EVALUATION OF WAVEGUIDE MODAL APPROXIMATION APPLIED TO ACOUSTIC ABSORPTION ANALYSIS OF POROELASTIC MATERIALS

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Abstract. *In order to analyze and design insulation systems, numerical methods, such as the finite element method and the boundary element method, are intensively used. In particular, for poroelastic materials models based on the Biot-Allard theory, mathematical relations in a mixed formulation were developed using the structural displacement u and the fluid pressure p as state variables. In this work, the simulation of the absorbing performance of poroelastic samples, as if placed in impedance tube, is carried out using coupled poroelastic and acoustic finite element models. The goal is to evaluate two methodologies in order to obtain the acoustic absorption response of foam materials found on typical insulating systems. For the first procedure, the porous material model is coupled to a waveguide using a modal expansion technique. For the second procedure, the full acoustic domain is solved using acoustic finite elements and the acoustic absorption function is determined by evaluation of the acoustic pressure field. A systematic comparison of the influence of waveguide modes on the absorption response in the frequency domain was made and the validity of the modal approximation in coupled acoustic-poroelastic analysis is discussed.*

Keywords: *Poroelasticity, finite element modeling, waveguide*

1. INTRODUCTION

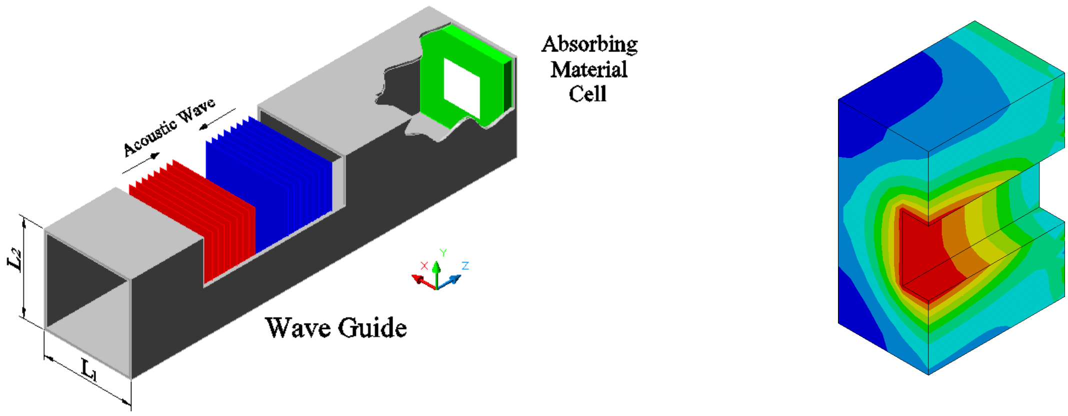
The goal of this study is to evaluate methodologies for estimation of absorbing characteristics employed in design acoustic insulating systems. The problem consists of a porous material placed into a semi-infinite waveguide with rigid walls and acoustically excited by plane waves. The perforated material is introduced as periodic porous-acoustic cell and can be formulated in meso-scale, Olny (1999). In this scale, we have the acoustic and porous phases. The new material properties used in the acoustic absorbing material design was tested in Atalla *et al.* (2001) and Olny and Boutin (2003). In this paper, we evaluated modal techniques and finite element discretization for modeling the acoustic domain in the tube. In order to compare the methodologies, the acoustical surface impedance and the absorption frequencies functions are performed. Several finite element formulations for sound absorbing materials have been developed in the last thirty years. Coupled fluid-structure models based on the Biot theory have been introduced and improved by Kang and Bolton (1995), Panneton and Atalla (1996) and Lamary *et al.* (2001). In Atalla *et al.* (1998), the classical Biot-Allard equations have been rewritten in terms of the solid phase macroscopic displacement vector and interstitial fluid phase macroscopic pressure. In this paper, a symmetric mixed formulation (u,p) developed by Panneton and Atalla (1996) is used. This way, we can find a solid phase macroscopic displacement vector u_i and the interstitial fluid macroscopic pressure p as the state variables. The resultant coupled system is similar to the classical Fluid-Structure (u,p) system Panneton and Atalla (1997a). The outline of the rest of the paper is as follows. In Section 2, the problem and some concepts are stated. In the next section, the poroelastic model is described. Moreover, the basic assumptions are presented and the governing equations for acoustic and solid phases are presented. A weak integral formulation is used. In this context, numerical approaches are applied to solve the equilibrium problem. The methods for measurement the vibroacoustic parameters are presented in Section 4 and 5. A full finite element description and a modal superposition are adapted to model the acoustic domain in the tube. In Section 6, numerical demonstrations of the computational implementation are presented and the performance of the method is illustrated. The conclusions are outlined in Section 7.

2. PROBLEM STATEMENT

The classical hypothesis for linear acoustic and elastic behavior are assumed Allard (1993). In this approach, the wave propagation theory for the coupled medium is valid for low frequency range and fully saturated conditions. In this case, all dependent quantities represent small fluctuations around a static reference value and the poroelastic properties (porosity, tortuosity, etc) are continues in the domain.

The interface conditions of the porous-acoustic system take into account the continuity of fluid normal displacements, continuity of pressures, mass flow conservation and internal forces equilibrium. The mixed formulation (u,p) simplifies the assemblage process of the porous-acoustic systems. Therefore, for coupling among poroelastic and acoustic medias is not necessary calculate any interface matrix, Debergue *et al.* (1999).

Using the system geometric symmetry feature and the periodic system properties, we can adopt the hypothesis of the plane wave excitation and solve the Kundt tube problem like is represented in detail in Fig. (1a). In Figure (1b), a typical solution for the acoustic pressure diffusion in the porous-acoustic cell is presented.



(a)Waveguide Model and Absorbing Material Cell (b)Pressure Distribution in the Porous-Acoustic Cell

Figure 1. Porous-Wave Guide Coupled System

In this conditions, the Porous-Acoustic System is excited by waveguide. A modal superposition technique is used to model the pure acoustic domain. The development of the waveguide modal superposition theory can be found in the works Atalla *et al.* (2001) and Sgard *et al.* (2005). The Porous-Waveguide interface Γ condition involving porous coatings can be expressed as an inhomogeneous mixed Dirichlet-Neumann boundary condition, as described in Eq. (1).

$$k\frac{\partial p}{\partial n} = A(p - p_b) \tag{1}$$

where p is the acoustic pressure on the interface, k is a coefficient linked to the acoustic media impedance, A is the admittance term of the porous-waveguide interface and p_b is the blocked pressure amplitude and related by:

$$\left.\frac{\partial p_b}{\partial n}\right|_{\Gamma} = 0 \tag{2}$$

3. POROELASTIC FINITE ELEMENT MODEL

Using the Galerkin method, taking δp as the admissible virtual variation of the fluid phase pressure field (p), for the acoustic domain, we can find the weak integral form for the acoustic domain, as follows in Eq. (3).

$$\int_\Omega \frac{1}{\omega^2 \rho_0} p_{,i}\delta p_{,i} d\Omega - \int_\Omega \frac{1}{\rho_0 c_0^2} p\delta p d\Omega - \int_\Gamma \frac{1}{\omega^2 \rho_0}\frac{\partial p}{\partial n}\delta p d\Gamma = 0 \tag{3}$$

where ω is the frequency, ρ_0 is the mass density of the fluid (air), c_0 is the speed of the sound propagation in the fluid phase.

For the poroelastic domain, taking δu_i as the admissible virtual variation of the solid phase displacement vector (u_i), the weak integral form results in the following relations for the solid and fluid phase, Eq. (4) and (5), respectively.

$$\int_\Omega \widehat{\sigma}_{ij}^s \varepsilon_{ij}^s(\delta u_i) d\Omega - \omega^2 \int_\Omega \widetilde{\rho} u_i \delta u_i d\Omega - \int_\Omega \widetilde{\gamma} p_{,i} \delta u_i d\Omega - \int_\Gamma \widehat{\sigma}_{ij}^s \cdot n_j \cdot \delta u_i d\Gamma = 0 \tag{4}$$

$$\int_\Omega \frac{h^2}{\omega^2 \widetilde{\rho}_{22}} p_{,i}\delta p_{,i} d\Omega - \int_\Omega \frac{h^2}{\widetilde{R}} p\delta p d\Omega - \int_\Omega \widetilde{\gamma} u_i \delta p_{,i} d\Omega + \int_\Gamma \left(\widetilde{\gamma} u_n - \frac{h^2}{\omega^2 \widetilde{\rho}_{22}}\frac{\partial p}{\partial n}\right)\delta p d\Gamma = 0 \tag{5}$$

where the Ω and Γ denote the poroelastic domain and its boundary, respectively. The vector n_j is the unitary normal vector and pointing outward the boundary Γ, $\partial p/\partial n$ is the directional derivative of the fluid phase pressure. The term $\widehat{\sigma}_{ij}^s$ represent the elastic linear skeleton stress tensor in vacuum, ε_{ij}^s is elastic strain tensor, $\widetilde{\rho}_{22}$ is the fluid mass coefficient that take into account the fact that the relative flow in the pores is not uniform and $\widetilde{\rho}$ is the complex effective density of the porous domain, $\widetilde{\gamma}$ is a new coupling term and defined in the works Panneton and Atalla (1997b) and Atalla *et al.* (1998).

The discrete form of dynamic equation of the coupled system is done by Eq. (6).

$$\begin{bmatrix} [K]-\omega^2[M] & -[C] \\ -[C]^t & 1/\omega^2[H]-[Q]+[A] \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ [A]\,p_b \end{Bmatrix} \tag{6}$$

where $[K]$ is the phase solid stiffness matrix and $[M]$ is the phase solid mass matrix of the Porous-Acoustic material. the matrix $[H]$ is the volumetric matrix of the Porous-Acoustic material domain and $[Q]$ are the phase fluid compressibility matrix. The rectangular matrix $[C]$ is the coupling matrix among the solid and fluid phases in the poroelastic material. In a compact form, the Eq. (6) can be rewritten as:

$$[D_A]\{U\} = \{F\} \tag{7}$$

where $[D_A]$ is the dynamic matrix coupled to waveguide contribution and can be expressed as $[D_A] = [D] + [A]$. The dynamic matrix of poroelastic phase is $[D]$ and the contribution of the waveguide is expressed for $[A]$.

The Equation System (7) is solved to determine the dynamic response $\{U\}$, using direct solution method for each frequency value. In this formulation, $[D_A]$ is complex, symmetric and frequency dependent. Each mode has four degrees of freedom: three displacements and one pressure. The computational cost of this solution can be prohibited for large 3D problems. In this case, partitioning techniques can be used. The porous material acoustic performance is evaluated by the absorption coefficient:

$$\alpha(\omega) = \frac{\Pi_{diss}}{\Pi_{inc}} \tag{8}$$

where α is the acoustic absorption coefficient, Π_{diss} represents the dissipated potential and Π_{inc} is the incidence potential (Sgard *et al.*, 2005). The absorption coefficient α is a real and frequency dependent function. In a finite element formulation, the dissipative potential Π_{diss} is determined as follows:

$$\Pi_{diss} = \frac{1}{2}\omega\,\Im\left(\{U\}^t[D]\{U\}\right) \tag{9}$$

and the incident potential can be calculated as:

$$\Pi_{inc} = \frac{S|p_0^2|}{2\rho_0 c_0} \tag{10}$$

where S represents the transversal section of the tube.

Another important result of vibroacoustic is the surface acoustic impedance Z_n. The acoustic impedance is the ratio of acoustic pressure p and the normal velocity v_n. In a frequency domain, the impedance function over the surface S is done by:

$$Z_n(\omega) = \frac{\int_\Gamma p\,(x,y)\,dS}{\int_\Gamma v_n\,(x,y)\,dS} \tag{11}$$

In a dynamic context, the normal velocity can be calculated as:

$$v_n = j\omega\,(hU_n + (1-h)u_n) \tag{12}$$

where h is the porosity of the porous material sample, U_n and u_n represent the normal displacements of the fluid and solid phase over the surface S, respectively.

In the next section, the modal decomposition technique is presented.

4. POROUS-WAVEGUIDE APPROACH

For the waveguide domain, one analytical model defined in Eq.(13) and determined by a discrete formulation described in (Castel, 2005) is used. The coupling of the porous-acoustic numeric model with the semi-analytic waveguide model is obtained by the imposition of pressure continuity at the porous-acoustic interface. Using the concept of the admittance operator A from point P_1 to point P_2, as defined by Eq.(13), (Sgard *et al.*, 2005), it is possible to represent the waveguide domain using a modal superposition for the Admittance operator A given by:

$$A\left(P_1, P_2\right) = \sum_{m,n} \frac{k_{mn}}{\rho_0 \omega N_{mn}} \psi_{mn}\left(P_1\right) \psi_{mn}\left(P_2\right) \tag{13}$$

where ω is the frequency, ψ_{mn} represents the orthogonal modal shape of the transversal section of the waveguide, N_{mn} is the quadratic norm of the modal shape and k_{mn} is the wavenumber of the propagation wave in the Kundt tube. The acoustic pressure p is written as the sum of the incidence average pressure $p_b = 2p_0$ and the radiated pressure from the porous-acoustic interface.

In Figure (2), it is presented the harmonic mode shapes $\psi_{mn}(x, y)$ of the rectangular tube section. The modes are plotted by (m, n) series order and specific normalization N_{mn}.

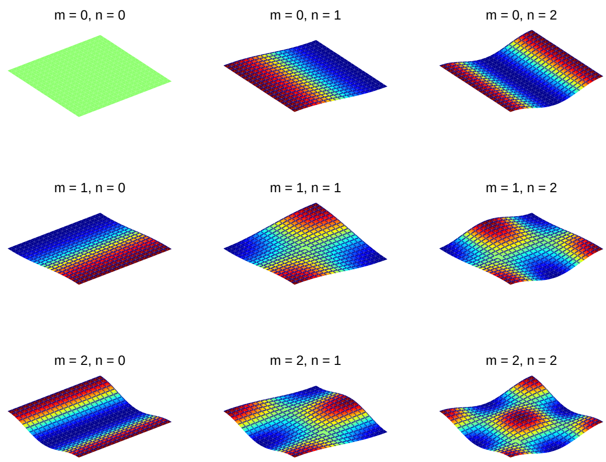


Figure 2. Orthogonal Geometric Modes of rectangular section $L_x \times L_y$

The boundary integral form of the acoustic interface, shown in Eq. (3), can be calculated using the modal discretization of the waveguide, shown as follows:

$$\int_{\Gamma_a} \frac{1}{\omega^2 \rho_0} \frac{\partial p}{\partial n} \delta p \, d\Gamma = \frac{1}{i\omega} \int_{\Gamma_{pa}} \int_{\Gamma_{pa}} A\left(P_1, P_2\right) \, p\left(P_2\right) \, \delta p\left(P_1\right) \, d\Gamma_2 \, d\Gamma_1 - \frac{1}{i\omega} \int_{\Gamma_{pa}} \int_{\Gamma_{pa}} A\left(P_1, P_2\right) \, p_b\left(P_2\right) \, \delta p\left(P_1\right) \, d\Gamma_2 \, d\Gamma_1 \tag{14}$$

The admittance operator A is projected on the element mesh of the porous-acoustic interface. The effects of the local admittance matrix can be assembled in the global numeric model, resulting in an equivalent fluid coupling represented by coupling waveguide matrix $[A]$.

5. FULL ACOUSTIC MODEL

This method is based on the Norm ASTM-E-1050 (1998). That uses an impedance tube with a sound source connected to one end and the test sample mounted at the other end. In this application, the acoustic pressure is determined by finite element discretization of the Kundt tube environment. After the pressure field in the tube is calculated, it is proceed the estimation of the absorption coefficient. The method is based on the transfer function value H among two pressure responses provided from two points in the domain acoustic inside the tube. This way, the coefficient of reflection R can be determined by Eq. (15) as follows:

$$R = \left(\frac{H - e^{-jks}}{e^{jks} - H} \right) e^{2k(l+s)} \quad (15)$$

where k is the wave number, s is the distance of the microphones and l is the distance among the surface of absorbing material and the microphone nearest the sample. Therefore, the coefficient of absorption α can be evaluated by:

$$\alpha = 1 - |R^2| \quad (16)$$

In Equation (17), it is described a relevant relationship of specific impedance and the coefficient of reflection.

$$\frac{Z_n}{\rho_0 c_0} = \frac{1 + R}{1 - R} \quad (17)$$

In the next section, the methodologies for determination of acoustic absorption function are evaluated by numerical simulations.

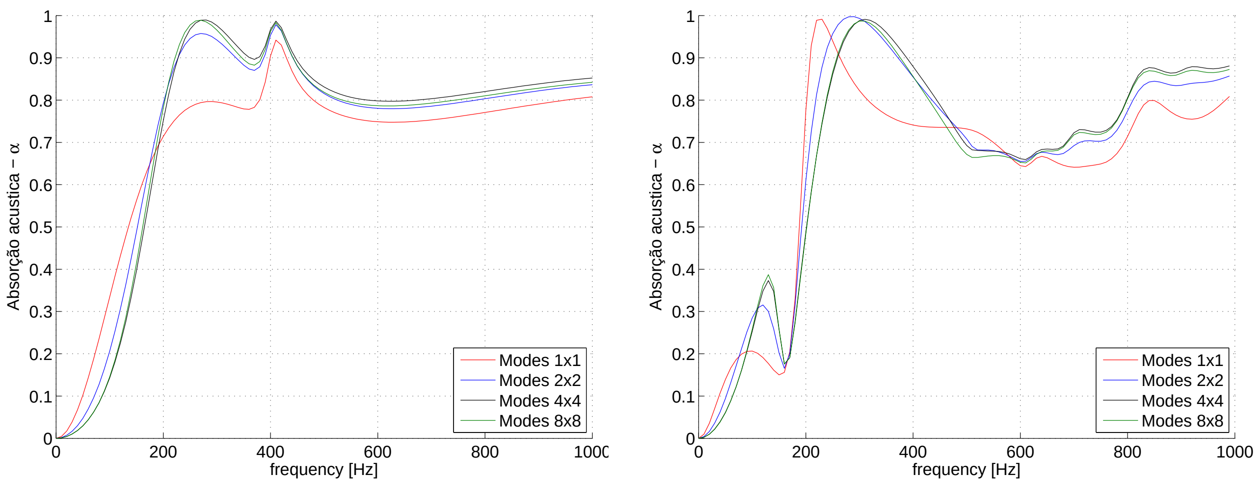
6. NUMERICAL TESTS

The evaluation of the methods consists in the absorption and impedance frequency functions of a rectangular section cell with dimensions 0.085 x 0.085 m and thickness is 0.115m. There is a rectangular perforation of 0.0283 m in all extension of the sample. For the present procedure, the domain is divided in meshes of eight-node tree-dimensional elements. It is implemented a porous-acoustic system coupled to the acoustic waveguide excitation and a full acoustic domain. In this paper, the numeric tests were performed using porous material properties given in Table (1).

Table 1. Properties of typical Poroelastic Materials.

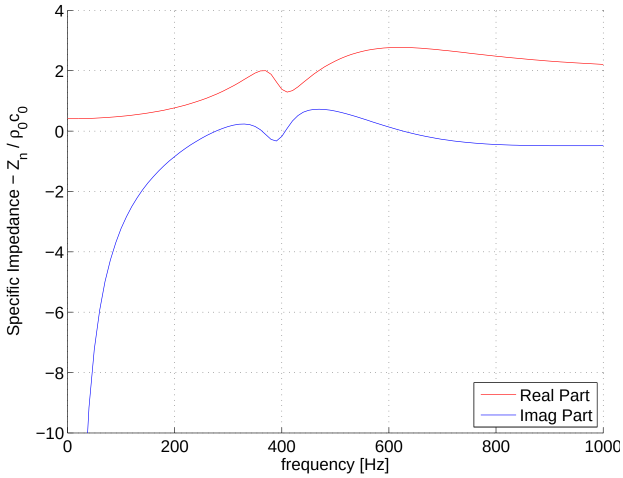
Material	ν	η_s	N $[kN/m^2]$	h	$\rho_s[kg/m^3]$	α_∞	$\sigma[Ns/m^4]$	$\Lambda[\mu m]$	$\Lambda'[\mu m]$
GlassWool	0.3	0.055	55	0.97	31	2.52	87000	37	119
RockWool	0.0	0.1	2200	0.94	130	1.06	40000	56	110

In Figure (3), it is presented the influence of the modes employed in the modal superposition approach. By convergence analysis, it was calculated several acoustic absorption responses for two different material samples. The average acoustic impedance over the porous-acoustic interface is used to validate the modal approach with respect the full acoustic domain discretization. The same result of impedance function was found for both methodologies and it is presented in Fig. (4).

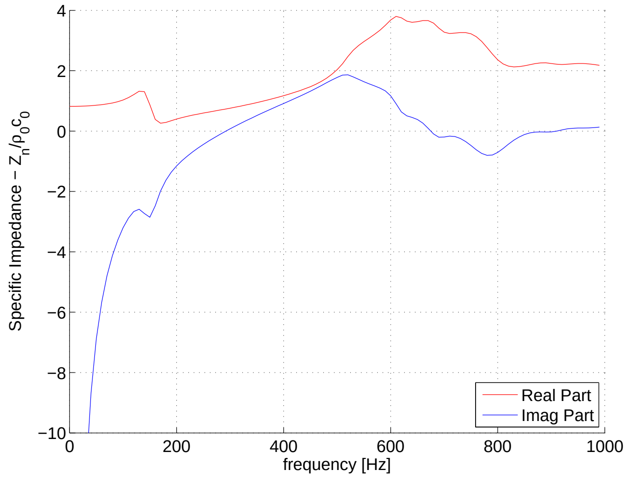


Rock Woll   Glass Woll

Figure 3. Convergence of absorption response with the modal superposition approximation



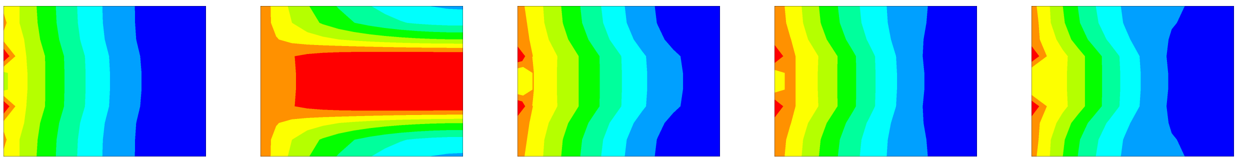
Rock Woll



Glass Woll

Figure 4. Acoustic Impedance of the Poroelastic Material Samples

The next results consist in the evolution of pressure and solid displacement in the longitudinal section of porous-acoustic cell systems with the variation of frequency value. In this context, the pressure field of the Rock Woll perforated sample is plotted in Fig. (5) and the displacement resultant is presented in Fig. (6). The results are presented in real and imaginary part solutions.



Real - 200 Hz | Real - 400 Hz | Real - 600 Hz | Real - 800 Hz | Real - 1000 Hz

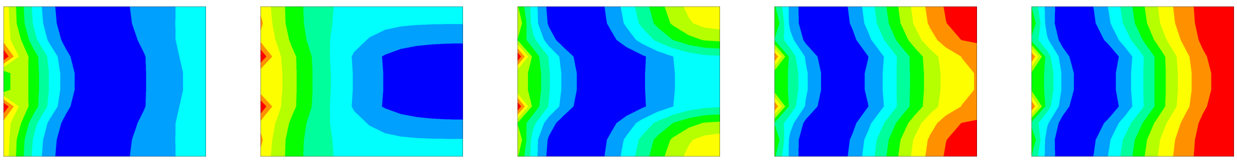


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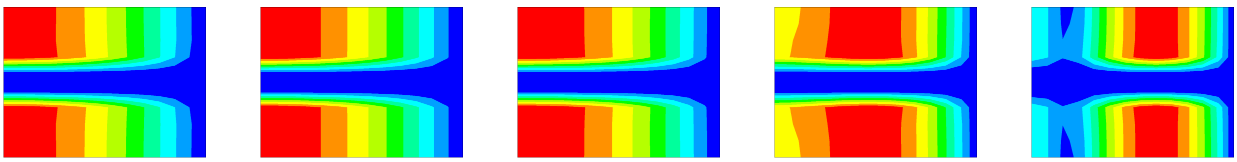
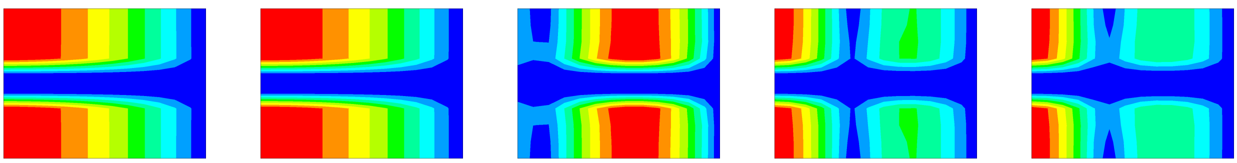
Figure 5. Evolution of the Pressure with frequency



Real - 200 Hz | Real - 400 Hz | Real - 600 Hz | Real - 800 Hz | Real - 1000 Hz



Imag - 200 Hz | Imag - 400 Hz | Imag - 600 Hz | Imag - 800 Hz | Imag - 1000 Hz

Figure 6. Evolution of the Solid Displacement with frequency

7. CONCLUSIONS

In this work, typical absorbing materials were performed in a Kundt tube by numerical methods. A convergence analysis of modal superposition for rectangular acoustic cavities was evaluated. For the numerical tests presented in this work, a few number of modes is necessary to reach a tolerable convergence. It can be noted, the number of modes is dependent of domain size and frequency range of analysis. The semi-analytical model of waveguide propagation is adapted in a finite element mesh and this approach was validated with a full acoustic discretization model. In this context, several numerical analysis were done in order to determine the dynamic response of acoustic pressure and solid phase displacement in the porous-acoustic system. For the poroelastic materials perforated samples, the absorbing curves obtained presented interesting evolutions. Thus, the absorbing cell system proposed has a good performance of acoustic absorption in the low frequency domain. Further works will include experimental analysis, axi-symmetric analysis for circular section tubes and studies about the influence of the material parameters in acoustic absorption process.

8. ACKNOWLEDGEMENTS

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