to the natural river, it is considered. The results show that this methodology had a great acting and it constitutes in a good way for the study of this class of problem.
Keywords: Fuzzy Theory; Hydrodynamic Models; River Mechanics.

1. Introduction

The intense human activity, present in the proximities of rivers, lakes or estuaries, result of great urban areas, has been causing considerable alterations in the urban river systems and producing great impacts, of the environmental point of view, considering the occurrence of the improper use of the bodies of water. In Brazil, many are the rivers that have their waters in complete degradation state, as a consequence of the lack of a public politics and of the disordered occupation of the urban areas.

In this context, there is a necessity to study forms of urban planning, so that these occupations can be structured and, consequently, be reduced the risks of inundations in used areas, mainly, for house. To begin the studying of urban planning, in the proximities of any natural river system, it is necessary to know the dynamics of the river. In other words, it is important to know as the river answers to an inundation, considering that the propagation of a flood wave, in the space and in the time, it is a complex problem.

Usually, the mathematical models that describe the unsteady flow in open channels are composed by the momentum and the continuity equations, developed by Saint-Venant, that are partial differential equations, strongly no lineal, whose solution can be obtained by numerical scheme.

However, if one tries to make some analysis concerning to flood risk analysis, there is a necessity of a more necessary evaluation of the present uncertainties in the flow processes. Such analysis can be developed through the use of the probability theory, or the fuzzy theory fuzzy. The first one demand a solid database for its application and analysis,

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that most of time is not available for use. The present work used of the fuzzy theory, whose main advantage is the not necessity of great groups of data to reach their main objectives.

This theory was applied to a hydrodynamic model, by the transformation of the classic momentum and continuity equations into a new class of partial differential equations called fuzzy hydrodynamic modeling, in such way that the solution could be represented by membership functions of the control variables. This way, it is possible to establish a methodology to allow the calculation of the flood risk for any natural river system. The results have shown that the fuzzy theory became an alternative way to work with uncertainty analysis and any king of engineering risk analysis that can be applied in the environmental system.

2. Mathematical Modeling

The flow field, in the river, is obtained through the numeric solution of the Saint-Venant equations. Those equations, of the continuity and of the momentum, are described, according to Chow (1988):

- Continuity Equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

- Momentum Equation

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(Q^2 / A\right)}{\partial x} + gA(\frac{\partial y}{\partial x} - S_0) + gAS_f = 0 \tag{2}$$

where x is the longitudinal distance along the channel (m), t is the time (s), A is the cross section area of the flow (m^2), y is the surface level of the water in the channel (m), S_0 is the slope of bottom of the channel, S_f is the slope of energy grade line, B is the width of the channel (m), and g is the acceleration of the gravity ($m.s^{-2}$).

In order to calculate S_f, the Manning formulation will be used. Thus,

$$V = \frac{1}{n} R^{2/3} S_f^{\ 1/2} \tag{3}$$

where V is the mean velocity (m/s), R is the hydraulic radius (m) e n is the roughness coefficient.

- Initial Conditions

$$Q(x,0)=Q_0 \tag{4}$$

$$A(x,0)=A_0 \tag{5}$$

where Q_0 is the steady state flow of the channel, and the A_0 is the cross section area for the steady state conditions.

- Boundary Conditions:

$$Q(0,t) = f(t) \tag{6}$$

where *f(t)* is the hydrograph.

3. Fuzzy Set Theory

One way to evaluate the risk of collapse of any system is through the Fuzzy Set Theory. This theory, appeared in the sixties, it represents an important tool in the science or technology problem concerning with high degree of uncertainties. In this case, the theory in subject develops an important game, mainly, because its application does not demand a rigorous database.

In the field of water resources engineering, especially, in the subjects of water resources planning, the application of fuzzy number theory is beginning growing. At this time, there are not works in the literature that treat with this methodology. However, some works that were already developed show the great potential that this theory to investigate all problem concerning with flood risk analysis.

The concept of uncertain or fuzzy number may be presented in many ways. As point out Vieira (2005), the fuzzy number can be defined as an extension of the concept of an interval of confidence. This extension is based on a natural and very simple idea. Instead of considering the interval of confidence at one unique level, it is considered as a several levels from 0 to 1. Mathematically speaking, one can define fuzzy number as follow.

Let F be a referential set. An ordinary subset A. of this referential set is defined by it characteristic function, called the membership function, which takes its values in the interval [0,1]. In such way, for any x, belonging to F, $\mu_A(x) \in F$, that is, the elements of F belongs to A with a level located in [0,1].

A pair of equation defined into some interval can represent a membership function. For example, a membership function of a fuzzy number may be described mathematically by means of two strictly function L and R defined by;

$$\mu_x(x) = L\left(\frac{x_m - x}{x_1}\right), \quad x < x_m, \quad x_1 > 0 \tag{7}$$

$$\mu_x(x) = R\left(\frac{x - x_m}{x_2}\right), \quad x > x_m, \quad x_2 > 0 \tag{8}$$

where, x_1, x_2 and x_m are number with defined level or confidence.

The application of that theory on the hydrodynamic equation brings the fuzzy hydrodynamics modeling that was proposed in this research. Therefore, applying the fuzzy theory into the equations (1-6), the new formulation for the model is represented below through the formulation,

- Continuity Fuzzy Equation

$$\frac{\partial \tilde{A}}{\partial t} + \frac{\partial \tilde{Q}}{\partial x} = \tilde{q} \tag{9}$$

- Momentum Fuzzy Equation

$$\frac{1}{\tilde{A}}\frac{\partial \tilde{Q}}{\partial t} + \frac{1}{\tilde{A}}\frac{\partial}{\partial x}\left(\frac{\tilde{Q}^2}{\tilde{A}}\right) + g\frac{\partial \tilde{y}}{\partial x} - g\left(\tilde{S}_0 - \tilde{S}_f\right) = 0 \tag{10}$$

where $\tilde{A}$ is the membership function for the transversal area of the river; $\tilde{Q}$ is the membership function for the flow; $\tilde{y}$ is the membership function for the depth; $\tilde{q}$ is the membership function for lateral flow; $\tilde{S}_0$ is the membership function for the bed slope of the river; and $\tilde{S}_f$ is the membership function for the headline slope.

- Boundary Conditions:

$$\tilde{Q}(0,t) = \tilde{Q}_0(t) \tag{11}$$

$$\left.\frac{\partial \tilde{Q}}{\partial x}\right/_{x=L} = 0 \tag{12}$$

- Initial Conditions:

$$\tilde{Q}(x,0) = \tilde{Q}_1(x) \tag{13}$$

- Energy Gradeline equation

$$\tilde{Q} = \frac{1}{\tilde{n}}\tilde{A}\tilde{R}^{2/3}\tilde{S}_f^{\ 1/2} \tag{14}$$

This is the new model in its fuzzy form. It is important to remind that the solution of this model will supply four membership functions, one for each control variable. Therefore, a membership function will exist for the flow, the velocity, the flow depth and the cross section area of the channel. These membership functions can be used in the evaluation of the flood risk analysis.

4. Fuzzy Risk Analysis

The solution of this group of equations, shown previously, allows determining the dependent variables, in the form of membership functions. Those functions are calculated, along of the river, for different time. Like this, the hydrodynamic fuzzy equation produces a flow field in fuzzy form, defined by its membership functions, as membership function for the transverse area, membership function for the velocity, and a membership function for the depth, being this last one the most important for the present work.

The subject is to define, in the context of the risk analysis, the importance of the heights of water, according to the membership functions. Obtained these relative membership functions to the heights, the same ones will be compared with other membership function, also necessary to evaluate the flood risk of a certain area, that represents the of levels of the land in study, in other words, the allowed maximum heights. This membership function acts, in their characteristics fuzzy way, the maximum limits of the line of water in the channel, after a given flow. Besides this limit, the body of water should begin to overflow, flooding the neighboring areas. This membership function is called of resistance and, once defined, it allows that the risk can be calculated.

Consequently, let the resistance membership function be defined by the maximum limits of shipment allowed by a course of water. Let yet the membership function of the height that one calculated by the mathematical model proposed in this study, and that it represents the answer of the receiving system to the arrival of a flood wave.

In such way, the safety margin $\widetilde{M}$, of the body of water can be represented by the difference among the resistance membership function $\widetilde{R}$, and the membership function calculated of the heights $\widetilde{H}$, that it represents the answer to the possible waves of floods or intense rains in the river. In such way, the fuzzy risk can be defined by, see Ganoulis, (1994),

$$R_f = \frac{\int_{-\infty}^{0} \mu_{\widetilde{M}}(m)dm}{\int_{-\infty}^{\infty} \mu_{\widetilde{M}}(m)dm} \tag{15}$$

where $\mu_{\widetilde{M}}$ is the safety margin membership function, and m is an element of the M. Yet the fuzzy Reliability can be defined by

$$R_c = \frac{\int_{x>0} \mu_{\widetilde{M}}(m)dm}{\int_{-\infty}^{\infty} \mu_{\widetilde{M}}(m)dm} \tag{16}$$

It is important to note that R_f and R_c are real functions, defined in the close interval [0,1], and they depend on the hydraulics and hydrologic parameters of the channel.

5. Results

The results obtained through the solution of the model fuzzy for the equations of Saint-Venant were in membership function form for the all control variables. To do so, it was defined a set of membership function for the boundary conditions and for the initial conditions. In such way, the membership functions have dynamic characteristic that varies from section to section of the channel and for deferential intervals of time.

The studies grew considering a sequence of variations of the several hydraulic parameters that were transformed in membership functions as entrance data for the hydrodynamic fuzzy model. Therefore, the roughness coefficient, the bed slope and the entrance flow were transformed in membership functions, with the level of pertinence 0, equal to 25% of the level of pertinence 1, for the right and for the left.

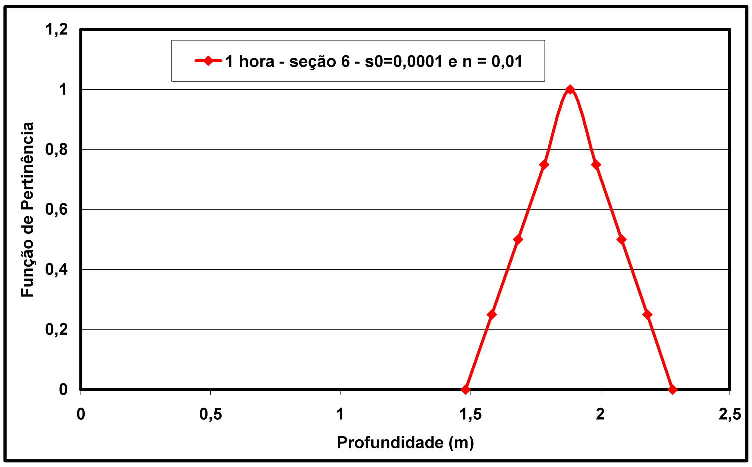

Figure 1. Membership function of the depth for time observation of 1 hour.

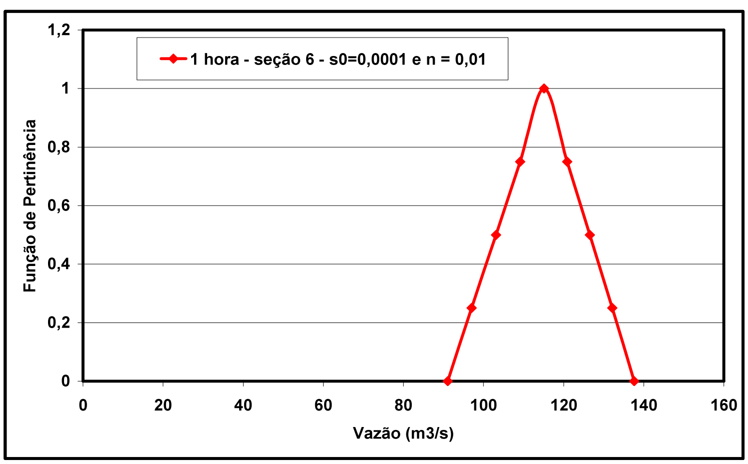

Figure 2. Membership function of the flow for time observation of 1 hour

The figures 1 and 2 show the results obtained for the membership function of the flow depth and the membership function for the flow starting from a simulation, where a channel is 50 Km long, 40 m width. The bed slope of the channel, with the highest level of confidence was used as 0.0001 and the Manning roughness coefficient equal to 0.01. This function was calculated for time of simulation equal to 1 hour, and for a distant section 5 Km from the origin. As one can see, the membership functions either for the flow or for the depth change from time to time and from section to section. In such way, it is possible to concluded that, at any section, if one has a flood wave propagation, the membership functions goes to right or to the left side according with the propagation of the wave. Through the figures 3 and 4 ones can see that fact.

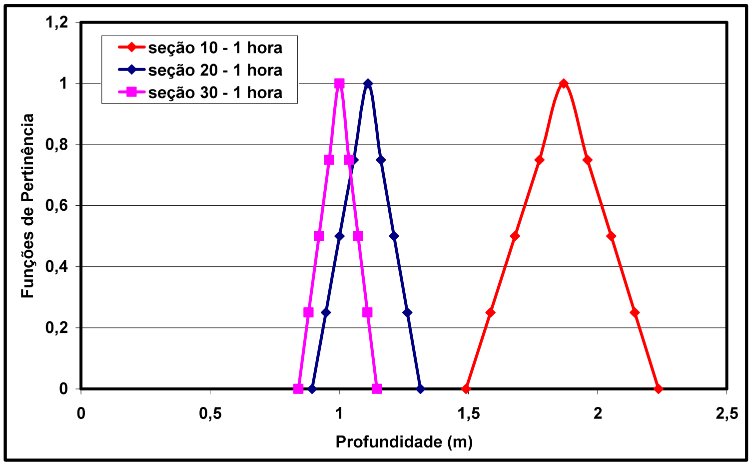

Figure 3. Membership functions of the depth for different sections.

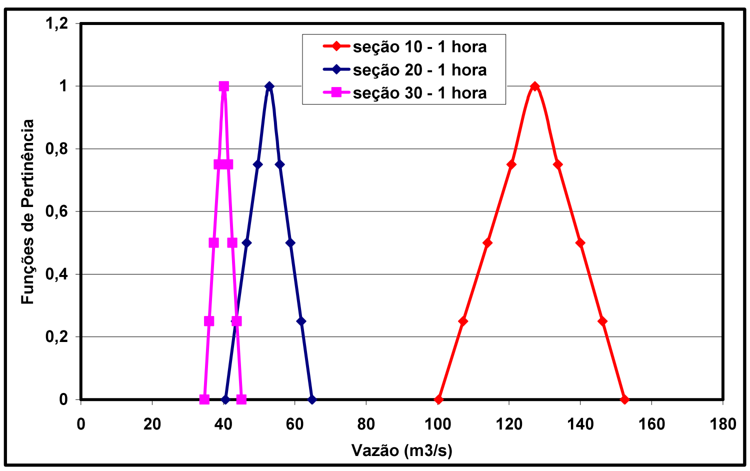

Figure 4. Membership function of the flow for different sections.

Still showing the dynamic characteristic of the membership functions for this class of problem, the figures 5 and 6 show the results of a simulation, for different times, in a section to 5 Km of the beginning of the channel. The results confirm the previous analysis. For instance, in the figure 6, the membership function for a time of 4 hours, is to the right of the membership function for the time of 1 hour. This happens because, in 1 hour, the flood wave is going by the section corresponding to 5 Km of the origin, making the membership function to move to the right side. This results show that the membership function can play a very strong game on the behavior of the flood risk analysis.

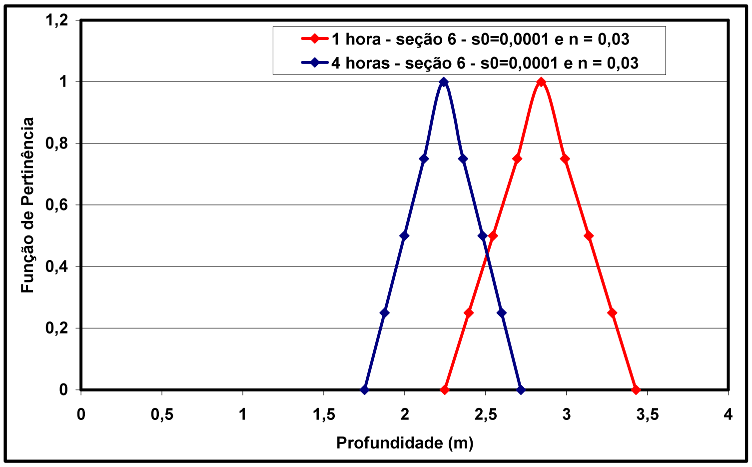

Figure 5. Membership functions of the depth for different time.

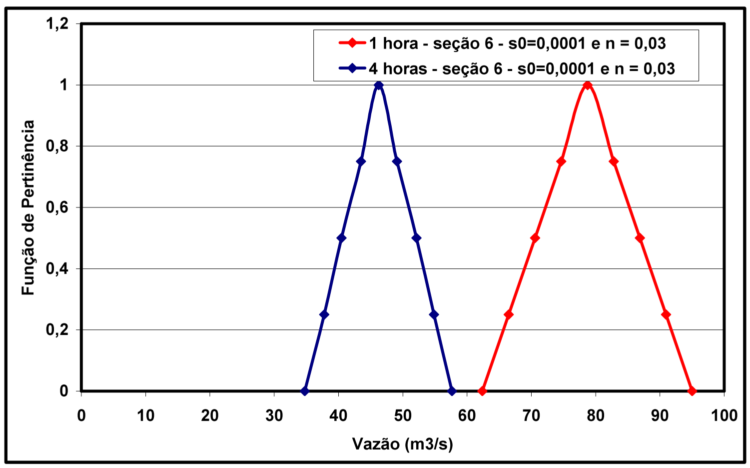

Figure 6. Membership functions of the flow for different times.

6. Conclusions

The results analysis permitted to conclude that this methodology can become a good way in order to calculate flood risk in Natural River. The capacity of the fuzzy hydrodynamic model to solve the Saint Venant Equation, in fuzzy formulation, as membership functions, brings the possibility of the determination of fields of risk for different environmental situations. In the present studying the results have showed that membership functions can be determinate for different sections and different time, showing, in such way the versatility and the potential of this methodology. However, it is fact that this research is just beginning and much more should be done with respect to the advances on these processes.

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