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RODRIGO DE OLIVEIRA MAYORGA

AN APPLICATION OF VALUE AT RISK AND EXPECTED SHORTFALL

FORTALEZA
2016

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Tese apresentada ao Curso de Doutorado em Economia do Departamento de Pós-Graduação em Economia - CAEN da Universidade Federal do Ceará, como parte dos requisitos para obtenção do título de Doutor em Economia. Área de concentração: Econometria Aplicada.

Orientador: Prof. Dr. Andrei Gomes Simonassi.
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BANCA EXAMINADORA

Prof. Dr. Andrei Gomess Simonassi (Orientador)
Universaidade Federal do Ceará - CAEN

Prof. Dr. Rafael Bráz Azevedo Farias (Coorientador)
Universaidade Federal do Ceará - DEMA

Prof. Dr. Paulo Rogério Faustino Matos
Universaidade Federal do Ceará - CAEN

Prof. Dr. Luiz Ivan de Melo Castelar
Universaidade Federal do Ceará - CAEN

Prof. Dr. Roberto Tatiwa Ferreira
Universaidade Federal do Ceará - CAEN

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“Stay Hungry, Stay Foolish”

Steve Jobs

ABSTRACT

The last two decades have been characterized by significant volatilities in financial world marked by few major crises, market crashes and bankruptcies of large corporations and liquidations of major financial institutions. In this context, this study considers the Extreme Value Theory (EVT), which provides well established statistical models for the computation of extreme risk measures like the Value at Risk (VaR) and Expected Shortfall (ES) and examines how EVT can be used to model tail risk measures and related confidence interval, applying it to daily log-returns on four market indices. These market indices represent the countries with greater commercial trade with Brazil for last decade (China, U.S. and Argentina). We calculate the daily VaR and ES for the returns of IBOV, SPX, SHCOMP and MERVAL stock markets from January 2nd 2004 to September 8th 2014, combining the EVT with GARCH models. Results show that EVT can be useful for assessing the size of extreme events and that it can be applied to financial market return series. We also verified that MERVAL is the stock market that is most exposed to extreme losses, followed by the IBOV. The least exposed to daily extreme variations are SPX and SHCOMP.

Keywords: Extreme Value Theory; Value at Risk; Expected Shortfall.

RESUMO

As duas últimas décadas têm sido caracterizadas por volatilidades significativas no mundo financeiro em grandes crises, quebras de mercado e falências de grandes corporações e liquidações de grandes instituições financeiras. Neste contexto, este estudo considera a evolução da Teoria do Valor Extremo (EVT), que proporciona modelos estatísticos bem estabelecidos para o cálculo de medidas de risco extremos, como o *Value at Risk (VaR)* e *Espected Shortfall (ES)* e examina como a EVT pode ser usada para modelar medidas de risco raros, estabelecendo intervalos de confiança, aplicando-a aos log-retornos diários a quatro índices de mercado. Estes mercados representam os países com maior intercâmbio comercial com o Brasil (China, U.S. e Argentina). Calculamos o VaR e ES diários dos índices IBOV, SPX, SHCOMP e MERVAL, com dados diários entre de 02 de janeiro de 2004 e 08 de setembro de 2014, combinando a EVT com modelos GARCH. Os resultados mostram que EVT pode ser útil para avaliar o tamanho de eventos extremos e que ele pode ser aplicado a séries de retorno do mercado financeiro. Verifica-se ainda que MERVAL é o mercado de ações que está mais exposta a perdas extremas, seguido do IBOV. Os menos expostos a variações extremas diárias são SPX e SHCOMP.

Palavras Chave: Teoria dos Valor Extremos; Valor em Risco; Expected Shortfall.

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SUMMARY

1. Introduction

After a series of economic collapse in the past three decades including Black Monday in 1987, Asian market crisis in 1997, Subprime crisis in 2008 and the most recent China Aviation Oil Singapore (CAO) incident, financial markets analysts started to investigate the impacts of these breakdowns on financial markets and how it affects the economy as a whole. The financial crisis of 2008 caused a worldwide economic collapse that is considered the most severe since the 1930s. It was triggered by bad investment decisions by major banks in the U.S. on potentially unplayable mortgages.

Globally the major impact was felt in Britain, European Union, Russia, Japan, the oil countries of the Middle East, and the Third World. The crisis affected almost every sector of the economy, including housing, construction, office buildings, automobiles, retail sales, and government (Recession of 2008). According to the International Monetary Fund (IMF), the global financial crisis gave impact on $3.4 trillion losses from financial institutions around the world between 2007 and 2010 (Dattels and Kodres, 2009).

The risk manager must be aware of the possibility of occurrence of extremes events in other markets, and the magnitude of changes in the markets in which it operates, reducing unexpected sever losses. Empirical work using Extreme Value Theory (EVT) approach to model spillovers in financial markets has grown in recent years.

In the EVT approach, financial crises are viewed as rare, i.e., extreme events whose occurrence is governed by different laws than those governing the entire domain of asset return distributions studied. The focus is on the tails of the distributions. This allows the avoidance of some typical misassumptions, of which the most commonly made are that the analyzed empirical distributions follow normal distributions.

Value at Risk (VaR) was introduced by JP Morgan in the mid 1990s who introduced the RiskMetrics methodology. In 1999, Artzner et al., underlines about VaR being used as a risk measure, since it is not necessarily sub-additive (it's not coherent) which means there is no guarantee that merger of two portfolios do not create extra risk. In fact, diversification of portfolio, containing more than one asset, can reduce risk. Artzner et al. (1999) also argues that, VaR measures only percentiles of profit-loss distributions, disregarding any loss beyond the VaR level ("tail risk" problem). In their study, they proposed the Expected Shortfall (ES) as coherent measure.

China, U.S. and Argentina have been the three countries with greater commercial trade with Brazil for last decade. The fact that these economies are strong economic partners, with several agreements, maintaining strong commodity exchanges relations, turn out to be extremely possible that economic crisis in one of these countries could be an indicator that the other may be the next to go into crisis. This paper deals with the behavior of the tails of financial series of these four countries, focusing on the use of extreme value theory to compute tail risk measures and the related confidence intervals. We model and estimate the dynamic next day VaR and ES for the Brazilian stock index (IBOV Index), the American S&P 500 index (SPX Index), the Chinese stock index (SHCOMP Index) and the Argentina stock index (MERVAL Index) series of daily log-return data using EVT.

This paper is organized as follows. Market evaluation of the countries studied is outlined in Section 3. Section 4 describes the literature review of GARCH and EVT. In section 5, we present an overview of the theoretical framework of EVT, describe the measures of extremes risks (VaR and ES), present the GARCH models and explain how conditional EVT is applied on VaR and ES. In Section 6 we explain the methodology for the dynamic approach and measure of accuracy by backtesting models. We discuss in Section 7 the tail modeling of the Brazilian, American, Chinese and Argentinian return series, assess the outcomes and provide the estimates of the risk measures. Finally, we conclude the study in section 8.

2. Market Evaluation

Between 2004 and 2015 the Brazilian import market grew about 172%, but during this period it had two major drops when compared with the year before. One was in 2009 with 26% (decrease) and the other was in 2015 with 25%. The three main countries that Brazil imports goods since 2004 are respectively (in 2015), China (18%), U.S. (15%) and Argentina (6%), representing approximately 39% of Brazilian total imports. The goods that are most imported are, petroleum oils, petroleum gases, crude oil from petroleum, motorcars and other motor vehicles for passengers and electrical apparatus. During the same period (2004-2015), the Brazilian export market grew nearly 98%. In 2009 there was a major fall of 22% when compared to 2008, and since 2012 the exports have been decreasing. In 2012 the drop was of approximately

5% (when compared to 2011), and continued decreasing, 22% in 2013, 7% in 2014 and 15% in 2015. The major goods that Brazil exports are soyabeans, iron ore and its concentrates, crude oil from petroleum, crude sugar cane and raw coffee beans (MDIC, 2016).

For the last 15 years, Argentina has been through economic issues. In December 2001 and July 2014 it defaulted on its debt, and until April 2015 its leaders have refused to pay its creditors, including some U.S. hedge funds. The nonpayment has meant the country can't borrow money from foreign governments, so it's essentially had to self-fund its own operations. Any gains could quickly vanish the country's stock market is smaller than others in the region, so shares could tumble if even a small amount of money leaves the market.

The Sao Paulo Stock Exchange (Bovespa) was founded in 1890, and was initially linked to the government, especially to the financial departments of state governments. At that time, stockbrokers were nominated by the government. With the reform of the financial system in the 1960s, the Bovespa ended up becoming a self-regulatory organization, and operates only with the supervision of the Securities and Exchange Commission – CVM (Comissão de Valores Mobiliários). In 2008 the merger of the Brazilian Mercantile & Futures Exchange (BM&F) and the Sao Paulo Stock Exchange (Bovespa) was created. The integrated BM&F Bovespa offers a host of products for trading such as stocks, ETFs, futures, commodities, forwards, options, corporate and government bonds, etc. It also provides indices based on market capitalization, liquidity, industry, corporate governance and sustainability.

Chinese stock markets are fundamentally different than Western markets. They don't send the same signals and don't have the same effects when they rise and crash. Unlike every other major stock market in the world, China's markets are almost completely closed to foreign investors. There's heavy involvement from the government in the market, state-owned companies dominate China's Shanghai Composite (the top ten valued companies are all state-owned). The Communist Party floats only a small percentage of a company's balance sheet on the stock exchange while keeping control of the rest. But slowly, China has been liberalizing its capital markets to foreign investor, given access to it's stock market. In November 17, 2014 global investors were able to purchase shares of companies listed on the Shanghai Stock Exchange. Previously, only a select group of institutional investors that met certain qualifications had access to Shanghai's $2 trillion market.

The S&P 500 is one the most commonly used benchmarks for the overall U.S. stock market, and the most important index to investors. The index includes 500 of the largest (not necessarily 500 largest) companies whose stocks trade on either the NYSE (New York Stock Exchange) or NASDAQ (National Association of Securities Dealers Automated Quotations). These 500 stocks are chosen by market size, liquidity and industry grouping, among other factors. The index is not only used to track the performance of the broad market, it also informs on which hundreds of billions of investors' money is invested in through mutual funds and exchange-traded funds.

3. Literature Review: GARCH – EVT

Since risk measurement methodologies used to estimate the VaR of financial assets assuming that the market behavior was stable, extreme market events required a special approach from risk managers. Among the first researchers who introduced EVT models to estimate extreme risks were Danielsson and De Vries (1997), McNeil (1998) and Longin (2000, 2001). In there research, the authors focused on estimating unconditional (stationary) asset returns. McNeil (1998) and Longin (2000) used EVT estimation models based on limit theorems for block maxima, whereas Danielsson and de Vries (1997) used semi-parametric approach based on Hill-estimator. Since these authors, several researchers have tested real market data. These studies concern several financial assets (stocks, bonds, hedge funds, commodities, among others), different returns distributions (normal, t-student's, distributions from EVT, such as Fréchet, Weibull, Gumbel or Generalized Pareto Distributions - GPD) and also estimate both VaR and ES.

Neftci (2000) used the maximum likelihood approach to fit the Generalized Pareto Distribution (GPD) to extreme changes for a number of foreign exchange rates and U.S. dollar interest rates and uses the resulting estimates to compute the 1% tail probability for distributions. Both the in-sample data and out-of-sample data show that tails estimated with extreme distribution theory perform surprisingly well in capturing the rate of occurrence and the (average) extent of extreme events. These results are encouraging, in that they indicate that the GPD accurate 99% confidence VaR forecast for a portfolio with a (single) linear exposure to any of the exchange or interest rates studied.

The method developed by McNeil and Frey (2000) involved an approach that combined the volatility adjustment by a Generalized Autoregressive Conditional

Heteroskedasticity (GARCH) process and elements of the EVT of five time series of log returns, the S&P 500 index (USA), the DAX index (Germany), the shares of BMW, and the U.S. dollar British pound exchange rate. The conditional GPD based VaR estimates perform well at all quantiles, and significantly better than other approaches to which they compare it at 99 and 99,5% quantiles. They also showed that the GPD of EVT better estimates for Expected Shortfall than the Gaussian model

Silva and Mendes (2003) used the EVT to analyze ten Asian stock indices (such as of China, India, Japan, Indonesia, Korea, Malaysia, Singapore, Philippines, Taiwan and Thailand during the period of 1990 to 1999), testing which type of extreme value and asymptotic distribution best fits into extreme historical market events in Asia. The results showed show the accuracy of EVT of estimating VaR is a more conservative approach to determining capital requirements than traditional/historical methods.

To better understand the financial system of Chile, Fernandez (2003), estimates the Chilean and U.S. stocks returns series with GARCH-type models and compute tails distributions of GARCH innovations by EVT, allowing to analyze conditional quantiles (VaR) comparing it to other alternatives, such as conditional normal, conditional t, and nonparametric quantiles. The result is that the conditional-EVT approach is the best to compute VaR

Gençay et al. (2003) compared the performance of EVT in VaR calculation to other modeling techniques, such as GARCH, variance-covariance and the historical simulation method applied to the Istanbul Stock Exchange Index (ISE-100) 1987 to 2001. The models were classified into two groups. The first group consisted of GARCH (1,1) normally distributed and with student's t-distribution. The second group comprised historical simulation, the Var-Cov approach, adaptable to the GPD and non-adaptive GPD models. The quantile forecasts of GARCH (1,1) proved to be excessively volatile relative to the GPD quantile forecasts. That made the GPD model to be a more robust quantile forecasting tool, being more practical for implementation and presenting a more regular performance for VaR measurements.

Brooks et al. (2005) compared different models based on EVT to determine VaR of three LIFFE (London Financial Futures Exchange's) futures contracts referring to the period from 1991 to 1997. A semi-nonparametric approach was also proposed, where the tail events were modeled using the GPD, and normal market conditions were captured by the empirical distribution function. The VaR estimates from this approach are compared with those of standard nonparametric extreme value tail estimation approaches, with a small sample bias-corrected extreme value approach, and with those calculated from bootstrapping the unconditional density and bootstrapping from a GARCH(1,1) model. The results suggest that, for a holdout sample, the proposed semi-nonparametric extreme value approach produced better results to other methods analyzed, and they also verified consistent results for small sample tail index technique.

Assaf (2009) analyzed four emerging financial markets belonging to the Middle East and North African (MENA) region (Egypt, Jordan, Morocco, and Turkey). These markets presented fatter tails than the normal distribution and therefore introduce the EVT to evaluate daily loss by computing VaR in each market and explore the implications for portfolio diversification and risk management. He found that, in general the VaR estimates based on the tail-index are higher than those based on a normal distribution for all markets. They state that a proper risk assessment should not neglect the tail behavior in these markets, since that may lead to an improper evaluation of market risk.

Singh et al. (2011) apply dynamic EVT (focusing on Peaks Over Threshold - POT method) approach to the to Australian stock market return series for predicting next day VaR. They model VaR in a dynamic two-stage extreme value process with a GARCH (1,1) to forecast one day ahead 1% and 5% VaR estimates. With historical data in a moving window of the last 1000 days log returns for ASX-All ordinaries and S&P-500 indices. They verified that the dynamic-EVT performs better than the other widely used methods of normal GARCH(1,1) and RiskMetric, and has the advantage of reacting to extreme market conditions (such as the Global Financial Crises of 2008), therefore getting better VaR forecasts.

Brooks and Persand (2003) state that there is consensus in the relevant literature that equity return volatility raises more following negative than positive shocks. Pagan and Schwert (1990), Nelson (1991), Campbell and Hentschel (1992), Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993), Henry (1998) and Engle and Lee (1999) are some of the researches that demonstrate the existence of asymmetric effects in stock index returns. Recent studies have been developed using asymmetric GARCH models to evaluate extreme risk:

Alberg et al. (2008) uses various GARCH models to analyze the mean return and conditional variance of Tel Aviv Stock Exchange (TASE) indices. The authors investigate the forecasting performance of GARCH, Exponential GARCH (EGARCH), (Glosten, Jagannathan and Runkle) GARCH (the GJR-GARCH) and Asymmetric Power ARCH (APARCH) models together with the different density functions: normal distribution, student's t-distribution and asymmetric student's t-distribution, and also compare between symmetric and asymmetric distributions using these three different density functions. The results showed that asymmetric GARCH models improve the forecasting performance

Mokni et al. (2009) used the GARCH family models such as, GARCH, Integrated GARCH (IGARCH), and GJR-GARCH (each of which were adjusted based on three residuals distributions: normal, student's-t and skewed student's-t) models to investigate the effects of subprime crisis on the VaR estimation. They separated their sample into two periods: the first covers the stability period (calm period) from 1st of January 2003 to 16th of July 2007 and the second period covers the crisis period (turbulent period) from 17th of July 2007 to 10th of July 2008. They verified that the amount of VaR is different during these two time periods. And they state that, this finding could be explained by the volatility clustering effect. There empirical results showed also that GJR-GARCH model performs better in both sub-sample periods, in comparison with GARCH and IGARCH models. And they finish concluding that student's-t and skewed student's t-distributions are preferred in the stable period while the normal distribution is recommended during the turbulent period.

Bucevska (2013) used the daily returns of the Macedonian Stock Exchange Index (MBI 10), to test the performance of the symmetric GARCH (1,1) and the GARCH-in-mean (GARCH-M) model as well as of the asymmetric EGARCH (1,1) model, the GJR-GARCH model and the APARCH (1,1) model with different residual distributions form 2005 to 2011. The results indicated that the most adequate GARCH family models for estimating and forecasting volatility in the Macedonian stock market are the asymmetric EGARCH model with student's t-distribution, the EGARCH model with normal distribution and the GJR- GARCH model, which are robust with regard to the estimation.

The majority of these studies, using GARCH models to estimate the current volatility of the log returns series, showed the empirical superiority of EVT for VaR and ES estimation. Moreover, recent studies indicate that the use of asymmetric GARCH models improve results in estimating extreme risks.

4. Theoretical Framework

One of the most important tasks of financial institutions is evaluation of exposure to market risks, which arises from variations in prices of equities, commodities, exchange rates and interest rates. The dependence on market risks can be measured by changes in the portfolio value or profits and losses. Regulators and the financial industry advisory committees recommend VaR as a way of risk measuring, which is also recommended in the Basel II accord[1]. Therefore, VaR is used to ensure that the financial institutions can still be in business after a catastrophic event.

According to Jorion (2007), VaR summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence. While VaR can be used by any entity to measure its risk exposure, it is used most often by commercial and investment banks to capture the maximal loss of a financial position from adverse market movements over a specified period. This can then be compared to their available capital and cash reserves to ensure that the losses can be covered without putting the firms at risk.

VaR measurement is widely applied to estimate exposure to market risks. However, many authors claim that VaR has several conceptual problems. Artzner et al. (1999) shows that VaR fails on coherency since it's not sub-additive[2]. Another issue encountered on VaR measurements is that informs nothing about the extent of the losses that could be incurred in the event that the VaR is exceeded. A number of alternative risk measures have been proposed to overcome the problem of lack of sub-additivity in the VaR and/or provide more information about the tail shape (Danielsson, 2011). One method of risk measure that overcomes these weaknesses is

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[1]Since the global financial crisis (2008), trading books at a number of financial organizations around the world began to show significant losses. The magnitude of these losses prompted questions over whether financial institutions had been holding adequate capital reserves, as calculated using risk measures such as VaR. Since then, the Bank for International Settlements' (BIS) Basel Committee has searched for ways to improve the capital positions of financial organizations by changing existing guidelines and requirements for risk measurement. In July 2009, the BIS issued its final copy of the "Revisions to the Basel II market risk framework", which was the fist steps for the Basel III standards. These revisions were developed upon the BIS's Basel II framework published in June of 2006. Hence, the purpose of Basel II was to create standards and regulations on how much capital reserve financial institution must have, to reduce the risks associated with its investing and lending practices. And Basel III seeks to improve the banking sector's ability to deal with financial and economic stress, improve risk management and strengthen the banks' transparency. The focus is to promote greater resilience at the individual bank level in order to reduce the risk of the system wide shocks (BIS, 2015).

[2]Whereby the risk of a combined portfolio cannot be greater than the sum of the risks associated with any possible division of that portfolio.

called Expected Shortfall (ES). Artzner et al. (1999) demonstrate that ES is sub-additive, and it's defined as the average level of losses, given that the VaR is exceeded, from conditional VaR (Alexander, 2008).

On the other hand, one of the most significant criticisms to VaR approach is the common assumption in quantitative financial risk modeling that assets return series present normal distribution, i.e., focus on the central observation or, in other words, on returns under normal conditions. This makes the evaluation inefficient if the data exhibit heavy tails (common in financial data), the risk of high quantiles are underestimated. Investors and risk managers have become more concerned with events occurring under extreme market conditions. To overcome this problem recent studies propose VaR based on the Extreme Value Theory (EVT), since it has the ability to accurately estimate probability and quantile at the extremes of the sample as well as outside it. EVT provides well-established statistical models for the computation of extreme risk measures like the Return Level, Value at Risk and Expected Shortfall. Originally, EVT concepts were applied mainly to the study of natural extreme and rare events, such as floods and earthquakes. However, EVT quickly became popular in the financial literature.

Generally there are two main related ways of identifying extremes in real data. The most traditional models are Block Maxima models based on Generalized Extreme Value (GEV), and a more modern and powerful group of models for Threshold Exceedances, based on Generalized Pareto Distribution (GPD). The method of looking only at observations above a certain threshold and fitting a GPD to these exceedances is called the Peak Over Threshold (POT) method (McNeil, Frey and Embrechts, 2005). Although they are related, each of them treats extreme data in a different manner.

Block Maxima Method (BM) focuses in the largest values (maxima) taken from samples of independent and identically distributed (iid) observations. The asymptotic distribution of a series is modeled and the distribution of the standardized maximum is shown to follow to extreme value distributions of Gumbel, Fréchet or Weibull distributions. The Generalized Extreme Value distribution (GEV) is a standard form of these three distributions, and hence the series is shown to converge to GEV. It has a major defect that it is very wasteful of data, because it only uses periodical maxima and, therefore, requires wide datasets.

The Generalized Pareto Distribution (GPD) gives a good model for the upper tail, providing reliable extrapolation for exceedances over a sufficiently high threshold. This method, which is defined on the excesses, is called Peaks Over Threshold (POT). The choice of threshold is crucial (involves balancing bias and variance), as it defines which part of the data can be considered as extreme, or more formally where the asymptotically justified extreme value models will provide a reliable approximation to the GPD. It is generally considered to be the most useful for practical applications, due to their more efficient use of the (often limited) data on extreme outcomes (McNeil, Frey and Embrechts, 2005).

According to Bhattacharrya and Ritolia (2006), the Block Maxima modeling approach is extensively used in hydrology and other engineering applications, but it's not practically suited for financial time series because of volatility clustering. Peaks Over Threshold utilizes data more efficiently and therefore has become the method of choice in financial applications. Fernandez (2003) states that an additional advantage of POT is that provides VaR and ES estimates that are easy to compute. We focus on the POT method to the losses on the Brazilian, American, Chinese and Argentinian stock indexes.

It is important to mention that some EVT methods assume that the data to be studied are iid (independent and identically distributed), which is not always the case for most financial log returns series, since it presents certain characteristics, such as, changing volatility, clustering, asymmetry, leverage effect and long memory properties. These properties are usually approached by modeling the price process with Autoregressive Conditional Heteroskedastic (ARCH) - type model, originated by Engle (1982) and later extended by Bollerslev (1986) as Generalized Autoregressive Conditional Heteroskedastic (GARCH) - type model. McNeil and Frey (2000) proposed conditional EVT model to estimate the tails of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) residuals, before estimating VaR. They found that this methodology gives better estimates, than methods that ignore the heavy tails of the innovations or the stochastic nature of the volatility.

GARCH models capture volatility clustering and leptokurtosis, but as their distribution is symmetric, they fail to model the leverage effect[3]. It assumes that the positive or negative information have the same impact on the volatility, which show

[3] Black (1976) pioneered the asymmetric volatility study and attributed it to firms' leverage effect.

symmetric effect in the variance equation of the model. To address this problem, many extensions of GARCH have been proposed, such as the Exponential GARCH (E-GARCH) model by Nelson (1991), and the so-called GJR-GARCH model by Glosten, Jagannathan and Runkle (1993). Engle and Lee (1993) later claimed that the volatility could be decomposed into a transitory or short-run and a permanent or long run component, they applied the Component GARCH (C-GARCH) model.

4.1. Asset Returns and Losses

Let financial asset prices be denoted by P_t, where $t \in \mathbb{Z}$, usually refers to a day, but can indicate any frequency (e.g., year, week, hour). The simple net return, is the price variation relation to the previous price:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \tag{4.1.1}$$

The simple gross return is the ratio between current and previous price:

$$\log(1 + R_t) = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{4.1.2}$$

An alternative return measure is continuously compounded return. The gross return can be approximated by the log returns, which is given by:

$$r_t = \log(P_t) - \log(P_{t-1}) \tag{4.1.3}$$

The loss is defined by X_t at day t as the negative of the log return, i.e. $X_t = -r_t$. Negative (positive) log returns of financial asset prices are defined for the left tail of distribution, assuming traders at long position (short position)[4]. And Positive (negative) log returns of financial asset prices are defined for the right tail of distribution considering trader at long position (short position).

Let $(X_t, t \in \mathbb{Z})$ be a strictly stationary time series representing losses on financial asset price. Dynamics of X is given by:

$$X_t = \mu_t + +\varepsilon_t \tag{4.1.4}$$

$$\varepsilon_t = \sigma_t Z_t \tag{4.1.5}$$

where $\mu_t = \varphi_0 + \varphi_1(X_{t-1})$, $|\varphi_1| < 1$, the innovations Z_t are iid continuous random variables with mean zero, unit variance, and comes from a location-scale family distribution, $F_z(z)$[5], and where μ_t and σ_t are measurable with respect to the return

[4] Long position is when the investor buys and holds a traded asset, in this case the risk comes from a drop in the price of the asset. And short position is when the investor borrows a traded asset, it's not owned by seller, in this case the risk comes from a rise in price of the asset.

[5] As McNeil and Frey (2000), instead of assuming $F_z(z)$ to be standard normal, we apply the POT estimation procedure to this distribution of residuals.

process up to time $t-1$[6]. The concept of Efficient Market Hypothesis (EMH) by Fama (1964, 1970), proposes that stock returns themselves do not have predictive power for their future returns and suggests conditional mean dynamics become negligible. Consequently, the linear first order Autoregressive model (or AR(1)) is dropped, and is modified to be GARCH*(m,s)* since the higher order terms turn out to be necessary for correct specifications.

4.2. Value at Risk (VaR)

The VaR measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence level $q \in \{0,1\}$ and time t. The VaR at the confidence level q is given by the smallest number x_q such that the probability that the loss X_{t+1} at time t will fall below x_q (McNeil et al. 2005). Formally:

$$\begin{aligned} VaR_q^t &= \inf\{x_q \epsilon R: P(X_{t+1} \leq x_q) \geq q\} \\ &= \inf\{x_q \epsilon R: P(X_{t+1} > x_q) \leq 1-q\} \end{aligned} \tag{4.2.1}$$

In probabilistic terms, VaR is thus simply a quantile of the loss distribution. Typical values for q are $q = 0.95$ or $q = 0.99$.

VaR is a risk measure that ask the question "How bad can things get?". However, it is often more of interest to know "If things do get bad, how bad can it get?" (Hull, 2012). Expected Shortfall answers this last question.

4.3. Expected Shortfall (ES)

ES is also known as expected tail loss or conditional VaR (CVaR), which is defined as the expected loss given that we have a loss larger than VaR. The ES at level $q \in \{0,1\}$, is the expected value at time t of the loss in the next period X_{t+1}, conditional on the loss exceeding VaR_q^t:

$$ES_q^t = E_t(X_{t+1} | X_{t+1} > VaR_q^t) \tag{4.3.1}$$

The two models considered (VaR and ES) assumes that the log returns are iid, the mean and variance are constants, which may not be realistic in practice, since they are time dependent.

[6] It starts by estimating μ_t and σ_t, usually by means of quasi-maximum likelihood (QML), and applies the classical POT method to the residuals (Brodin and Klueppelberg, 2008).

4.4. Symmetric GARCH *(m,s)* model

4.4.1 The Standard GARCH*(m,s)* model

The GARCH model, proposed by Bollerslev (1986) adds a sum of lagged conditional variances to the definition of the ARCH process. From losses on financial asset price, X_t (equation 4rício.1.4), let $\varepsilon_t = X_t - \mu_t$ be the innovation at time t. Then ε_t follows a GARCH*(m,s)* process if it satisfies the equations:

$$\varepsilon_t = \sigma_t Z_t \tag{4.4.1.1}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{4.4.1.2}$$

where $\omega > 0$ and $\alpha_i \geq 0$ for $i = 1, \dots, m,$ and $\beta_j \geq 0$ for $j = 1, \dots, s.$ If $\sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_j\right) < 1,$ then the process ε_t is covariance stationary. If ε_{t-i} are large in magnitude, ε_t will likely be large in magnitude, which provides a reasonable explanation of volatility clusters.

4.5. Asymmetric GARCH*(m,s)* models

4.5.1 The GJR-GARCH*(m,s)* model

Glosten, Jaganathan and Runkle (1994) introduced an extension of the GARCH model by introducing an indicator variable in the sum of the ARCH-terms (shocks) ε_{t-1}. Its generalized version is given by:

$$\varepsilon_t = \sigma_t Z_t \tag{4.5.1.1}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} (\alpha_i + \gamma_i I_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{4.5.1.2}$$

where $\omega, \alpha_i, \beta_j > 0$ guarantees positivity, $\sum_{i=1}^{m} \alpha_i + c \sum_{i=1}^{m} \gamma_i + \sum_{j=1}^{s} \beta_j < 1$ is necessary for stationarity (where c is the expected values of standardized residuals Z_t below zero[7]), and I_{t-i} is an indicator variable such that:

$$I_{t-i} = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases} \tag{4.5.1.4}$$

If $\varepsilon_{t-1} \geq 0$, $I_{t-i} = 0,$ the effect of a ε_{t-1} shock on σ_t^2 is $\alpha_i \varepsilon_{t-i}^2$. When $\varepsilon_{t-1} < 0,$ $I_{t-i} = 1,$ the effect on σ_t^2 is $(\alpha_i + \gamma_i)\varepsilon_{t-i}^2$. The model uses zero as its threshold to

[7] Effectively the probability of being below zero: $c = \mathrm{E}\left(I_{t-i} {Z_t}^2\right) = \int_{-\infty}^{0} f(Z, 0, 1, \dots)\, dZ$ where f is the standardized conditional density with any additional skew and shape parameters. If the distribution is symmetric $c = 0.5$ (Ghalanos, 2015).

separate the impacts of past shocks. The coefficient γ_i dictates the correlation between volatility and returns, if it's statistically different from zero, the data contain a threshold effect. Observe that if $\gamma > 0$, negative shocks will have larger effects on volatility than positive shocks. The opposite will happen if $\gamma < 0$ (provided that $\alpha_i + \gamma_i \geq 0$).

4.5.2 The Component GARCH*(m,s)* model

In the Component GARCH model, the conditional variance is decomposed into two parts corresponding to transitory and permanent effects. Let q_t represent the permanent (or trend) component in the conditional variance. The variance of this model is given by:

$$\sigma_t^2 = q_t + \sum_{i=1}^{m} \alpha_i(\varepsilon_{t-i}^2 - q_{t-i}) + \sum_{j=1}^{s} \beta_j\left(\sigma_{t-j}^2 - q_{t-j}\right) \qquad (4.5.2.1)$$

$$q_t = \omega + \rho q_{t-1} + \phi\left(\varepsilon_{t-i}^2 - \sigma_{t-j}^2\right) \qquad (4.5.2.2)$$

where $0 < \alpha + \beta < \rho < 1$ and $0 < \phi < \beta$, and ρ is the speed of mean reversion. Typically ρ is between 0.9 and 1, so the q_t converges to ω very slowly. For $\rho = 1$, the long-term volatility process is integrated. The forecasting error term $\varepsilon_{t-1}^2 - \sigma_{t-1}^2$ is the zero-mean and serial uncorrelated, which drives the evolution of the permanent component. The difference between σ_{t-j}^2 and q_{t-j} represents the transitory part of the conditional variance.

4.6. The Extreme Value Theory

The basic objective of any VaR approach is to provide an estimate of the largest expected loss in a given investment position for a given level of confidence and investment period. The focus of interest has been on the analysis of very rare (low probability) risk events, i.e., extreme risk returns, which causes high effects on the economy.

The branch of mathematical statistics that emerged with the study of such problems is called Extreme Value Theory (EVT). In essence, EVT determines the nature of the tail of the distribution without having to make assumptions on the distribution from which the observations are obtained. In this sense, the EVT is instrumental basis for analysis of statistical properties of extreme returns.

As we reject normality for distribution of studied asset return, we need to adequately fit the tail of the return distributions to estimate its risk parameters with EVT. Modeling extremes can be done in two different ways. One way consists of dividing the observation period into non-overlapping periods of equal size and restricts attention to the maximum observation in each period. And the other way is by modeling the largest observations that exceed a certain high threshold, known as The Peak Over Threshold (POT). We use the latter as it uses data more efficiently.

4.6.1 Generalized Extreme Value (GEV) Distribution

Consider a iid sequence of random variables, $Z_1, Z_2, \ldots, Z_n$ whose common cumulative distribution function is F, i.e.

$$F(z) = P\{Z_i \leq z\} \tag{4.6.1.1}$$

Let $M_n = \max(Z_1, Z_2, \ldots, Z_n)$ denote the *nth* sample maximum of the process, then:

$$P\{M_n \leq z\} = F(z)^n \tag{4.6.1.2}$$

This result (4.6.1.2) is of no immediate interest, since the distribution function F is unknown. The distribution of M_n degenerates to a point mass for z upper end-point z_+ $(z < z_+)$[8] of F, as n tends to infinity. This difficulty is avoided by allowing a linear renormalization of the variable M_n. Fisher and Tippett (1928) theorem (which result was latter derived rigorously by Gnedenko, 1943) states that if there exist constants $a_n > 0$ and $b_n \epsilon \mathbb{R}$, such that:

$$P\left\{\frac{M_n - b_n}{a_n} \leq z\right\} = F^n(a_n z + b_n) \to H(z) \text{ as } n \to \infty \tag{4.6.1.3}$$

for some non-degenerate distribution H[9], then H must belong to the type of one of the three so-called standard extreme value distributions:

$$H(z) = \exp\left\{-\exp\left[-\left(\frac{z-b}{a}\right)\right]\right\}, \quad -\infty < z < \infty \tag{4.6.1.4}$$

$$H(z) = \begin{cases} 0, & z \leq 0 \\ \exp\left\{-\left(\dfrac{z-b}{a}\right)^{-\alpha}\right\}, & z > 0 \end{cases} \tag{4.6.1.5}$$

$$H(z) = \begin{cases} \exp\left\{-\left[-\left(\dfrac{z-b}{a}\right)^{\alpha}\right]\right\} & z < 0 \\ 1, & z \geq 0 \end{cases} \tag{4.6.1.6}$$

[8] The point $z_+ = \sup\{z \epsilon \mathbb{R}: F(z) = 1\} \leq \infty$.
[9] If (4.6.1.3) holds for some non-degenerate density function H then F is said to be in the maximum domain of attraction of H, written $F \in \text{MDA}(H)$.

Equation (4.6.1.4) is called the Gumbel type, (4.6.1.5) the Fréchet type and (4.6.1.6) the Weibull type. In (4.6.1.5) and (4.6.1.6), $\alpha > 0$.

The Jenkinson-von Mises theory[10] states that, by taking a reparametrization $\xi = \alpha^{-1}$ one obtains a continuous, unified model, named the Generalized Extreme Value (GEV) distribution:

$$H(z) = \begin{cases} \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} & \text{if } \xi \neq 0 \\ \exp\left\{-\exp\left\{-\left(\frac{z-\mu}{\sigma}\right)\right\}\right\} & \text{if } \xi = 0 \end{cases} \tag{4.6.1.7}$$

defined on the set $\{z \in \mathbb{R}: 1 + \xi\,(z-\mu)/\sigma > 0\}$ where $\mu \in \mathbb{R}$ is a location parameter, $\sigma > 0$ is a scale parameter and $\xi \in \mathbb{R}$ is a shape parameter. The case $\xi = 0$ is interpreted as the limit $\xi \to 0$ and corresponds to the Gumbel distribution, $\xi > 0$ to the Fréchet distribution and $\xi < 0$ to the Weibull distribution.

4.6.2 Generalized Pareto Distribution (GPD)

Consider a sequence of iid observations $Z_1, ,\dots, Z_n$ from an unknown distribution function F. The interest is in the excess losses over a high threshold u. Let z_+ be the upper end-point of a distribution F $(z_+ \leq \infty)$. Then the corresponding distribution function of the excesses over the threshold u denoted as $Y_1, ,\dots, Y_{N_u}$, $N_u =$ card=$\{i : i = 1, \dots, n, Z_i > u\}$ is given by:

$$F_u(y) = P\{Y = Z - u \leq y | Z > u\}, 0 \leq y \leq z_+ - u \tag{4.6.2.1}$$

Were F_u can be written as:

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} \tag{4.6.2.2}$$

Balkema and de Haan (1974) and Pickands (1975) posed that for a large class of underlying distribution function F, the GDP is the limiting distribution for the conditional excess distribution function $F_u(y)$, as the threshold u tends to the right end point, formally:

$$\lim_{u \to z_+} \sup_{0 \leq z < z_+ - u} |F_u(y) - G(y)| = 0 \tag{4.6.2.3}$$

If and only if, F is in the maximum domain of attraction of the Generalized Extreme Value distribution, $H(z)$. The GPD is defined as:

[10] Due to Von Mises (1936) and Jenkinson (1955).

$$G(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\sigma}\right) & \text{if } \xi = 0 \end{cases} \quad (4.6.2.4)$$

defined on $\{y \in \mathbb{R}: y > 0 \text{ and } 1 + (\xi\, y/\sigma) > 0\}$. The parameters μ, σ and ξ are uniquely determined by those of the associated GEV. The duality between the GEV and the Generalized Pareto families means that the shape parameter ξ is dominant in determining the qualitative behavior of the GPD, just as for the GEV distribution (Coles, 2001). If Z is defined as $Z = u + y$, the GPD can also be expressed as a function of Z, i.e., $G(Z) = 1 - \left(1 + \frac{\xi}{\sigma}(Z - u)\right)^{-\frac{1}{\xi}}$.

If $\xi > 0$ indicates a distribution with a thick tail (decrease polynomially), $\xi = 0$ a tail with medium thickness (decrease exponentially) and if $\xi < 0$ tail with finite endpoint. The advantage of this model is the more efficient use of data and the disadvantage is how to choose threshold u is not so evident. The method analyzing observation above a determined threshold and fitting a GPD to this exceedance is called the Peak Over Threshold (POT) method.

4.6.3 Extreme Value Approach to Value at Risk and Expected Shortfall

Once the distribution of excesses over a threshold is estimated, an approximation of the unknown original distribution (log return loss distribution, that generates the extreme observations) and an estimation of the p-quantile from it can be used to estimate the extreme VaR and ES. Let $F^{\leftarrow}$ denote the generalized inverse of distribution F:

$$F^{\leftarrow}(q) = \inf\{z \in \mathbb{R}: F(z) \geq q\}, \qquad 0 < q < 1 \quad (4.6.3.1)$$

which is called the quantile function of F. The q-quantile of F is $z_q = F^{\leftarrow}(q)$. Using the distribution of excesses beyond u in equation (5.6.2.2):

$$F_u(y) = \frac{F(u + y) - F(u)}{1 - F(u)} \quad (4.6.3.2)$$

If n is the total observations, N_u the number of observations above a threshold u, $F(u)$ (proportion of samples below the threshold u) can be estimated from the empirical distribution of observations:

$$\widehat{F(u)} = 1 - \frac{1}{n}\sum_{i=1}^{n} I_{\{Z_i > u\}} = 1 - \frac{N_u}{n} \quad (4.6.3.4)$$

and considering (4.6.2.3), $F_u(y)$ can be replaced by the GPD, we get a estimator for tail probabilities[11] (Smith, 1987):

$$F(\widehat{u+y}) = 1 - \frac{N_u}{n}\left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(z-u)\right)^{-\frac{1}{\xi}} \quad (4.6.3.5)$$

The POT estimator of z_q (the VaR_q) is obtained by inverting the tail distribution (quantile of the log return loss distribution) given in (4.6.3.5):

$$\hat{z}_q = \widehat{VaR_q^t}(Z) = u + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-q)\right)^{-\hat{\xi}} - 1\right] \quad (4.6.3.6)$$

Similarly, ES can be estimated, provided that log returns have finite expectations, assuming that $\xi < 1$, and we obtain:

$$\widehat{ES_q^t}(Z) = q + \frac{\hat{\sigma} + \hat{\xi}\left(\widehat{VaR_q^t}(Z) - u\right)}{1-\hat{\xi}} = \frac{\widehat{VaR_q^t}(Z)}{1-\hat{\xi}} + \frac{\hat{\sigma} + \hat{\xi}u}{1-\hat{\xi}} \quad (4.6.3.7)$$

5. Methodology

5.1. EVT, VaR and ES – A dynamic approach

By use of GARCH models to forecast the estimates of conditional volatility, the model provides dynamic one-day ahead forecasts of VaR and ES for the financial time series. As in section 4.1, one can define a model for the losses X_t as:

$$X_t = \mu_t + \varepsilon_t \quad (5.1.4)$$

$$\varepsilon_t = \sigma_t Z_t \quad (5.1.5)$$

with σ_t (volatility) given by the standard GARCH (4.4.1.2), GJR-GARGH (4.5.1.2), C-GARCH (4.5.2.1). Again, μ_t is the expected return on day t and Z_t gives the noise distribution $F(z)$, which we will assume either has a normal distribution, a student-t distribution, generalized error distribution or their respective skewed forms.

The dynamic risk modeling (using EVT), models the conditional return distribution (conditioned on the historical data) to forecast the loss over the next $t \geq 1$ days. If we follow the GARCH models, the one-day forecast of VaR is calculated as:

[11] If $z = u + y$, the GPD can also be expressed as a function of z, i.e., $G(z) = (1 - \xi(z-u)/\sigma)^{-\frac{1}{\xi}}$.

$$VaR_q^t(X_{t+1}) = \mu_{t+1} + \sigma_{t+1}.VaR_q^t(Z) \tag{5.1.3}$$

$$ES_q^t(X_{t+1}) = \mu_{t+1} + \sigma_{t+1}.ES_q^t(Z) \tag{5.1.4}$$

5.2. Exploratory Data Analysis

The threshold in the POT models are generally set equal to the $(n-k)$th order statistic, i.e., set $u = Z_{n-k,n}$. This allows consideration of exactly k exceedances. The problem, however, is the choice of k. A threshold needs to achieve balance between bias and variance. If a threshold is chosen very low (the larger the value of k) the more observations will be used to construct parameter and quantile estimates. The estimates will consequently have lower variance. However, the GPD may not be a good fit to the excesses over the threshold and consequently there will be a bias in the estimates. Conversely, If the threshold is chosen very high (smaller value of k), then there are not enough exceedances over the threshold to obtain good estimates of the extreme value parameters, and consequently, the variances of the estimators are high.

5.2.1 Quantile-Quantile (QQ) Plots

The QQ-plot is a graphical technique, which allows comparing the quantiles of the empirical distribution to those of a reference distribution.

Let $Z_1, \dots, Z_n$ be an iid sequence of random variables from a common population with unknown distribution F, and define $\hat{F}$ as its estimate. Let $Z_{n,n} \leq \cdots \leq Z_{1,n}$ denote the ordered sample. For any one of the $Z_{k,n}$, exactly k of the n observations have a value less than or equal to $Z_{k,n}$, so an empirical estimate of the probability of an observation being less than or equal to $Z_{k,n}$ is $\tilde{F}(Z_{k,n}) = n - k + 1/(n+1)$. From a population with estimated distribution function $\hat{F}$, the graph of quantiles (QQ – plots) is defined by the set of points (Coles, 2001):

$$\left\{\left(\hat{F}^{-1}\left(\frac{n-k+1}{n+1}\right), Z_{k,n}\right): \quad k = 1, \dots, n\right\} \tag{5.2.1.1}$$

If $\hat{F}$ is a reasonable estimate of F, the plot should be roughly a straight line of unit slope through the origin.

5.2.2 Mean Excess Function (MEF)

The Mean excess function represents the conditional mean of the exceedance size over threshold (given that an exceedance occurred). The first approach for threshold selection utilizes the empirical mean excess function. It also checks if it's reasonable to assume that the underlying distribution falls in the Fréchet domain.

Based on linearity of the mean excess function $e(u)$, suppose Z has a GPD with parameters $\xi < 1$ and β. Then for a given threshold $u < z_+$, MEF is defined as:

$$e(u) = E(Z - u \mid Z > u) = \frac{\sigma + \xi u}{1 - \xi} \qquad \sigma + \xi u > 0 \qquad (5.2.2.1)$$

hence $e(u)$ is linear. A graphical test for tail behavior can be based on the empirical mean excess function of a given sample $Z_1, \ldots, Z_n$ given by (Embrechts et al, 1997):

$$\hat{e}_n(u) = \frac{\sum_{i=1}^{n}(Z_i - u) I_{Z_i > u}}{\sum_{i=1}^{n} I_{Z_i > u}} \qquad u > 0 \qquad (5.2.2.2)$$

where I is an indicator function. The set of points $\{Z_i, \hat{e}_n(Z_i)\}$ creates the Mean Excess plot. It is an estimate of the mean excess function that describes the expected overshoot of a threshold once an exceedance occurs. If the empirical MEF is a positively sloped plot, indicates that the data follows the GPD with a positive shape parameter ξ (heavy tailed). On the other hand, exponentially distributed data would show a horizontal MEF (ξ is near zero) while short tailed data would have a negatively sloped line ($\xi < 0$).

5.2.3 Hill Plots

The Hill (1975) estimator is a classic tail index estimator for the Pareto type distribution ($\xi > 0$). Define the ordered sample $Z_{n,n} \leq \cdots \leq Z_{1,n}$. Hill estimate is calculated as:

$$\hat{\xi} = H_{k,n} = \frac{1}{k} \sum_{i=j}^{k} \ln \frac{Z_{j,n}}{Z_{k,n}} \qquad (5.2.3.1)$$

where k is the number of exceedence above the threshold.

Hill plots, which is based on Hill Estimator, are another way to determine the threshold for GPDs. The Hill plot involves plotting the Hill estimators against k, i.e.,

$$\{(k, H_{k,n}) : k = 2, \ldots, n\} \qquad (5.2.3.2)$$

A suitable threshold may be chosen based on the criterion of stability of the estimated shape parameter ξ. Stability would imply a relatively flat part of the graph, i.e., k is chosen in a region where the plot seems constant.

5.3. Backtesting Value at Risk

The principal of backtesting is the comparison of actual trading results with model-generated risk measures. Backtesting is a formal statistical framework that consists of verifying if actual losses are in line with projected losses. This involves systematically comparing the history of VaR forecast with their associated portfolio returns (Jorion, 2007). According to Daníelson (2011), backtesting is a procedure that is used to compare the various risk models. It aims to take ex-ante VaR forecasts from a specific model and compare them with ex-post realized return (i.e., historical observations).

We measure accuracy of our risk models using the Proportion of Failure test, the Kupiec (1995) test for unconditional coverage and Christoffersen (1998) test for conditional coverage and Bayes Information Criterion (BIC). The Kupiec (1995) unconditional coverage test, fails to detect violations of independence property of an accurate VaR measure. An accurate VaR model must exhibit both the unconditional coverage and independence property. Both properties are jointly tested based on Christoffersen's (1998) conditional coverage test.

The Proportion of Failures test verifies the proportion of times in which the estimated VaR is exceeded in certain sample. The main focus is on a particular transformation of the reported VaR and realized losses. Let $N = \sum_{t=1}^{T} I_{t+1}$ be the number days over a sample size T, in which the loss on a portfolio was higher than the respective VaR estimation. Denoting the losses on the portfolio over a fixed time interval, i.e. daily, as X_{t+1} then define the "hit" functions as follows:

$$I_{t+1} = \begin{cases} 1 & \text{if } X_{t+1} < VaR_{t+1} \\ 0 & \text{if } X_{t+1} \geq VaR_{t+1} \end{cases} \tag{5.3.1}$$

so that the hit function sequence, accounts the history of whether or not a loss in excess of the reported VaR has been realized. N is the number of observed exceptions and $p = N/T$ is the proportion of failures. The purpose of this test is to examine whether the failure rate p is statistically equal to the expected one.

5.3.1 Unconditional Coverage

According to Kupiec (1995) the probability of observing N violations over a sample size T is modeled by a binomial distribution with a probability of occurrence equaling p. The null hypothesis is $H_0 = \frac{N}{T} = p$ and can be verified through a Likelihood Ratio (LR) test of the form:

$$LR_{uc} = 2ln\left(\frac{\left(\frac{N}{T}\right)^N \left(1 - \frac{N}{T}\right)^{T-N}}{p^N(1-p)^{T-N}}\right) \tag{5.3.1.1}$$

and is asymptotically (under H_0) distributed as a chi-squared with one degree of freedom, $\chi^2(1)$.

5.3.2 Conditional Coverage

The unconditional coverage test does not give any information about the temporal dependence of violations, and the Kupiec (1995) test ignores conditioning coverage, since violations could cluster over time, which should also invalidate a VaR model. Christoffersen (1998) points out that the problem of determining the accuracy of a VaR model can be reduced to the problem of determining whether the hit sequence I_{t+1}, satisfies unconditional coverage and independence property. The previous LR test given by (5.3.1.1) to specify the hit sequence is extended by a test statistic to verify independence over time.

The proposed test statistic is based on the mentioned hit sequence I_t, and on T_{ij} that is defined as the number of days in which a state j occurs, while state i occurred the previous day, with $i, j \in \{0,1\}$. It is also assumes that the hit sequence follows a discrete-time Markov chain with transition probability matrix:

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \tag{5.3.2.1}$$

where π_{ij} is the probability that $I_{t+1} = j$ (observing a violation in one day) conditional on $I_t = i$ the previous day. The null hypothesis that the hit sequence is independent is $\pi_{01} = \frac{T_{01}}{T_{00}+T_{11}} = \pi_{11} = \frac{T_{11}}{T_{00}+T_{11}} = \pi = \frac{T_{01}T_{11}}{T}$. With T observations, the LR function of this process is given by:

$$LR_{ind} = 2ln\left(\frac{(1-\pi_{00})^{T_{00}}{\pi_{01}}^{T_{01}}(1-\pi_{11})^{T_{10}}\pi_{11}^{T_{11}}}{(1-\pi)^{(T_{00}+T_{01})}\pi^{(T_{01}+T_{11})}}\right) \tag{5.3.2.2}$$

6. Results

6.1. Data and Descriptive Statistics

Table 1: List of data sets tested.

Acronym	Index Name	Country	Observations (n)
IBOV	Ibovespa Brasil São Paulo Stock Exchange Index	Brazil	2027
SPX	S&P 500 index	USA	2108
SHCOMP	Shanghai Stock Exchange Composite Index	China	2050
MERVAL	Buenos Aires Stock Exchange Merval Index	Argentina	2040

Source: Elaborated by authors.

The data are daily closing prices of stock exchange indexes of four countries[12], for a period of ten years (from January 2nd 2004 to September 8th 2014). All the prices are in USD. Table 2 presents market indicators for 2014. For market capitalization as indicator of market size, U.S. is by far the largest market. China is also large when compared to Brazil and Argentina. The number of listed companies provides information of the choice of firms available to an investor. In this sense, U.S. is the market with most listed companies (505). However, combining the value of market capitalization of each stock exchange with the number of listed companies will provide the average market value for listed companies. In that case, China has the highest average market value of listed companies, about 120.09 billion dollars, followed by U.S. with 46.19 billion, Brazil with 13.18 billion, and the lowest average market value is Argentina with 4.62 billion dollars. The turnover ratio indicates the market liquidity, in which, China stands to be the more liquid and active market (199.15%), then comes U.S. (148.03%) and Brazil (76.33%), while Argentina showed a very low ratio (5.86%). But, the value of stocks traded in 2014 was more than three times higher in the U.S.'s financial market than China's market. Argentina presented a very low market trade value, about 3.52 million dollars[13].

Table 2: Market Indicators, 2014.

Stock Index	Market Capitalization (Billions .$)	No. of Listed Companies	Average Market Value (Billions US$)	Turnover Ratio	Stock Traded (Millions US$)
IBOV	843.89	64	13.18	76.33%	644.17
SPX	23,330.60	505	46.19	148.03%	38,976.64
SHCOMP	6,004.95	50	120.09	199.15%	11,959.33
MERVAL	60.14	13	4.62	5.86%	3.52

Source: http://databank.worldbank.org.

[12] Data were downloaded from https://economatica.com. For the development of analyses we used R statistical software.
[13] Argentina is considered a frontier market (or pre-emerging market), i.e., less advanced capital markets, with lower market capitalization and poor liquidity, than emerging markets.

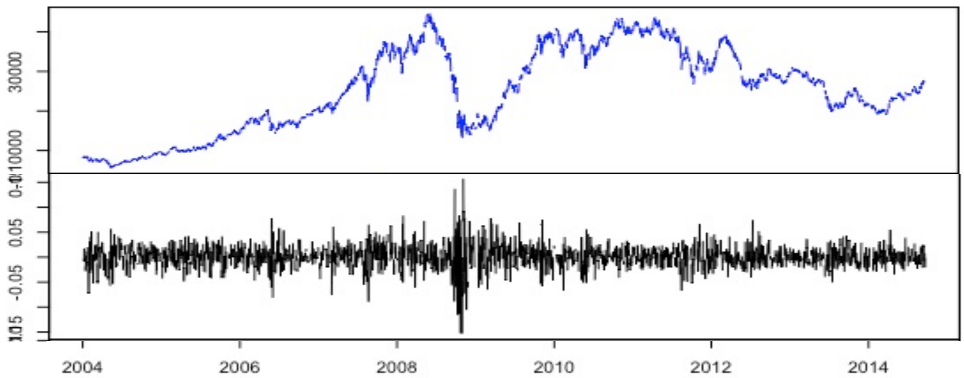
Daily prices and respective return series for countries under focus are plotted in Figure 1. The Daily prices demonstrate that there is in upward trend form 2004 until late 2007 in all markets. In late 2008 there is the subprime crisis. Among the indexes, IBOV was the one that presented strongest recovery right after breakdown, but since 2012 the Brazilian stock index is in decline. This full recovery was also presented in the GPD (7.5% growth), after the Brazilian government implemented the New Economic Matrix[14]. Since 2010 the Shanghai Stock Index is in slow rate decline. On the other hand, S&P 500 index shows slow, but constant recovery. The Argentine Index recovered well from the crisis until late 2011, and then suffered a depression until 2013, which since shows a trend of growth. This growth was due the investor's consideration of potential change in presidential elections of October 2015, from which Cristina Kirchner is constitutionally barred. Contenders for her post have vowed to work toward exiting default and to adopt policies aimed at righting the economy.

The returns series shows that the period of higher volatility for all four indexes was at the end 2008. And continued to be a problem until 2010. Argentina and Brazil have returns around (-0.15, 0.15) whereas for China and U.S. was over (-0.10, 0.10). The plot of the return series reveals the presence of volatility clusters, supporting the existence of heteroskedasticity.

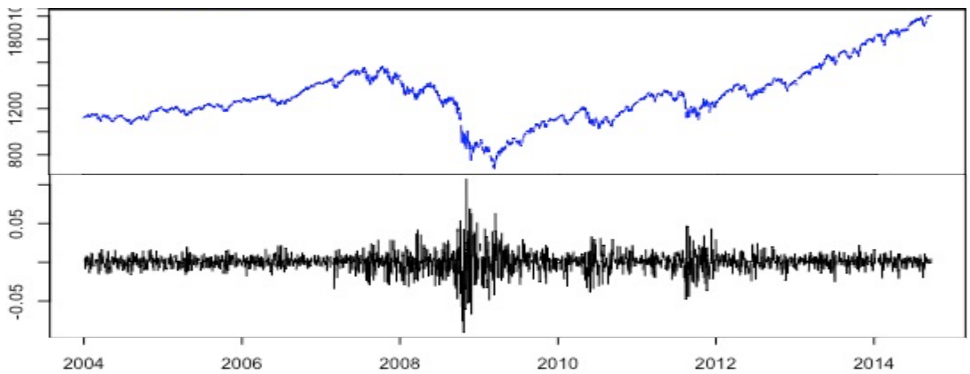
[14]This policy was based on five pillars: expansionary fiscal policy, low interest rates, cheap credit provided by state banks, undervalued exchange rate and increase import tariffs to stimulate the domestic industry.

Figure 1: Daily prices and return series (data from January 2nd 2004 to September 8th 2014).

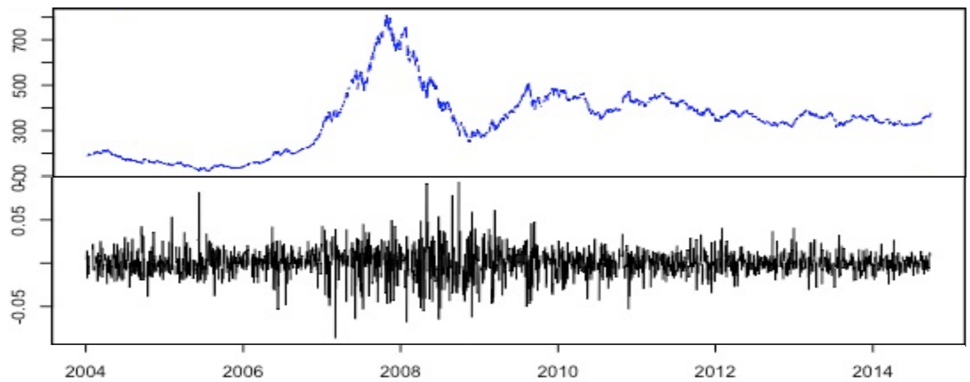
IBOV



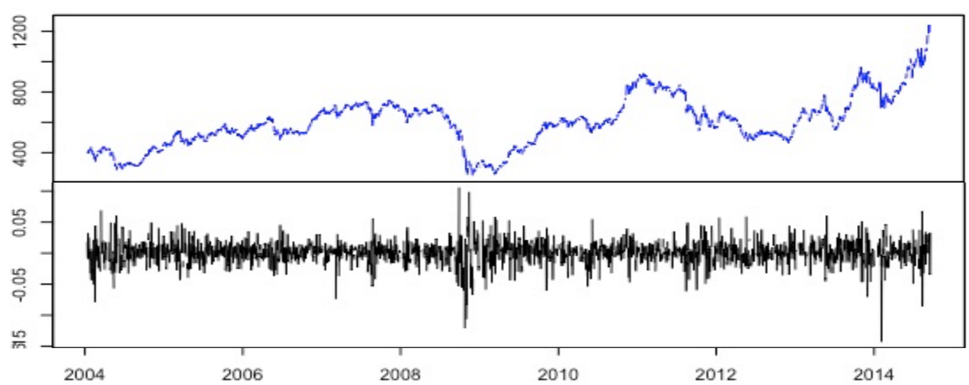
SPX



SHCOMP



MERVAL



Analyzing Table 3, the Augmented Dickey-Fuller and Phillips-Perron unit test reject the null hypothesis of unit root in the series, moreover the KPPS test (trend and level) does not reject the null hypothesis for stationarity, indicating that the series can overall be assumed to be stationary. Although the series are stationary, they do not follow a normal distribution, as indicated by the large excess kurtosis and negative skewness, suggesting that the return distributions are leptokurtic relative to normal and are fat-tailed. This indicates that the distributions of these stock exchanges tend to contain extreme values. The Jarque-Bera test for normality was rejected for all series. The Ljung-Box statistics indicate presence of serial correlation, and hence are not iid. Additionally, the Engle‘s ARCH-test confirms the presence of ARCH effects.

Table 3: Descriptive Statistics of the returns.

Statistics	IBOV	SPX	SHCOMP	MERVAL
Mean	0.0008361	0.0003725	0.0001146	0.0007544
Standard Deviation	0.02316	0.01212	0.01577	0.01892
Minimum	-0.1526	-0.0903	-0.0867	-0.1426
Maximum	0.1573	0.1079	0.0940	0.1056
Skewness	-0.2297	-0.1449	-0.0223	-0.5671
Ex. Kurtosis	4.925	9.200	3.766	5.201
Aug. Dickey-Fuller	-12.765 lag = 12	-14.323 lag = 12	-13.325 lag = 12	-14.322 lag = 12
	(< 0.01)	(< 0.01)	(< 0.01)	(< 0.01)
Phillips-Perron	-45.792 lag = 8	-50.613 lag = 8	-49.732 lag = 8	-43.069 lag = 8
	(0.01)	(0.01)	(0.01)	(0.01)
KPSS (trend)	0.036 lag = 10	0.024 lag = 10	0.080 lag = 10	0.054 lag = 10
	(>0.1)	(>0.1)	(>0.1)	(>0.1)
KPSS (level)	0.252 lag = 10	0.119lag = 10	0.133 lag = 10	0.073 lag = 10
	(>0.1)	(>0.1)	(>0.1)	(>0.1)
Jarque Bera test	2073.3 ($< 2.2 \times 10^{-16}$)	7461.2 ($< 2.2 \times 10^{-16}$)	1215.8 ($< 2.2 \times 10^{-16}$)	2415.7 ($< 2.2 \times 10^{-16}$)
Ljung-Box	140.1 ($< 2 \times 10^{-16}$)	131 ($< 2 \times 10^{-16}$)	58.45 (9.36×10^{-10})	69.81 (5.36×10^{-12})
ARCH-LM test	715.7 ($< 2 \times 10^{-16}$)	675.5 ($< 2 \times 10^{-16}$)	138.8 ($< 2 \times 10^{-16}$)	205.6 ($< 2 \times 10^{-16}$)
Analysis of Squared Returns				
Ljung-Box	2008 ($< 2 \times 10^{-16}$)	1451 ($< 2 \times 10^{-16}$)	210.3 ($< 2 \times 10^{-16}$)	465.2 ($< 2 \times 10^{-16}$)
ARCH-LM test	722.3 ($< 2 \times 10^{-16}$)	444.4 ($< 2 \times 10^{-16}$)	138.83 (0.503)	68.49 (6.14^{-10})

Source: Elaborated by authores.
p-value are in parentheses;
Augmented Dickey-Fuller null hypothesis: has unit root;
Phillipe-Perron null hypothesis: has unit root;
KPSS null hypothesis: is stationary;
Jarque Bera null hypothesis: normality;
Ljung-Box null hypothesis: no serial correlation;
ARCH-LM null hypothesis: serially independent.

Skewness statistics shows the lack of symmetry in the distributions of especially Argentina and Brazil. The negative skewness is to be expected for an index of share prices, since extreme negative returns are more likely than extreme positive returns. According to mean realized returns, Brazil and Argentina yield significantly greater returns than China and U.S. Brazil and Argentina exhibits the largest standard deviation (0.02316) and (0.01892) respectively, followed by China (0.01577) and U.S. (0.01212). This shows that Brazil and Argentina stock markets undergoes higher fluctuations from the mean return, corroborated by the difference between the maximum and the minimum returns, which are respectively (0.31) and (0.25).

The plot (Figure 2) of the ACF indicates a nonexistent MA (moving Average) order for the returns of IBOV index, while the plot of the PACF indicates a low AR (Autoregressive) order. However, in Figure 3, the ACF and the PACF of the squared returns are highly significant for all lags and decay slowly, which signals persistence in variance. Similar results are confirmed for the other markets as seen in Appendix A. This test confirms that there is autocorrelation in the second moment. We filter the returns series using the GARCH models to remove the autocorrelation and to capture the conditional heteroskedasticity.

In summary, the log returns series demonstrates the defining characteristics of the financial series, such as volatility clustering effect, heavy tailed and exhibiting excess kurtosis distributions. These findings support the need for GARCH model to filter the data series and then to apply the EVT.

Figure 2: Autocorrelation and Partial Autocorrelation of returns of IBOV index.

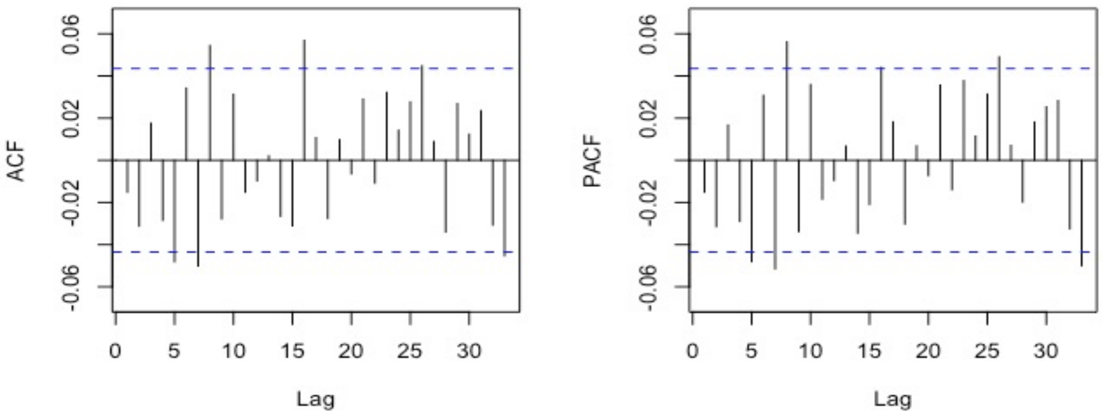
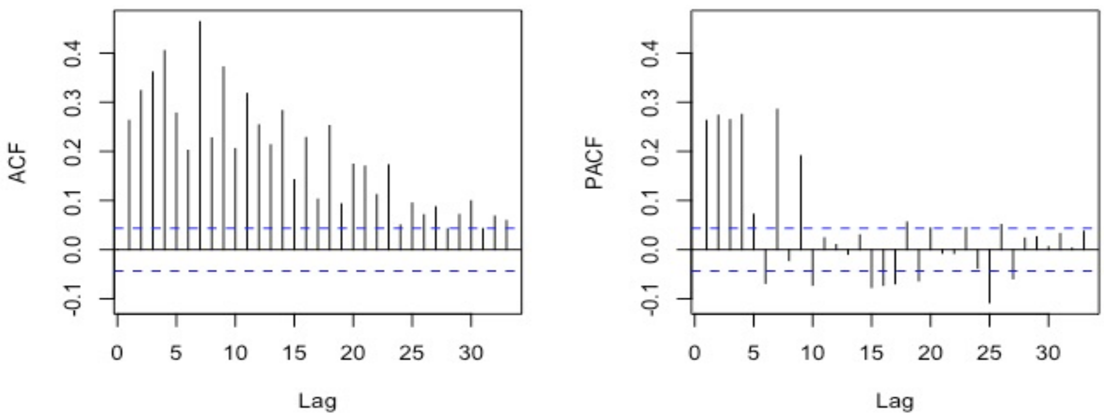


Figure 3: Autocorrelation and Partial Autocorrelation of squared returns of IBOV index.



6.2. GARCH models

In order to compare the adjustment of AR(1)-GARCH (m, s) specifications, we assumed m and $s = 1, 2$, and used for estimation the Normal Distribution (norm), Student Distribution (std) and the Generalized Error Distribution (ged). We also used the Skewed Normal (snorm), Skewed Student (sstd) and the Skewed Generalized Error Distribution (sged). The Generalized Error and Student distributions and their skewed versions have additional shape parameters, which are changed in the estimation. First we report the ranking of these models based on Bayesian Information Criteria (BIC), this method penalizes models with more parameters, and therefore the models will be more parsimonious. After having previously selected the top GARCH models (GARCH, GJR-GARCH C-GARCH with respective distribution), we verified the parameters that are not significant and re-estimated the model without those parameters. Then, the models where adjusted considering the Bayes Information Criteria (once again), the proportion of failures and the unconditional and a conditional test of coverage developed by Kupiec and Christoffersen.

The tests confirms that estimation of AR(1) for mean equation does not enhance accuracy for VaR results, therefore we present the 12 best adjusted models in Table 4 and 5 at 95% confidence level ($\alpha = 5\%$). The skewed distribution version, which admits asymmetric effects (occurrence of extreme movements), improves all 3 GARCH models for all indexes. For Brazil and U.S. the best results were the GJR-GARCH(1,1) model with Skewed Generalized Error Distribution (sged). The Brazilian market presented proportion of failures of 4.49% with significant p-values for Kupiec and Christoffersen tests respectively 33.4% and 19.6%. For the American index the proportion of failures nearest to α, is 5.91% while the Kupiec and Christoffersen tests are statistically significant for α at 5% presenting p-value of 9.2% and 8.9% respectively. The Chines stock market presented the Standard GARCH(1,1) with Skewed Generalized Error Distribution (sged), with p-value of 46.8% for Kupiec test and 6.9% for Christoffersen test and proportion of failures of 4.97%. This result for Chinas' stock market must be because it suffers low impact from external conditions since it's slowly opening for foreign investors and most of the market is state-owned. The best adjusted model for the index of Argentina is the GJR-GARCH(1,1) with the Skewed Normal Distribution (snorm). The proportion of failures is 5.67% while the p-value for Kupiec (22.2%) and Christoffersen (45%) are significant at 95% confidence level.

Table 4: Fit diagnostic of the GARCH models for the IBOV, SPX stock markets at 5% significance level.

			IBOV			
GARCH Models	**BIC**	**Parameters equal to Zero**	**BIC after adjustment**	**Proportion of failures**	**Kupiec**	**Christoffersen**
Standard GARCH						
GARCH(1,1) norm	-4.9196	None		0.0553	0.937 (0.333)	1.952 (0.377)
GARCH(1,1) std	-4.9251	None		0.0541	0.558 (0.455)	0.701 (0.704)
GARCH(1,1) ged	-4.9251	None		0.0541	0.558 (0.455)	1.386 (0.5)
GARCH(1,1) snorm	-4.9185	None		0.0535	0.404 (0.525)	1.146 (0.564)
GARCH(1,1) sstd	-4.9235	None		0.0541	0.558 (0.455)	1.386 (0.5)
GARCH(1,1) sged	**-4.9238**	**None**		**0.0510**	**0.035 (0.852)**	**0.477 (0.788)**
GJR-GARCH						
GJR-GARCH(1,1) norm	-4.9306	$\mu = 0$	-4.9337	0.0473	0.249 (0.618)	3.148 (0.207)
GJR-GARCH(1,1) std	-4.9344	$\mu = 0$	-4.9369	0.0467	0.378 (0.539)	3.13 (0.209)
GJR-GARCH(1,1) ged	-4.9341	$\mu = 0$	-4.9367	0.0461	0.535 (0.464)	3.143 (0.208)
GJR-GARCH(1,1) snorm	-4.9300	$\mu = 0$	-4.9332	0.0455	0.72 (0.396)	3.188 (0.203)
GJR-GARCH(1,1) sstd	-4.9333	$\mu = 0$	-4.9364	0.0455	0.72 (0.396)	3.188 (0.203)
GJR-GARCH(1,1) sged	***-4.9334***	$\boldsymbol{\mu = 0}$	***-4.9365***	***0.0449***	***0.933 (0.334)***	***3.264 (0.196)***
C-GARCH						
C-GARCH(1,1) norm	**-4.9131**	$\mu = 0$	**-4.9139**	**0.0504**	**0.005 (0.941)**	**0.01 (0.995)**
C-GARCH(1,1) std	-4.9182	None		0.0608	3.781 (0.052)	3.781 (0.151)
C-GARCH(1,1) ged	-4.9184	None		0.0596	2.993 (0.084)	3.119 (0.21)
C-GARCH(1,1) snorm	-4.9122	None		0.0596	2.993 (0.084)	3.119 (0.21)
C-GARCH(1,1) sstd	-4.9167	None		0.0590	2.632 (0.105)	2.724 (0.256)
C-GARCH(1,1) sged	-4.9171	None		0.0559	1.162 (0.281)	1.164 (0.559)
			SPX			
GARCH Models	**BIC**	**Parameters equal to Zero**	**BIC after adjustment**	**Proportion of failures**	**Kupiec**	**Christoffersen**
Standard GARCH						
GARCH(2,1) norm	**-6.4895**	$\boldsymbol{\alpha_1 = 0}$	**-6.4928**	**0.0644**	**6.864 (0.009)**	**7.413 (0.025)**
GARCH(2,1) std	-6.5201	$\omega = 0; \alpha_1 = 0$	-6.5070	0.0796	26.959 (0)	27.972 (0)
GARCH(2,1) ged	-6.5280	$\omega = 0; \alpha_1 = 0$	-6.5144	0.0779	24.049 (0)	26.24 (0)
GARCH(2,1) snorm	-6.5022	$\omega = 0; \alpha_1 = 0$	-6.482	0.0697	12.464 (0)	15.155 (0.001)
GARCH(2,1) sstd	-6.5269	$\omega = 0; \alpha_1 = 0$	-6.5154	0.0720	15.428 (0)	15.592 (0)
GARCH(2,1) sged	-6.5373	$\omega = 0; \alpha_1 = 0$	-6.526	0.0673	9.788 (0.002)	10.011 (0.007)
GJR_GARCH						
GJR-GARCH(1,1) norm	-6.5059	$\mu = 0; \omega = 0$	-6.4738	0.0738	17.836 (0)	17.896 (0)
GJR-GARCH(1,1) std	-6.5321	$\mu = 0; \omega = 0$	-6.5107	0.0714	14.661 (0)	14.671 (0.001)
GJR-GARCH(1,1) ged	-6.5384	$\mu = 0; \omega = 0$	-6.5135	0.0708	13.91 (0)	13.956 (0.001)
GJR-GARCH(1,1) snorm	-6.5212	$\mu = 0; \omega = 0$	-6.4959	0.0644	6.864 (0.009)	7.065 (0.029)
GJR-GARCH(1,1) sstd	-6.5432	$\mu = 0; \omega = 0$	-6.5271	0.0644	6.864 (0.009)	8.654 (0.013)
GJR-GARCH(1,1) sged	***-6.5525***	$\boldsymbol{\mu = 0; \omega = 0}$	***-6.5370***	***0.0591***	***2.841 (0.092)***	***4.848 (0.089)***
C-GARCH						
C-GARCH(2,1) norm	-6.4849	$\omega = 0; \alpha_1 = 0$	-6.4572	0.0685	11.089 (0.001)	5.991 (0.002)
C-GARCH(2,1) std	-6.5140	$\omega = 0; \alpha_1 = 0$	-6.5063	0.0732	17.016 (0)	17.428 (0)
C-GARCH(2,1) ged	-6.5222	$\omega = 0; \alpha_1 = 0$	-6.514	0.0738	17.836 (0)	20.288 (0)
C-GARCH(2,1) snorm	-6.4974	$\omega = 0; \alpha_1 = 0$	-6.4841	0.0644	6.864 (0.009)	6.994 (0.03)
C-GARCH(2,1) sstd	**-6.5206**	$\boldsymbol{\alpha_1 = 0}$	**-6.5243**	**0.0644**	**6.864 (0.009)**	**6.866 (0.032)**
C-GARCH(2,1) sged	-6.5312	$\alpha_1 = 0$	-6.5349	0.0632	5.83 (0.016)	5.95 (0.051)

Source: Elaborated by authors.
p-value are in parentheses;
Kupiec null hypotheses: no serial correlation;
Christoffersen null hypotheses: serially independent.

Table 5: Fit diagnostic of the GARCH models for the SHCOMP, MERVAL stock markets at 5% significance level.

SHCOMP						
GARCH Models	**BIC**	**Parameters equal to Zero**	**BIC after adjustment**	**Proportion of failures**	**Kupiec**	**Christoffersen**
Standard GARCH						
GARCH(1,1) norm	-5.6768	$\mu = 0;\ \omega = 0$	-5.6777	0.0582	2.214 (0.137)	5.571 (0.062)
GARCH(1,1) std	-5.7366	$\mu = 0;\ \omega = 0$	-5.7391	0.0618	4.527 (0.033)	8.945 (0.011)
GARCH(1,1) ged	-5.7392	$\mu = 0;\ \omega = 0$	-5.7420	0.0552	0.893 (0.345)	3.471 (0.176)
GARCH(1,1) snorm	-5.6732	$\mu = 0;\ \omega = 0$	-5.6740	0.0539	0.526 (0.468)	5.343 (0.069)
GARCH(1,1) sstd	-5.7339	$\mu = 0;\ \omega = 0$	-5.7365	0.0600	3.274 (0.07)	7.145 (0.028)
GARCH(1,1) sged	***-5.7363***	$\boldsymbol{\mu = 0;\ \omega = 0}$	***-5.7396***	***0.0497***	***0.526 (0.468)***	***5.343 (0.069)***
GJR_GARCH						
GJR-GARCH(1,1) norm	-5.2406	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
GJR-GARCH(1,1) std	-5.3099	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
GJR-GARCH(1,1) ged	-5.3055	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
GJR-GARCH(1,1) snorm	-5.2467	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
GJR-GARCH(1,1) sstd	-5.3101	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
GJR-GARCH(1,1) sged	-5.3059	$\mu = 0;\ \omega = 0;\ \gamma = 0.$	The model reduces to Standard GARCH (1,1).			
C-GARCH						
C-GARCH(1,1) norm	-5.6716	$\mu = 0;\ \omega = 0$	-5.6688	0.0533	0.378 (0.539)	2.534 (0.282)
C-GARCH(1,1) std	-5.7313	$\mu = 0;\ \omega = 0$	-5.7326	0.0588	2.546 (0.111)	6.07 (0.048)
C-GARCH(1,1) ged	-5.7338	$\mu = 0;\ \omega = 0$	-5.7355	0.0545	0.698 (0.403)	3.131 (0.209)
C-GARCH(1,1) snorm	**-5.6680**	$\boldsymbol{\mu = 0;\ \omega = 0}$	**-5.6651**	**0.0539**	**0.003 (0.955)**	**3.594 (0.166)**
C-GARCH(1,1) sstd	-5.7286	$\mu = 0;\ \omega = 0$	-5.7301	0.0570	1.617 (0.203)	7.412 (0.024)
C-GARCH(1,1) sged	-5.7311	$\mu = 0;\ \omega = 0$	-5.7317	0.0533	0.378 (0.539)	5.009 (0.082)
MERVAL						
GARCH Models	**BIC**	**Parameters equal to Zero**	**BIC after adjustment**	**Proportion of failures**	**Kupiec**	**Christoffersen**
Standard GARCH						
GARCH(1,1) norm	-5.2370	None		0.0497	4.33 (0.037)	6.576 (0.037)
GARCH(1,1) std	-5.3109	$\omega = 0$	-5.2923	0.0707	13.221 (0)	21.918 (0)
GARCH(1,1) ged	-5.3059	None		0.0604	3.489 (0.062)	6.097 (0.047)
GARCH(1,1) snorm	-5.2427	None		0.0564	1.492 (0.222)	2.879 (0.237)
GARCH(1,1) sstd	-5.3108	None		0.0579	2.069 (0.15)	5.488 (0.064)
GARCH(1,1) sged	**-5.3058**	**None**		**0.0564**	**1.006 (0.316)**	**2.674 (0.263)**
GJR_GARCH						
GJR-GARCH(1,1) norm	-5.2406	None		0.0618	4.782 (0.029)	4.856 (0.088)
GJR-GARCH(1,1) std	-5.3099	None		0.0671	9.134 (0.003)	9.518 (0.009)
GJR-GARCH(1,1) ged	-5.3055	None		0.0640	6.262 (0.012)	6.523 (0.038)
GJR-GARCH(1,1) snorm	***-5.2467***	$\boldsymbol{\mu = 0}$	***-5.2492***	***0.0567***	***1.492 (0.222)***	***1.599 (0.45)***
GJR-GARCH(1,1) sstd	-5.3101	None		0.0604	3.489 (0.062)	3.49 (0.175)
GJR-GARCH(1,1) sged	-5.3012	None		0.0576	2.069 (0.15)	2.117 (0.347)
C-GARCH						
C-GARCH(1,1) norm	-5.2335	None		0.0594	3.102 (0.078)	3.906 (0.142)
C-GARCH(1,1) std	-5.3065	None		0.0677	9.767 (0.002)	10.634 (0.005)
C-GARCH(1,1) ged	-5.3007	None		0.0612	4.33 (0.037)	5.586 (0.061)
C-GARCH(1,1) snorm	-5.2468	None		0.0594	3.102 (0.078)	3.906 (0.142)
C-GARCH(1,1) sstd	**-5.3070**	**None**		**0.0585**	**2.391 (0.122)**	**3.408 (0.182)**
C-GARCH(1,1) sged	-5.2982	$\omega = 0$	-5.2959	0.0594	3.102 (0.078)	4.769 (0.092)

Source: Elaborated by authors.
p-value are in parentheses;
Kupiec null hypotheses: no serial correlation;
Christoffersen null hypotheses: serially independent.

The parameters results for each time series are summarized in Table 6. In the cases of the GJR-GARCH models (Brazil, U.S. and Argentina), the volatility persistence $(\alpha_1 + \beta_1 + c\gamma)$ rate are estimated to be less than one, which supports the argument of stationary variance. This instability measure varies between 0.97 in for the Argentina index to 0.99 for the American market. This means that, transitory volatility resulting from shock movements is larger in the SPX financial market. The leverage effect (γ) presented to be positive for the three stock indexes, implicating that negative shocks tend to influence future volatility more than positive shocks. China's stock index presented symmetric GARCH model in which $\alpha_1 + \beta_1 < 1$. And $\alpha_1 < \beta_1$ showing that volatility appears to be persistent, remaining around the same level for longer.

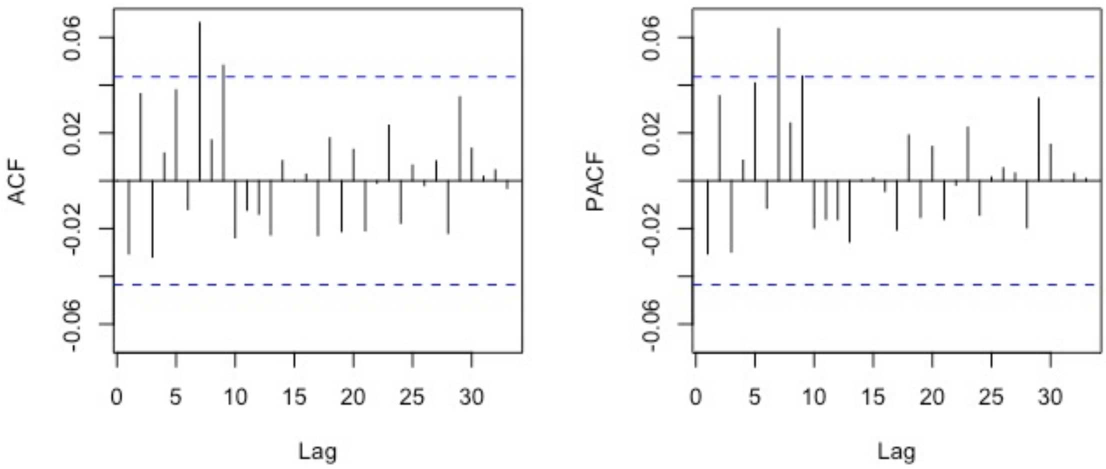
Table 6: Summary Statistics of the log returns.

Parameter	IBOV	SPX	SHCOMP	MERVAL
μ	-	-	-	-
ω	0.000012 (0.0000)	-	-	0.000024 (0.0008)
α_1	0.023130 (0.0010)	0.019014 (0.02344)	0.028443 (0.0000)	0.055233 (0.0006)
β_1	0.902299 (0.0000)	0.935839 (0.0000)	0.970557 (0.0000)	0.829095 (0.0000)
γ	0.103715 (0.0000)	0.096379 (0.0000)	-	0.096558 (0.0000)
Skew	0.921138 (0.0000)	0.840652 (0.0000)	0.962917 (0.0000)	0.887834 (0.0000)
Shape	1.678159 (0.0000)	1.340145 (0.0000)	1.250673 (0.0000)	-
	$\alpha_1 + \beta_1 + 0.47\gamma = 0.97$	$\alpha_1 + \beta_1 + 0.44\gamma = 0.99$	$\alpha_1 + \beta_1 = 0.99$	$\alpha_1 + \beta_1 + 0.49\gamma = 92$
Analysis of Squared Standardized Residuals				
Ljung-Box	19.077 (0.0145)	10.102 (0.258)	2.89 (0.941)	5.55 (0.698)
ARCH-LM test	6.5395 (0.8865)	0.1063 (1.00)	0.514 (1.00)	0.064 (1.00)

Source: Elaborated by author.
p-value are in parentheses;
Ljung-Box null hypotheses: no serial correlation;
ARCH-LM null hypotheses: serially independent.

Analyzing the standardized squared residuals in Table 6, only for IBOV the Ljung-Box test presented to not reject the presence of serial correlation, on the other hand the ARCH-LM test confirms no ARCH effects for all indexes. And by comparing Figure 3 with 4 we see that all the autocorrelation in squared residuals is smaller than squared returns, verifying that data are approximately iid. Similar results for SPX, SHCOMP and MERVAL are respectively presented in Appendix A, Figures 3.A, 6.A and 9.A. Now we have the data in the required form for applying the Extreme Value Theory to estimate VaR and ES and apply the Granger causality in risk between markets.

Figure 4: Autocorrelation and Partial Autocorrelation of squared residuals of IBOV index.



6.3. Determination of Threshold

Figure 5 plots the Mean Excess Function (MEF) of the Brazilian stock index of both negative and positive returns are estimated to choose thresholds, jointly with Table 7 that shows the tradeoff between standard error of the parameters (shape and scale) and Bayes Information Criterion (BIC) with respective percentile. Thus, by Figure 5 and Table 7, the chosen thresholds are u = 1.6135 for negative returns and u = 1.6096 positive returns. Under these thresholds, the VaR and ES quantiles (at 5% and 1%) obtained from e equations (4.6.3.6) and (4.6.3.7) are reported in Table 8. Similar observations are made for the SPX, SHCOMP and MERVAL stock markets presented in Appendix B and C.

Table 7: Shape and Scale estimates for GJR-GARCH(1,1) model under different thresholds and percentiles. IBOV log returns.

Percentile	$-u$	ξ	σ	**BIC**	u	ξ	σ	**BIC**
99	2.4809	-0.06526 (0.1761)	0.4112 (0.1504)	25.1573	2.2931	-1.008 (0.0000)	1.175 (0.0000)	19.3005
98	2.3819	-0.008907 (0.1465)	0.470528 (0.1014)	32.2026	2.0472	-0.01866 (0.2181)	0.42032 (0.1132)	22.3961
97	1.9285	0.06556 (0.1079)	0.55025 (0.0924)	53.6748	1.8665	-0.04267 (0.1525)	0.44245 (0.0883)	30.2543
96	1.7285	0.1001 (0.0851)	0.6117 (0.0853)	79.3955	1.7286	-0.05606 (0.1213)	0.45871 (0.0754)	39.8954
95	***1.6135***	***0.08223 (0.0825)***	***0.59651 (0.0769)***	94.2507	***1.6096***	***-0.09144 (0.1006)***	***0.49746 (0.07025)***	***55.7136***

Source: Elaborated by author.
Numbers in parentheses are the Standard Errors.

Figure 5: Mean Excess Function of IBOV index.

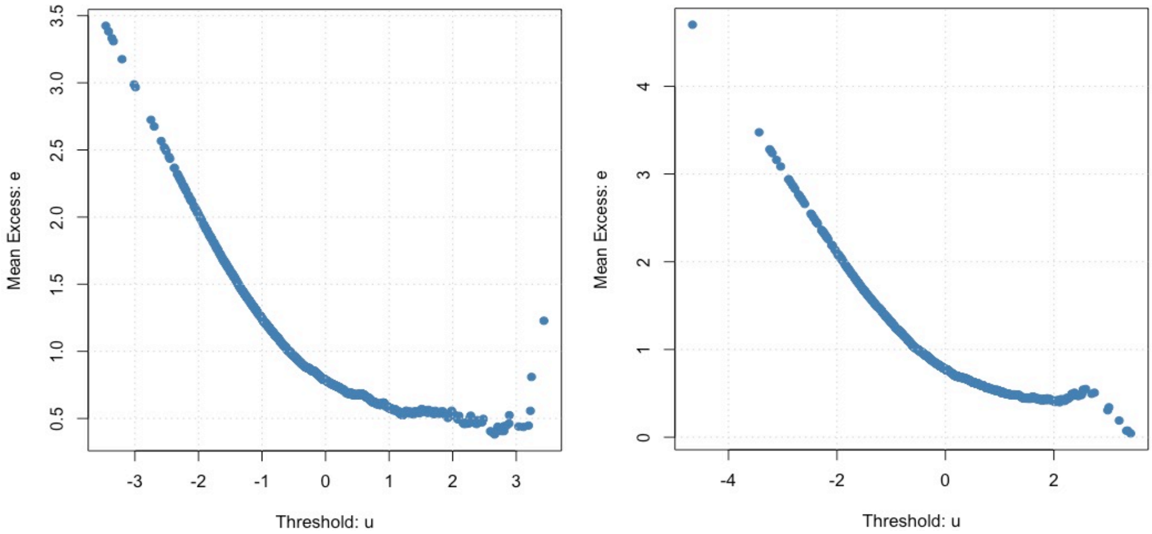


Table 8: Parameter estimates for GARCH model, IBOV log returns.

	Negative returns	Positive Returns
	u = 1.6135	u = 1.6096
Total in sample observation n	2027	2027
Number of exceedance N_u	101	101
% of exceedance in sample N_u/n	4.9	4.9
GPD shape parameter ξ	0.08223 (0.0825)	-0.09144 (0.1006)
GPD scale parameter σ	0.5965 (0.0769)	0.49746 (0.07025
VaR quantile:		
$VaR^t_{0.05}(Z)$ / $VaR^t_{0.95}(Z)$	-1.6113	1.6078
$VaR^t_{0.01}(Z)$ / $VaR^t_{0.99}(Z)$	-2.5108	2.3526
ES quantile:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-2.1627	2.0638
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-2.9938	2.7461

Source: Elaborated by author.
Numbers in parentheses are the Standard Errors.

6.4. Risk Measure Estimation

Table 9 shows the conditional VaR and ES for a one-day horizon (equations 5.1.3 and 5.1.4) for all four indexes at 5 and 1 percentile for the left (long position) and right tail (short position). An interesting observation is that, for any given threshold and quantile level, the corresponding VaR and ES estimate in the left tail are larger than that in the right tail, i.e., left tail is heavier than the right for all indices.

For the IBOV index, the conditional VaR is estimated as -2.62% at 5th percentile for the left tail (long position). This denotes that, for the lower 5% negative daily returns, the worst daily loss in the Brazilian Market value may exceed -2.62% in expectation, i.e., if we invest in market portfolio, we are 95% confident that our daily loss at worst will not exceed -2.62% during one trading day. Assuming that the loss is greater than expected (5th percentile), the average amount that is lost over a one-day period is -3.51%. On the other hand, VaR is estimated as 2.61% at the 95th percentile for the right tail. We expect that a daily change in the market portfolio would not increase by more than 2.61%. Put differently, we are 95% confident that our daily loss will not exceed 2.61% if we take short position of market portfolio. But if this value is exceeded, the average loss is -3.35%. At 1% for long position the value of conditional VaR is around 4%, implying that for the lower 1% negative the IBOV daily returns may exceed 4%, in other words we are 99% confident that loss of this daily trading asset at worst will not exceed -3.35% for the next day. However, in case of the loss is greater than expected (1 %), the average amount that is lost over a one-day period is -4.86%. Now considering the right tail (short position) at 99%, the value of the conditional VaR is 3.82%, which means that at probability of 99%, tomorrow's loss is expected to be lower than 3.82%, and the corresponding expected shortfall is 4.46%.

The American stock market reported the lowest values of VaR and ES, reveling to be the asset with lower rates of risk. With 5% confidence level we can predict tomorrow's loss (left tail) to exceed -0.95% and that the corresponding expected loss, that is the average loss in situations where the losses exceed -0.95%, is -1.29%. Assuming short position, with 95% confidence level the expected loss is not to exceed 0.86 for the next trading day, and the expected average loss in case of exceedance, is around 1.09%. Corresponding results are verified at a lower quantile of 1% (long position) and 99% level (short position), but in higher rates.

Looking at the Chinese market, the results indicate that, with probability 0.05, the next day loss on long position will exceed the value -1.18 and the corresponding expected average loss is -1.64. At short position, with probability of 95%, tomorrow's loss will not exceed the value of 1.14%, but if exceeded the corresponding expected average loss is 1.64%. The same inference can be drawn for 1% percentile.

The index of Argentina reported the higher values of VaR and ES prediction for the next day, i.e., presenting to be the riskier asset among the four markets analyzed. The conditional VaR at 5th percentile, assuming long position is not to exceed -3.68% and the average loss in case of exceedance is -5%. Taking short position, at 95% confidence level, the daily loss will not exceed 3.49%, and the expected average loss is 4.73%. At probability of 1% for long position the next day loss will not exceed -5.65%, implying that for the lower 1% negative the MERVAL stock market may exceed -5.65%, in other words we are 99% confident that loss of this daily trading asset at worst will not exceed 5-.65% for the next day. Conversely, in case of the loss is greater than expected (at 1 %), the average amount that is lost over a one-day period is -7.55%. Now considering the right tail (short position) at 99%, the value of the conditional VaR is 5.54%, which means that at probability of 99%, tomorrow's loss is expected to be lower than 5.54%, and the corresponding expected average loss is 6.47%.

Table 9: Conditional VaR and Conditional ES for IBOV, SPX, SHCOMP and MERVAL log returns at 5% percentile.

IBOV	Negative returns	Positive Returns
	u = 1.6135	u = 1.6096
Conditional VaR:		
$VaR^t_{0.05}(X_{t+1})$ / $VaR^t_{0.95}(X_{t+1})$	-0.0262	0.0261
$VaR^t_{0.01}(X_{t+1})$ / $VaR^t_{0.99}(X_{t+1})$	-0.0408	0.0382
Conditional ES:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-0.0351	0.0335
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-0.0486	0.0446
SPX	Negative returns	Positive Returns
	u = 1.8277	u = 1.6391
Conditional VaR:		
$VaR^t_{0.05}(X_{t+1})$ / $VaR^t_{0.95}(X_{t+1})$	-0.0095	0.0086
$VaR^t_{0.01}(X_{t+1})$ / $VaR^t_{0.99}(X_{t+1})$	-0.0148	0.0125
Conditional ES:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-0.0129	0.0109
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-0.0185	0.0139
SHCOMP	Negative returns	Positive Returns
	u = 1.6506	u = 1.5920
Conditional VaR:		
$VaR^t_{0.05}(X_{t+1})$ / $VaR^t_{0.95}(X_{t+1})$	-0.0118	0.0114
$VaR^t_{0.01}(X_{t+1})$ / $VaR^t_{0.99}(X_{t+1})$	-0.0194	0.0192
Conditional ES:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-0.0164	0.0164
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-0.0234	0.0257
MERVAL	Negative returns	Positive Returns
	u = 1.6782	u = 1.5882
Conditional VaR:		
$VaR^t_{0.05}(X_{t+1})$ / $VaR^t_{0.95}(X_{t+1})$	-0.0368	0.0349
$VaR^t_{0.01}(X_{t+1})$ / $VaR^t_{0.99}(X_{t+1})$	-0.0565	0.0554
Conditional ES:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-0.0500	0.0473
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-0.0755	0.0647

Source: Elaborated by author.

7. Conclusion

We calculate the daily VaR for four market returns by combining the EVT with GARCH models. By this method we get standardized residuals that are close to iid so that EVT models can be applied. Applying conditional EVT, the POT method has its advantages in modelling the available data more efficiently than BMM as it uses excesses over a threshold and can be more effective if we have limited data sets. The estimates of conditional VaR and conditional ES computed under different high quantile levels exhibit stability through the selected thresholds, implying the accuracy and reliability of the estimated quantile-based risk measures. The VaR and ES measures based on conditional EVT model provide quantitative information for analyzing the extent of potential extreme risks in the market portfolio of IBOV, SPX, SHCOMP and MERVAL. These countries represent the ones with stronger economic relations with Brazil. Looking at estimated VaR and ES values, we observe that MERVAL is the stock market that is most exposed to extreme losses, followed by the IBOV. The least exposed to daily extreme variations are SPX and SHCOMP. Then we examined the extreme risk spillover effect in international financial markets during the recent global financial turmoil. We employ a statistical testing procedure for Granger causality test in risk to examine the joint dynamics of extreme values in the left and right tail of distribution. This test enables us to investigate how risk spills over between stock market and foreign exchange market in Brazil. The results of testing the Granger causality in risk show that non-expected but positive signals (short position) were weaker than the corresponding negative signals (long position) for all risk measures. Among the countries that were studied in this paper, the American stock market presented to be the one with stronger influence in case of crisis. Therefore, understanding its mechanism of transmission is of extreme importance for economic agents and policymakers to soften its impact. Although China is the Brazilian latest spurt of growth driven by its appetite for the Brazilian commodities, we did not observe evidence of causality among these markets. The Brazilian market reported wide contagion effect to the regional market (Argentina), but no contagion effect to the global dominant market. Finally, the results show that the fact that China, US and Argentina are the strongest Brazilian trading partners for the past decade doesn’t necessarily reflect extreme spillover effects to the Brazilian stock market.

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Appendix A

Figure 1.A: Autocorrelation and Partial Autocorrelation of returns of SPX index.

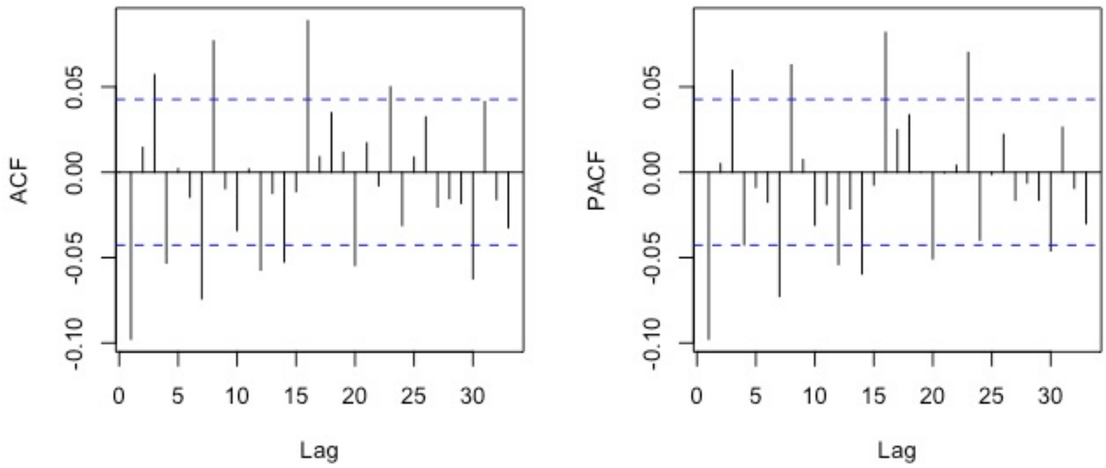


Figure 2.A: Autocorrelation and Partial Autocorrelation of squared returns of SPX index.

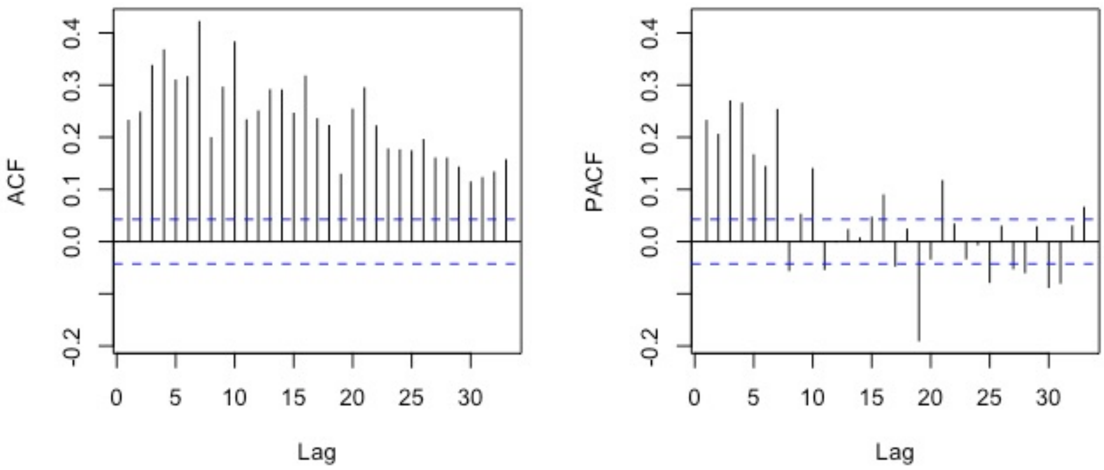


Figure 3.A: Autocorrelation and Partial Autocorrelation of squared residuals of SPX index.

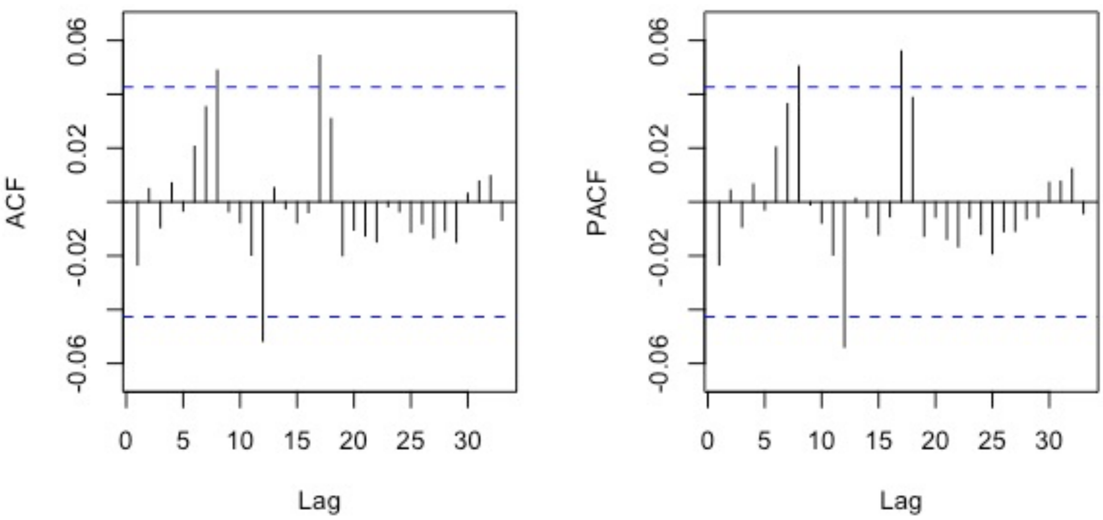


Figure 4.A: Autocorrelation and Partial Autocorrelation of returns of SHCOMP index.

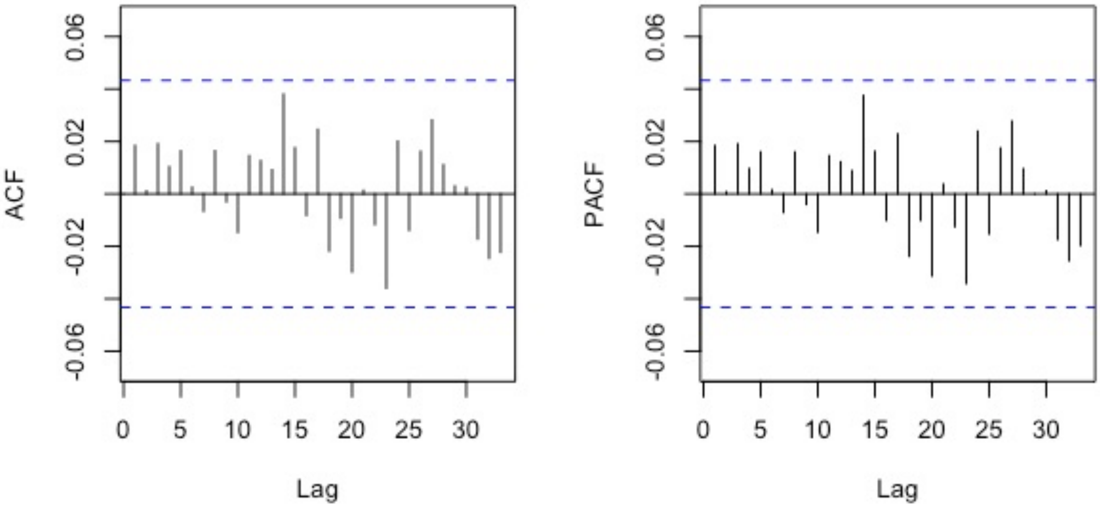


Figure 5.A: Autocorrelation and Partial Autocorrelation of squared returns of SHCOMP index.

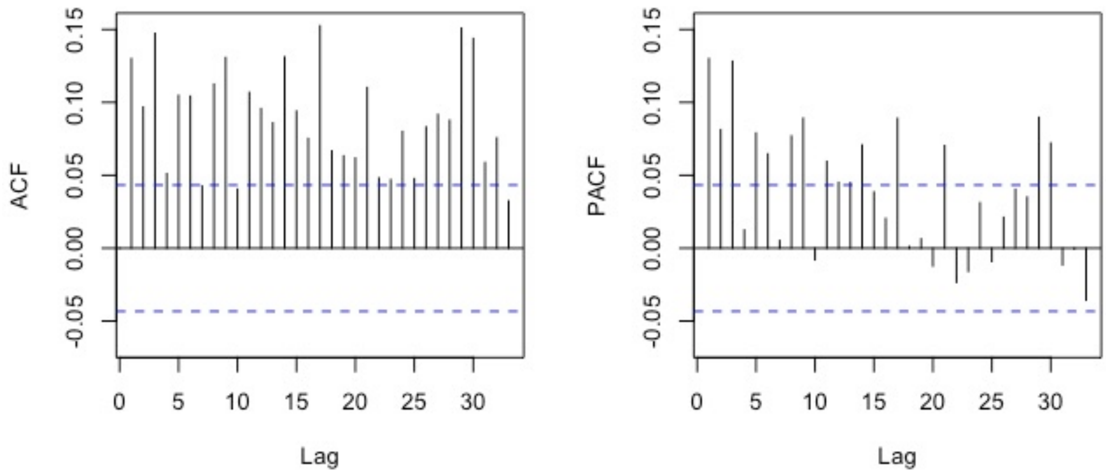


Figure 6.A: Autocorrelation and Partial Autocorrelation of squared residuals of SHCOMP index.

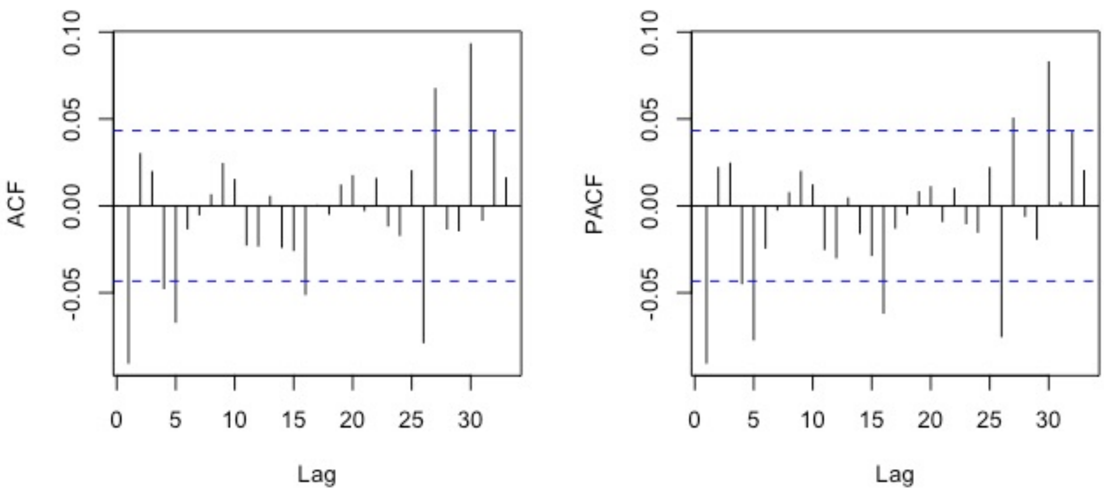


Figure 7.A: Autocorrelation and Partial Autocorrelation of returns of MERVAL index.

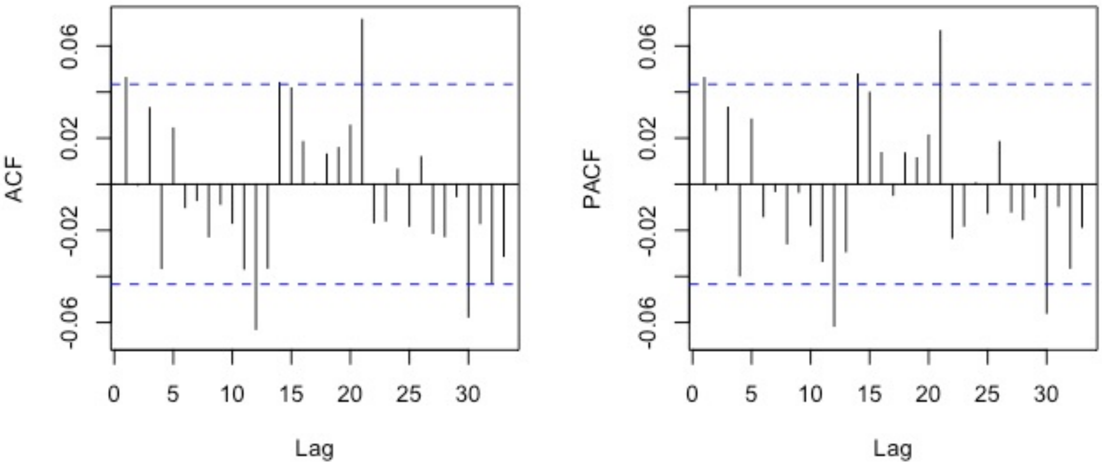


Figure 8.A: Autocorrelation and Partial Autocorrelation of squared returns of MERVAL index.

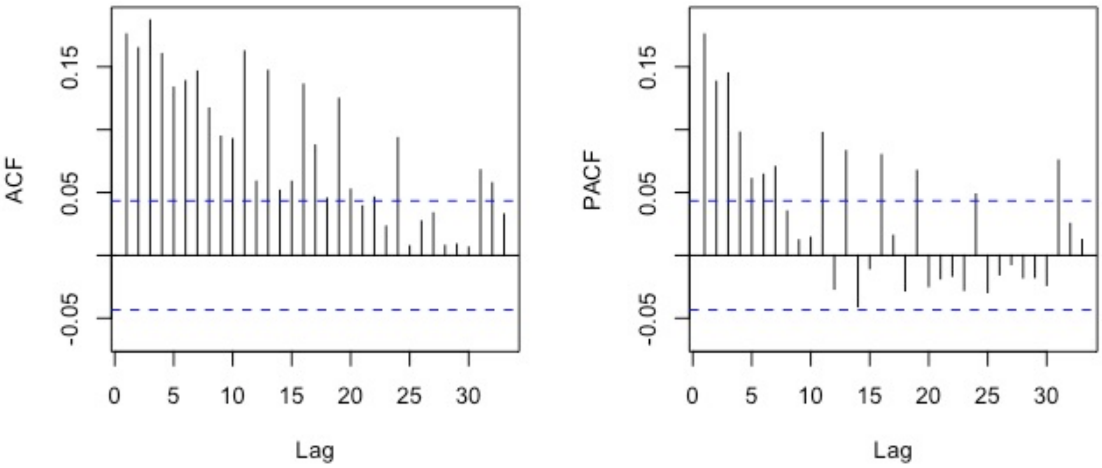
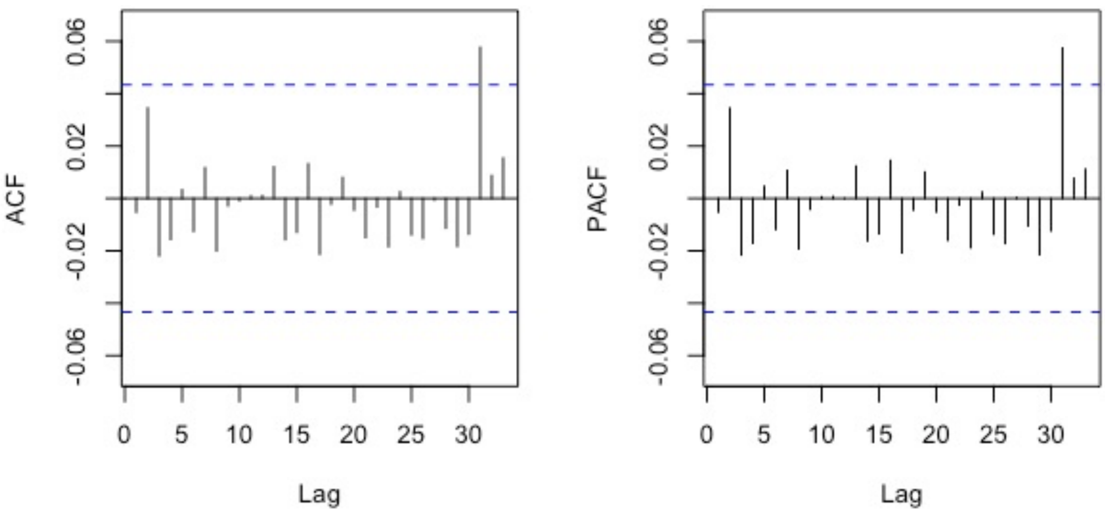


Figure 9.A: Autocorrelation and Partial Autocorrelation of squared residuals of MERVAL index.



Appendix B

Table 1.B: Shape and Scale estimates for GJR-GARCH(1,1) model under different thresholds and percentiles. SPX log returns.

Percentile	$-u$	ξ	σ	BIC	u	ξ	σ	BIC
99	2.7042	0.468 (0.3396)	0.4112 (0.1606)	37.6301	2.3429	-0.6013 (0.2639)	0.535 (0.1692)	3.7701
98	2.3819	0.2431 (0.1598)	0.4614 (0.101)	52.7406	2.1142	-0.3054 (0.1004)	0.4394 (0.1749)	2.5562
97	2.2033	0.4188 0.1402)	0.2423 (0.0779)	60.1640	1.9204	-0.2804 (0.1345)	0.4827 (0.0873)	12.1853
96	1.9856	0.1232 (0.0948)	0.529 (0.0761)	95.0209	1.782	-0.2841 (0.1061)	0.5228 (0.0779)	24.5959
95	***1.8277***	***0.0761 (0.0757)***	***0.5828 (0.0718)***	***125.8849***	***1.6391***	***-0.316 (0.0873)***	***0.5955 (0.0758)***	***48.0703***

Numbers in parentheses are the Standard Erros.

Figure 1.B: Mean Excess Function of SPX index.

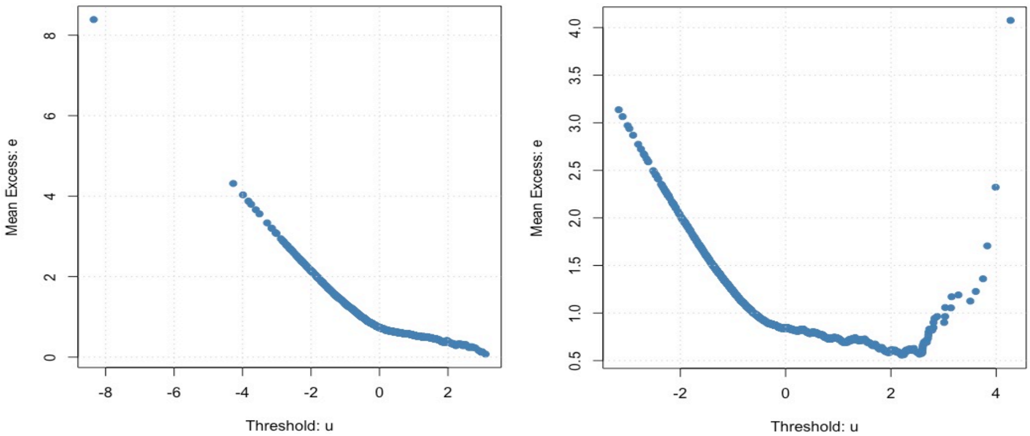


Table 2.B: Shape and Scale estimates for Standard GARCH(1,1) model under different thresholds and percentiles. SHCOMP log returns.

Percentile	$-u$	ξ	σ	BIC	u	ξ	σ	BIC
99	2.4809	-0.3853 (0.2645)	1.0541 (0.3536)	39.9424	2.7552	0.1212 (0.2885)	0.7565 (0.2754)	46.9335
98	2.3004	0.2235 (0.2411)	0.4186 (0.1194)	42.1655	2.1856	0.1611 (0.2449)	0.6736 (0.1741)	76.0601
97	2.0649	0.0432 (0.1445)	0.5309 (0.1025)	53.6944	1.9733	0.1802 (0.1525)	0.1189 (0.0883)	90.7342
96	1.7959	0.0853 (0.1036)	0.6941 (0.1049)	103.3758	1.6969	0.09188 (0.1201)	0.69012 (0.1124)	131.4847
95	***1.6506***	***0.0812 (0.0926))***	***0.6990 (0.0946)***	***127.6437***	***1.5517***	***0.1572 (0.1217)***	***0.5957 (0.09304)***	***143.6427***

Numbers in parentheses are the Standard Erros.

Figure 2.B: Mean Excess Function of SHCOMP index.

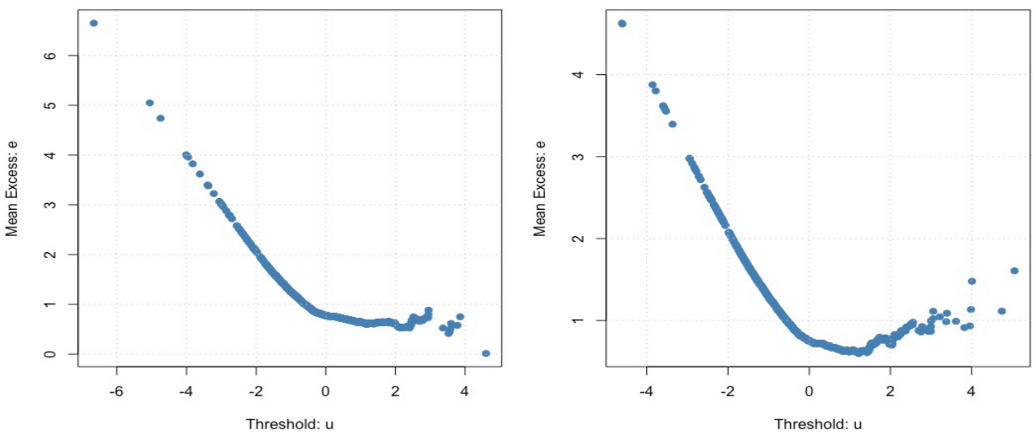
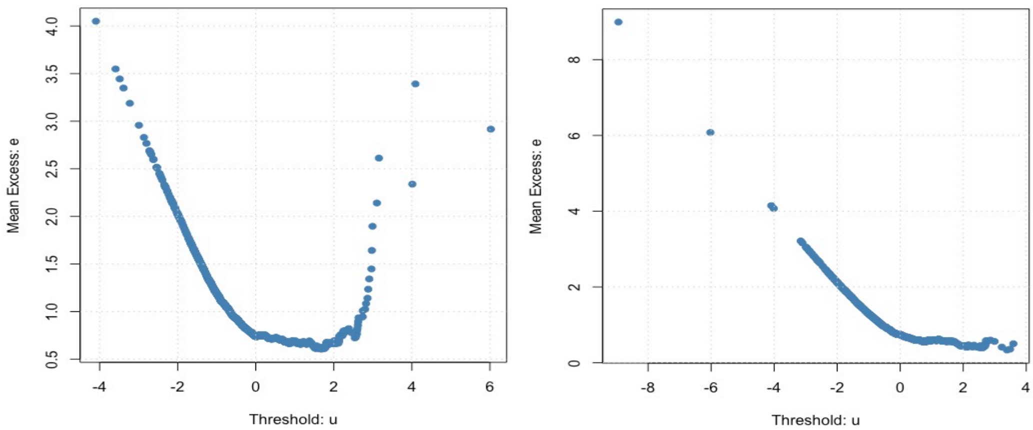


Table 3.B: Shape and Scale estimates for GJR-GARCH(1,1) model under different thresholds and percentiles. MERVAL log returns.

Percentile	$-u$	ξ	σ	BIC	u	ξ	σ	BIC
99	2.6057	0.6929 (0.3613)	0.3218 (0.1273)	35.6008	2.4688	-0.02558 (0.282)	0.44059 (0.1585)	19.4298
98	2.1556	0.2963 (0.1734)	0.118 (0.1194)	63.6065	2.1574	-0.04058 (0.1695)	0.1067 (0.1741)	27.9364
97	1.9389	0.2494 (0.1354)	0.4953 (0.0909)	79.9445	1.9718	-0.04583 (0.12829)	0.46741 (0.0846)	36.8602
96	1.7933	0.2441 (0.1233)	0.4702 (0.0769)	92.5258	1.7347	-0.1883 (0.08223)	0.6523 (0.08912)	***75.5072***
95	***1.6782***	***0.2274 (0.10898)***	***0.4608 (0.0670)***	***105.5592***	***1.5882***	***-0.1815 (0.07533)***	***0.6698 (0.0824)***	***98.4507***

Numbers in parentheses are the Standard Errors.

Figure 3.B: Mean Excess Function of MERVAL index.



Appendix C

Table 1.C: Parameter estimates for GARCH model, SPX, SHCOMP and MERVAL log returns.

SPX	Negative returns	Positive Returns
	u = 1.8277	u = 1.6391
Total in Sample observation n	2108	2108
Number of exceedance N_u	105	105
% of exceedance in sample N_u/n	4.98	4.98
GPD shape parameter ξ	0.0761 (0.0757)	-0.3160 (0.0873)
GPD scale parameter σ	0.5828 (0.0718)	0.5955 (0.0758)
VaR quantile:		
$VaR^t_{0.05}(Z)$ / $VaR^t_{0.95}(Z)$	-1.8255	1.6368
$VaR^t_{0.01}(Z)$ / $VaR^t_{0.99}(Z)$	-2.8230	2.3889
ES quantile:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-2.4561	2.0898
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-3.5357	2.6613
SHCOMP	Negative returns	Positive Returns
	u = 1.6506	u = 1.5920
Total in Sample observation n	2050	2050
Number of exceedance N_u	102	102
% of exceedance in sample N_u/n	4.98	4.98
GPD shape parameter ξ	0.0812 (0.0926)	0.1572 (0.1217)
GPD scale parameter σ	0.6990 (0.0946)	0.5957 (0.09304)
VaR quantile:		
$VaR^t_{0.05}(Z)$ / $VaR^t_{0.95}(Z)$	-1.6471	1.5891
$VaR^t_{0.01}(Z)$ / $VaR^t_{0.99}(Z)$	-2.7022	2.6792
ES quantile:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-2.2939	2.2954
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-3.2697	3.5888
MERVAL	Negative returns	Positive Returns
	u = 1.6782	u = 1.5882
Total in Sample observation n	2040	2040
Number of exceedance N_u	102	102
% of exceedance in sample N_u/n	5.0	5.0
GPD shape parameter ξ	0.2274 (0.1089)	-0.1815 (0.0753)
GPD scale parameter σ	0.4608 (0.0670)	0.6698 (0.0824)
VaR quantile:		
$VaR^t_{0.05}(Z)$ / $VaR^t_{0.95}(Z)$	-1.6781	1.5882
$VaR^t_{0.01}(Z)$ / $VaR^t_{0.99}(Z)$	-2.5736	2.5230
ES quantile:		
$ES^t_{0.05}(Z)$ / $ES^t_{0.95}(Z)$	-2.2745	2.1551
$ES^t_{0.01}(Z)$ / $ES^t_{0.99}(Z)$	-3.4335	2.9463

Numbers in parentheses are the Standard Error.