Flood Risk Analysis of Cocó Urban River in Fortaleza, Brazil

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Abstract: A mathematical model based on the Saint-Venant hydrodynamic equations combined with fuzzy set theory is formulated for the flood risk analysis of Cocó urban river in Fortaleza, Ceará, Brazil, which is subjected to the propagation of a flood wave. The model is capable of evaluating the behavior of the control variables related to the flow in terms of the hydraulic and hydrological parameters of the basin. The model is also capable of evaluating the fuzzy risk for such area subjected to the flood process during intense rains. The governing partial differential equations are solved with the aid of finite differences, and for the solution of the system of nonlinear algebraic equations the iterative Newton-Raphson algorithm is employed. A computer program QUARIGUA (Risk Quantitative Analysis of Flooding in Urban Rivers) is used to perform the simulations. The computer program QUARIGUA is organized in a modular manner, with two main modules: the deterministic module, where the depth of the water in the river and the flow of the channel are calculated as discreet values; and the fuzzy module, based on the fuzzy set theory, where the depth of the water and the flow are calculated as membership functions. To evaluate the behavior of the control variables, several scenarios for the main channel as well as for the flood waves are considered and different simulations are performed. The simulations demonstrate the reliability, versatility and computational efficiency of the proposed model.
Keywords: Flood Control; Fuzzy Risk Analysis; River Mechanics.

1. Introduction

The Saint-Venant equations represent a good way to describe problems concerning with flood waves propagations in open channels. This is a physical process of high complexity, caused by an intense rain or the breaking of any control structure, which represents an interesting problem to be studied.

The solution of this kind of problem passes, invariably, for the development of methods that allow solving the equations of Saint-Venant. These non linear equations contain, in its mathematical representation, all elements that, directly or indirectly, are related with the behavior of the flow in the channel. Through those equations it can be determined all the hydrodynamic of the system and it can be verified the possible risk of occurrence of inundations. It is enough that, for that, the function is modeled appropriately. Evidently, the modeling of such functions implicates in more efforts in the process of solution of the equations.

On the other hand, the application of fuzzy theory on the hydrodynamics problems to calculate the flood risk becomes a challenge to be gotten. As one knows this theory has became an alternative for the solution of conventional problems of subjectivity. In this

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case, the theory in subject develops an important paper, mainly, for the reason that its application do not demand a rigorous database.

In this research a fuzzy hydrodynamic model was developed to simulate the flood risk in the Cocó River that passes through the City of Fortaleza, Brazil. The model solves the equations of Saint-Venant using the difference method, with an explicit discretization, proposed by Chow (1988). The results have shown that the computational program developed to this research represents an alternative way to be applied in the uncertainty problem.

2. Methodology

The basic equations used for the mathematical modeling of the superficial drainages are obtained by the application of the law of the conservation of the mass and of the second law of Newton to a volume of representative control of the drainage. In general, those equations are expressed in the differential form and, with the application of numeric methods, the differential equations are solved.

The drainage in rivers possesses a predominant direction along the longitudinal axis of the channel, considered in the direction of the current, and the process is represented by equations with one dimension. The continuity and the momentum equations, knowing as Saint-Venant Equations are used to represent the superficial drainage in basins and the drainage in rivers.

In the process of solution of the model some fundamental conditions will be observed. The flow will be considered undimensional, so that, the momentum equation will be applied just in the x direction, along the longitudinal channel. The pressure has a hydrostatic distribution, and the channel will be considered with a rectangular section. Thus, the modeling that will be used to simulate the flow field is:

- Continuity Equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

- Momentum Equation

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(Q^2 / A\right)}{\partial x} + gA(\frac{\partial y}{\partial x} - S_0) + gAS_f = 0 \tag{2}$$

where x is the longitudinal distance along the channel (m), t is the time (s), A is the cross section area of the flow (m^2), y is the surface level of the water in the channel (m), S_0 is the slope of bottom of the channel, S_f is the slope of energy grade line, B is the width of the channel (m), and g is the acceleration of the gravity ($m.s^{-2}$).

In order to calculate S_f, the Manning formulation will be used. Thus,

$$V = \frac{1}{n} R^{2/3} S_f^{\ 1/2}$$

where V is the mean velocity (m/s), R is the hydraulic radius (m) e n is the roughness coefficient.

In order to calculate the flood risk in the river, it is necessary first to transform the model into a fuzzy model. This is done by the transformation of the boundary conditions of the problem in membership functions, so that the all control variables in the model can be solved and the form of membership functions. Thus, the new formulation for the model can be writing by,

- Continuity Fuzzy Equation

$$\frac{\partial \tilde{A}}{\partial t} + \frac{\partial \tilde{Q}}{\partial x} = \tilde{q} \tag{3}$$

- Momentum Fuzzy Equation

$$\frac{1}{\tilde{A}} \frac{\partial \tilde{Q}}{\partial t} + \frac{1}{\tilde{A}} \frac{\partial}{\partial x} \left(\frac{\tilde{Q}^2}{\tilde{A}} \right) + g \frac{\partial \tilde{y}}{\partial x} - g \left(\tilde{S}_0 - \tilde{S}_f \right) = 0 \tag{4}$$

where $\tilde{A}$ is the membership function for the transversal area of the river; $\tilde{Q}$ is the membership function for the flow; $\tilde{y}$ is the membership function for the depth; $\tilde{q}$ is the membership function for lateral flow; $\tilde{S}_0$ is the membership function for the bed slope of the river; and $\tilde{S}_f$ is the membership function for the headline slope.

- Boundary Conditions:

$$\tilde{Q}(0,t) = \tilde{Q}_0(t) \tag{5}$$

$$\left. \frac{\partial \tilde{Q}}{\partial x} \right/_{x=L} = 0 \tag{6}$$

- Initial Conditions:

$$\tilde{Q}(x,0) = \tilde{Q}_1(x) \tag{7}$$

- Energy Gradeline equation

$$\tilde{Q} = \frac{1}{\tilde{n}} \tilde{A} \tilde{R}^{2/3} \tilde{S}_f^{\,1/2} \tag{8}$$

This is the new model in its fuzzy form. It is important to remind that the solution of this model will supply four (4) membership functions, one for each control variable. Therefore, a membership function will exist for the flow, the speed, the height (depth of the drainage) and the wet area of the section of the channel. These membership functions will be used in the evaluation of the risk.

3. Fuzzy Risk Analysis

The solution of this group of equations, shown previously, allows determining the dependent variables, in the form of membership functions. Those functions are calculated, along of the river, for different time. Like this, the hydrodynamic fuzzy equation produces a flow field in fuzzy form, defined by its membership functions, as membership function for the transverse area, membership function for the velocity, and a membership function for the depth, being this last one the most important for the present work.

The subject is to define, in the context of the risk analysis, the importance of the heights of water, according to the membership functions. Obtained these relative membership functions to the heights, the same ones will be compared with other membership function, also necessary to evaluate the flood risk of a certain area, that represents the of levels of the land in study, in other words, the allowed maximum heights. This membership function acts, in their characteristics fuzzy way, the maximum limits of the line of water in the channel, after a given flow. Besides this limit, the body of water should begin to overflow, flooding the neighboring areas. This membership function is called of resistance and, once defined, it allows that the risk can be calculated.

Consequently, let the resistance membership function be defined by the maximum limits of shipment allowed by a course of water. Let yet the membership function of the height that one calculated by the mathematical model proposed in this study, and that it represents the answer of the receiving system to the arrival of a flood wave.

In such way, the safety margin $\widetilde{M}$, of the body of water can be represented by the difference among the resistance membership function $\widetilde{R}$, and the membership function calculated of the heights $\widetilde{H}$, that it represents the answer to the possible waves of floods or intense rains in the river. In such way, the fuzzy risk can be defined by, see Ganoulis, 1994,

$$R_f = \frac{\int_{-\infty}^{0} \mu_{\widetilde{M}}(m)dm}{\int_{-\infty}^{\infty} \mu_{\widetilde{M}}(m)dm} \tag{9}$$

where $\mu_{\widetilde{M}}$ is the safety margin membership function, and *m* is an element of the M. Yet the fuzzy Reliability can be defined by

$$R_c = \frac{\int_{x>0} \mu_{\widetilde{M}}(m)dm}{\int_{-\infty}^{\infty} \mu_{\widetilde{M}}(m)dm} \tag{10}$$

It is important to note that R_f and R_c are real functions, defined in the close interval [0,1], and they depend on the hydraulics and hydrologic parameters of the channel.

4. Application of the Model to the Cocó River

For the application of the model, the data from the Cocó river were used. This river is located in the State of Ceará. Its basin is placed at the east area district of Fortaleza. It is one of the main axes of natural drainage of the city of Fortaleza, capital of the State of Ceará, and it has an extension of 50 km from the Mountain of Aratanha, where it born, until through its mouth in the coast east of Fortaleza. The principal data that were used from this river are:

Uniform flow of entrance = 10 m / s
Length = 20 Km
Medium width of the channel = 40 m
Manning Roughness Coefficient (n) = 0.03
Steepness of the line of water = 0.000005 m/m

5. Results

The figures 1 and 2 show the results of the simulation for different flood waves, for one section 5 Km distant from the origin. It is observed that the maximum risk happens approximately 0,8 hour after the entrance of the wave, being its value of approximately 29% when the pick of the wave is 5 times larger than the medium flow. The results show that the flood risk, for these data of Rio Cocó, is significant, where its risk reliability value, approximately 71%, showing, like this, a certain fragility for the close areas to the course of water, when this basin is subject to intense rains.

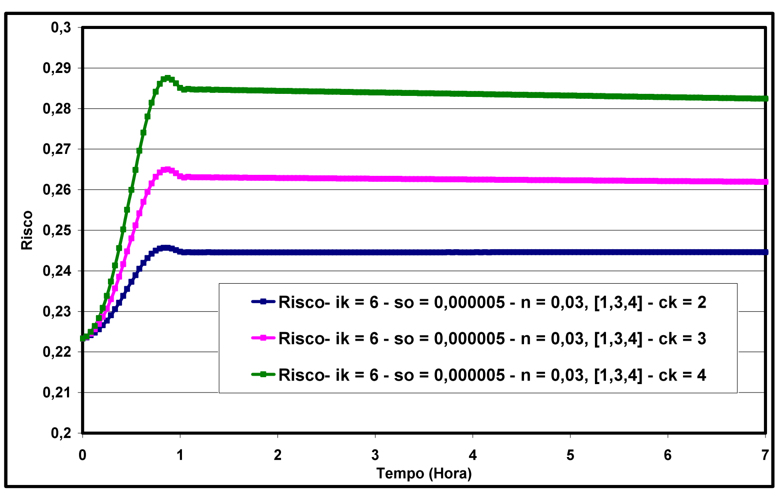

Figure 1. Fields of Flood Risk for different intensities of flood waves.

The figures 3 and 4 show the risk and the reliability for different sections of the main channel. The results show that, the more far way is the section, from the origin, where the wave arrives, the less is the risk. For instance, in a section to 15 Km from the origin, the maximum pick of the risk is around 24%, while, form 5 Km from the origin, the maximum pick of the risk is of 26,5%, for an flood wave pick 4 times larger than the normal medium flow. These results show yet that the flood waves lose energy in its propagation, where your amplitude suffers a considerable reduction, making that the risk maximum gets reduces in the same proportion.

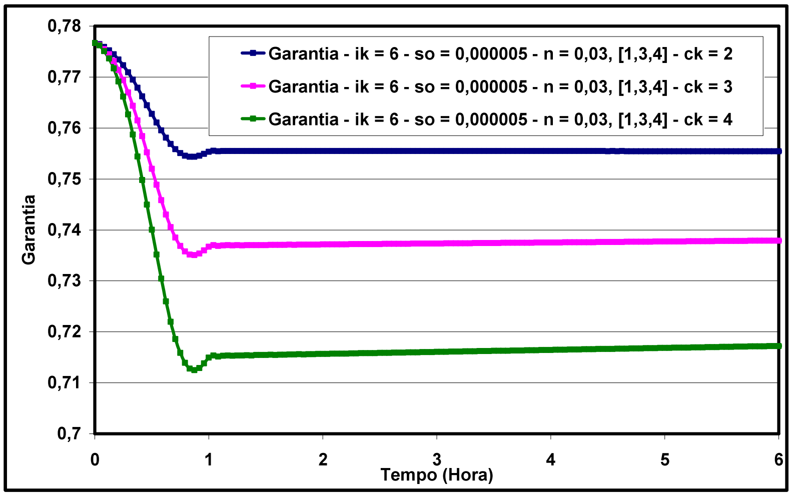

Figure 2. Fuzzy Reliability for different intensities of flood waves.

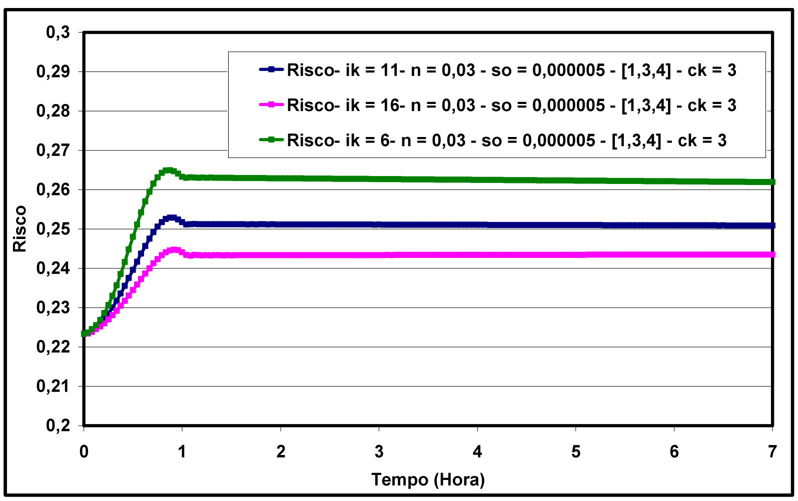

Figure 3. Fields of Risk for different sections for n = 0.03 and S_0 = 0,000005

6. Conclusions

The computation mathematical modeling developed has answers fully to the objectives of the research, with versatility and efficiency, in the solution of the most varied practical situations found in the field of its applicability, producing compatible results with observation fields. In the determination of the fuzzy risk, the computational program lowed the solution of the fuzzy model for the Saint-Venant Equation, using, as boundary conditions and hydraulic parameters, membership functions.

In the application of the model into the Cocó river, located in the Metropolitan Area of Fortaleza, it was verified that the fuzzy model proposed is capable to establish the risk and the reliability through the entire part of the river considered. In this simulation, it was possible to calculate the flood risk for some scenery and to verify the influence of the hydraulic parameters in the risk functions. The results allowed to conclude that the proposed methodology satisfies the objectives of the research, and it represents an alternative to evaluate flood plains and, therefore, to supply more solid subsidies in the environmental planning of the metropolitan areas.

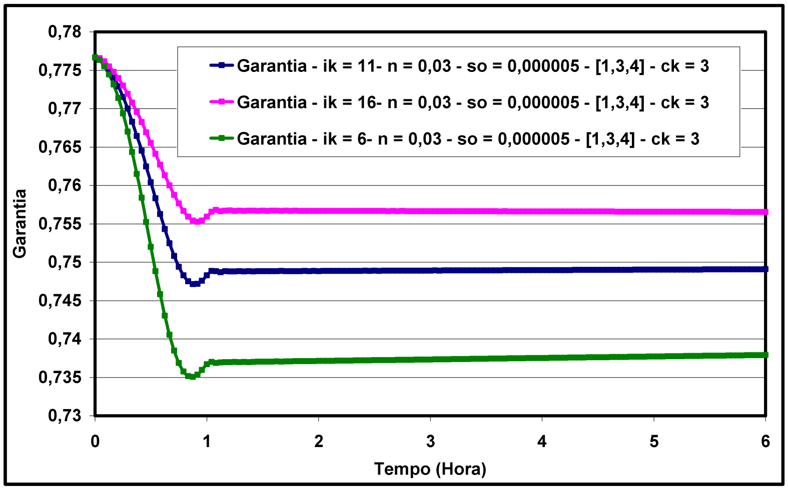

Figure 4. Fields of Risk for different sections for n = 0.03 and S_0 = 0.000005

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