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FACULDADE DE ECONOMIA, ADMINISTRAÇÃO, ATUÁRIA E CONTABILIDADE
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA - CAEN
DOUTORADO EM ECONOMIA

LUIZ ALMEIDA SAMPAIO FILHO

ESSAYS ON THE PUBLIC DEBT IN EUROPE

FORTALEZA
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Tese apresentada ao Programa de Pós-Graduação em Economia - CAEN, da Universidade Federal do Ceará - UFC, como parte dos requisitos para obtenção do título de Doutor em Economia.

Orientador: Prof. Dr. Paulo Rogério Faustino de Matos

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Aprovada em:___/___/_____

BANCA EXAMINADORA

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A Deus, por sua Graça e infinita Misericórdia.

Aos meus pais Luiz e Francisca (*in memoriam*)

AGRADECIMENTOS

A Deus, pois sem Ele nada seria possível.

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RESUMO

A literatura sobre a dívida pública europeia é extensa e controversa. Assim, o presente estudo fez uso de uma abordagem sistemática buscando superar as principais dificuldades nesta área. Deste modo, pretende-se responder três perguntas principais. Primeiro, se há alguma classificação de grupos entre os países europeus baseada no comportamento de dívida, com algum grupo de núcleo; caso exista tal grupo de núcleo, o que pode ser dito a respeito da solvência/sustentabilidade da dívida para os países desse grupo; e, se a maior integração econômica entre os países pertencentes a tal grupo de núcleo se traduz em transbordamentos de choques de uns para os outros e, em caso afirmativo, em qual magnitude. Para responder à primeira pergunta utilizou-se técnicas de cluster o que tornou possível distinguir três grupos com um bom nível de precisão. Em seguida, a fim de detectar possíveis clubes de convergência, se aplicou a metodologia de testes de convergência proposta por Phillips e Sul (2007, 2009) e os algoritmos propostos por von Lyncker and Thoennessen (2017). Os resultados confirmam a existência de dois clubes de convergência de dívida na União Europeia. Tomando a interseção dos resultados de cluster e convergência, foi definida a subamostra de doze países para os quais se aplicou uma análise de solvência. De modo a responder à segunda pergunta, se fez uso de wavelets para avaliar a solvência/sustentabilidade da dívida pública dos doze países da subamostra. Os resultados apontam para evidências de comportamento insolvente em oito dos doze países analisados. A fim de responder à terceira pergunta, se fez uso da abordagem de transbordamentos de conectividade proposta por Diebold e Yilmaz (2009, 2012, 2014) aplicada em uma estrutura de um modelo VAR com parâmetros variantes no tempo (TVP-VAR) conforme Antonakakis, Chatziantoniou, e Gabauer (2020). Os resultados mostram que cerca de 60% da conectividade total da rede formada pelos doze países da subamostra é proveniente de transbordamentos, neste caso, de choques de dívida entre esses países. Os transbordamentos aumentam em períodos de crise com destaque para o período da pandemia da COVID-19. Os resultados do presente estudo se mostram úteis para a avaliação da política fiscal e mapeamento da transmissão de choques, o que permite adotar ações preventivas visando reduzir os impactos negativos de uma possível crise, principalmente para uma região com histórico fiscal desfavorável e ameaçada por recentes turbulências como a União Europeia.

Palavras-chave: Dívida Pública; União Europeia; Solvência; Sustentabilidade; Transbordamentos

ABSTRACT

The literature on European public debt is extensive and controversial. This study has therefore used a systematic approach to overcome the main difficulties in this area. In this way, it aims to answer three main questions. First, whether there is any classification of groups among European countries based on debt behavior, with some core group; if there is such a core group, what can be said about the solvency/sustainability of the debt for the countries in this group; and, whether the greater economic integration among the countries belonging to this core group translates into spillovers of shocks from one to another and, if so, in what magnitude. Cluster techniques were used to answer the first question, making it possible to distinguish three groups with a good level of precision. Next, to detect possible convergence clubs, the convergence testing methodology proposed by Phillips and Sul (2007, 2009) and the algorithms proposed by von Lyncker and Thoennessen (2017) were applied. The results confirm the existence of two debt convergence clubs in the European Union. Taking the intersection of the cluster and convergence results, a sub-sample of twelve countries was defined for which a solvency analysis was applied. To answer the second question, wavelets were used to assess the solvency/sustainability of the public debt of the twelve countries in the sub-sample. The results point to evidence of insolvent behavior in eight of the twelve countries analyzed. To answer the third question, the connectivity spillover approach proposed by Diebold and Yilmaz (2009, 2012, 2014) was applied to a VAR model structure with time-varying parameters (TVP-VAR) according to Antonakakis, Chatziantoniou, and Gabauer (2020). The results show that around 60% of the total connectivity of the network formed by the twelve countries in the sub-sample comes from spillovers, in this case from debt shocks between these countries. Spillovers increase during periods of crisis, especially during the COVID-19 pandemic. The results of this study are useful for evaluating fiscal policy and mapping the transmission of shocks, which makes it possible to adopt preventive actions aimed at reducing the negative impacts of a possible crisis, especially for a region with an unfavorable fiscal history and threatened by recent turbulence such as the European Union.

Keywords: Public Debt; European Union; Solvency; Sustainability; Spillovers

SUMÁRIO

1. INTRODUCTION

Fiscal policy is (or at least should be) a concern for any government, and within this, public debt plays a crucial role. In this context, it is of interest to question when public debt can be considered sustainable. This issue is a matter of debate in both the political and academic environments.

The concept becomes even more important when considering an economic and political union such as the European Union (EU), where, in addition to greater economic integration, 20 of the 27 members share the same currency, forming what is known as the Euro Area (EA). In this way, fiscal policy becomes the primary tool under the control of each member country for economic stabilization, as monetary policy is centralized at the European Central Bank (ECB).

The European Union is a unique case, as it consists of members with very distinct fiscal and monetary histories. In fact, during the process of forming the group, convergence criteria were established in the Maastricht Treaty[1] and the Stability and Growth Pact[2], related to interest rates, exchange rates, budget deficits, debt levels, and currency stability. For the purposes of this work, it is worth highlighting that, regarding the deficit and debt criteria, the countries that signed the treaty were required to maintain a public deficit of no more than 3% of GDP, and public debt could not exceed 60% of GDP.

The topic has garnered increased attention, particularly after the European sovereign debt crisis[3], as well as the recent turbulences in Europe due to the pandemic and the Russia-Ukraine conflict.

It is important to note that this study frequently refers to the concepts of solvency and sustainability together. While these concepts are not equivalent, they are closely related in that, generally, solvency is a necessary condition for public debt sustainability, as demonstrated, for example, in the formulation by Hamilton and Flavin (1986). Furthermore, the concept of sustainability may vary depending on the methodology employed and the time horizon considered (NECK; STURM, 2008). Nevertheless, even if implicitly, the notion of solvency is incorporated into the analysis of sustainability.

This work aims to apply a systematic approach to address common issues found in the literature on public debt solvency/sustainability, while also providing results regarding the grouping of countries in Europe as well as spillover effects.

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[1] Glossary:Treaty on European Union - Statistics Explained
[2] Glossary:Stability and growth pact (SGP) - Statistics Explained (europa.eu)
[3] To find details about such a crisis, see for example Gourinchas, Martin e Messer (2023).

Therefore, this study aims to answer three main questions: 1) Is there a grouping of countries within Europe based on debt behavior, specifically the existence of a debt core group? 2) If such a core group exists, what can be said about the solvency/sustainability of debt for the countries within this group? 3) Does greater economic integration among the countries within this core group translate into spillovers of shocks from one to another, and if so, to what magnitude?

The results provide the following answers to the previous questions: 1) Based on two complementary clustering methodologies, it is possible to affirm that there is a grouping of countries within Europe, with a debt core group composed of twelve countries that share similar characteristics in the trajectory of the debt-to-GDP ratio. 2) Among the twelve countries in the debt core group, evidence of insolvent behavior was found in eight of them, suggesting an unsustainable fiscal policy during specific periods and frequencies of the sample. 3) Indeed, greater economic integration also leads to higher spillovers of debt shocks between these countries, with particular emphasis on the influence exerted by Belgium, Italy, and France. In turn, Greece appears as the most affected member by the others within the group.

The remainder of the study is organized as follows. The next section presents the related literature. Section 3 describes the data and provides descriptive statistics. Section 4 details all the methodologies used, while Section 5 presents the results obtained. Section 6 concludes.

2. LITERATURE

The classification of countries into different groups based on common characteristics is a practice widely employed across various fields of economic research.

Studies that aim to examine debt behavior in the European Union often divide countries into groups based on an assumed core-periphery relationship (PANIAGUA; SAPENA; TAMARIT, 2017; TOVAR; JALLES, 2017; FINCKE; GREINER, 2012; BALDI; STAEHR, 2015). Such classification is also observed in studies on the theory of optimum currency areas (ARTIS; ZHANG, 2001, 2002) and business cycle synchronization (AGUIAR-CONRARIA; SOARES, 2011; PENTECÔTE; HUCHET-BOURDON, 2012; ARESTIS; PHELPS, 2016; BELKE; DOMNICK; GROS, 2017; GADEA-RIVAS; GÓMEZ-LOSCOS; BANDRÉS, 2018). The criteria for defining these groups range from geographic proximity and economic performance to arbitrary classifications such as the acronym PIIGS.[4].

The core-periphery classification is considered arbitrary by Paniagua, Sapena, and Tamarit (2017) and even problematic by Ahlborn and Wortmann (2018). Furthermore, Belke, Domnick, and Gros (2017) argue that there is no exact definition that allows for a clear determination of which countries belong to the core or the periphery. The main motivation of this study in applying cluster analysis to define the groups is to overcome the challenges identified in literature, such as subjectivity and the issue of selecting the criteria to be used for classification. This is possible within the cluster framework employed here since multiple representative groups are considered by the algorithm, ensuring that the outcome is data-driven rather than influenced by any *ad hoc* assumptions made by the researcher.

Although there are applications of cluster methodology in economics, such as those by Artis and Zhang (2001, 2002), Boreiko (2003), Camacho, Perez-Quirós, and Saiz (2006), Quah (2014), and Ahlborn and Wortmann (2018), to the best of our knowledge, the present study is the first to apply this methodology specifically to the study of solvency/sustainability of European public debt.

The grouping of countries based on common characteristics is also closely related to the concept of convergence clubs, which emerged in neoclassical growth theory (SOLOW, 1956; BARRO; SALA-I-MARTIN, 1991; MANKIW; ROMER; WEIL, 1992), considering

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[4] The term has a somewhat pejorative origin and includes Portugal, Ireland, Italy, Greece, and Spain. Some authors, such as Pragidis et al. (2015) and Ando, Greenwood-Nimmo, Shin (2022), already use "GIIPS" as a modification. Others, like Mink and de Hann (2013), consider the "GIPS" without Italy.

variables such as per capita income, capital stock, and technological progress. Since then, literature has explored various methodological approaches to either support or challenge the idea of convergence clubs. Specifically for the European Union, both at the national level, as in the studies by Monfort, Cuestas, and Ordoñez (2013) and Apergis, Panopulu, and Tsoumas (2010), and at the regional level, as seen in the works of Fischer and Stirböck (2006), Badinger, Müller, and Tondl (2004), as well as Battisti and Vaio (2008).

However, the evaluation of convergence clubs for countries grouped according to debt behavior remains unexplored. Thus, the present study aims to fill this gap by examining whether there is evidence of debt convergence clubs within the European Union.

Regarding the study of public debt solvency/sustainability, two main approaches stand out. The first focuses on unit root tests of public debt series and cointegration relationships between government revenues and expenditures to verify whether budgetary constraints are being met. In this approach, the concept of sustainability is related to the ability to service the current stock of debt based on the present value of expected future surpluses.

The second approach uses so-called fiscal reaction functions and aims to assess whether the authorities responsible for fiscal policy administration take corrective actions. More specifically, it evaluates how surpluses (or deficits) respond to changes in the debt stock. The intuition behind this approach is that, by implementing corrective measures, fiscal authorities prevent public debt from following an explosive trajectory, thereby reducing the risks associated with illiquidity and insolvency.

In the empirical literature, the earliest works focus on the U.S. economy. The seminal paper by Hamilton and Flavin (1986) plays a prominent role by incorporating public debt theory into a framework in which solvency could be tested empirically. Using data from the U.S. official budget from 1960 to 1981, the authors conclude that the government's budget constraint is consistent with the data, despite the presence of deficits throughout the period. In other words, it is concluded that the government's budget constraint was satisfied, implying the sustainability of fiscal policy.

Motivated by Hamilton and Flavin (1986), several other studies have empirically tested public debt sustainability. Wilcox (1989), considering the occurrence of structural breaks, finds evidence of the unsustainability of U.S. deficit policy only after 1974.

Based on the occurrence of exogenously determined structural break points in the cointegration relationships between government revenues and expenditures, Hakkio and Rush

(1991) conclude that U.S. deficits between the early 1970s and 1988 follow an unsustainable trajectory, corroborating the evidence found by Wilcox (1989).

Haug (1991) and Trehan and Walsh (1991), also based on cointegration tests, find evidence supporting the results of Hamilton and Flavin (1986), as well as Ahmed and Rogers (1995), who study whether the present value budget constraints, both governmental and external, remain valid in the presence of structural breaks. Using data from the United States and the United Kingdom, after conducting unit root tests and verifying cointegration relationships, the evidence points to a sustainable fiscal policy in the U.S., while the results for the United Kingdom are considered inconsistent.

However, in a study on the theoretical foundations of sustainability formulations, Bohn (1995) identifies issues with previous approaches. Therefore, Bohn (1998, 2007) proposes evaluating public debt solvency/sustainability through fiscal reaction functions. Since then, several studies have adopted this approach, primarily applied to the public debt of European countries. In this regard, Fincke and Greiner (2012) examine the response of the primary surplus as a percentage of GDP to changes in the debt-to-GDP ratio. The results indicate sustainable fiscal policies for all the countries analyzed, even for Portugal and Italy, countries that have historically faced budgetary challenges.

Baldi and Staehr (2015) examine the differences in fiscal behavior among various groups of European Union countries before and after the 2008 global financial crisis. The estimations of fiscal reaction functions for the crisis period reveal a stronger response of primary surpluses to changes in debt for all country groups, particularly among those that faced greater difficulties. This indicates a prudent (sustainable) fiscal policy in the sense of Bohn (1998).

Afonso and Jalles (2017) study the relationship between primary surplus and the debt-to-GDP ratio for a sample of eleven euro area countries. The results indicate fiscal policy sustainability in Belgium, France, Germany, and the Netherlands. In contrast, for Portugal, Ireland, Italy, Greece, and Spain, fiscal policy was considered unsustainable. Paniagua, Sapena, and Tamarit (2017) apply the fiscal reaction function methodology to a panel of eleven European Union countries. They conclude that most EU member states respond to increases in debt by adjusting their fiscal policies to ensure sustainability, although in some cases, the responses are weaker.

It is clear, therefore, that the literature remains inconclusive in many cases due to factors such as methodological approach, different specifications, and the time period considered (BAJO-RUBIO; DÍAZ-ROLDÁN; ESTEVE, 2006). Thus, the present study seeks to contribute to literature by addressing the problem from a different perspective. By using wavelet-based tools, which allow for the consideration of information in both time and frequency domains, this approach considers the dynamics of fiscal policy at different frequencies, providing a significant informational advantage in interpreting the results. Additionally, this approach does not require stationarity of the time series, a particularly important advantage in economics.

The application of wavelets in economics and finance has been growing in recent years, spanning various research fields. Since the pioneering work of Ramsey and Lampart (1998), which uses wavelets to decompose economic time series, several other applications have been explored. Gençay, Selçuk, and Whitcher (2001) and Wong et al. (2003) apply wavelets to exchange rates; studies on business cycle synchronization include Jagrič and Ovin (2004), Aguiar-Conraria and Soares (2011); monetary policy applications are found in Aguiar-Conraria, Azevedo, and Soares (2008) and Aguiar-Conraria, Martins, and Soares (2018); financial market applications are discussed in Lin et al. (2018), Aguiar-Conraria, Soares, and Sousa (2018), Matos Costa e Silva (2021), Matos et al. (2021); public debt is addressed in Lo Cascio (2015), Matos and Monteiro (2023), among others. The list is extensive, making it difficult to map all these applications. However, the studies mentioned above illustrate the growing popularity of wavelets. This study specifically employs Continuous Wavelet Transform (CWT).

The study of spillovers addressed in this work is based on connectivity measures. This methodology was initially developed by Diebold and Yilmaz (2009) and applied to study spillovers of volatility and returns across stock market indices. It is based on the well-known concept of forecast error variance decomposition (FEVD) from a VAR model. Since Diebold and Yilmaz (2009) used a Cholesky factorization to orthogonalize the shocks, the results depend on the ordering of the variables in the VAR. To overcome this issue, Diebold and Yilmaz (2012) extended the connectivity measures to a generalized VAR framework, where the results are independent of the ordering of the variables. However, the shocks are no longer orthogonal. Diebold and Yilmaz (2014) demonstrate the relationship between spillover indices and network literature concepts, showing that a group of related objects can be viewed as a connectivity network.

Since then, the literature on spillover indices has been applied to a variety of objects and research fields, ranging from market index returns and exchange rates to relationships between sectoral indices, among others (BUBÁK; KOČENDA; ŽIKEŠ, 2011; AKHTARUZZAMAN; BOUBAKER; SENSOY, 2021; CHATZIANTONIOU; GABAUER;

STENFORS, 2021; COSTA; MATOS; SILVA, 2022). In addition to these applications, there have also been modifications, such as the calculation of these measures within a quantile VAR (QVAR) framework (ANDO; GREENWOOD-NIMMO; SHIN, 2022).

All the studies cited earlier evaluate the dynamics of spillover indices based on rolling windows. However, this technique has two main disadvantages. The first is that the results are sensitive to the chosen window length, which is determined with a certain degree of arbitrariness, requiring the testing of several other potential window lengths. The second disadvantage is the loss of observations in the estimation process. For the present study, the latter is a critical issue due to the small sample size (as will be detailed in the next section).

To overcome these difficulties, this study applies connectivity measures within a time-varying parameter VAR (TVP-VAR) model framework. In addition to addressing the two disadvantages associated with using rolling windows, the TVP-VAR approach offers two additional advantages: a) changes in parameter values can be determined more precisely, and b) the results are less sensitive to outliers due to the estimation process being based on multivariate Kalman filters (ANTONAKAKIS; CHATZIANTONIOU; GABAUER, 2020).

The use of time-varying parameters allows for observing the different fiscal practices adopted by various governments over the sample period (NETO, 2020), as well as the adjustment mechanisms that characterize the government's dynamic response to changes in debt (AFONSO; JALLES, 2017). Antonakakis et al. (2018) and Gabauer and Gupta (2018) have already applied connectivity measures in a TVP-VAR model to economic policy uncertainty indices. Additionally, Balcilar, Gabauer and Umar (2021) and Antonakakis, Gabauer, and Gupta (2019) propose other alternative measures that also utilize time-varying parameters.

The next section describes the data used and highlights some descriptive statistics.

3. DATA

The data used refer to the time series of public debt stock (consolidated gross), surplus, revenues, and expenditures (total) of the general government sector [5] of the 27 European Union countries plus Norway. The series are quarterly, covering the period from Q1 2002 to Q4 2023, totaling 88 quarters, all expressed as a proportion of GDP.

The inclusion of Norway is due to its membership in the European Free Trade Association (EFTA)[6]. Thus, it has strong economic integration with the European bloc. Furthermore, Norway is the only EFTA country with available data on both debt and revenues and expenditures, which enables the application of the analyses with the largest dataset available.

All data were collected from Eurostat, the statistical office of the European Union. Additionally, the version of the dataset used refers to the update from April of this year. The surplus was calculated as the difference between total revenues and total expenditures, following the Eurostat methodology.[7] The concept of debt refers to the general government gross debt, also known as Maastricht debt, or simply public debt. This concept is preferred as it is based on a set of mandatory criteria that must apply to all countries, thereby enabling comparisons among the member states[8].

The choice to use quarterly data is driven by several factors. First, it allows for the inclusion of the greatest number of available observations. Second, it enables the consideration of fiscal maneuvers to correct potential imbalances (NETO, 2020). Third, quarterly data are more suitable for capturing adjustments within the fiscal year, which facilitates the detection of potential deviations from a sustainable trajectory (AFONSO; JALLES, 2017).

Below is a debt correlation graph. The darker the shade of blue and the larger the circle, the stronger the positive correlation. Conversely, the darker the shade of orange and the larger the circle, the stronger the negative correlation. An interesting pattern that can be observed is that countries with high debt-to-GDP ratios also exhibit a strong positive correlation. This suggests that such countries may be classified into groups based on their

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[5] Further information is available in: Glossary:General government sector - Statistics Explained

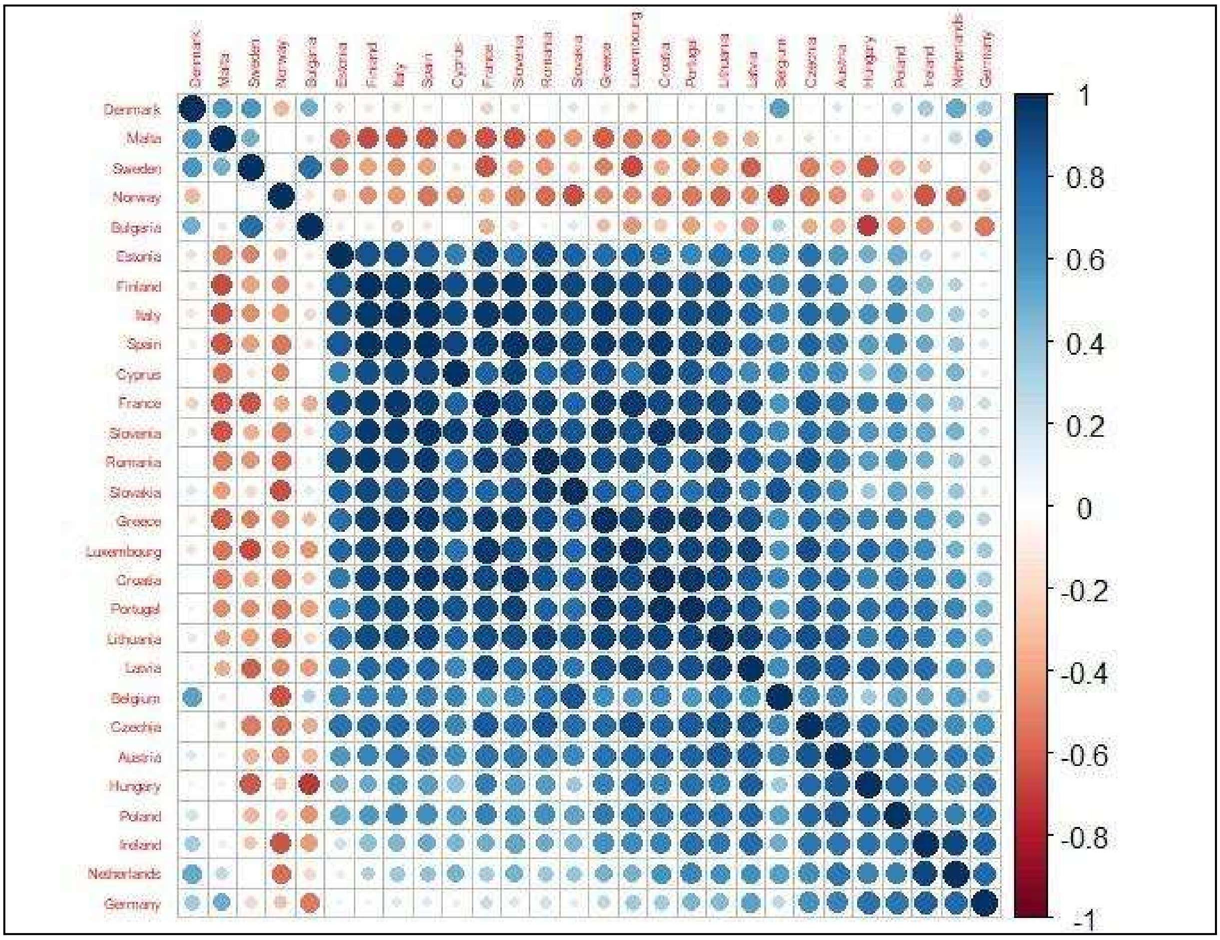
[6] It is an organization currently composed of the following countries: Iceland, Norway, Switzerland, and Liechtenstein. Further information is available at: Glossary:European Free Trade Association (EFTA) - Statistics Explained.

[7] Details can be found in: Glossary:Government revenue and expenditure - Statistics Explained

[8] For a detailed explanation, see: Glossary:Government debt - Statistics Explained (europa.eu)

debt behavior. Additionally, it indicates the potential for spillover effects from debt shocks, which will be addressed later.

Figure 1 – Correlation plot debt.



Source: Own elaboration.

Table 1 presents a summary of the descriptive statistics. Notably, 13 out of the 28 countries exhibit average debt-to-GDP ratios above the 60% threshold established by the Stability and Growth Pact. The last two columns of the table show the results of the ADF (Augmented Dickey–Fuller) test.

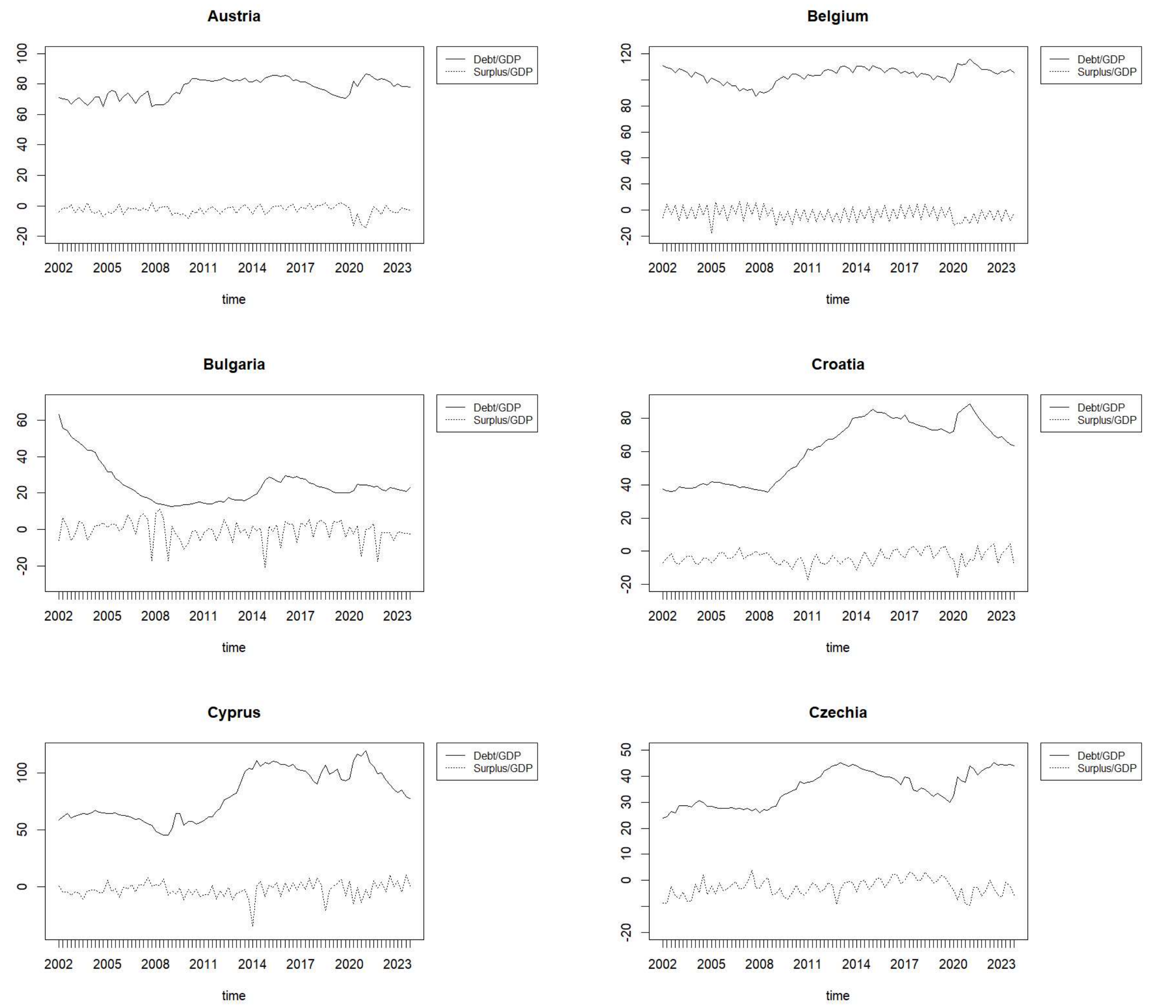
Table 1 – Descriptive Statistics for Debt.

Country	Observations	Mean	Standard Deviation	Variance	Kurtosis	Skewness	ADF	ADF (First difference)
Belgium	88	103.75	5.94	35.27	0.24	-0.77	-0.040	-2.75***
Bulgaria	88	24.52	10.73	115.20	2.42	1.60	-3.78**	-4.21***
Czechia	88	35.39	6.59	43.42	-1.48	-0.04	0.63	-4.22***
Denmark	88	39.39	6.78	45.96	-0.98	-0.16	-1.20	-5.28***
Germany	88	68.91	6.24	38.99	-0.64	0.58	-0.24	-4.98***
Estonia	88	9.01	4.70	22.10	0.07	1.08	1.01	-6.25***
Ireland	88	62.09	30.63	938.34	-0.75	0.56	-2.87*	-1.89*
Greece	88	150.30	36.03	1298.12	-1.54	-0.23	0.76	-5.18***
Spain	88	78.79	30.02	900.90	-1.70	-0.13	0.75	-3.85**
France	88	88.36	17.73	314.45	-1.29	-0.15	1.69	-5.66***
Croatia	88	61.14	18.18	330.36	-1.62	-0.18	0.37	-4.13***
Italy	88	125.85	15.20	230.98	-1.21	0.11	-3.49**	-4.32***
Cyprus	88	80.60	21.52	462.92	-1.50	0.12	-0.10	-5.67***
Latvia	88	31.41	13.74	188.83	-1.41	-0.61	-0.03	-2.75***
Lithuania	88	31.45	9.99	99.71	-1.31	-0.52	0.41	-5.20***
Luxembourg	88	17.42	7.01	49.08	-1.35	-0.43	1.11	-7.32***
Hungary	88	71.62	8.04	64.67	-0.72	-0.61	0.45	-7.16***
Malta	88	60.20	8.72	75.98	-0.67	-0.67	-1.01	-5.49***
Netherlands	88	55.05	7.31	53.49	-0.98	0.40	-0.37	-5.44***
Austria	88	77.21	6.26	39.14	-1.21	-0.36	0.25	-7.45***
Poland	88	50.11	4.36	19.03	-0.64	-0.10	-0.17	-5.38***
Portugal	88	103.65	27.77	771.27	-1.55	-0.37	0.26	-4.43***
Romania	88	30.55	11.63	135.20	-1.04	-0.17	0.73	-3.53**
Slovenia	88	52.99	23.14	535.32	-1.76	-0.06	0.51	-4.43***
Slovakia	88	46.76	9.81	96.20	-0.94	-0.47	0.38	-5.21***
Finland	88	55.51	13.69	187.30	-1.49	-0.09	1.06	-2.83*
Sweden	88	40.46	4.99	24.93	-0.63	0.29	-1.74*	-3.06**
Norway	88	38.15	6.03	36.35	-0.71	0.09	-0.22	-7.18***

Source: Own elaboration. *** Significant at the 1% level; ** Significant at the 5% level; * Significant at the 10% level.

Figure 2 presents the line graphs of debt and surplus, both as a percentage of GDP. Notable countries such as Belgium, France, Italy, Portugal, and Ireland stand out with high levels of debt and persistent deficits for much of the period. Also noteworthy is the sharp increase in the debt-to-GDP ratio during the sovereign debt crisis for all countries, with the exception of Bulgaria, Sweden, and Norway, which experienced a decline during this period. This is accompanied by a significant divergence between debt and surplus in Ireland, Latvia, Portugal, and Slovenia. Finally, it is interesting to note that Norway, being a country with lower debt levels and higher surpluses, also shows considerable variation throughout the entire period of analysis.

Figure 2 – Graphs for debt and surplus.



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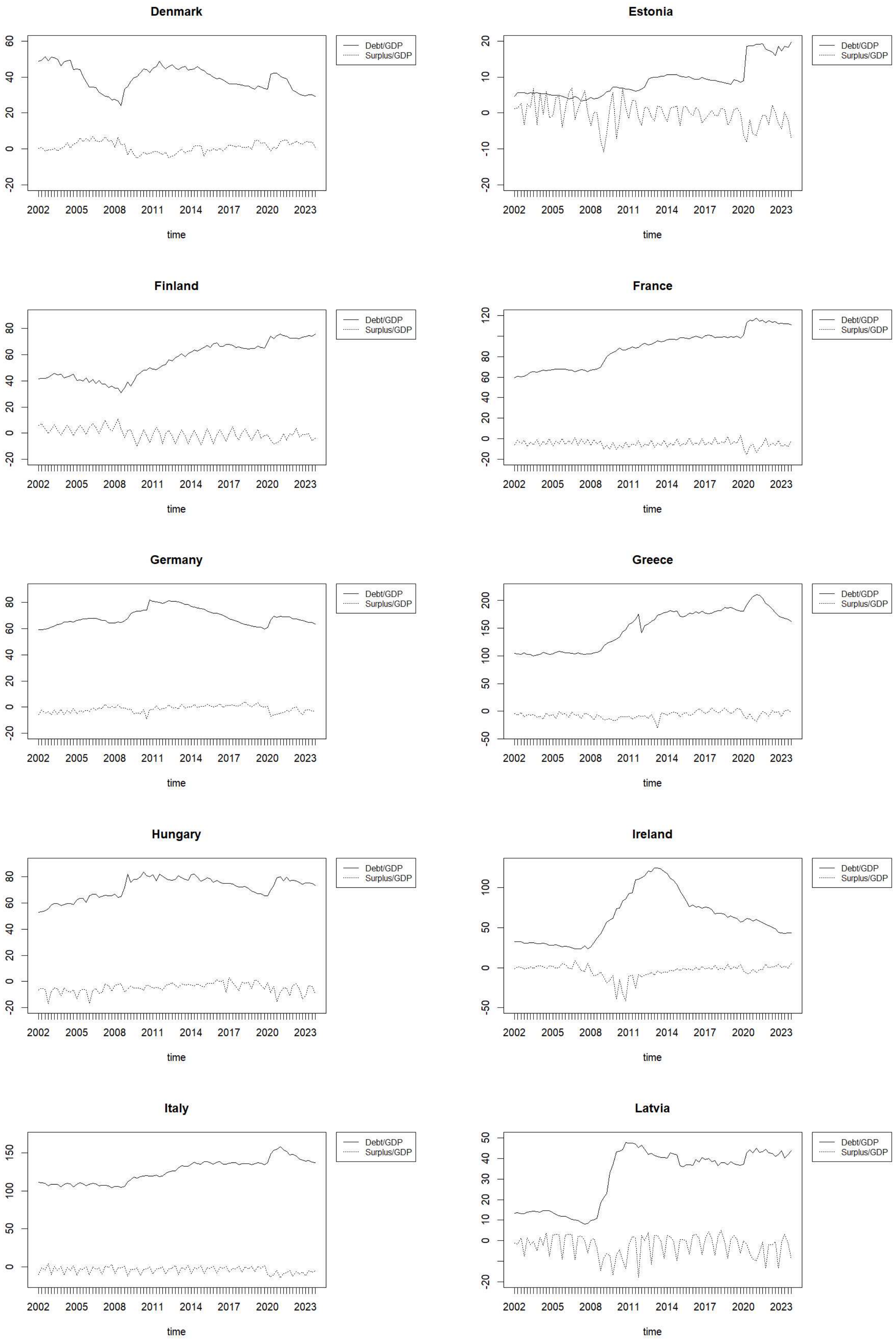
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Estonia
Finland
France
Germany
Greece
Hungary
Ireland
Italy
Latvia
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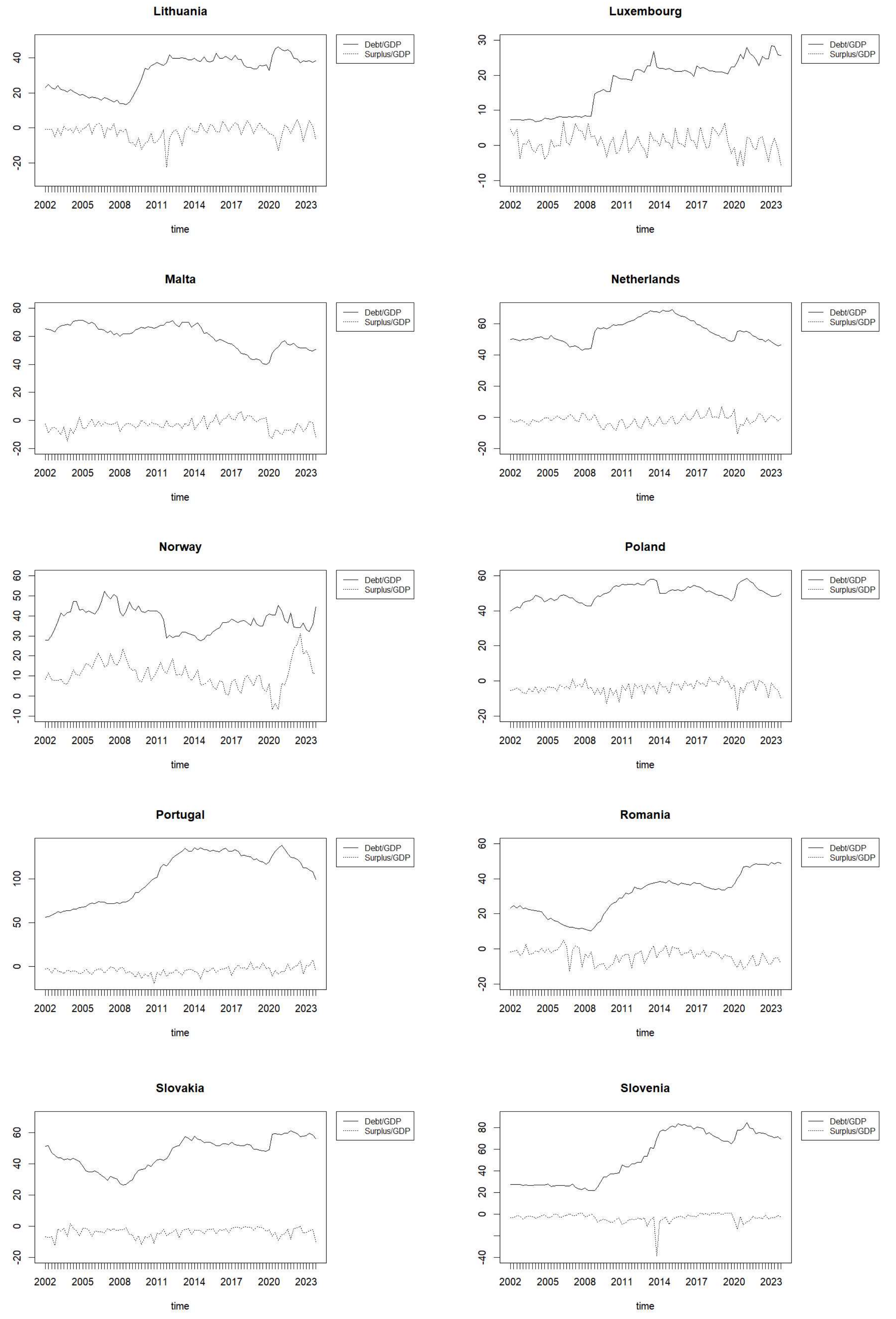
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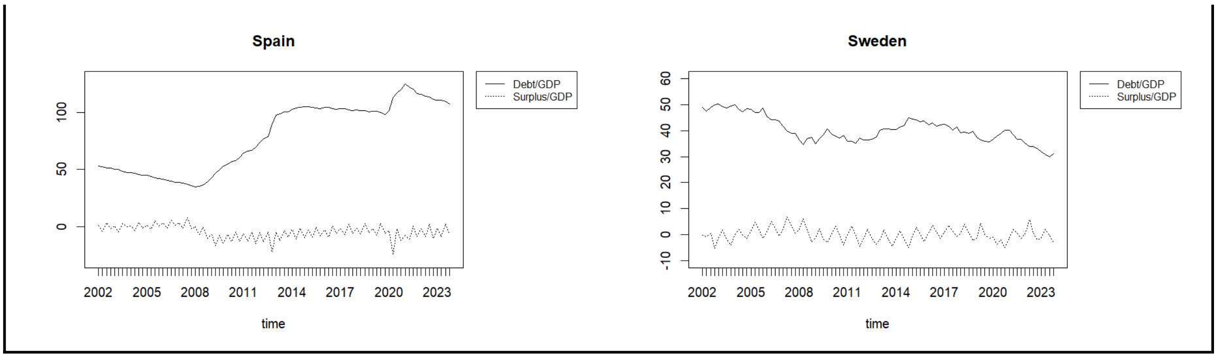


Source: Own elaboration.

4. METHODOLOGY

4.1 Cluster Analysis

By proposing the formation of distinct groups from a collection of objects, cluster analysis emerges as a widely explored alternative in the literature (see, for example, Maharaj, D'Urso, and Caiado, 2019). Indeed, cluster analysis is a technique for grouping similar objects based on some pre-established criterion. Given any set or sample of objects (such as data points), the objective is to organize the observations in such a way that those within each group are as similar as possible to one another, while the groups themselves are as distinct as possible. The concept of distance between groups is linked to the criteria used for grouping and may vary depending on the specific application and the type of object being clustered (MAHARAJ; D'URSO; CAIADO, 2019).

Cluster analysis is a broad research field with a wide variety of available methods. This study employs the *fuzzy c-means* algorithm, which belongs to the class of fuzzy algorithms, in contrast to hard algorithms. Fuzzy algorithms differ from hard algorithms in that the latter assign each object to a single group, while the former assign a 'degree of membership' or 'membership coefficient' to each object, with a value ranging from 0 to 1. This allows an object to belong to multiple groups simultaneously, with varying degrees of membership (MAHARAJ; D'URSO; CAIADO, 2019; KAUFMAN; ROUSSEEUW, 1990). The main advantages of this approach include the provision of more detailed information about the structure (pattern) inherent in the data, as well as its ability to effectively handle uncertainties and inaccuracies associated with real-world data (KAUFMAN; ROUSSEEUW, 1990). Additionally, the clustering process is less influenced by outliers (AHLBORN; WORTMANN, 2018).

The following subsection presents the algorithm employed, along with the validation criteria used to assess the quality of the clustering.

4.1.1 Fuzzy c-Means Clustering

The fuzzy c-means algorithm is based on 'centroids' or 'prototype' clusters, which are initially chosen either based on prior knowledge or, more commonly, randomly. The centroids can be regarded as representatives of the groups.

For example, if a group is initially formed with n objects, the average value of these n objects can be computed and used as a representation of the group. Subsequently, the distance of other objects from the centroid of this group can be calculated, thus assigning a degree of membership to each object. The closer an object is to the centroid of a given group, the higher its degree of membership to that group.

The location of the centroids is adjusted through an iterative process aimed at minimizing an objective function that depends on the distances between each observation and the centroids of the groups. During each iteration, the degrees of membership of the objects to each group are calculated. These degrees of membership are then used to recalculate the centroids, and the iterative process continues until the optimal solution is reached.

Following what was said in Maharaj, D'Urso and Caiado (2019) and Kaufman and Rousseeuw (1990), the fuzzy c-means algorithm can be described from a set $X = \left\{x_{ij}; i = 1, ..., I; j = 1, ..., J\right\}$ where x_{ij} indicates the j-th quantitative variable of object i.

According to Bezdek (1981), the algorithm seeks to solve the following constrained optimization problem

$$min: \sum_{i=1}^{I}\sum_{c=1}^{C} u_{ic}^{m} d_{ic}^{2} = \sum_{i=1}^{I}\sum_{c=1}^{C} u_{ic}^{m} \|x_i - h_c\|^2; \ \sum_{c=1}^{C} u_{ic} = 1, \ u_{ic} \geq 0 \qquad (1)$$

where u_{ic} represents the degree to which the i-th object belongs to the c-th cluster, d_{ic}^{2} is the square of the Euclidean distance between the i-th object and the centroid of the c-th cluster, given by $h_c = (h_{c1}, ..., h_{cj}, ..., h_{cJ})$; $m > 1$ is the parameter responsible for controlling the fuzziness of the cluster. In this context x_i represents the time series of the debt/GDP ratio of a specific country while h_c is the centroid time series of cluster c.

It can be shown that the solution to the above problem is given by (BEZDEK, 1981; KAUFMAN; ROUSSEEUW, 1990)

$$u_{ic} = \frac{1}{\sum_{c'=1}^{C}\left[\frac{\|x_i - h_c\|}{\|x_i - h_{c'}\|}\right]^{\frac{2}{m-1}}}, \ h_c = \frac{\sum_{i=1}^{I} u_{ic}^{m} x_i}{\sum_{i=1}^{I} u_{ic}^{m}} \qquad (2)$$

Finally, the algorithm can be described by the following steps.

1. Randomly initialize (usually) the u_{ic}.
2. Calculate the centroids and the distances of the objects from the centroids.
3. Compute the objective function.
4. Update u_{ic} according to the equation (2) above.

5. Return to step 2 and repeat the process until the optimum is reached.

The algorithm's stopping rule is as follows: if $max_{ic}\left\{\left|u_{ic}^{(k+1)} - u_{ic}^{(k)}\right|\right\} < \varepsilon$ for a given $\epsilon > 0$, stop. Otherwise, return to step 2; $u_{ic}^{(k+1)}$ denotes the degree to which the i-th object belongs to the c-th cluster at step $k + 1$ of the iterative process. In other words, if the maximum change in the degrees of membership between steps k and $k + 1$ is sufficiently small, the composition of the groups remains stable, indicating that the distances of the objects from their respective centroids are not undergoing significant changes. Otherwise, the degrees of membership would also change. This, in turn, implies that the objective function is experiencing minimal variations, which, for numerical purposes, can be considered as convergence. In this study, $\epsilon = 10^{-15}$, which is the value used in the computer packages used for calculations.

The validation criteria used in this study are Dunn's partition coefficient and its normalized version, as well as the silhouette. Here, we opt for the use of two validation criteria, in contrast to the use of only one, as in Ahlborn and Wortmann (2018) and Artis and Zhang (2001, 2002). The use of two validation criteria provides more information about the quality of the clustering, both regarding the optimal number of clusters and their composition.

Dunn's partition coefficient provides an indication of how close the fuzzy clustering is to a hard clustering, that is, how close the fuzzy clustering is to a partition. The coefficient is given by

$$F_C = \sum_{i=1}^{I} \sum_{c'=1}^{C} \frac{u_{ic}^2}{I} \quad (3)$$

If $u_{ic} = \frac{1}{C}$ for all c, the coefficient reaches its minimum value of $\frac{1}{C}$, which implies full fuzziness, meaning that objects cannot be clearly grouped. On the other hand, if $u_{ic} = 1$ for a given $c = c'$ and $u_{jc} = 0, \forall\, c \neq c'$, the coefficient reaches its maximum value of 1. This implies that the fuzzy process resulted in a hard clustering, meaning that each object was assigned to a single group. The normalized version of Dunn's coefficient is given by

$$F_C^{'} = \frac{F_C - (1/C)}{1 - (1/C)} = \frac{CF_C - 1}{C - 1} \quad (4)$$

Note that $0 \leq F_C^{'} \leq 1,\ \forall C > 1$. The closer the value is to 0, the fuzzier the process. The closer it is to 1, the closer the result is to a hard clustering.

Let $i \in (1, ..., I)$ and $p \in (1, ..., C)$ be the sets of objects and clusters, respectively. Given that object i belongs to cluster p, let a_{ip} denote the average distance (calculated by the squared Euclidean distance) from object i to all objects belonging to cluster p. Let d_{iq} be the average distance of object i to all objects belonging to any other cluster q, $q \neq p$. Finally, let b_{ip} be the minimum d_{iq} calculated for all $q \in (1, ..., C)$, $q \neq p$. In words, d_{iq} denotes the distance of object i to its nearest neighbor cluster. With this notation, the sillhouette of object i is defined as (ROUSSEEUW, 1987)

$$S_i = \frac{b_{ip} - a_{ip}}{max\{a_{ip}, b_{ip}\}} \quad (5)$$

The denominator is just a normalization term. The higher the value of S_i, the further away object i is from its nearest neighboring cluster and therefore the better the association of object i with cluster p. A negative value of S_i indicates that the association of object i with cluster p is wrong. The *sample silhouette* is defined as the average of the individual silhouettes

$$SIL = \frac{1}{I} \sum_{i=1}^{I} S_i \quad (6)$$

The higher the SIL value, the better the quality of the clustering process. A high SIL value indicates a lower intracluster distance and a higher intercluster distance, which is the goal of the clustering process.

The choice of m follows Nikkil and Bezdek (1995) that $1.5 \leq m \leq 2.5$. The standard value used is usually $m = 2$, which is the value adopted in this work. As a robustness test, the method was applied to values of $m = 1.5$ and $m = 2.5$ and the results are provided in Appendix A.

It should be noted that the algorithm requires initially defining how many clusters will be considered. Nikhil and Bezdek (1995) recommend that $2 < C < \sqrt{I}$, where C is the number of clusters and I is the number of objects to be grouped. In this case, as $I = 28$ we have $C \in (3, 4, 5)$. According to the validation criteria, the best results are achieved with $C = 3$.

4.2 Convergence

Based on the understanding that the concept of public debt solvency/sustainability inherently incorporates a long-term perspective, it is important to specifically consider the long-term dynamics in the relationship between countries. This becomes even more relevant when applied to the UE, as member states must meet the convergence criteria of the Stability and Growth Pact, including the debt criterion.

For this purpose, the present study employs the convergence approach developed by Phillips and Sul (2007, 2009) and extended by von Lyncker and Thoennessen (2017). Phillips and Sul (2007, 2009) propose a clustering algorithm based on a convergence test in a panel, accounting for heterogeneity across units and over time. Additionally, von Lyncker and Thoennessen (2017) propose two further procedures to refine the algorithm.

The algorithm is based on a convergence test for all units in the panel. Based on the intuition that convergent behavior, if present, should manifest in the most recent observations of the time series for each individual in the panel, the convergence test and its respective clustering algorithm are particularly suited to the purposes of the present study.

In summary, Phillips and Sul (2007) develop the original method, hereinafter referred to as the basic algorithm. Phillips and Sul (2009) refine the original approach by introducing an algorithm to test whether the groups identified by the basic algorithm can be merged (*merge algorithm*). Finally, von Lyncker and Thoennessen (2017) extend Phillips and Sul (2009) in two directions. First, by modifying the merge algorithm to improve consistency. Second, by proposing an algorithm to test for divergent units.

The convergence approach of Phillips and Sul (2007) is based on panel data and considers a time-varying parameter structure. The approach begins by considering a factor model represented by

$$X_{it} = \delta_i \mu_t + \varepsilon_{it} \tag{7}$$

where X_{it} denotes the i-th variable under study (in the present context, a country), μ_t represents a common factor, ε_{it} is an error term, and δ_i is a parameter that measures the idiosyncratic distance between the common factor μ_t and the systematic part of X_{it}. The model seeks to understand the evolution of the individual X_{it} in relation to μ_t through its two idiosyncratic components, δ_i and ε_{it}. Then, Phillips and Sul (2007) extend the model by allowing the idiosyncratic systematic component (δ_i) to vary over time. This extension allows for the consideration of heterogeneous behavior of the agent, as well as the evolution of such behavior over time. Additionally, the time-varying idiosyncratic systematic component, δ_{it}, now includes a random component that absorbs ε_{it}, thus enabling a potential convergence behavior of δ_{it} toward μ_t over time. The model is now represented by

$$X_{it} = \delta_{it}\mu_t \quad (8)$$

The above representation is used by Phillips and Sul (2007) to separate common components from idiosyncratic components in the panel through the decomposition of the data as

$$X_{it} = g_{it} + a_{it} \quad (9)$$

With g_{it} incorporating the systematic components and a_{it} containing the transitory components, this decomposition enables the separation of the components through transformation.

$$X_{it} = \left(\frac{g_{it}+a_{it}}{\mu_t}\right)\mu_t = \delta_{it}\mu_t, \forall\, i,\ t \quad (10)$$

where μ_t represents a single common component, and δ_{it} is a time-varying idiosyncratic element. Phillips and Sul (2007) refer to δ_{it} as the loading coefficient or factor and propose modeling it by constructing a relative measure of transition coefficients as:

$$h_{it} = \frac{X_{it}}{\frac{1}{N}\sum_{i=1}^{N} X_{it}} = \frac{\delta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}} \quad (11)$$

which measures the loading coefficient relative to the panel average at time *t*; N denotes the number of individuals in the panel. h_{it} is referred to as the relative transition path, outlining the transition trajectory of country i relative to the panel average. To proceed with the convergence test, Phillips and Sul (2007) construct the cross-sectional variance ratio H_1/H_t, where H_t is the cross-sectional variance of h_{it} defined as:

$$H_t = \frac{1}{N}\sum_{i=1}^{N}\left(h_{it} - 1\right)^2 \quad (12)$$

In this context, convergence implies that an individual unit approaches the sample mean over time. Specifically, if $\delta_{it} \rightarrow \delta\ then\ h_{it} \rightarrow 1$. Consequently, this implies that $H_t \rightarrow 0$ $t \rightarrow \infty$. To establish the null hypothesis of the test, the authors propose the following semi-parametric specification for δ_{it}:

$$\delta_{it} = \delta_i + \sigma_i \xi_{it} L(t)^{-1} t^{-\alpha} \quad (13)$$

In the last equation, δ_i is fixed, $L(t)$ is a slowly varying increasing function, which in this context means that $L(t) \rightarrow \infty$ as $t \rightarrow \infty$, α represents a decay rate (or speed of convergence), $\sigma_i > 0$ and ξ_{it} is an i.i.d. random error term with $E\left(\xi_{it}\right) = 0$ and $Var\left(\xi_{it}\right) = 1$, weakly dependent on *t*. According to Monfort, Cuestas, and Ordoñez (2013), weak dependence on t ensures that the stochastic component declines asymptotically, such that the trend vanishes, and each coefficient converges to δ_i.

Following Phillips and Sul (2007), $L(t) = log(t)$, ξ_{it} introduces time-varying components specific to each country, and the magnitude of α determines whether δ_{it} exhibits convergent or divergent behavior. This formulation guarantees convergence of δ_{it} for all $\alpha \geq 0$. The same authors further highlight that even if $\delta_i = \delta_j$, $\forall\, i \neq j$, the model still allows for transitional periods during which $\delta_{it} \neq \delta_{jt}$, accommodating both heterogeneity and transitional divergence among countries.

The procedure for grouping countries is based on the following convergence test, known as the *log t*-test, with the following hypotheses:

$$H_0: \delta_i = \delta,\ \alpha \geq 0;\ H_1: \delta_i \neq \delta,\ \forall i\ ou\ \alpha < 0 \quad (14)$$

Phillips and Sul (2007) outline the following steps for implementing the test:

1. Compute the cross-sectional variance H_1/H_t.
2. Run the following regression and compute a conventional robust t-statistic, $t_{\hat{b}}$, for the coefficient $\hat{b}$ using an estimate of the long-run variance of the regression residuals

$$log\left(\frac{H_1}{H_T}\right) - 2log(L(t)) = \hat{a} + \hat{b} \log log\ (t) \ + \hat{u}_t \qquad (15)$$

For $t = [rT], [rT] + 1, ..., T; r > 0, L(t) = \log log\ (t)$ and $\hat{b} = 2\hat{\alpha}$, where $\hat{\alpha}$ is the estimate of α under H_0. Additionally, $[rT]$ denotes the integer part of rT for some fraction $r > 0$, meaning that $r\%$ of the data is discarded. This procedure is crucial for addressing issues related to limiting distributions and the test's power properties. Based on Monte Carlo simulations, Phillips and Sul (2007) recommend setting $r = 0.3$.

3. Apply a one-sided *t*-test robust to heteroskedasticity and autocorrelation for the null hypothesis α≥0, using $\hat{b}$ and HAC (Heteroskedasticity- and Autocorrelation-Consistent) standard errors. At the 5% significance level, the null hypothesis is rejected if $t_{\hat{b}} <- 1.65$.

The rejection of the null hypothesis of convergence means that the panel of countries, as a whole, does not converge. However, it is still possible for there to be convergence clubs or even divergent groups. To assess these possibilities, Phillips and Sul (2007) develop the following algorithm, which is based on the existence of a core group, G_k, with k members and convergent behavior. The steps of the algorithm are as follows:

1. Order the individuals in the panel (in this case, countries) according to the last observation. This is because when there is evidence or suspicion of convergence, this behavior is more apparent in the final observations of the time series. The individuals are ordered in descending order according to their respective last observations.
2. The core subgroup G_k is formed for some $N > k≥2$ by executing the *log t*-test and calculating the statistic $t_k = t(G_k)$. The size of the core group, k^*, is chosen based on the following criterion:

$$k^* = argmax_k\left\{t_k\right\} st. t_k >- 1.65 \qquad (16)$$

In words, what is being done is selecting the core group as the one with the highest test statistic value among all the groups that showed evidence of convergence, given by the condition $t_k >- 1.65$. Intuitively, this means choosing the group that presents the "strongest" evidence of convergence. If there is a convergence group with all the individuals included, then the size of the core group is N. If $min\{t_k\} >- 1.65$ does not hold for $k = 2$, then the largest individual in G_k is removed from each subgroup, and new subgroups $G_{2j} = \{2, ..., j\}$ for $2 \leq j \leq N$ are formed. The step is repeated with the statistic $t_j = t(G_{2j})$. If again the condition $min\{t_j\} > -1.65$ is not satisfied for $j = 2$, the process is repeated, removing the largest individual from G_j and proceeding as before. If $t_j > -1.65$ is not observed for any two chosen units, it is concluded that there are no convergent subgroups in the panel. Otherwise, the core group G_{k^*} is identified.

3. Let $G^c_{k^*}$ denote the complement of G_{k^*}. A member of $G^c_{k^*}$ is added to G_{k^*}, and the *log* t-test is executed, with the test statistic for this regression denoted by $\hat{t}$. If $\hat{t} > c$, the individual is included in a convergence subgroup, where c is a critical value. This procedure is repeated with each of the remaining units to form the first convergence subgroup. Intuitively, the first convergence subgroup is the "strongest" convergence group after the core group. Next, the *log* t-test should be performed for this first convergence subgroup, and $t_{\hat{b}} > -1.65$ should be observed for the group as a whole. If not, the critical value c is increased to enhance the discriminatory power of the test, and the process is repeated until $t_{\hat{b}} > -1.65$ for the first convergence subgroup.
4. Stopping rule. A subgroup is formed with the individuals for whom $\hat{t} < c$ in step 3. The *log* t-test is executed for this subgroup. If $t_{\hat{b}} > -1.65$, it is concluded that there are two convergent subgroups in the panel. If not, steps 1 to 3 are repeated for this subgroup in search of smaller convergence subgroups. If in step 2 of this case there is no k such that $min\{t_k\} > -1.65$, it is concluded that these remaining individuals diverge.

In the present study, $c = 0$ is used, following the recommendation of Phillips and Sul (2007) based on Monte Carlo simulations. The higher the value of c, the more conservative the search for new members of the groups. This is because a higher c value makes it more difficult to meet the criterion for belonging to the group being evaluated, which also implies a lower chance of assigning an individual to the wrong group. The intuition behind this is that an individual who achieves $\hat{t} > c$ for a higher c value means that the individual indeed shows strong evidence of convergence to the group in question.

However, Phillips and Sul (2009) highlight that this procedure may result in more convergence groups than actually exist. To address this issue, Phillips and Sul (2009) propose applying the *log t*-test to adjacent groups, i.e., to groups 1 and 2, 2 and 3, and so on. If $t_{\hat{b}} > -1.65$, the groups are merged at the 5% significance level. If P groups (or clubs) are initially identified, $P - 1$ convergence tests would be conducted between these groups.

However, von Lyncker and Thoennessen (2017) argue that in the presence of transition between groups, as discussed by Phillips and Sul (2009), it is possible that a given group contains elements converging to the next higher group, while other elements converge to the next lower group. For example, group 2 could have some of its elements converging to group 1 and others converging to group 3. In this way, von Lyncker and Thoennessen (2017) state that simply merging all adjacent groups with significant values of the test statistic may form groups in cases where the hypothesis of convergence is rejected. Furthermore, if many similar groups are identified, *ex-post* manual merging can be ambiguous. To address this issue, von Lyncker and Thoennessen (2017) propose the following algorithm (*merge algorithm vLT*):

1. *Merging Vector*. Starting with P groups, a *log t*-test is executed with the adjacent groups in order to obtain a vector $(M \times 1)$ of test statistics $t_{\hat{b}}(m)$, where $m = 1, 2, \ldots, M = P - 1$. Note that m denotes the number of adjacent group pairs considered for the convergence test.
2. *Merging Rule*. The rule starts with the first element of the vector from the previous step. If $t_{\hat{b}}(m) > -1.65$ and $t_{\hat{b}}(m) > t_{\hat{b}}(m + 1)$, then the two groups used to calculate $t_{\hat{b}}(m)$ are merged, and the algorithm restarts from step 1. If $t_{\hat{b}}(m) < -1.65$ and/or $t_{\hat{b}}(m) < t_{\hat{b}}(m + 1)$, the merging rule is applied to all subsequent pairs of $t_{\hat{b}}(m)$. To clarify, consider groups 1, 2, and 3. Groups 1 and 2 are used to form $t_{\hat{b}}(1)$, while groups 2 and 3 are used to form $t_{\hat{b}}(2)$. If $t_{\hat{b}}(1) > -1.65$, there is evidence that groups 1 and 2 converge with each other, and in this case, it would make sense to merge them. A similar interpretation applies for the case when $t_{\hat{b}}(2) > -1.65$. The criterion $t_{\hat{b}}(m) > t_{\hat{b}}(m + 1)$ determines whether group 2 will be merged with group 1. If so, it indicates that group 2 is closer to converging to group 1 than to group 3. Note also that if $t_{\hat{b}}(2) < -1.65$ groups 1 and 2 should be merged because, in this case, $t_{\hat{b}}(1) > -1.65 > t_{\hat{b}}(2)$.
3. *Last Element*. If $t_{\hat{b}}(m = M) > -1.65$, then the last two groups should be merged.

Finally, von Lyncker and Thoennessen (2017) argue that while the previous algorithms provide more statistical significance for the groups formed and address the problem of overdetermination of the number of groups, there is still an issue to be dealt with. Units considered divergent in the basic Phillips and Sul (2007) algorithm may no longer remain divergent after the application of the *merge algorithm vLT*. This is because divergent units in the basic Phillips and Sul (2007) algorithm may end up being associated with some of the new groups merged by the *merge algorithm vLT*. Therefore, it is necessary to test whether the remaining divergent units can be included in the new groups or if they form their own convergence group. To address this, von Lyncker and Thoennessen (2017) propose the following algorithm:

1. *Divergence Club*. A *log t* test is conducted for all divergent units. If $t_{\hat{b}} >- 1.65$, the divergent units form their own group, and the algorithm stops.
2. *Merging Table*. A *log t* test is conducted for each divergent unit and for each group one at a time. The results are saved in a matrix $(d \times p)$, where each row d represents a divergent unit and each column p represents a convergence group.
3. *Merging Rule*. If the largest value of $t_{\hat{b}}$ in the table is greater than a given critical value e^*, the respective divergent unit is added to the corresponding club, and the algorithm restarts from step 1.
4. *Stopping rule*. The algorithm stops when the *merging table* no longer contains any $t_{\hat{b}} > e^*$. All remaining regions are truly divergent.

The value of e^* used in the present study is $e^* =- 1.645$, at the 5% significance level, as recommended by von Lyncker and Thoennessen (2017).

4.3 Solvency

Since the present study is based on the works of Bohn (1998, 2007) to assess the solvency/sustainability of public debt through fiscal reaction functions, the wavelet approach adequately serves these purposes. This is because the tools to be presented below provide information on both lead-lag relationships between debt and surplus, as well as the magnitude of these relationships, with the advantage of capturing comovements across different frequencies and associating them with significant events occurring in the sample period. Applied to this context, it embodies the essence of the fiscal reaction function method while providing additional information and handling well the well-known characteristics of economic variables such as cycles at different frequencies.

According to Aguiar-Conraria and Soares (2011), the wavelet transform of a time series estimates the spectral characteristics of the series, decomposing its periodic components and evaluating the evolution of these components over time.

The Continuous Wavelet Transform (CWT) approach offers several advantages over alternatives in the time domain, such as standard time series methods. One of the main advantages is that it does not require the stationarity of the series, which is especially important in economics and finance. Compared to other frequency domain alternatives, such as the Fourier Transforms, wavelets stand out due to their redundancy, meaning they preserve the temporal information of the original series while allowing for the assessment of spectral characteristics and associating them with the sample period, thus enabling a better interpretation of the results. Aguiar-Conraria and Soares (2014) further emphasize that CWTs offer greater flexibility in the choice of wavelet functions and can detect hidden patterns in the data that may not be identified using other approaches.

The following presents the methodology for the case of two variables and its extension to the multivariate case. The presentation here follows the approach outlined in Aguiar-Conraria and Soares (2011) and Aguiar-Conraria, Martins, and Soares (2018).

4.3.1 Continuous Wavelet Transform (CWT)

Starting from a "mother wavelet" function ψ, a family of functions can be constructed from it through scaling and translation:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \qquad s, \tau \in R,\ s \neq 0 \tag{17}$$

where s is a scaling factor that determines the width of the wavelet, and τ is a time localization parameter. For $|s| < 1$, the wavelet is compressed, which allows capturing high-frequency variations. On the other hand, for $|s| > 1$, the wavelet is stretched, enabling the capture of low-frequency variations. The factor $\frac{1}{\sqrt{|s|}}$ ensures that $||\psi|| = 1$ where $||.||$ denotes the Euclidean norm.

Given a function $x(t)$ such that $\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$, the Continuous Wavelet Transform (CWT) corresponding to ψ is defined by

$$W_{x;\psi}(\tau, s) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{|s|}}\overline{\psi}\left(\frac{t-\tau}{s}\right)dt \tag{18}$$

It is evident that the Continuous Wavelet Transform (CWT), $W_{x;\psi}(\tau, s)$, is a function of the location and scale parameters. Therefore, it can be observed in the time-scale plane (the relationship between scale and frequency will be discussed later), where τ provides the time location, and s provides the location in the scale. The overline on ψ denotes the complex conjugate.

From now on, all wavelet measures that will be presented are constructed based on the CWT and, therefore, are also functions of τ and s. To simplify the notation, the argument (τ, s) will be omitted.

4.3.2 Wavelet Power Spectrum

The first important measure to be presented is the Wavelet Power Spectrum (WPS), which provides the distribution of the variance of the original series $x(t)$ in the time-scale plane. According to Aguiar-Conraria, Martins, and Soares (2018), the WPS (local as it depends on τ and s) can be defined as

$$(WPS)_x = W_x\overline{W_x} = \left|W_x\right|^2 \tag{19}$$

When the wavelet ψ is complex-valued, it can be divided into real part $R\{W_x\}$ and imaginary part $J\{W_x\}$, and represented in its polar form as $W_x = \left|W_x\right|e^{-i\phi_x}$, with $\phi_x \in (-\pi, \pi]$ denoting the phase.

4.3.3 Bivariate Wavelet Measures

Given two time series $x(t)$ and $y(t)$, Aguiar-Conraria, Martins, and Soares (2018) define the Cross-Wavelet Transform (XWT) as

$$W_{yx} = W_y\overline{W_x}, \tag{20}$$

whose absolute value is the Cross-Wavelet Power, $\left|W_{yx}\right|$; W_y and W_x denote the wavelet transforms of $x(t)$ and $y(t)$, respectively. The bar on top denotes, as before, the complex conjugate. This measure provides the local covariance between the two series at each time and scale location. The following is the Complex Wavelet Coherency, defined by

$$\varrho_{yx} = \frac{S(W_{yx})}{\left[S\left(|W_y|^2\right)S\left(|W_y|^2\right)\right]^{1/2}} \,, \tag{21}$$

where S denotes a smoothing operator in time and scale. The absolute value of the Complex Wavelet Coherency is called the Wavelet Coherency and is denoted by

$$R_{yx} = \frac{|S(W_{yx})|}{\left[S\left(|W_y|^2\right)S\left(|W_y|^2\right)\right]^{1/2}} \,, \tag{22}$$

with $0 \le R_{yx} \le 1$. The Complex Wavelet Coherency can also be written in its polar form as $\varrho_{yx} = |\varrho_{yx}| e^{i\phi_{yx}}$, where the angle ϕ_{yx} is called the phase-difference and is obtained through

$$\phi_{yx} = Arctan\left(\frac{J(S(W_{yx}))}{R(S(W_{yx}))}\right) \tag{23}$$

A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{yx} \in (0, \frac{\pi}{2})$, the series move in phase, but the time-series y leads x, while if $\phi_{yx} \in (-\frac{\pi}{2}, 0)$ then it is x that is leading. A phase-difference of $\phi_{yx} = \pm\pi$ indicates an anti-phase relation. If $\phi_{yx} \in (\frac{\pi}{2}, \pi)$ then x is leading and time-series y is leading if $\phi_{yx} \in (-\pi, -\frac{\pi}{2})$. The phase-difference gives information about the possible delays of the oscillations of the two series, as a function of time and frequency.

Following Aguiar-Conraria, Martins, and Soares (2018), we define $S_{yx} = S\left(W_{yx}\right)$ as the smoothed cross-wavelet transform of the series y and x, $\sigma_x = \sqrt{S\left|W_x\right|^2} = \sqrt{S_{xx}}$ denotes the square root of the smoothed power spectrum of the series xxx, with analogous notation for the series y. With this notation, the complex wavelet gain of y over x can be denoted by:

$$G_{yx} = \frac{S_{yx}}{S_{xx}} = \varrho_{yx} \frac{\sigma_y}{\sigma_x} \tag{24}$$

Mandler and Scharnagl (2014) define the wavelet gain as the modulus of the complex wavelet gain, expressed as:

$$G_{yx} = \frac{|S_{yx}|}{S_{xx}} = R_{yx} \frac{\sigma_y}{\sigma_x}. \tag{25}$$

According to Engle (1976), the wavelet gain can be interpreted as the coefficient of a regression of y on x.

4.3.4 Multivariate Wavelet Measures

Since this study is interested in evaluating the relationships between surplus and debt while controlling for the effects of revenues and expenditures, the use of a multivariate framework becomes necessary. In this case, Aguiar-Conraria and Soares (2011) extend wavelet measures to the case of multiple variables.

For the time series $x_1, ..., x_p$, Aguiar-Conraria and Soares (2011) denote the complex partial wavelet coherency between x_1 and x_j (where $1 \leq j \leq p$) by $\varrho_{1j.q_j}$, and the partial wavelet coherency between x_1 and x_j as the absolute value of $\varrho_{1j.q_j}$, $r_{1j.q_j} = \left|\varrho_{1j.q_j}\right|$, where $q_j = \{2, ..., p\}\backslash\{j\}$, i.e., an abbreviated notation to indicate all indices of the series except for index j. With this notation, the phase-difference of x_1 over x_j, given all other series, is also defined as $\phi_{1j.q_j}$, which represents the angle of $\varrho_{1j.q_j}$.Thus, we have:

$$\varrho_{1j.q_j} = -\frac{L^d_{j1}}{\sqrt{L^d_{11}L^d_{jj}}} \tag{26}$$

With L^d_{ij} denoting the cofactor of the element in position (i, j) of the smoothed cross-wavelet spectra matrix, the expression for $r_{1j.q_j}$ is given by:

$$r_{1j.q_j} = \frac{\left|L^d_{j1}\right|}{\sqrt{L^d_{11}L^d_{jj}}} \tag{27}$$

Thus, the phase-difference becomes

$$\phi_{1j.q_j} = Arctan\left(\frac{J(\varrho_{1j.q_j})}{R(\varrho_{1j.q_j})}\right) \tag{28}$$

Finally, the squared multiple wavelet coherency between the series x_1 and all other series $x_2, ..., x_p$ is denoted in terms of the squared partial coherencies as follows: $R^2_{1(2,...,p)} = 1 - \left(1 - r^2_{12}\right)\left(1 - r^2_{13.2}\right)...(1 - r^2_{1p.2...(p-1)})$, where, for example, $r^2_{1p.2...(p-1)}$ is the partial coherency between x_1 and x_p, controlling for the effects of the other series. For details of the derivations of the measures above, one should refer to Appendix A of Aguiar-Conraria and Soares (2018).

4.3.5 Location and Scale-Frequency Relation

Since the CWT is a function of the location parameters in both time and scale, there is a trade-off between the precision of the measures in scale and in time, due to the application of Heisenberg's Uncertainty Principle (Aguiar-Conraria and Soares, 2011). This principle states that for certain pairs of related quantities (such as position and momentum in quantum mechanics), the more precise the measurement of one of these quantities, the less precise the measurement of the other. There is also a lower limit to the uncertainty of knowledge of these measures. In the context of the CWT, this means that the more precise the location in the time domain, the less precise the location in the frequency domain, and vice versa.

Given that, technically, the CWT provides information in the time-scale plane, it is necessary to convert scales into frequencies. Lilly and Olhede (2009) highlight three ways to accomplish this, associating three frequency measures to each wavelet ψ, namely, energy-frequency, peak frequency, and central instantaneous frequency. Each of these frequency measures is linked to the scale by the formula $\omega(s) = \frac{\omega_\psi}{s}$, where ω_ψ denotes one of the aforementioned frequency measures.

As highlighted by Aguiar-Conraria and Soares (2011), the choice of the wavelet function is of utmost importance. For the study of cycles, it is essential to have information about both amplitude and phase. Therefore, a complex-valued wavelet should be chosen. Within this class, Morlet wavelets play a special role. This is because they possess two fundamental characteristics. First, the three frequency measures (energy-frequency, peak frequency, and central instantaneous frequency) used for scale conversion are identical. Second, Morlet wavelets achieve the lower bound of the Heisenberg Uncertainty Principle. This means they reach the lowest possible level of uncertainty in time-frequency localization. Following Aguiar-Conraria, Martins, and Soares (2018), this work employs the Morlet wavelet, given by

$$\psi_{\omega_0(t)} = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2} \quad (29)$$

selecting $\omega_0 = 6$ (TORRENCE; COMPO, 1998). Using the Fourier frequency formula $f(s) = \frac{\omega_\psi}{2\pi s}$ and knowing that for the Morlet wavelet $\omega_\psi = \omega_0$, it follows that $f(s) = \frac{6}{2\pi s}$, which implies $f(s) \approx \frac{1}{s}$. This approximation simplifies the conversion of scales into frequencies and, consequently, the interpretation of results, giving them economically relevant meaning.

4.3.6 Significance and COI

So far, no significance tests are known for multiple and partial coherence measures. In this study, the recommendation of Aguiar-Conraria, Martins, and Soares (2018) is followed, relying on Monte Carlo simulations. An ARMA (1,1) model is fitted to each series, extracting errors from a Gaussian distribution with variance equal to the estimated error terms. This procedure is repeated 5.000 times for each series to derive critical values at the 5% and 10% significance levels, which are represented in the heat maps by black and gray contour lines, respectively.

For the partial phase-difference and gain measures, average values are shown for each point in time within the selected frequency bands. Since the development of significance tests for these measures remains an open question, Aguiar-Conraria, Martins, and Soares (2018) is once again followed, interpreting these measures only within the significant regions corresponding to the partial coherence graphs.

Another important point to highlight is that, since the CWT is applied to a finite dataset, edge distortions will occur due to artificially generated values at the beginning and end of the transform. Thus, there is a region on the heatmaps known as the Cone of Influence (COI) where these edge effects are observed. In such regions of the time-frequency plane, results should be interpreted with caution. This region is identified in the graphs as the area outside the U-shaped black line boundary.

4.4 Connectedness

In order to address the final question posed at the beginning of this study, whether greater integration among European countries translates into debt shock spillovers and, if so, to what extent, the connectedness approach proposed by Diebold and Yilmaz (2009, 2012, 2014) is utilized, based on spillover indices. This approach is now applied within a TVP-VAR (Time-Varying Parameter Vector Autoregressions) framework as presented in Antonakakis, Chatziantoniou, and Gabauer (2020).

It is important to note that within a group of entities, what happens to one member can affect the entire group. This is no different when considering political and/or economic groups such as the European Union (EU). For instance, this dynamic was evident during the sovereign debt crisis. A key motivation behind the bailout packages for Greece at the onset of the crisis was the concern that the effects of Greece's collapse could spread to the rest of the group, jeopardizing its survival. This initial evidence of crisis spillovers was also reflected in financial market movements at the time, such as the rising sovereign debt interest rates in other countries (MINK; de HAAN, 2013), as well as bailout packages for other severely affected members, including Portugal, Ireland, and Spain. Moreover, Collignon (2012) emphasizes that in a context of greater economic integration within a single-currency area, a liquidity crisis impacting the sovereign debt of some member states can threaten the solvency of the entire group.

Given the observations above, it is crucial to identify which countries most influence and/or are most influenced within the EU group. This information is particularly relevant for policymakers, as understanding which member(s) are most sensitive to shocks and which will be most affected enables strategies to map the transmission of shocks and preemptively implement measures to minimize negative impacts. It is akin to observing a domino effect, where one knows the exact position of each piece and the intensity with which one piece affects another.

In this context, the connectedness methodology is well-suited to the proposed objectives.

The methodology used is presented below. The presentation follows the framework outlined in Antonakakis, Chatziantoniou, and Gabauer (2020).

4.4.1 TVP-VAR

The proposed TVP-VAR model allows the variance-covariance matrices to vary over time through the estimation of a Kalman Filter with forgetting factors, as per Koop and Korobilis (2014). A TVP-VAR(p) model can be expressed as

$$y_t = A_t z_{t-1} + \epsilon_t \qquad \epsilon_t | \Omega_{t-1} \sim N\left(0, \Sigma_t\right) \qquad (30)$$

$$vec(A_t) = vec(A_{t-1}) + \xi_t \qquad \xi_t|\Omega_{t-1} \sim N(0, \Xi_t) \tag{31}$$

$$z_{t-1} = (y_{t-1}\, y_{t-2}\, \vdots\, y_{t-p}) \qquad A_t^{'} = (A_{1t}\, A_{2t}\, \dots\, A_{pt}) \tag{32}$$

where Ω_{t-1} represents the information set up to period $t - 1$, y_{t-1} and z_{t-1} denote $m\times1$ and $mp\times1$ vectors, respectively, where m is the number of variables considered. The coefficient matrix A_t is $m \times mp$, composed of $m \times m$ submatrices A_{it}, and ϵ_t is an $m\times1$ vector of error terms with a time-varying variance-covariance matrix Σ_t of dimension $m \times m$. Lastly, $vec(A_t)$ and $vec(A_{t-1})$ are vectorizations of A_t with dimensions $m^2p\times1$, and ξ_t is $m^2p\times1$, while its time-varying variance-covariance matrix Ξ_t has dimensions $m^2p \times m^2p$.

Since the connectedness measures are based on generalized impulse response functions (GIRF) and generalized forecast error variance decompositions (GFEVD) as per Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998), it is necessary to consider the VAR in its vector moving average (VMA) representation given by:

$$y_t = \sum_{j=0}^{\infty} J^{'} M_t^j J \epsilon_{t-j} \qquad B_{jt} = J^{'} M_t^j J, \quad j = 0, 1, \dots \tag{33}$$

$$y_t = \sum_{j=0}^{\infty} B_{jt} \epsilon_{t-j} \tag{34}$$

where

$$M_t = (A_t \;\; I_{m(p-1)} \;\; 0_{m(p-1)\times m}) \qquad \eta_t = (\epsilon_t\, 0\, \vdots\, 0) = J\epsilon_t \qquad J = (I\, 0\, \vdots\, 0) \tag{35}$$

where M_t is an $mp \times mp$ matrix, η_t is an $mp\times1$ vector, and J is an $mp \times m$ dimensional matrix, with B_{jt} being an $m \times m$ matrix.

The following notation, $\Psi_{ij,t}(H)$, will be used to denote the GIRFs at a forecast horizon of H periods ahead. $\Psi_{ij,t}(H)$ represents the responses of all *j* variables to a given shock in variable *i*. In the absence of a structural model, a shock to variable *i* will be computed as the difference between the forecasts in a scenario where variable *i* experiences a shock and a scenario where it does not. Thus, we have:

$$GIRF_T(H, \delta_{j,t}, \Omega_{t-1}) = E(y_{t+H}|e_j = \delta_{j,t}, \Omega_{t-1}) - E(y_{t+H}|\Omega_{t-1}) \tag{36}$$

$$\Psi_{j,t}(H) = \frac{B_{H,t}\Sigma_t e_j}{\sqrt{\Sigma_{jj,t}}} \frac{\delta_{j,t}}{\sqrt{\Sigma_{jj,t}}} \qquad \delta_{j,t} = \sqrt{\Sigma_{jj,t}} \tag{37}$$

$$\Psi_{j,t}(H) = \Sigma_{jj,t}^{-\frac{1}{2}} B_{H,t} \Sigma_t e_j, \tag{38}$$

with e_j denoting a selection vector of dimensions $m\times1$, having a unit in the j-th position and zeros elsewhere, and $\Sigma_{jj,t}$ representing the j-th diagonal element of Σ_t, the next step involves calculating the generalized forecast error variance decompositions (GFEVD), denoted by $\tilde{\phi}_{ij,t}(H)$). This measure represents the pairwise directional connectedness from variable j to variable i. It indicates how much variable j influences variable i in terms of variance decomposition. In other words, $\tilde{\phi}_{ij,t}(H)$ shows the proportion of the forecast error variance of variable i, H steps ahead, that is attributable to shocks in variable j. These contributions to the variance are normalized by the row sum corresponding to each variable. This normalization ensures that all variables together (including variable i) are responsible for explaining the forecast error variance of variable i. Thus, we have:

$$\tilde{\phi}_{ij,t}(H) = \frac{\Sigma_{t=1}^{H-1}\Psi_{ij,t}^{2}}{\Sigma_{j=1}^{m}\Sigma_{t=1}^{H-1}\Psi_{ij,t}^{2}} \tag{39}$$

with $\sum_{j=1}^{m}\tilde{\phi}_{ij,t}(H) = 1$ and $\sum_{i,j=1}^{m}\tilde{\phi}_{ij,t}(H) = m$, the numerator denotes the cumulative effect of a shock on variable i, while the denominator illustrates the cumulative effect of all shocks. This measure is used to calculate connectedness indices. The first to be presented is the total connectedness index, which measures the contribution of shock spillovers among variables to the total forecast error variance. In other words, it quantifies how much of the total forecast error variance across all variables can be attributed to shocks transmitted among them.

$$C_t(H) = \frac{\Sigma_{i,j=1,i\neq j}^{m}\tilde{\phi}_{ij,t}(H)}{\Sigma_{i,j=1}^{m}\tilde{\phi}_{ij,t}(H)} * 100 = \frac{\Sigma_{i,j=1,i\neq j}^{m}\tilde{\phi}_{ij,t}(H)}{m} * 100. \tag{40}$$

The next index indicates how much a shock in variable i affects all other variables, referred to as the total directional connectedness to others, given by:

$$C_{i\rightarrow j,t}(H) = \frac{\Sigma_{j=1,i\neq j}^{m}\tilde{\phi}_{ji,t}(H)}{\Sigma_{j=1}^{m}\tilde{\phi}_{ji,t}(H)} * 100. \tag{41}$$

Notice that in the numerator, the contribution of variable i to itself is excluded. In this case, variable i is analyzed as a transmitter of shocks. The next index, referred to as the total directional connectedness from others, indicates how much variable i receives from shocks in all other variables and is obtained as:

$$C_{i\leftarrow j,t}(H) = \frac{\Sigma_{j=1,i\neq j}^{m}\tilde{\phi}_{ij,t}(H)}{\Sigma_{j=1}^{m}\tilde{\phi}_{ij,t}(H)} * 100. \tag{42}$$

The difference between the total directional connectedness to others and the total directional connectedness from others yields the net total directional connectedness. This measure indicates the influence of variable i within the connectedness network. Using this measure, one can determine whether variable i affects (or is more affected by) the other variables. A positive value indicates that variable i transmits more shocks than it receives from others. In terms of notation, it is represented by:

$$C_{i,t} = C_{i\rightarrow j,t}(H) - C_{i\leftarrow j,t}(H) \quad (43)$$

Finally, the spillover relationships can be analyzed bidirectionally by considering the variables in pairs. This allows for observing more specific effects of one variable on another. This measure is referred to as net pairwise directional connectedness and is given by:

$$NPDC_{ij}(H) = \left(\tilde{\phi}_{ji,t}(H) - \tilde{\phi}_{ij,t}(H)\right) * 100. \quad (44)$$

In words, it is the amount of shocks that variable *i* transmits to variable *j* minus the amount it receives from variable *j*. A value of $NPDC_{ij}(H) > 0$ and $(NPDC_{ij}(H) < 0)$ indicates that variable i dominates (is dominated by) variable j in the sense that it affects variable *j* more than it is affected by it.

In the next section, the results of each of the methodologies used will be presented.

5. RESULTS

5.1 Cluster Results

The results below consider the three options available for the number of clusters as mentioned in Section 3.1 of the methodology. In Frame 1, the following result is observed for the case of three clusters.

Frame 1 – Groups in the case of 3 Clusters.

Group	Countries
1	Belgium, Estonia, Greece, Spain, France, Croatia, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Portugal, Romania, Slovenia, Slovakia, and Finland
2	Bulgaria, Denmark, Malta, Sweden, and Norway
3	Czechia, Germany, Ireland, Hungary, Netherlands, Austria, and Poland

Source: Own elaboration.

The table 2 contain the membership coefficients of each country to each group for each possibility. The results are presented following the recommendation of Bezdek (1981), which suggests that it is always useful to evaluate the fuzzy cluster results by associating each object with its respective group according to the highest membership coefficient.

The results are also shown in Table 3. Given that the average silhouette value ranges from -1 to 1, a value of 0.5 for the case of 3 clusters represents a good quality of the process (KAUFMAN; ROUSSEEUW, 1990).

This can also be observed from the silhouette plot (Figure 4), which graphically provides the information available in Table 3. It shows only one negative value, referring to the Czechia, which should be associated with cluster 1, its closest neighbor in the case of 3 clusters. This table provides the information corresponding to the silhouette plot. It contains details about the groups to which each country belongs,

As for the values of the Dunn index and its normalized version, approximately 0.6 and 0.4 respectively, they indicate fuzziness in the clustering structure, which strengthens the justification for using a fuzzy approach for these data. The composition of the clusters can also be easily observed in Figure 3.

Table 2 – Cluster Results for 3 Groups.

Country	Cluster 1	Cluster 2	Cluster 3	Assigned Cluster
Belgium	**0.45**	0.20	0.36	1
Estonia	**0.69**	0.09	0.22	1
Greece	**0.86**	0.02	0.11	1
Spain	**0.94**	0.01	0.04	1
France	**0.86**	0.03	0.11	1
Croatia	**0.82**	0.03	0.15	1
Italy	**0.92**	0.02	0.06	1
Cyprus	**0.72**	0.08	0.20	1
Latvia	**0.47**	0.05	0.47	1
Lithuania	**0.72**	0.04	0.24	1
Luxembourg	**0.75**	0.04	0.21	1
Portugal	**0.66**	0.04	0.30	1
Romania	**0.84**	0.04	0.12	1
Slovenia	**0.87**	0.03	0.10	1
Slovakia	**0.73**	0.08	0.19	1
Finland	**0.90**	0.03	0.07	1
Bulgaria	0.13	**0.74**	0.13	2
Denmark	0.13	**0.65**	0.21	2
Malta	0.13	**0.63**	0.24	2
Sweden	0.06	**0.87**	0.07	2
Norway	0.25	**0.47**	0.28	2
Czechia	0.43	0.06	**0.51**	3
Germany	0.16	0.14	**0.69**	3
Ireland	0.18	0.08	**0.74**	3
Hungary	0.23	0.07	**0.71**	3
Netherlands	0.19	0.14	**0.68**	3
Austria	0.26	0.06	**0.68**	3
Poland	0.23	0.07	**0.69**	3
	Sample Average Silhouette		**0.51**	
	Dunn Index		**0.60**	
	Normalized		**0.40**	

Source: Own elaboration.

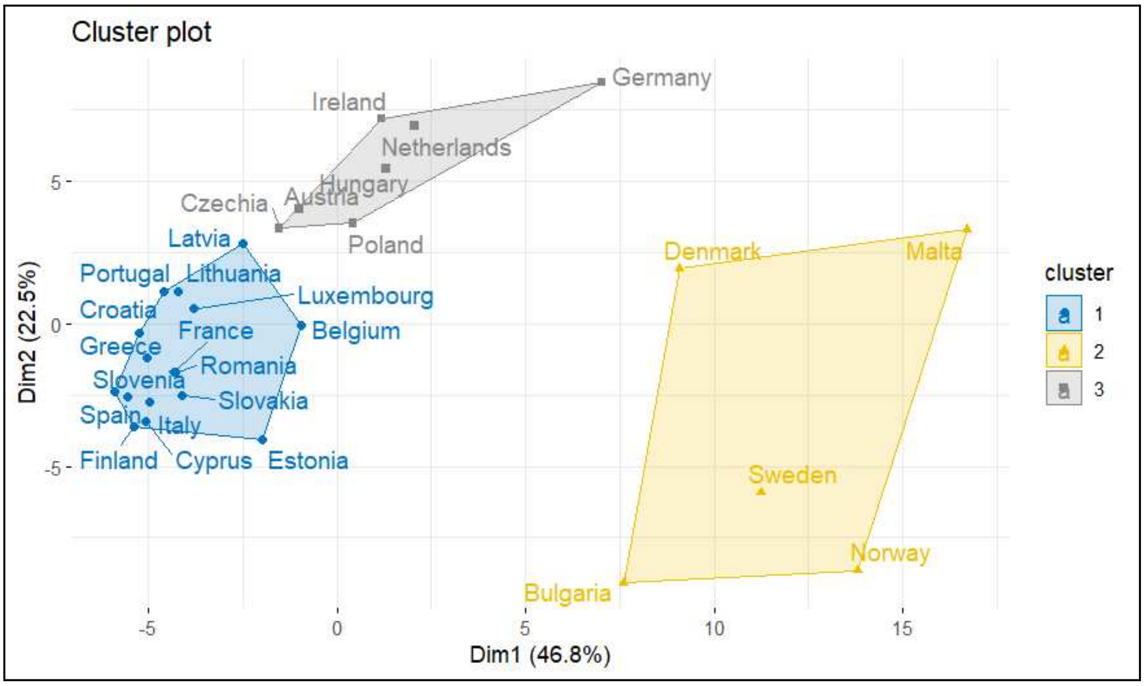
Table 3 – Silhouette Plot Information for 3 groups.

Country	Cluster	Neighbor	Silhouette Width
Spain	1	3	0.81
Finland	1	3	0.80
Italy	1	3	0.79
Romania	1	3	0.75
Slovenia	1	3	0.75
France	1	3	0.73
Greece	1	3	0.72
Slovakia	1	3	0.69
Croatia	1	3	0.67
Cyprus	1	3	0.66
Estonia	1	3	0.63
Luxembourg	1	3	0.60
Lithuania	1	3	0.59
Portugal	1	3	0.46
Belgium	1	3	0.33
Latvia	1	3	0.22
Sweden	2	3	0.60
Bulgaria	2	1	0.41
Malta	2	3	0.25
Norway	2	3	0.23
Denmark	2	3	0.13
Germany	3	1	0.63
Ireland	3	1	0.52
Netherlands	3	1	0.51
Hungary	3	1	0.42
Poland	3	1	0.32
Austria	3	1	0.20
Czechia	3	1	-0.23

Cluster	Number of Members	Size average Silhouette Width
1	16	0.64
2	5	0.33
3	7	0.34

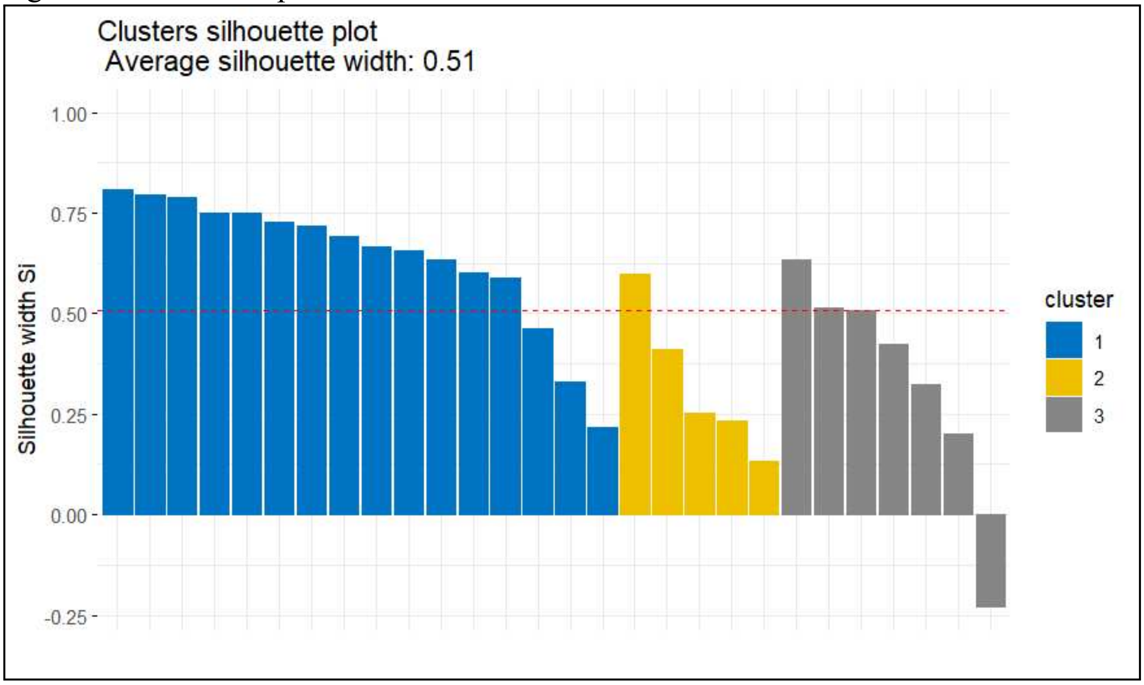
Source: Own elaboration.

Figure 3 – Cluster Results for 3 Groups.



Source: Own elaboration.

Figure 4 – Silhouette plot 3 clusters.



Source: Own elaboration.

For the case of four groups, the results are shown in Frame 2:

Frame 2 – Groups in the case of 4 Clusters.

Group	Countries
1	Belgium, Czechia, Latvia, Lithuania, Luxembourg, Austria and Portugal
2	Bulgaria, Denmark, Malta, Sweden and Norway
3	Germany, Ireland, Hungary, Netherlands, and Poland
4	Estonia, Greece, Spain, France, Croatia, Italy, Cyprus, Romania, Slovenia, Slovakia, and Finland

Source: Own elaboration.

It is observed that, for this case, there is no change in the composition of clusters 2 and 3 compared to the previous case. The difference is that, in this case, cluster 1 from the previous case has been split into two clusters. The average silhouette of approximately 0.3 indicates lower quality compared to the case with three clusters. From the silhouette plot, it is apparent that there are five incorrect associations, as also shown in Table 5. Lithuania, Belgium, Portugal, and Luxembourg should actually belong to cluster 4, their nearest neighbor, while Poland should be associated with cluster 1. The Dunn Index and its normalized version of 0.4 and 0.3, respectively, indicate greater imprecision in the clustering in this case compared to the previous case. The composition of the clusters can also be verified in Figure 5.

Table 4 – Cluster Results for 4 Groups.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Assigned Cluster
Belgium	**0.34**	0.13	0.21	0.32	1
Czechia	**0.53**	0.03	0.18	0.25	1
Latvia	**0.60**	0.03	0.14	0.23	1
Lithuania	**0.63**	0.02	0.07	0.29	1
Luxembourg	**0.54**	0.02	0.07	0.37	1
Austria	**0.43**	0.05	0.31	0.21	1
Portugal	**0.50**	0.02	0.11	0.37	1
Bulgaria	0.09	**0.71**	0.09	0.10	2
Denmark	0.15	**0.49**	0.24	0.12	2
Malta	0.14	**0.46**	0.28	0.12	2
Sweden	0.04	**0.87**	0.05	0.04	2
Norway	0.21	**0.36**	0.23	0.20	2
Germany	0.13	0.07	**0.72**	0.08	3
Ireland	0.17	0.04	**0.68**	0.10	3
Hungary	0.35	0.05	**0.41**	0.19	3
Netherlands	0.15	0.07	**0.69**	0.10	3
Poland	0.36	0.06	**0.39**	0.19	3
Estonia	0.34	0.06	0.11	**0.49**	4
Greece	0.34	0.02	0.05	**0.59**	4
Spain	0.10	0.01	0.02	**0.87**	4
France	0.35	0.02	0.05	**0.58**	4
Croatia	0.37	0.02	0.06	**0.55**	4
Italy	0.17	0.01	0.03	**0.79**	4
Cyprus	0.28	0.05	0.10	**0.57**	4
Romania	0.33	0.02	0.05	**0.59**	4
Slovenia	0.22	0.02	0.05	**0.72**	4
Slovakia	0.32	0.05	0.09	**0.54**	4
Finland	0.17	0.02	0.03	**0.78**	4
	Sample Average Silhouette			**0.30**	
	Dunn Index			**0.46**	
	Normalized			**0.28**	

Source: Own elaboration.

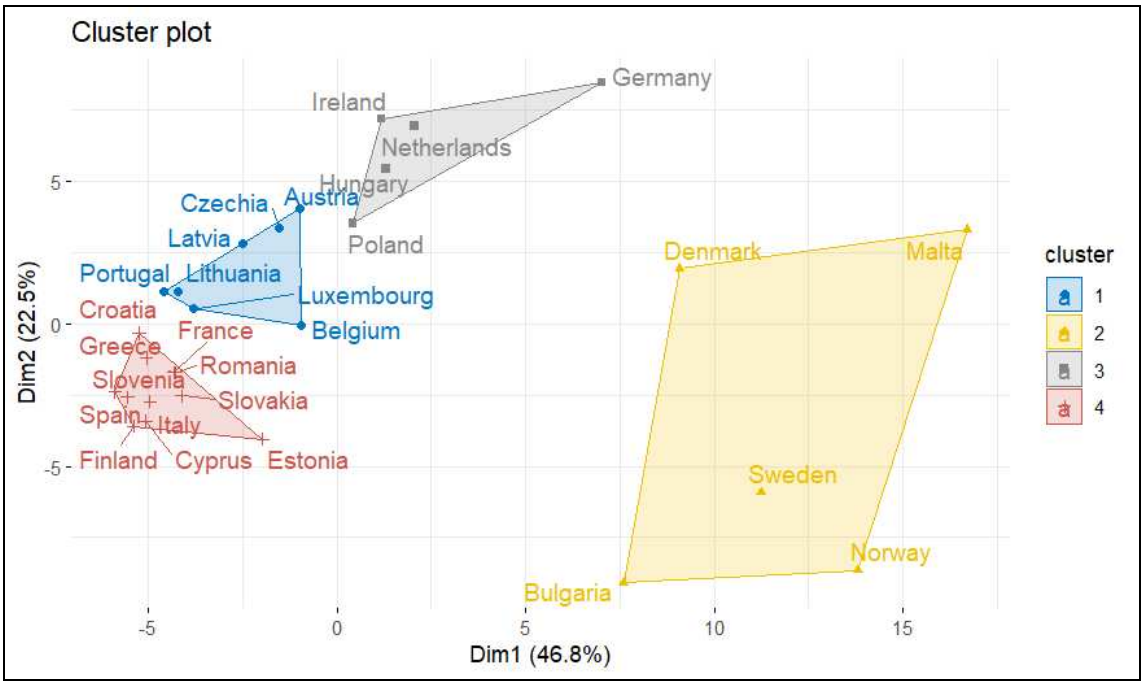
Table 5 – Silhouette Plot Information for 4 groups.

Country	Cluster	Neighbor	Silhouette Width
Latvia	1	4	0.27
Czechia	1	4	0.21
Austria	1	3	0.14
Lithuania	1	4	-0.04
Belgium	1	4	-0.10
Portugal	1	4	-0.23
Luxembourg	1	4	-0.25
Sweden	2	3	0.58
Bulgaria	2	4	0.40
Norway	2	3	0.22
Malta	2	3	0.19
Denmark	2	3	0.08
Germany	3	1	0.55
Netherlands	3	1	0.40
Ireland	3	1	0.36
Hungary	3	1	0.07
Poland	3	1	-0.04
Spain	4	1	0.68
Finland	4	1	0.67
Italy	4	1	0.64
Slovenia	4	1	0.58
Cyprus	4	1	0.51
France	4	1	0.44
Greece	4	1	0.43
Romania	4	1	0.43
Slovakia	4	1	0.40
Estonia	4	1	0.39
Croatia	4	1	0.34

Cluster	Number of Members	Size average Silhouette Width
1	7	0.00
2	5	0.29
3	5	0.27
4	11	0.50

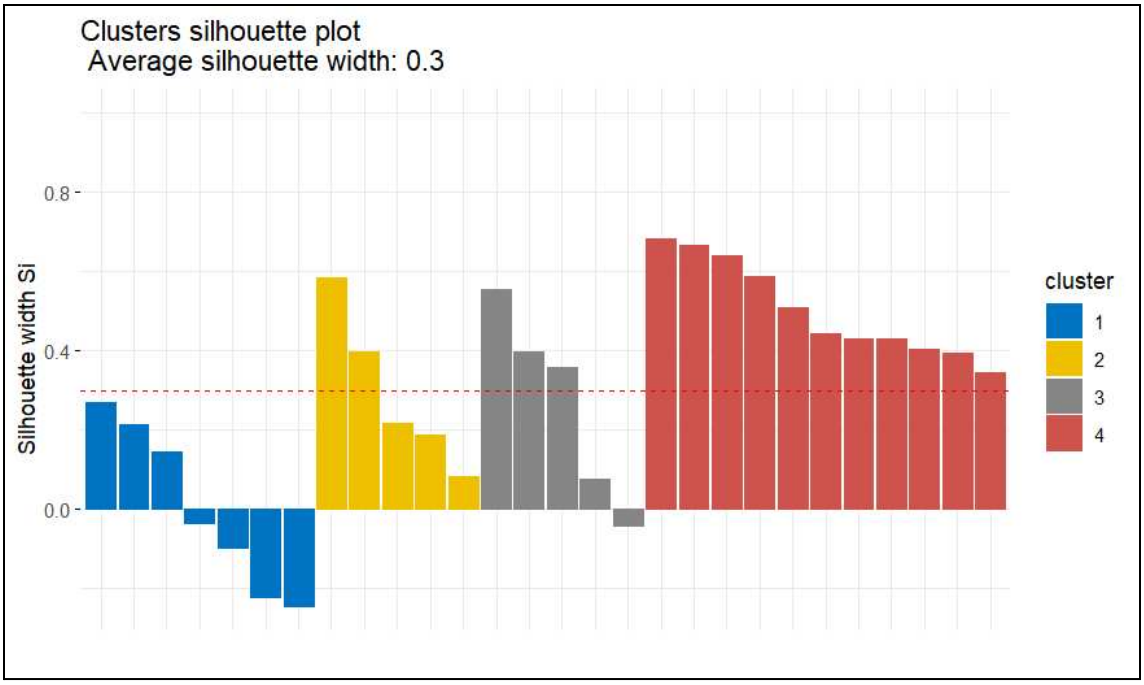
Source: Own elaboration.

Figure 5 – Cluster Results for 4 Groups.



Source: Own elaboration.

Figure 6 – Silhouette plot 4 clusters.



Source: Own elaboration.

For the case of five groups, the compositions are as follows:

Frame 3 – Groups in the case of 5 Clusters.

Group	Countries
1	Belgium, Czechia, Latvia, Lithuania, Luxembourg, Austria, and Portugal
2	Bulgaria, Denmark, Malta, and Sweden
3	Germany, Ireland, Hungary, Netherlands, and Poland
4	Estonia, Greece, Spain, France, Croatia, Italy, Cyprus, Romania, Slovenia, Slovakia, and Finland
5	Norway

Source: Own elaboration.

In this case, clusters 1, 3, and 4 are the same as in the previous case. Cluster 2 differs from the previous case by the absence of Norway, which now forms a separate cluster, the fifth in this case. The validation criteria show that the clustering result in this case is very similar to the case with four clusters, with the values of both criteria being quite close. Additionally, from the silhouette plot, the same incorrect associations as in the case with four clusters can be observed: Lithuania, Belgium, Portugal, and Luxembourg should actually belong to cluster 4, their nearest neighbor, while Poland should be associated with cluster 1.

The same results can be visually observed in Figures 7 and 8.

Table 6 – Cluster Results for 5 Groups.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Assigned Cluster
Belgium	**0.32**	0.12	0.20	0.30	0.05	1
Czechia	**0.51**	0.03	0.18	0.25	0.03	1
Latvia	**0.59**	0.03	0.14	0.22	0.02	1
Lithuania	**0.66**	0.01	0.06	0.26	0.01	1
Luxembourg	**0.56**	0.02	0.07	0.34	0.02	1
Austria	**0.41**	0.04	0.31	0.21	0.04	1
Portugal	**0.50**	0.02	0.11	0.35	0.02	1
Bulgaria	0.07	**0.70**	0.07	0.08	0.08	2
Denmark	0.13	**0.45**	0.22	0.11	0.09	2
Malta	0.12	**0.35**	0.23	0.11	0.19	2
Sweden	0.03	**0.84**	0.04	0.03	0.05	2
Germany	0.13	0.06	**0.67**	0.08	0.05	3
Ireland	0.14	0.03	**0.73**	0.08	0.02	3
Hungary	0.32	0.05	**0.40**	0.18	0.06	3
Netherlands	0.12	0.05	**0.71**	0.08	0.03	3
Poland	0.33	0.05	**0.37**	0.19	0.05	3
Estonia	0.33	0.05	0.10	**0.47**	0.05	4
Greece	0.36	0.02	0.05	**0.56**	0.01	4
Spain	0.09	0.01	0.01	**0.88**	0.00	4
France	0.37	0.02	0.05	**0.55**	0.02	4
Croatia	0.38	0.02	0.06	**0.52**	0.01	4
Italy	0.18	0.01	0.03	**0.78**	0.01	4
Cyprus	0.27	0.05	0.10	**0.55**	0.03	4
Romania	0.34	0.02	0.05	**0.57**	0.02	4
Slovenia	0.22	0.02	0.04	**0.70**	0.01	4
Slovakia	0.31	0.05	0.09	**0.52**	0.03	4
Finland	0.17	0.01	0.03	**0.77**	0.01	4
Norway	0.00	0.00	0.00	0.00	**0.99**	5
	Sample Average Silhouette			**0.31**		
	Dunn Index			**0.46**		
	Normalized			**0.33**		

Source: Own elaboration.

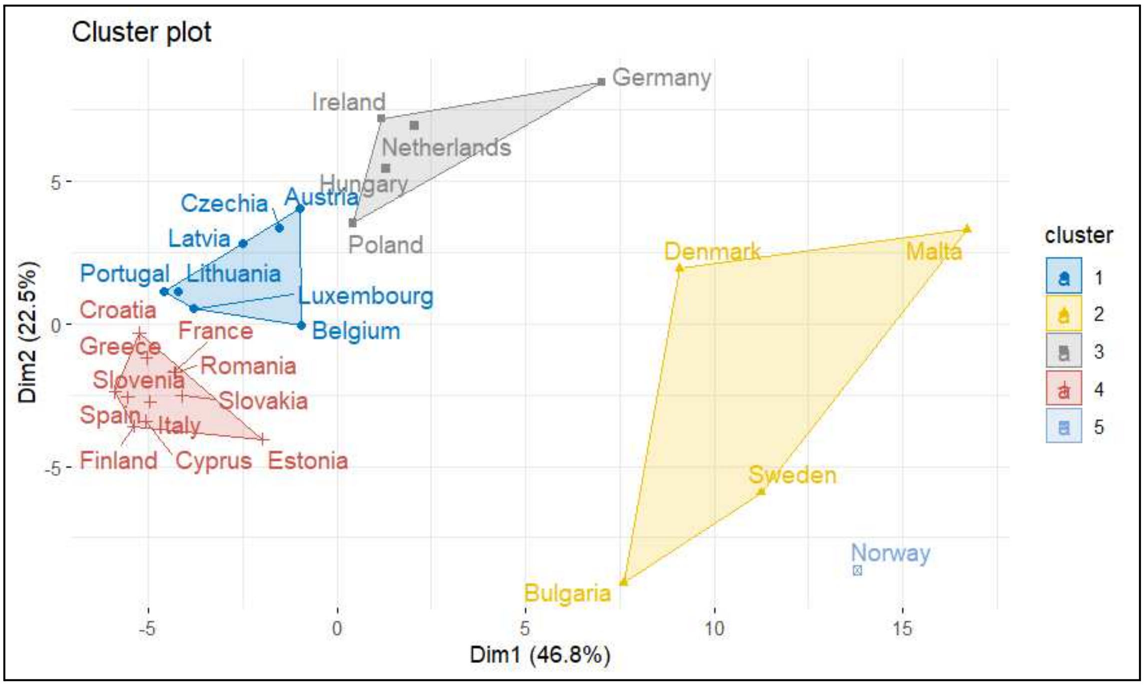
Table 7 – Silhouette plot information for 5 groups.

Country	Cluster	Neighbor	Silhouette Width
Latvia	1	4	0,27
Czechia	1	4	0,21
Austria	1	3	0,14
Lithuania	1	4	-0,04
Belgium	1	4	-0,10
Portugal	1	4	-0,23
Luxembourg	1	4	-0,25
Sweden	2	5	0,58
Bulgaria	2	5	0,53
Denmark	2	3	0,38
Malta	2	3	0,28
Germany	3	1	0,55
Netherlands	3	1	0,40
Ireland	3	1	0,36
Hungary	3	1	0,07
Poland	3	1	-0,04
Spain	4	1	0,68
Finland	4	1	0,67
Italy	4	1	0,64
Slovenia	4	1	0,58
Cyprus	4	1	0,51
France	4	1	0,44
Greece	4	1	0,43
Romania	4	1	0,43
Slovakia	4	1	0,40
Estonia	4	1	0,39
Croatia	4	1	0,34
Norway	5	2	0,00

Cluster	Number of Members	Size average Silhouette Width
1	7	0.00
2	4	0.44
3	5	0.27
4	11	0.50
5	1	0.00

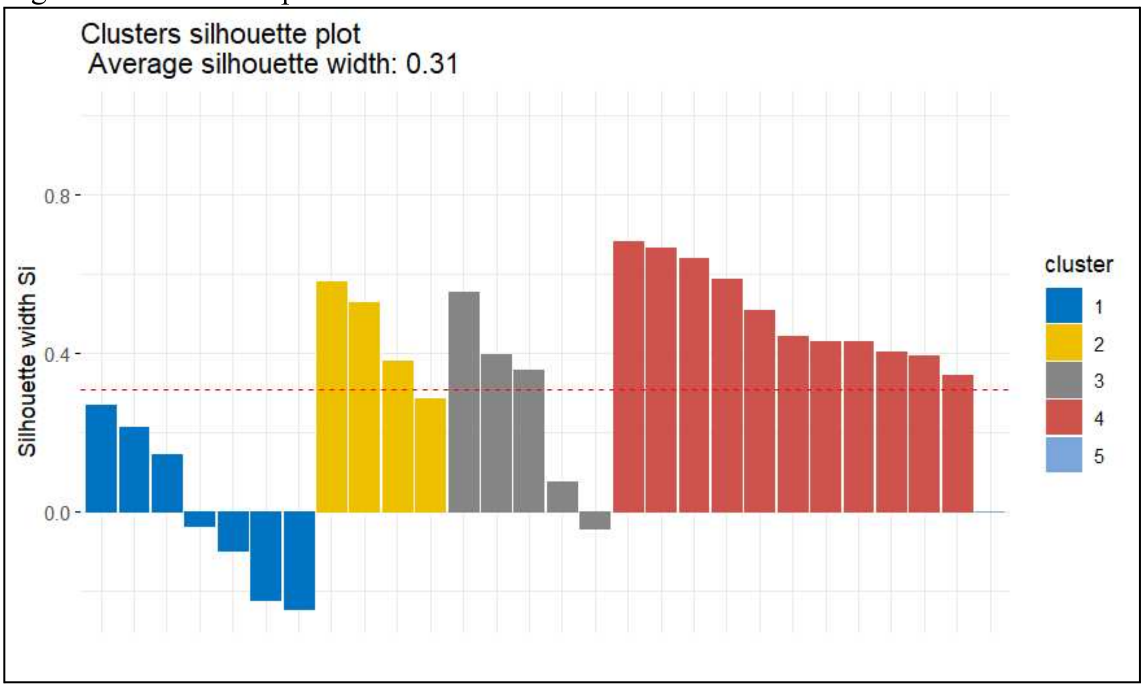
Source: Own elaboration.

Figure 7 – Cluster Results for 5 Groups.



Source: Own elaboration.

Figure 8 – Silhouette plot 5 clusters.



Source: Own elaboration.

According to the validation criteria, the solution that achieved the best results was the one with three groups. Based on the results from this section, the composition of the three clusters from the first case is retained. To complement this approach and make the sample selection more rigorous, considering long-term behavior, a convergence methodology is also applied to the same countries.

5.2 Results of the Convergence Analysis

Given that convergence analysis focuses on the long-term behavior of time series, Phillips and Sul (2007) recommend conducting tests on the trend components of the series. To this end, a filter must be applied to extract the trend component. In the present study, the Hodrick-Prescott (HP) filter was used on the debt series, applying a smoothing parameter of $\lambda = 1600$, the standard value recommended for quarterly data (HODRICK; PRESCOTT, 1997; RAVN; UHLIG, 2002).

Firstly, the Phillips and Sul (2007) $log\ t$ convergence test was applied to verify whether all twenty-eight countries form a single group (or club) of convergence. The result can be observed in the first row of the table below, where the null hypothesis of convergence can be rejected, as the t-value is significantly lower than -1.65.

Next, the basic Phillips and Sul (2007) algorithm was applied to verify the existence of convergence clubs. The results are shown in the subsequent rows of the same table. The algorithm identified two convergence groups with twelve and sixteen members, respectively. It is noteworthy that the convergence of Group 2 is much more evident than that of Group 1. The value of $\hat{b}$ for Group 1 is not statistically different from zero, which, according to Phillips and Sul (2009) and von Lyncker and Thoennessen (2017), can be interpreted as weak evidence of convergence for this group. On the other hand, Group 2 shows strong evidence of convergence, as highlighted by the significantly positive estimated coefficient and the high t-value.

Table 8 – Convergence Results.

	Number of Units	$\hat{b}$	**Std. Err**	**t-value**	**cstar**
Full Sample	-	-0.643	0.008	-77.012	-
club1	12	0.001	0.024	0.028	0
club2	16	0.200	0.042	4.822	0

Source: Own elaboration.

The composition of the two groups is presented in the frame below.

Frame 4 – Convergence Groups.

Group	**Countries**
1	Italy, France, Spain, Portugal, Greece, Belgium, Cyprus, Finland, Slovenia, Croatia, Slovakia, and Romania
2	Austria, Hungary, Germany, Malta, Poland, Netherlands, Czechia, Latvia, Ireland, Lithuania, Norway, Sweden, Denmark, Luxembourg, Bulgaria, and Estonia

Source: Own elaboration.

Next, the *merge algorithm* was applied, both the one proposed by Phillips and Sul (2009) and the one by von Lyncker and Thoennessen (2017). The results were identical, including the estimated values and, consequently, the composition of the groups. The calculation of robust standard errors, considering the presence of heteroskedasticity and autocorrelation, was performed following Andrews (1991). Additionally, a test was conducted to verify the existence of any group of divergent units, as proposed by von Lyncker and Thoennessen (2017). Once again, the results remained unchanged. This indicates that the group composition above is significant and robust to the choice of methods.

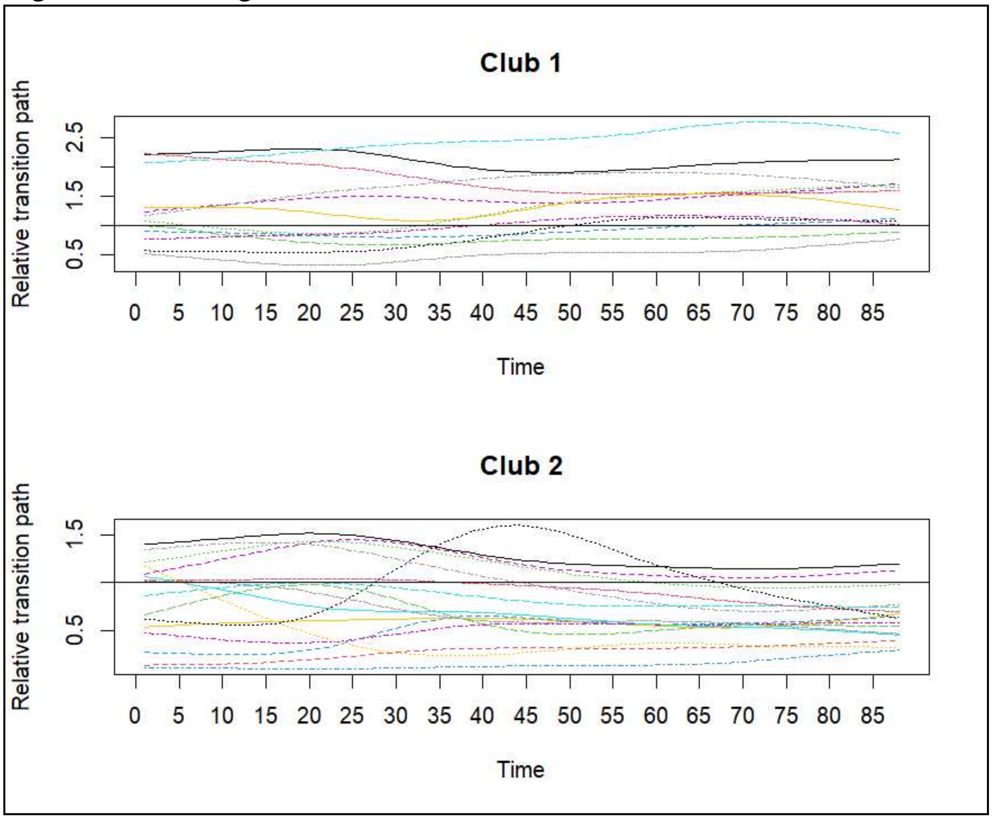
The first graph below represents the transition paths of the countries within each group. The second graph shows the overall transition paths (all members) for each group. The time axis represents the number of time units in the sample, measured in quarters. In this case, there are 88 quarters.

From the first figure, the convergence pattern within each group can be observed through the close co-movements throughout the entire sample period. For Group 1, a longer transition period is evident, resulting in weaker convergence, as shown by the smoother curves among all countries. Conversely, Group 2 demonstrates stronger convergence, as indicated by the closer alignment of trajectories starting around 2015 (approximately 65 quarters).

From the second figure, it is evident that the groups converge to distinctly different points. Until approximately 2013 (midway through the sample period), the relative transition curves of the two groups (or convergence clubs) exhibit similar behavior. However, from that point onward, there is strong evidence of divergence between the groups.

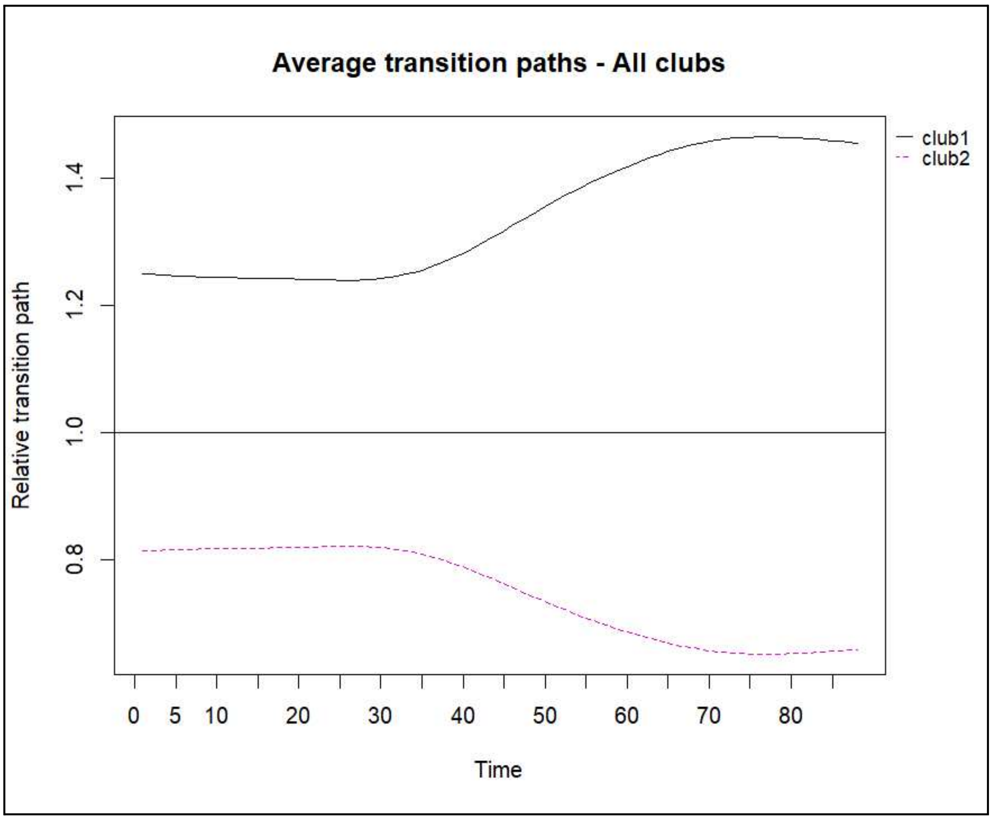
The trend of the first group is upward, while that of the second group is downward. This result reinforces the idea of clustering and the existence of two distinct steady-state trajectories.

Figure 9 – Convergence Results.



Source: Own elaboration.

Figure 10 – Convergence Results.



Source: Own elaboration.

In comparison with the previous clustering using the Fuzzy c-means technique, groups 2 and 3, plus Estonia, Latvia, Lithuania, and Luxembourg from the clustering analysis results, form group 2 in the convergence approach. When comparing, it can be seen that the results are very similar for the following reasons.

First, the best clustering solution with clusters was obtained with the smallest number of groups, in this case, three. Second, in the clustering approach, in addition to information about the group to which each country belongs, we also have information about which group is closest to each country. That is, country X might belong to group 1 and have group 2 as the closest, meaning that if country X were to belong to a group other than group 1, it would belong to group 2. This is exactly what happens with Estonia, Latvia, Lithuania, and Luxembourg in the clustering approach. These four countries belong to group 1, with group 3 as the closest. This means that by associating these countries with their closest group, we get the same results between both approaches.

In the *Fuzzy c-means* method, one of its main advantages is that an observation can belong to more than one group simultaneously with different degrees of membership. In the final result, the organization of the observations is made so that they are associated with the group where they have the highest degree of membership. Intuitively, this means the group to which the observations belong with the "greatest certainty."

The choice of a sub-sample aims to address the issue of grouping European countries based on common characteristics and, above all, to detect a group of countries that are more vulnerable in terms of fiscal policy. Within a group it is important to detect potential weaknesses to try to strengthen them for the good of the whole group.

Thus, the selection criterion used to define the subsample was to choose the group of countries that are in the intersection of the two clustering methods and that exhibit a stronger association among their members. Therefore, the subsample consists of the following countries: Italy, France, Spain, Portugal, Greece, Belgium, Cyprus, Finland, Slovenia, Croatia, Slovakia, and Romania.

5.3 Solvency Results (Wavelets)

The following results are based on the evaluation of the relationships between surplus, debt, and deviations in revenues and expenditures for the subsample of 12 countries. The deviations were obtained by applying the Hodrick-Prescott (HP) filter to the revenue and expenditure series, using the smoothing parameter λ=1600, the standard value recommended for quarterly data (HODRICK; PRESCOTT, 1997; RAVN; UHLIG, 2002).

To facilitate the presentation of the results, the multiple coherence is first presented, providing an overview of the relationships between the variables. Subsequently, the partial coherence measures, partial phase difference, and partial gain between surplus and debt are discussed, controlling for the effects of revenues and expenditures.

In each figure, the first chart displays the multiple coherence, followed by the partial coherence graphs, accompanied by corresponding diagrams of partial phase difference and partial gain. These diagrams are divided into three frequency intervals: periods of 1 to 2 years, considered short-term; 2 to 4 years, medium-term; and 4 to 6 years, long-term. These divisions align with the projection horizons used by the IMF (International Monetary Fund) in its public debt sustainability analysis tools (IMF, 2021). They are also based on the projections of stochastic DSA (Debt Sustainability Analysis) models used by the European Central Bank (ECB) and the European Commission (EC) for debt trajectory analysis, employing a five-year horizon (ALLOZA et al., 2024).

Both wavelet power and wavelet coherence are plotted as heatmaps, with colors ranging from blue (indicating low power/low coherence) to red (high power/high coherence).

The divisions into different frequencies are crucial as they allow for capturing the evolution of variations in cycles of different durations with greater flexibility. Indeed, some variations have a more pronounced short-term effect, while others dissipate over time and exhibit greater persistence (RAMSEY; LAMPART, 1998; AGUIAR-CONRARIA; AZEVEDO; SOARES, 2008).

It is important to note that the results from the partial phase difference and partial gain diagrams should only be interpreted for the significant regions in the partial coherence graphs. This limitation arises from the lack of appropriate significance tests for these measures (GE, 2008).

In the multiple coherence graph, significant regions indicate that debt and deviations in expenditures and revenues are jointly significant explanatory variables for the surplus during the analyzed period.

The phase difference diagrams allow us to observe variations across different frequency bands and associate these relationships with the time axis. Variations in frequency bands reveal the lagged response of the lag variable to changes in the leading variable. Leadership information in these variations is depicted in the phase difference graphs. Finally, partial gain provides an understanding of the magnitude of the response of the lag variable in relation to the lead variable.

The results below are organized by country and frequency range. Debt leading out-of-phase means that increases in debt are followed by reductions in the surplus, or alternatively, that positive changes in surplus cycles are preceded by negative changes in debt cycles. This situation indicates insolvency, as the surplus reacts negatively to changes in debt.

Indeed, the partial gain coefficient can be interpreted as a regression coefficient, allowing for the interpretation of the surplus' response to debt (AGUIAR-CONRARIA; MARTINS; SOARES, 2018). A similar interpretation applies in cases where debt leads in-phase.

The sample period begins in January 2002 (2002Q1) and ends in October 2023 (2023Q4).

The main results focus on the analysis of the partial coherence, partial phase difference, and partial gain graphs. When results pertain to the multiple coherence graph, this will be explicitly stated.

It is important to emphasize that the comovements observed across different frequencies have specific emphases, as noted in the literature on solvency/sustainability. Short-term variations are more closely related to liquidity risks (PANIAGUA; SAPENA; TAMARIT, 2017), while medium- and long-term variations relate to solvency, considering factors such as climate change, technological advancements, and population aging (ALLOZA et al., 2024).

Below are the results for the subsample of the 12 countries, listed in alphabetical order.

5.3.1 Belgium

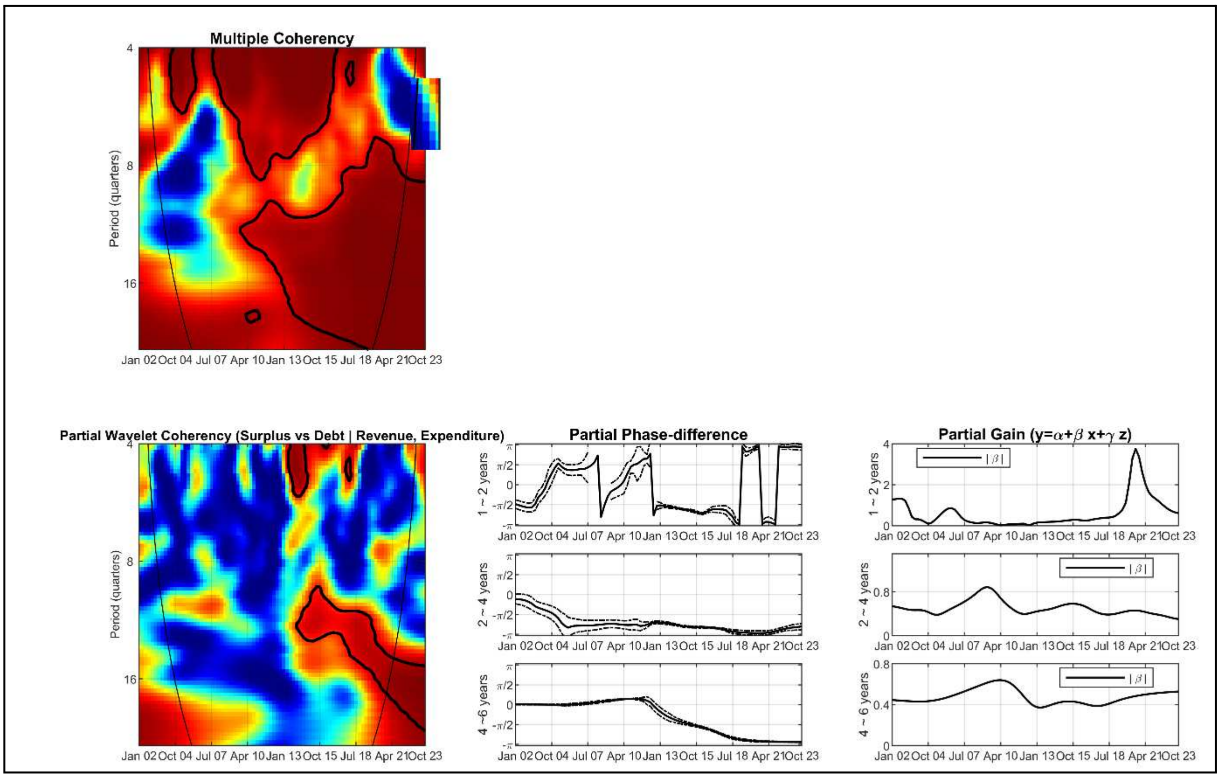
The first country evaluated is Belgium, for which there is no evidence of insolvency, as shown in Figure 11. Across all frequency ranges, the surplus leads out of phase. This indicates that increases in the surplus are followed by reductions in the debt stock.

In the 1- to 2-year cycles, there are two significant regions. The first spans from early 2013 to early 2015, while the second occurs during the second half of 2017. For the 2- to 4-year cycles, there is a single significant region from early 2014 to the end of the sample. In the 4- to 6-year cycles, significance is observed from the second half of 2020 to the end of the sample, with a partial gain of 0.5 in April 2021, which is a reasonable value in this context.

Despite high levels of debt and alternating surpluses and deficits, a significant fiscal consolidation effort has been observed since 2012 through the implementation of structural reforms aimed at reducing debt. The two main reforms focused on the pension system and fiscal federalism, forming part of a plan to achieve a balanced budget by 2015 (OECD, 2013). Additionally, starting in October 2014, further efforts targeted public expenditure reductions for the 2015–2019 period, primarily focusing on payroll costs, operational expenses, healthcare spending, and social transfers (OECD, 2015). These measures undoubtedly contributed to preventing a Belgian sovereign debt crisis.

The results confirm Afonso and Jalles (2017), who, using quarterly data from the first quarter of 1990 (1990Q1) to the fourth quarter of 2013 (2013Q4), also found fiscal sustainability in Belgium. Paniagua, Sapena, and Tamarit (2017), with annual data from 1970 to 2014, show that Belgium was one of the Eurozone countries that systematically responded to debt variations, thus providing evidence of sustainability in the sense of Bohn (1998, 2007).

Figure 11 – Wavelet results for Belgium.



Source: Own elaboration.

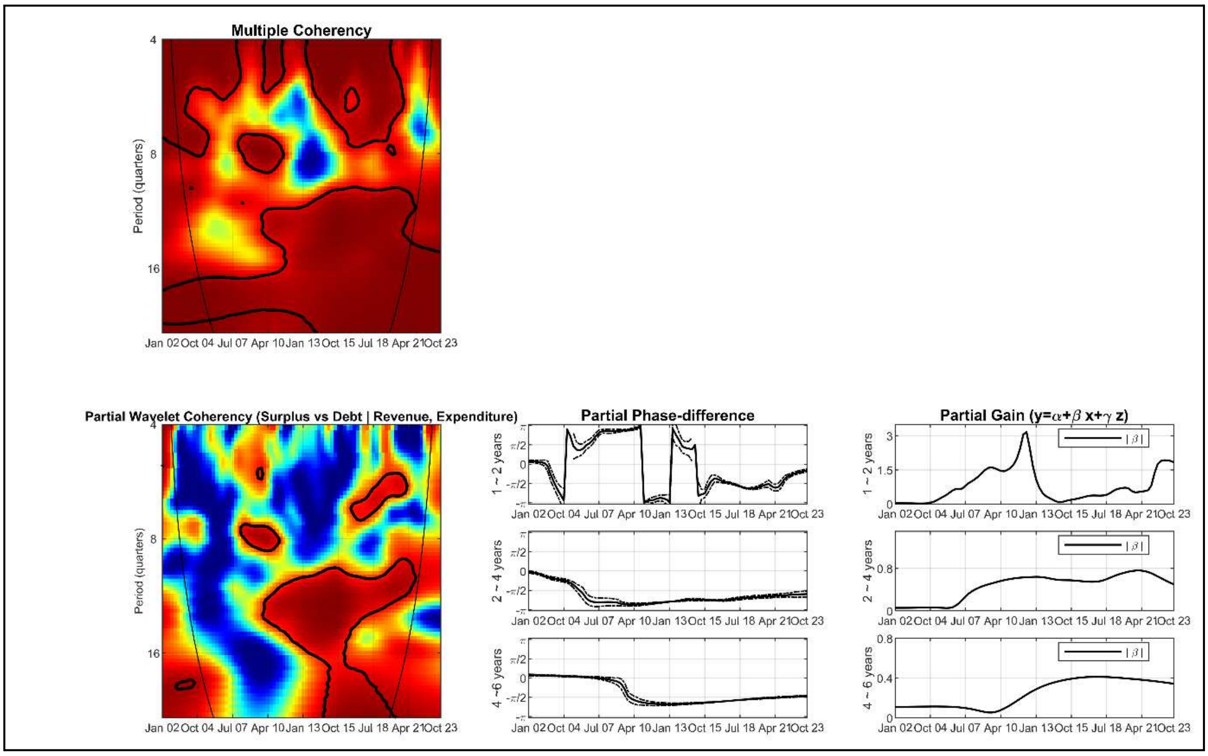
5.3.2 Croatia

Croatia shows signs of insolvency only in the shorter cycles (See Figure 12). The 1-2 year cycles present a region spanning from the first half of 2008 to the first half of 2011, where debt leads out of phase, meaning increases in debt are followed by reductions in the surplus approximately one year later. This period is marked by the development of the financial crisis in the US, followed by the onset of the sovereign debt crisis in Europe, during which Croatia experienced a sharp increase in the debt-to-GDP ratio compared to previous years, as well as worsening financing conditions and a lack of fiscal adjustment (BRKIĆ, 2016; ŠIMOVIĆ, 2018). The other region in this frequency range spans from the second half of 2016 to the first half of 2021, where the surplus leads out of phase, with a partial gain of 0.72 in January 2020, the highest during this period.

In the 2-4 year cycles, there is a large region extending from the first half of 2010 to the first half of 2021, where the surplus leads out of phase, with a maximum partial gain of 0.76 in April 2021. This shows effects of approximately equal magnitude in both the short and medium terms. For the 4-6 year cycles, there is a region from the second half of 2014 to the first half of 2020, where the surplus also leads out of phase, with an average partial gain of 0.4 during this period.

The medium and long-term cycles seem to highlight the effectiveness of the stabilization policies adopted by the European Union to contain the effects of the crises, as well as the consolidation policies adopted by Croatia (BRKIĆ, 2016).

Figure 12 – Wavelet results for Croatia.



Source: Own elaboration.

5.3.3 Cyprus

In relation to Cyprus, as shown in Figure 13, insolvency is also observed only in the short-term cycles (1 to 2 years). One region covers the year 2013, and another spans from the second half of 2018 to the second half of 2020, where debt leads out of phase with a partial gain of 0.87 in July 2018, indicating a strong relationship.

The effect observed in 2013 reflects the difficulties Cyprus faced during the sovereign debt crisis. The country had high debt-to-GDP ratios during this period, exceeding 80%, along with private sector and household indebtedness and a loss of competitiveness in the international market, which led to a decline in exports (ZENIOS, 2013). Moreover, Cyprus did not have national fiscal rules in place at the time (MOŻDZIERS, 2015). These factors contributed to the request for financial assistance from the IMF in June 2012.

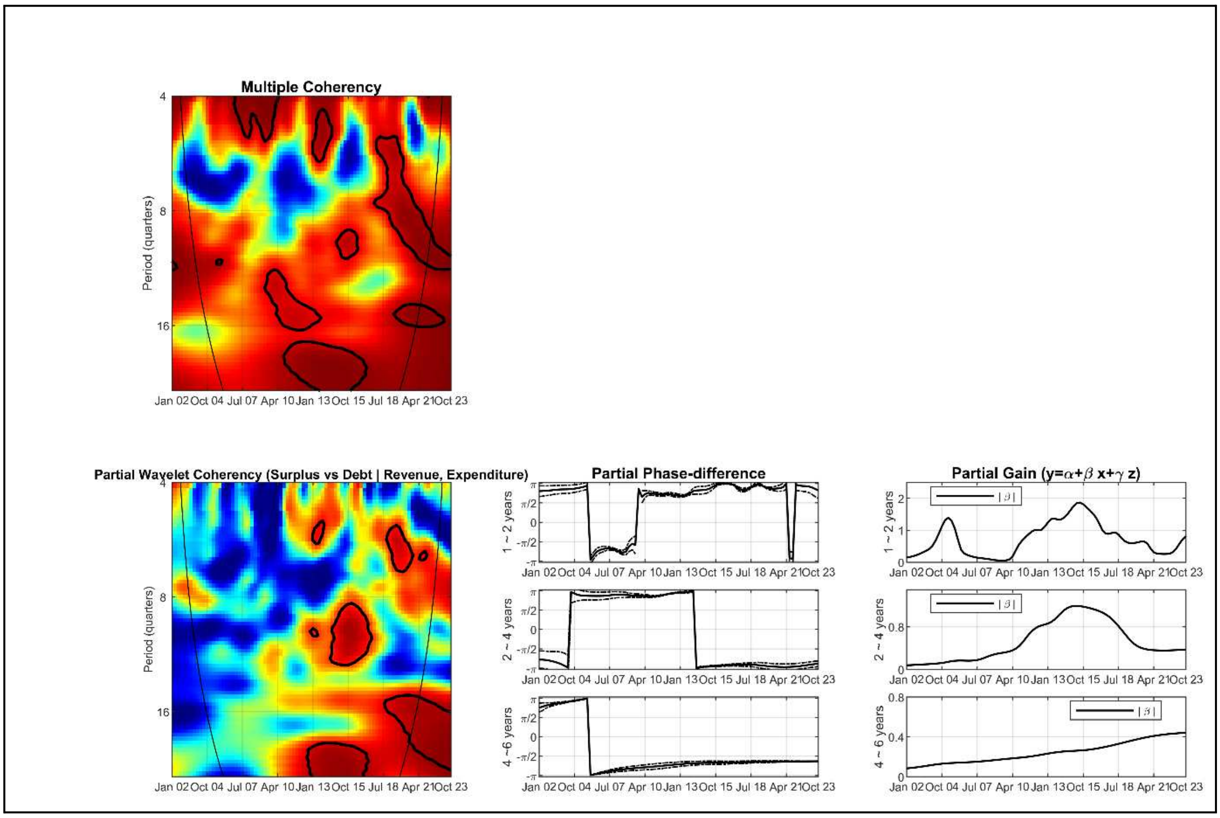
For the second region, we again observe high levels of debt. The average debt-to-GDP ratio during this period was 100.05%, well above the 60% limit set by the Stability and Growth Pact, partly reflecting the increased spending during the pandemic.

In the other frequency bands, it is observed that the variations in the surplus lead the variations in debt in an out-of-phase relationship. That is, positive changes in the debt cycles are preceded by negative changes in the surplus cycles. For the 2 to 4-year cycles, significant regions are observed from the first half of 2014 to the second half of 2017 and from the second half of 2018 to the end of the sample, with the highest partial gain of 1.19 in April 2015, showing a stronger relationship than in the short-term cycles. For the 4 to 6-year cycles, the regions span from the second half of 2012 to the second half of 2016 and from the second half of 2018 to the end of the sample, with the highest partial gain of 0.44 in July 2023.

The insolvency observed in the shorter cycles suggests that the measures taken for the country's restructuring took time to show positive effects. On the other hand, the absence of evidence of insolvency in the other cycles points to the effectiveness of the corrective actions taken during the crisis period, indicating that the policies were effective in the medium and long terms.

This finding underscores the advantage of using the wavelet approach, as it allows for identifying different relationships between variables depending on the frequency band considered. It enables, for example, the ability to observe whether specific policies achieved their expected objectives.

Figure 13 – Wavelet results for Cyprus.



Source: Own elaboration.

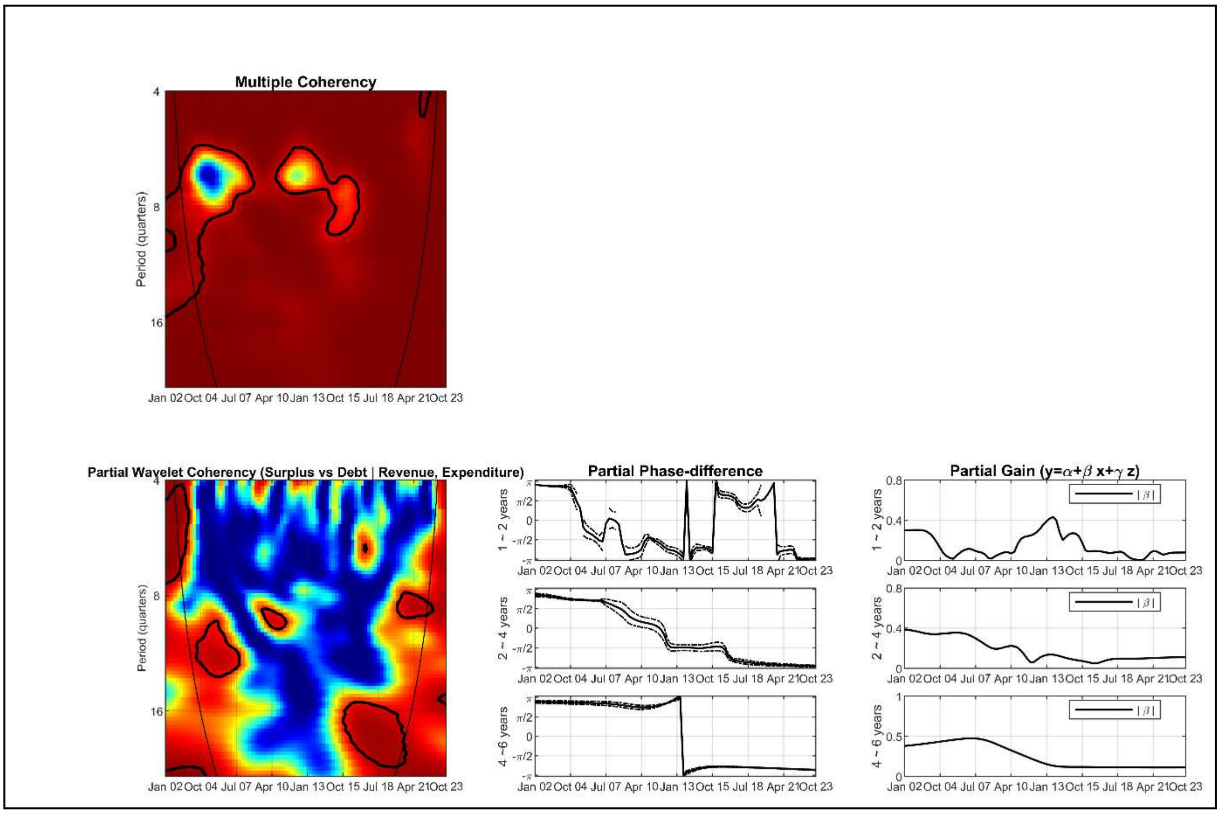
5.3.4 Finland

The majority of significant regions for Finland are observed in the longer cycles. The only region in the short term occurs between January 2002 and October 2003, with an average gain of 0.3 and debt leading out of phase, indicating a situation of insolvency. This period is marked by an expansionary fiscal policy, with increased public consumption and tax cuts (OECD, 2006).

In the medium term, three regions with different behaviors are identified. The first region spans from July 2004 to October 2007, with an average gain of 0.3, exactly the same magnitude as in the short-term cycles, with debt leading out of phase. From July 2009 to April 2011, the surplus leads in phase. A region from January 2016 to October 2022 represents an intersection of two regions, where the surplus leads out of phase with debt. Finally, a region from October 2015 to July 2020 shows the surplus leading debt in an out-of-phase relationship in the longer cycles.

Overall, the results for Finland suggest sustainability, despite ongoing concerns about budgetary pressures due to the rapid aging of the population, which was already an issue in 2007 (OECD, 2008).

Figure 14 – Wavelet results for Finland



Source: Own elaboration.

5.3.5 France

For France, the relationships observed in the 1-2 year cycles are weaker compared to those in other cycles. In this frequency band, the first significant region spans from April 2010 to the second half of 2013, where, for the most part, the surplus leads out of phase, but with significant variation — characteristic of short-term cycles. The second region extends from the second half of 2017 to the first half of 2021, where the surplus also leads out of phase.

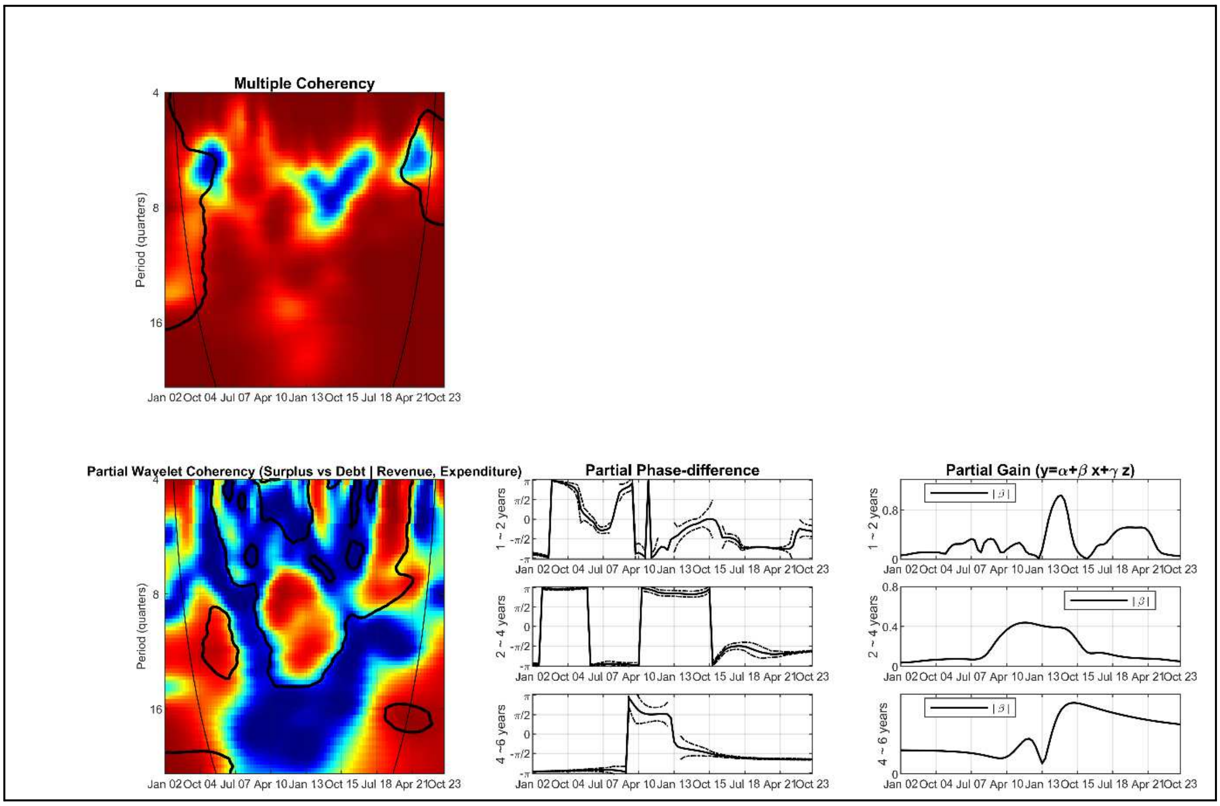
In the 2-4 year cycles, the first significant region is from January 2005 to October 2007, with an average partial gain of 0.07 — indicating a very weak impact of debt on the surplus. Until April 2006, debt leads out of phase, resulting in insolvency. After that, the surplus leads out of phase. The other two regions within these cycles exhibit weaker relationships, which can be noticed by the lighter red tones. These regions are from January 2009 to January 2016, with a maximum gain of 0.44 in October 2011, where debt leads out of phase for most of the time, resulting in insolvency, and from July 2017 to April 2021, where the surplus leads out of phase, with a decline in the partial gain, indicating a weakening relationship between surplus and debt during this period.

From January 2009 to January 2016, France experienced an increase of about 20 percentage points in the debt-to-GDP ratio, accompanied by successive deficits. During this period, a single surplus was observed in the last quarter of 2015, followed by another deficit. Notably, there was an accelerated rise in debt starting in early 2009, driven mainly by an economic stimulus package consisting of increased spending and tax cuts, lasting until 2010 (ANDRÉ et al., 2015).

These results align with the findings of Paniagua, Sapena, and Tamarit (2017), which show that, at least until 2014, France responded more slowly to increases in debt, reacting only after reaching a certain debt threshold. Additionally, the French recovery has been slower compared to other European countries that faced more severe situations, such as Portugal and Spain (OECD, 2019). This largely reflects the difficulty of the French government in reducing spending, especially on social assistance and pensions (ANDRÉ et al., 2015).

Finally, in the 4-6 year cycles, two regions are noted. One spans from the beginning of the sample to April 2007, and the other from January 2019 to January 2023. In both regions, the surplus leads out of phase, with the average gain remaining below 0.08, indicating that weaker relationships are also observed in the long term.

Figure 15 – Wavelet results for France.



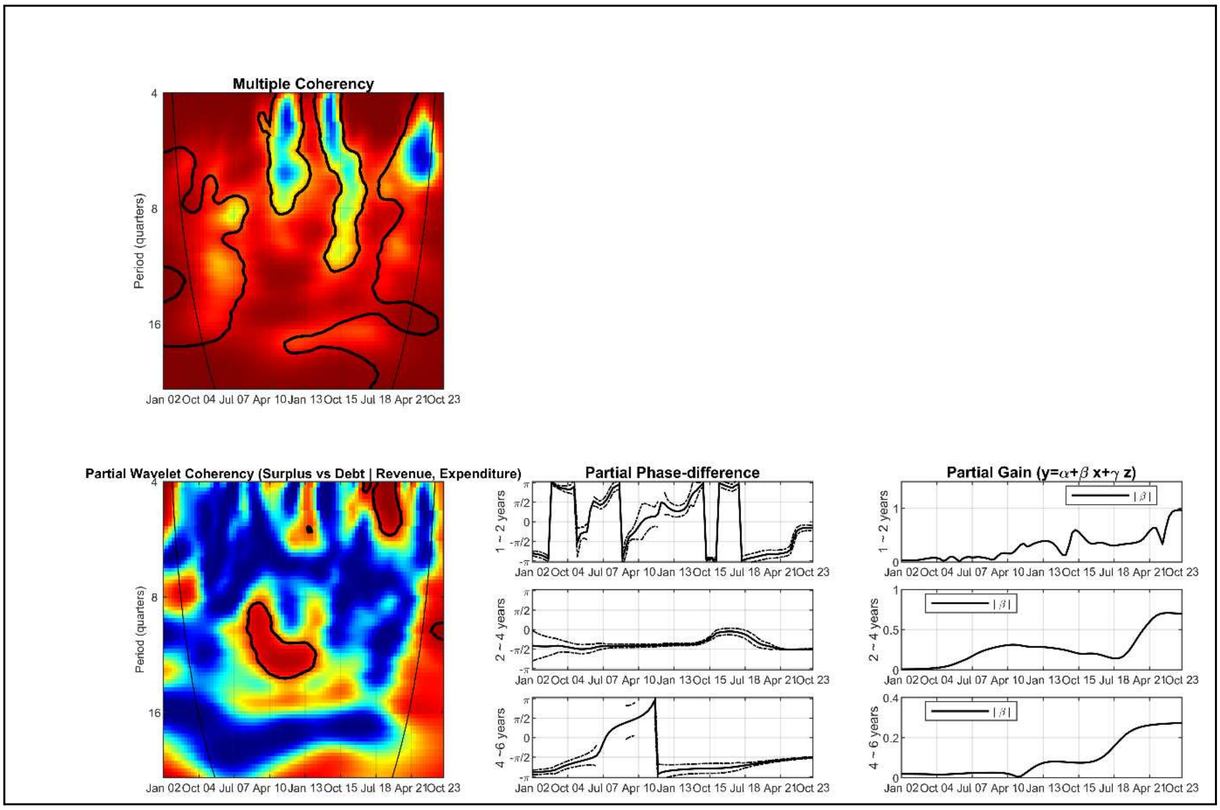
Source: Own elaboration.

5.3.6 Greece

Perhaps the most surprising result is that of Greece, where there is no evidence of insolvency. More than that, there are very few significant regions in the partial coherence graph. This suggests that the pattern observed in the multiple coherence graph is due to the effects of revenues and expenditures on the surplus. Aguiar-Conraria and Soares (2011), studying the synchronization of business cycles in Europe, found a similar result for the partial coherence graph of Greece's industrial production index. The country is among the least synchronized with the rest of Europe.

For the 1-2 year cycles, there is only one significant region, spanning from July 2017 to July 2020, with a partial gain of around 0.36, where the surplus leads out of phase. This means that increases in the surplus are followed by reductions in the debt stock. For the 2-4 year cycles, there is also a single significant region, from July 2008 to October 2013, with the partial gain peaking at 0.31 in October 2010, during the height of the Greek crisis. In this period, debt leads variations in phase, indicating a solvency behavior. This result may reflect the effects of the bailout packages and restructuring policies proposed by the European Commission and the IMF during this period. Between 2010 and 2016, Greece signed three bailout packages totaling approximately 289 billion euros. Finally, no significant regions are observed in the 4-6 year cycles.

Figure 16 – Wavelet results for Greece.



Source: Own elaboration.

5.3.7 Italy

The results for Italy show evidence of insolvency in both the medium and long-term cycles. However, the partial coherence graph shows lighter shades, indicating weaker relationships between the variables, similar to the case with France.

For the medium and long-term cycles, the significant regions coincide from January 2006 to the second quarter of 2010. In the medium term, debt leads the surplus in an out-of-phase relationship, indicating an insolvent behavior. The partial gain shows peaks of 0.44 in January 2011 and July 2014. The other region in these cycles spans from October 2017 to April 2019, with a partial gain averaging 0.33, and the surplus leads debt out of phase.

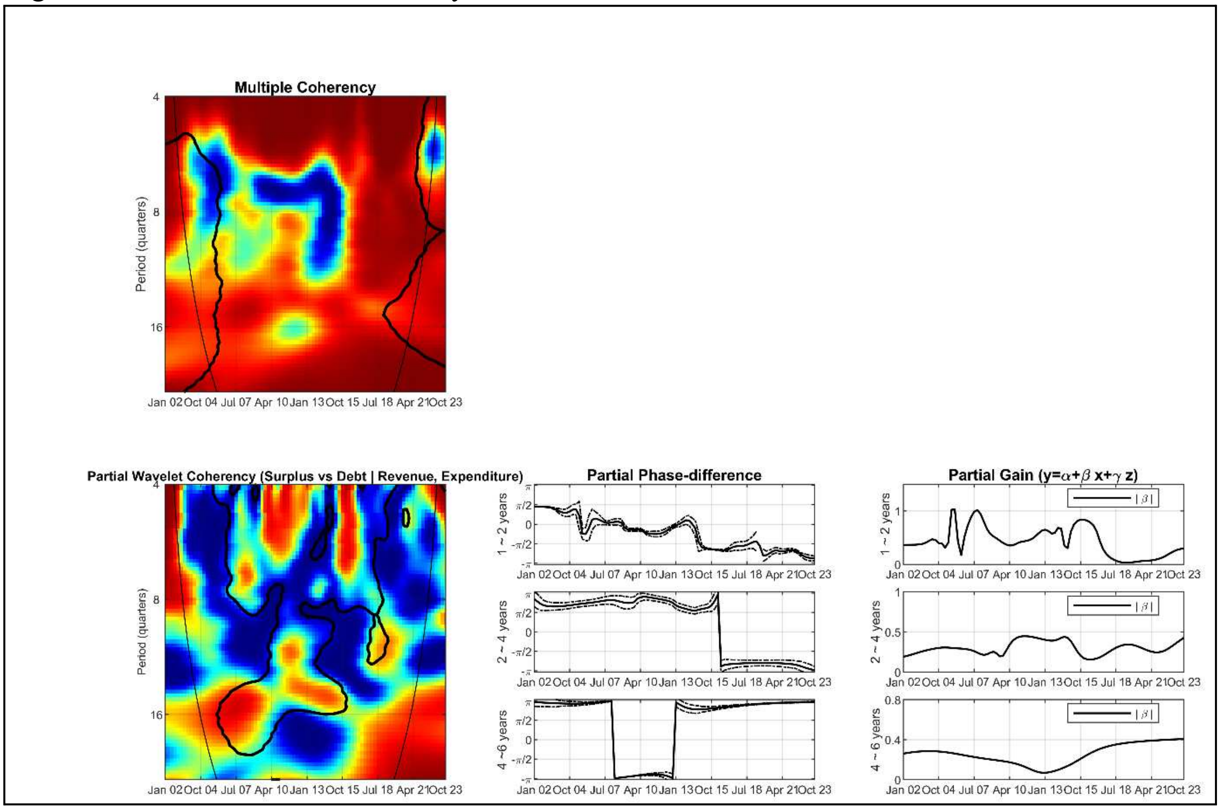
In the long term, the region from January 2006 to the second quarter of 2010 can be divided into two parts. Debt leads out of phase until January 2008, after which there is a shift to a situation where the surplus leads out of phase. In other words, increases in the surplus are followed by reductions in debt. In the short term, solvency is observed from October 2009 to July 2013, with an average gain of 0.5. From April 2015 to January 2017, the surplus leads out of phase.

Although the region from 2006 to 2010 is significant in both the short and medium terms, the behavior observed presents a small difference between 2008 and 2010, which coincides with the subprime crisis and the onset of the sovereign debt crisis. This is a common feature, as economic relationships can change when considering different time horizons (AGUIAR-CONRARIA, AZEVEDO, & SOARES, 2008).

In the literature on public debt sustainability, there is no consensus regarding the Italian case. For instance, while Afonso and Jalles (2017) observe unsustainability between 1999 and 2013, the results from Fincke and Greiner (2012) (between 1972 and 2009) and Paniagua, Sapena, and Tamarit (2017) (between 1970 and 2014) suggest sustainability for Italian fiscal policy.

The results of the present study shed light on these discrepancies by capturing both behaviors at different frequency bands. The medium-term results reflect the high levels of debt observed by Italy throughout the period and the growing trend of the debt stock due to the recession faced during that time (OECD, 2015). On the other hand, the long-term results capture the progress in strengthening Italy's fiscal policy, mainly through the reduction of the budget deficit between 2009 and 2012 and the implementation of austerity policies (OECD, 2015).

Figure 17 – Wavelet results for Italy.



Source: Own elaboration.

5.3.8 Portugal

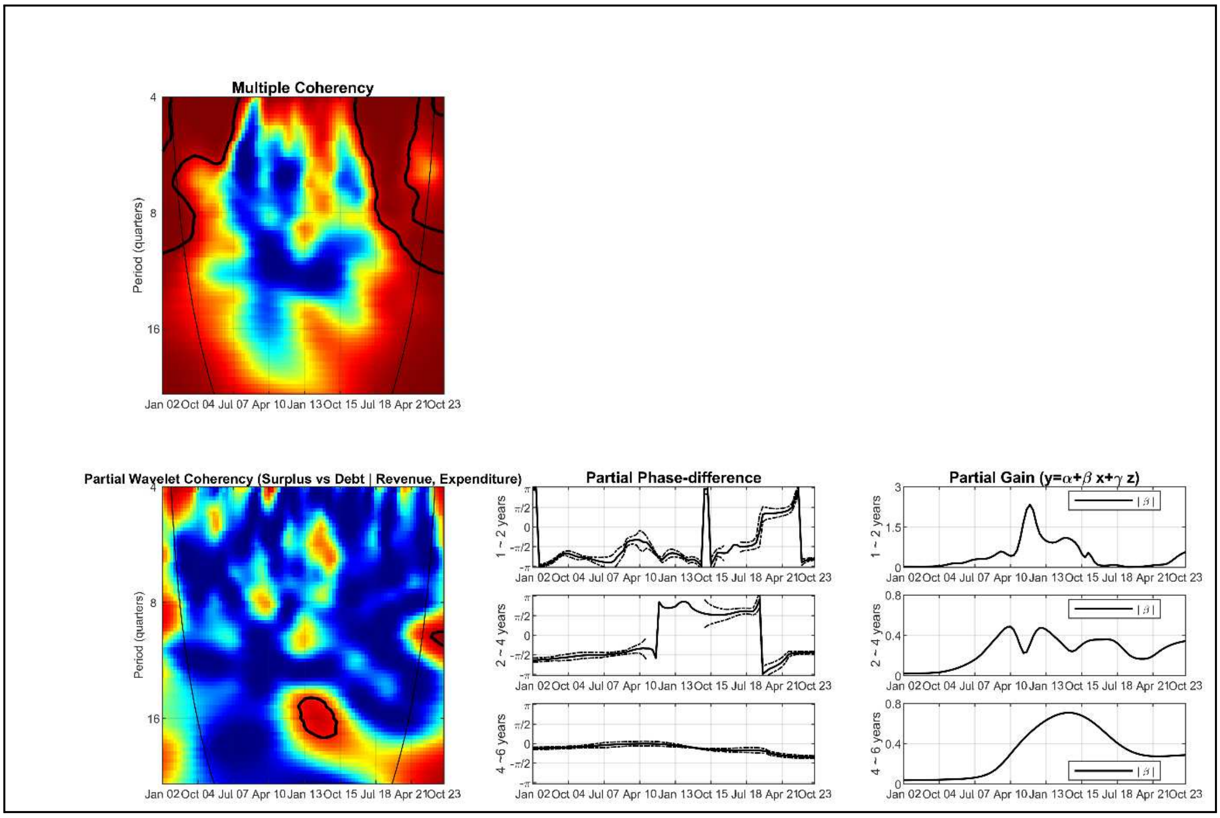
In the case of Portugal, there is also little significance, similar to the findings for Greece. Once again, the result mirrors that of Aguiar-Conraria and Soares (2011), who concluded that Portugal, like Greece, is poorly synchronized with the rest of Europe in terms of business cycles. This result is confirmed by Ahlborn and Wortmann (2018), using a cluster analysis methodology.

For the 1 to 2-year cycles, no significant regions were found. In the 2 to 4-year cycles, there is a single significant region from July 2012 to April 2015, where debt leads out of phase, indicating an insolvent behavior. The partial gain peaks at 0.47 in October 2012. In the 4 to 6-year cycles, there is also a single region extending from January 2013 to July 2015, which can be divided into two parts. Until October 2012, the surplus leads in phase. After that, debt leads in phase, showing a solvent behavior. The partial gain reaches its maximum value of 0.71 in October 2014.

Although there is a large overlap between the significant regions, the behavior is entirely different across frequencies. The medium-term results reflect the difficulties Portugal faced during the sovereign debt crisis, with high levels of debt and deficits, which led to the bailout request in April 2011. Neto (2020), using quarterly data from 1999Q4 to 2017Q2, concludes that there was a period of unsustainable debt for Portugal between 2006 and early 2016. From the second quarter of 2016, the debt seems to return to a sustainable trajectory. Afonso and Jalles (2017) also present evidence of unsustainability between 1999 and 2013.

However, in the long term, the effectiveness of the implemented measures is observed (BALDI & STAEHR, 2015). The difference in behaviors reflects the time it took for the measures to produce the expected effects. Indeed, Portugal became one of the most promising economies in Europe after the crisis, showing strong GDP growth and a reduction in the unemployment rate (NETO, 2020). Additionally, it reduced its deficits to 2% of GDP in 2016, which contributed to the country's exit from the Excessive Deficit Procedure in 2017 and allowed it to repay part of the IMF loan from 2011 ahead of schedule[9].

Figure 18 – Wavelet results for Portugal.



Source: Own elaboration.

5.3.9 Romania

Romania also presents evidence of insolvency, particularly for variations in the short term. In the 1 to 2-year cycles, there are two significant regions. The first is from January

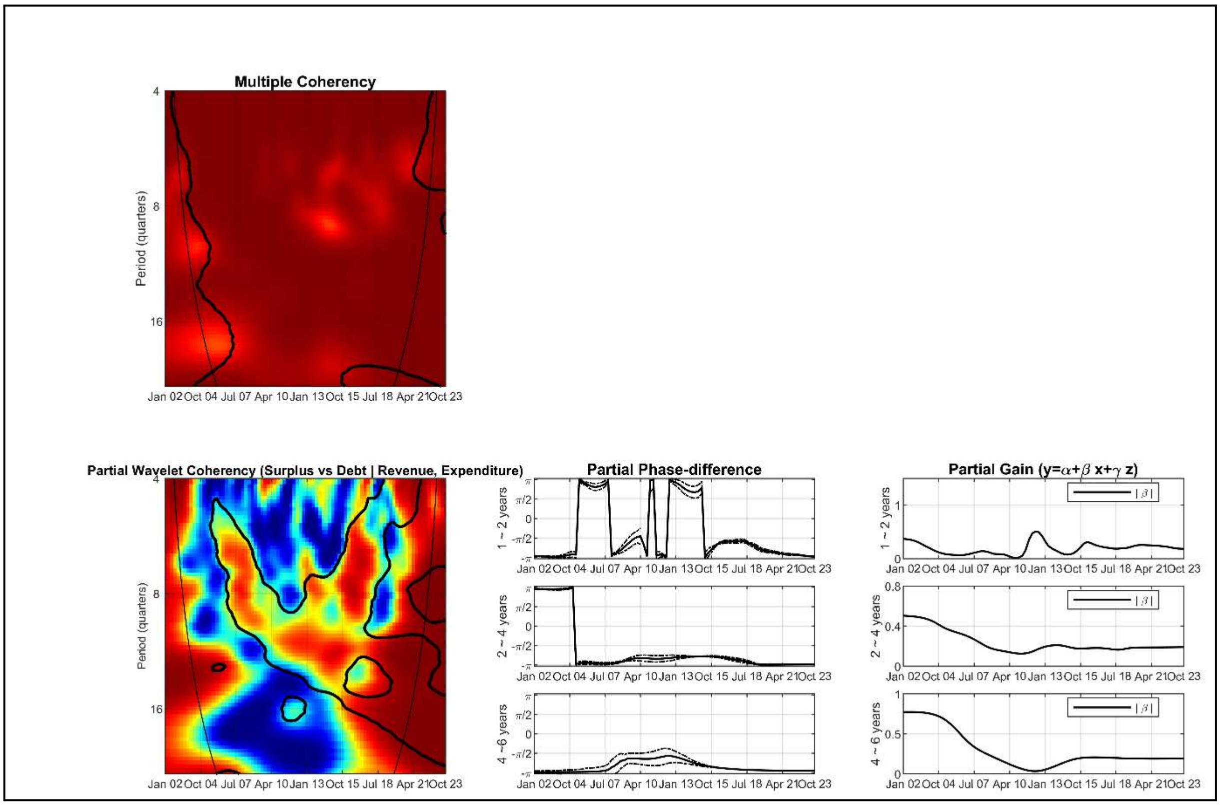
[9] See more at: Portugal freed from debt mechanism, as Spain threatens Greece | Euronews

2006 to July 2010, divided into two parts. Between January 2006 and the end of 2007, debt leads out of phase, indicating insolvency. After that, surplus leads out of phase. The second region spans from January 2013 to July 2021, where, until the first half of 2015, debt leads out of phase, signaling insolvency. After that, surplus leads out of phase.

For the other frequency ranges, there is only one significant region for each cycle, but with a large extension. In the 2 to 4-year cycles, significance is observed from October 2006 to the end of the sample, where, for a brief period, until January 2005, debt leads out of phase. After that, surplus leads out of phase. The partial gain peaks at 0.21 in October 2013. For the 4 to 6-year cycles, the region spans from April 2010 to the end of the sample, with surplus leading out of phase. The partial gain is, on average, 0.21.

The evidence of insolvency found only in the short-term cycles and for a brief period in the medium term reflects the rapid resolution of the debt crisis in Romania (BALDI & STAEHR, 2015). The frequency range also seems to indicate that this behavior is more associated with liquidity risks.

Figure 19 – Wavelet results for Romania.



Source: Own elaboration.

5.3.10 Slovakia

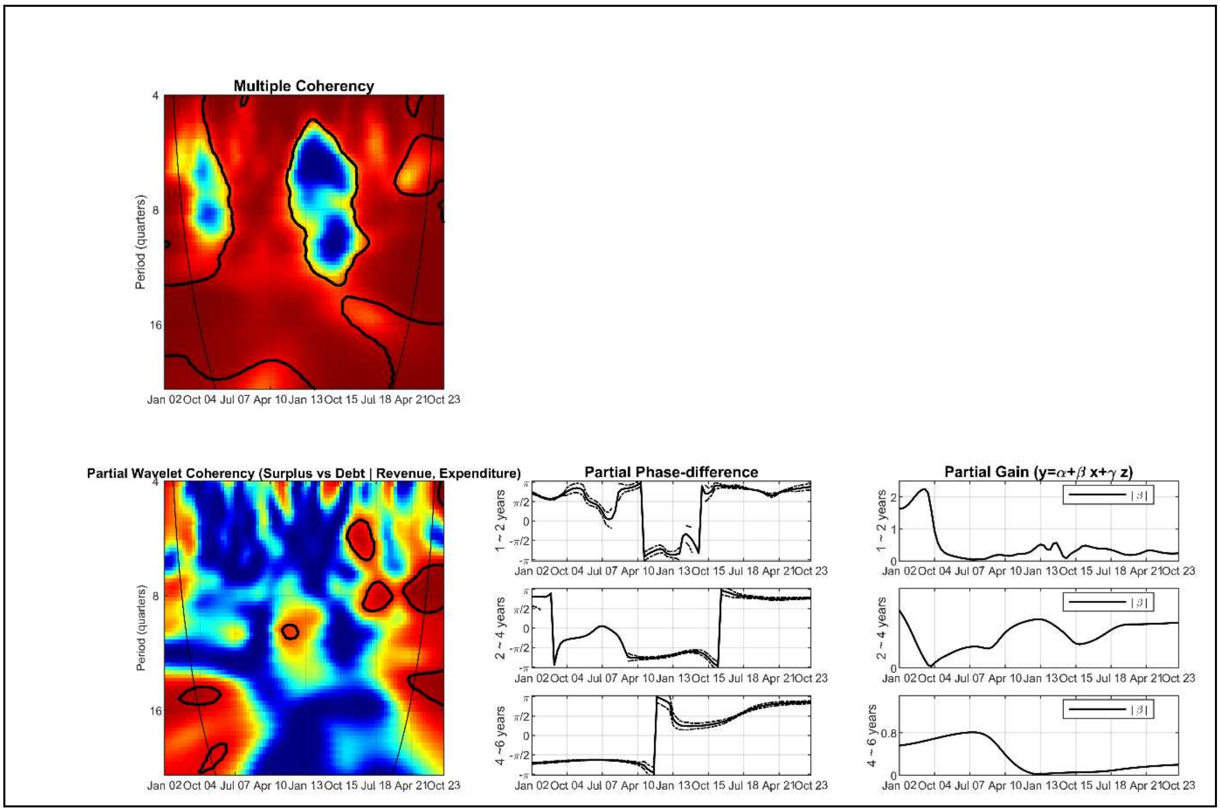
In Slovakia, regions demonstrating insolvent behavior are observed in both the short and medium-term cycles. In the short term (1 to 2 years), there are two intersecting regions: from April 2016 to October 2019 and from July 2020 to the end of the sample. In both, insolvency is evident. The partial gain reached its highest value in April 2016, at 0.46. The results in these regions can be better understood by examining the behavior in the other frequency ranges.

In the medium term (2 to 4 years), from April 2003 to April 2006, debt leads in phase, indicating solvency. This period saw a persistent decline in the debt-to-GDP ratio as part of the preparations for the adoption of the euro in 2009 (FIDRMUC et al., 2013). From April 2011 to April 2012, surplus leads out of phase. The insolvency period in these cycles spans from July 2017 to October 2019 and from July 2020 to the end of the sample, with debt leading out of phase and an average partial gain of 0.28, a weaker relationship than in the short-term cycles. In the 4 to 6-year cycles, there is a single significant region from January 2005 to October 2006, with surplus leading out of phase and an average gain of 0.75, indicating that the strongest relationships are observed in the long term.

Between 2020 and the end of the sample, the results reflect efforts in public spending to combat the effects of the pandemic and the recent energy crisis in Europe due to the Russia-Ukraine conflict, which worsened public finances (OECD, 2024). Starting in the first quarter of 2020, there was an abrupt rise in the debt-to-GDP ratio until the fourth quarter of 2021, when Slovakia first breached the 60% limit. This debt evolution was accompanied by successive deficits throughout the period.

Aguiar-Conraria and Soares (2011) highlight that, even after adopting the euro, Slovakia showed no signs of convergence in terms of business cycles, which could hinder the success of adopted policies. It is noteworthy that the insolvency periods coincide in both the short and medium term, suggesting a persistent behavior of debt in relation to surplus.

Figure 20 – Wavelet results for Slovakia.



Source: Own elaboration.

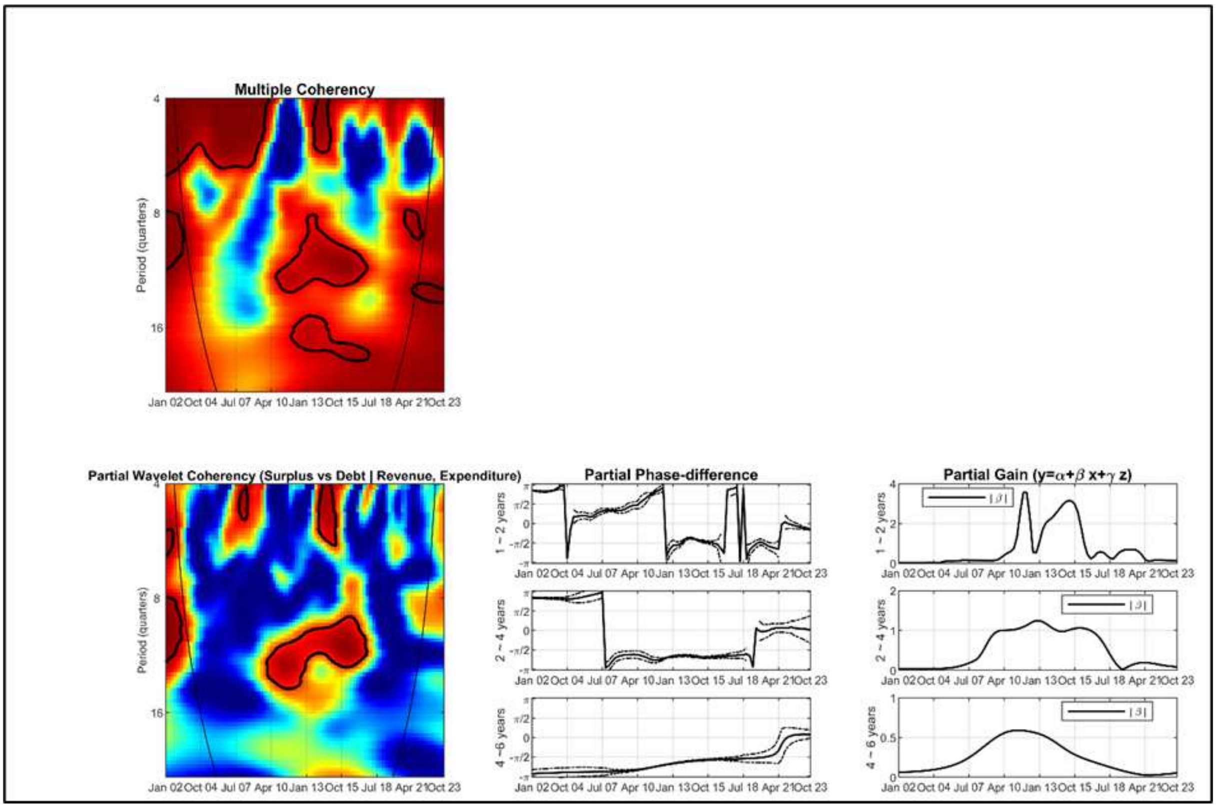
5.3.11 Slovenia

For Slovenia's economy, only one region appears to indicate insolvency, observed in the medium-term cycles, from January 2002 to April 2003. However, it is important to note that this region is completely outside the cone of influence, making it susceptible to edge effects. Therefore, this result should be viewed with caution. Another region in the medium-term cycle spans from January 2010 to October 2017, where surplus leads debt out of phase, indicating solvency.

In the short term (1 to 2 years), the first region extends from October 2007 to July 2008, with a low average partial gain of 0.14. In this period, the surplus cycles lead the debt cycles in a phase relationship, meaning increases in surplus are preceded by increases in debt. The second region in this frequency is between January 2014 and April 2015, with a peak partial gain of 3.14, a remarkably high value compared to other observations. In this period, debt leads in-phase, indicating solvency.

Finally, there are no significant regions in the 4 to 6-year cycles. Overall, the highest partial gain values are found in the significant regions, and there is an absence of strong evidence of insolvent behavior.

Figure 21: Wavelet results for Slovenia.



Source: Own elaboration.

5.3.12 Spain

In the case of Spain, the variability of the cycles tends to increase from shorter-duration cycles to longer-duration cycles, which can be interpreted as a focus on stabilization policies in the medium and long term.

In the short-term cycles (1 to 2 years), a region extends from the beginning of the sample to April 2006, with a maximum partial gain of 0.26, which is low compared to other periods. This region is divided into two parts. The first part, from the beginning of the sample to October 2004, shows debt leading out of phase, indicating insolvency. This is a brief period and the only instance of insolvency for Spain throughout the entire sample. There is a transition from insolvency to a surplus-led situation, with a consistent reduction in the debt-to-GDP ratio until the second quarter of 2008. By April 2006, surplus leads in-phase. Moreover, Spain had a favorable economic situation, with an average GDP growth of 3.8% from 2000 to 2007 (AFONSO; VERDIAL, 2020).

Also, in 2007, Spain adopted four fiscal rules at the national level, ranking sixth in the strength of such rules among EU countries (MOŻDZIERS, 2015). Afonso and Jalles

(2017) emphasize that fiscal rules, particularly those based on spending, significantly contribute to the fiscal sustainability of the euro area.

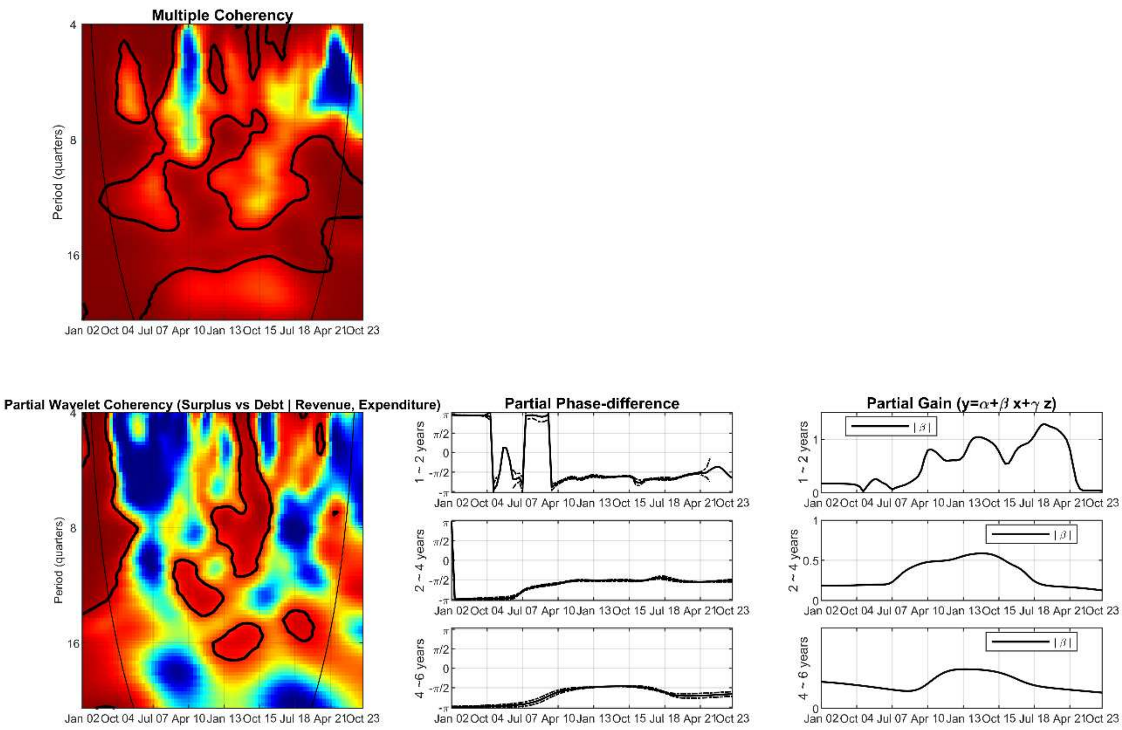
Additionally, a significant region is observed from January 2012 to July 2016, with the peak partial gain of 1.04 in January 2014, indicating a stronger relationship. In this region, surplus leads out of phase, meaning increases in surplus are followed by reductions in debt. This period is marked by a significant fiscal consolidation effort that began in 2010 with a contractionary fiscal policy and was reinforced over the years, alongside European Union assistance in 2012 via the European Stability Mechanism aimed at recapitalizing the banking system (AFONSO; VERDIAL, 2020).

For the 2 to 4-year cycles, a region from the beginning of the sample to April 2006 is observed, where surplus leads out of phase. An intersection of three regions extends from January 2009 to July 2016, a period during which the partial gain peaks at 0.59 in July 2014, with surplus leading out of phase for most of the time. Finally, a region from July 2017 to January 2020 is observed, divided into two parts. Until July 2019, debt leads in-phase, indicating solvency. After that, surplus leads out of phase.

In the 4 to 6-year cycles, there is a single significant region from January 2012 to October 2015, where debt leads in-phase, indicating solvency. This likely reflects the effects of the various measures adopted by the Spanish government since 2010 to combat the crisis, aligning with the trend in the multivariate coherence graph.

Despite the high levels of debt observed during this period, this behavior was closely monitored by the updated Stability Program. The program also outlined that the consolidation process would continue between 2013 and 2016, aiming to stabilize the debt around 100% of GDP (GORDO; HERNÁNDEZ de COS; PÉREZ, 2013), which was indeed achieved. The average debt-to-GDP ratio in 2016 was 103.8%. Therefore, these numbers seem to represent a two-step process. In the first step, efforts were made to slow the growth of debt and stabilize the initial impacts of the crisis. Once growth was controlled, the focus shifted to reducing the debt level, which was marginally achieved. The average debt-to-GDP ratio during 2018-2019 was 100.9%, with a value of 98.2% in the last quarter of 2019. The results demonstrate no strong evidence of fiscal unsustainability in Spain's fiscal policy.

Figure 22 – Wavelet results for Spain.



Source: Own elaboration.

Therefore, the results point to evidence of insolvent behavior in eight countries, namely Croatia, Cyprus, Finland, France, Italy, Slovakia, Romania and Portugal, suggesting an unsustainable fiscal policy in specific periods and frequencies of the sample. The results for Croatia, Cyprus and Spain show that insolvent behavior was more associated with liquidity risks. In addition, there is evidence that the policies adopted by these countries in times of crisis were effective.

The next section presents the results of the connectivity approach.

5.4 Connectedness results

The results below were obtained from the estimation of a TVP-VAR (1) model selected by the Bayesian Information Criterion with a two-year forecasting horizon (8 quarters). The choice of prediction horizon is based on the frequency ranges used in the solvency analysis, which, as explained in the previous section, is in line with what is used in the literature. Another point to note is that varying the predictive horizon in the connectivity approach does not constitute a robustness analysis. As argued by Diebold and Yilmaz (2014), the relationships between economic variables can change as different time intervals are considered. Longer time horizons increase the chances of connectivity appearing. Thus, there is no reason to expect connectivity measures to be “robust” to the forecast horizon considered.

The connectedness table is presented below. The elements in the columns represent the contribution of the country located in that column to the decomposition of the forecast error variance for the other countries. For example, Belgium accounts for 17.89% of the decomposition of the forecast error variance two years ahead for Italy. The "TO" row provides the total contribution of each country to the decomposition of the variance for other countries, excluding its own contribution.

The contribution of each country to the decomposition of the forecast error variance of the others is not limited to 100%, since imposing such a restriction does not make sense in the construction of the measures. Therefore, entries in the "TO" line can normally exceed 100%. It is the case for Spain, which transmits 117.55% of shocks to the others.

The "Inc. Own" row is the value of the "TO" row added to the country's contribution to itself (including its own contribution, hence the name "inc. own"). The "FROM" column corresponds to the sum of the rows excluding the elements on the main diagonal. It shows the participation of other countries in the decomposition of the variance of the country in that row. These values are normalized by the sum of the row, meaning that the sum of each row, including the elements on the main diagonal, is 100. For example, 66.58% of the two-year-ahead forecast error variance for Cyprus comes from shocks in other countries. By adding Cyprus's contribution to itself, we get 33.42 + 66.58 = 100. The "NET" row represents the difference between "TO" and "FROM." A positive (negative) value indicates that the country is a net transmitter (receiver) of shocks.

Finally, the "NPT" row indicates how many countries are "dominated" by the country in that column in terms of net dynamic pairwise connectedness. For example, Spain dominates all the countries in terms of connectivity transmission evaluated pairwise. It is the NPDC (Net Pairwise Dynamic Connectedness) graph represented in numbers (Figure 25). The total connectivity index (TCI) is 63.91. This means that 63.91% of all the connectivity in the network comes from shocks between the countries. It is a considerable value, indicating a high level of interrelations in the network.

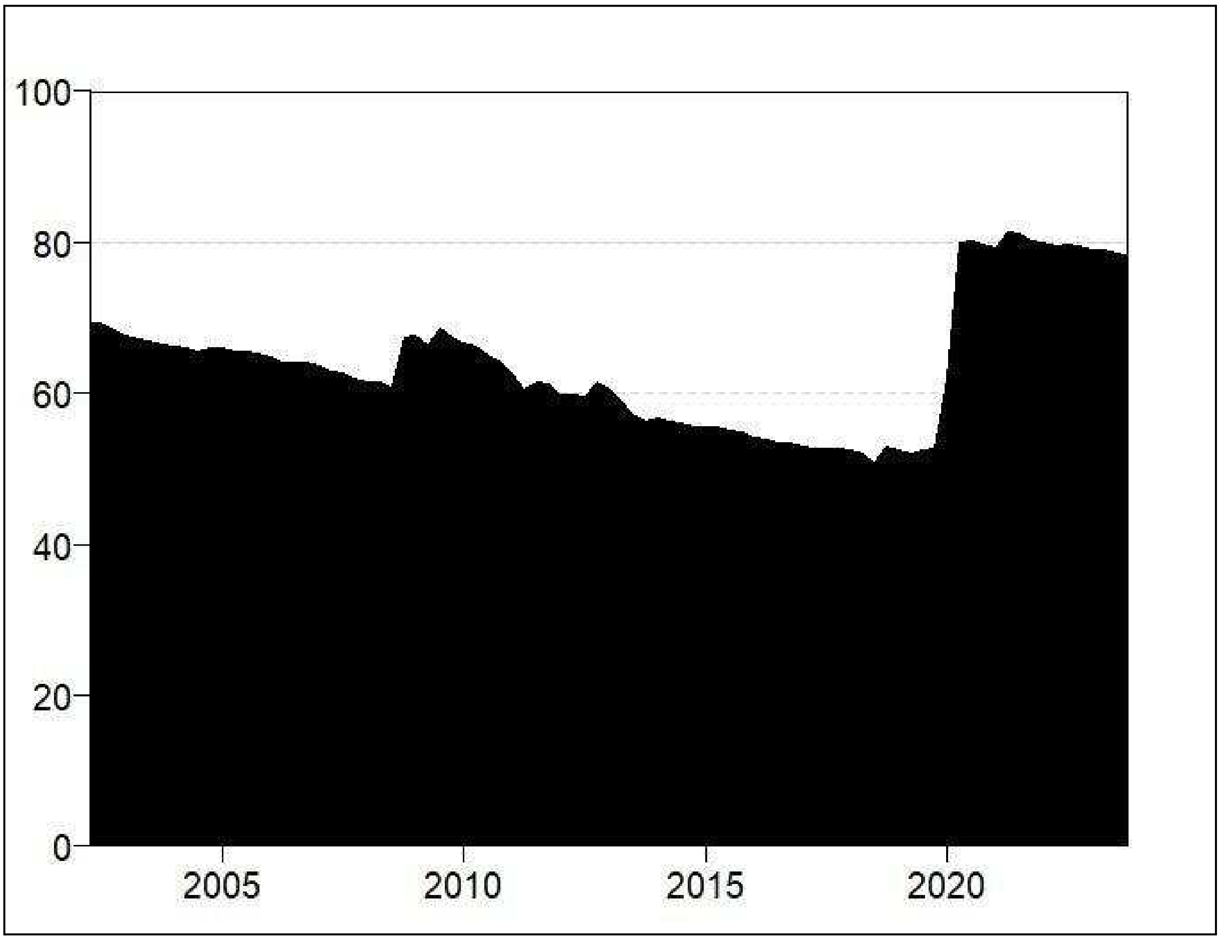
Table 9 – Connectedness results.

	Belgica	Cyprus	Croatia	Finland	France	Greece	Italy	Portugal	Romania	Slovakia	Slovenia	Spain	FROM
Belgica	**28.68**	6.00	3.82	8.32	12.15	0.96	19.38	2.12	4.07	3.70	3.21	7.59	71.32
Cyprus	8.47	**33.42**	6.72	3.82	7.83	1.28	8.61	5.21	4.41	4.96	6.99	8.28	66.58
Croatia	4.49	4.46	**35.06**	3.02	6.95	2.49	5.46	5.57	6.56	3.92	7.91	14.10	64.94
Finland	7.22	4.25	5.21	**47.55**	2.69	3.67	4.78	2.25	10.41	2.58	2.66	6.73	52.45
France	11.79	4.50	7.08	6.77	**24.07**	0.99	11.38	4.22	7.46	4.64	6.56	10.55	75.93
Greece	3.50	2.39	7.56	1.45	4.05	**46.24**	3.81	12.20	3.23	2.82	5.03	7.72	53.76
Italy	17.89	5.81	7.58	4.37	12.84	1.18	**26.20**	2.22	3.91	3.24	4.52	10.24	73.80
Portugal	3.99	2.97	11.35	2.12	7.83	2.33	4.86	**38.82**	5.06	4.21	4.01	12.46	61.18
Romania	3.07	2.04	5.08	6.76	5.78	4.16	2.06	2.65	**45.11**	6.03	3.59	13.67	54.89
Slovakia	7.37	2.19	4.02	4.96	8.33	1.18	3.88	3.33	8.51	**37.50**	3.06	15.67	62.50
Slovenia	5.63	7.04	9.87	4.17	9.66	2.78	5.57	3.89	3.08	3.71	**34.08**	10.53	65.92
Spain	4.37	2.53	8.36	5.30	8.85	1.86	4.38	5.19	7.38	8.99	6.41	**36.38**	63.62
To	77.79	44.18	76.64	51.05	86.94	22.88	74.16	48.85	64.08	48.81	53.94	117.55	766.87
Inc Own	106.47	77.60	111.70	98.60	111.01	69.12	100.36	87.68	109.19	86.31	88.02	153.94	**TCI**
NET	6.47	-22.40	11.70	-1.40	11.01	-30.88	0.36	-12.32	9.19	-13.69	-11.98	53.94	**63.91**
NPT	6	3	8	4	7	2	7	3	8	4	3	11	

Source: Own elaboration.

The chart below corresponds to the Total Dynamic Connectivity Index. It is clearly observable that there are spikes in connectivity during the subprime crisis (2007-2009), the sovereign debt crisis (2010-2012), the pandemic (2019-2021), and the Russia-Ukraine conflict followed by the European energy crisis (2022-2023). Such a result is expected, as total connectivity in the TVP-VAR model adjusts quickly to events (ANTONAKAKIS; CHATZIANTONIOU; GABAUER, 2020) and is highly dependent on events (CHATZIANTONIOU; GABAUER; STENFORS, 2021). During the crisis period corresponding to the pandemic, total connectivity reaches its highest values, a fact documented in recent literature (AKHTARUZZAMAN; BOUBAKER; SENSOY, 2020; COSTA; MATOS; SILVA, 2021).

Figure 23 – Total Dynamic Connectivity Index.



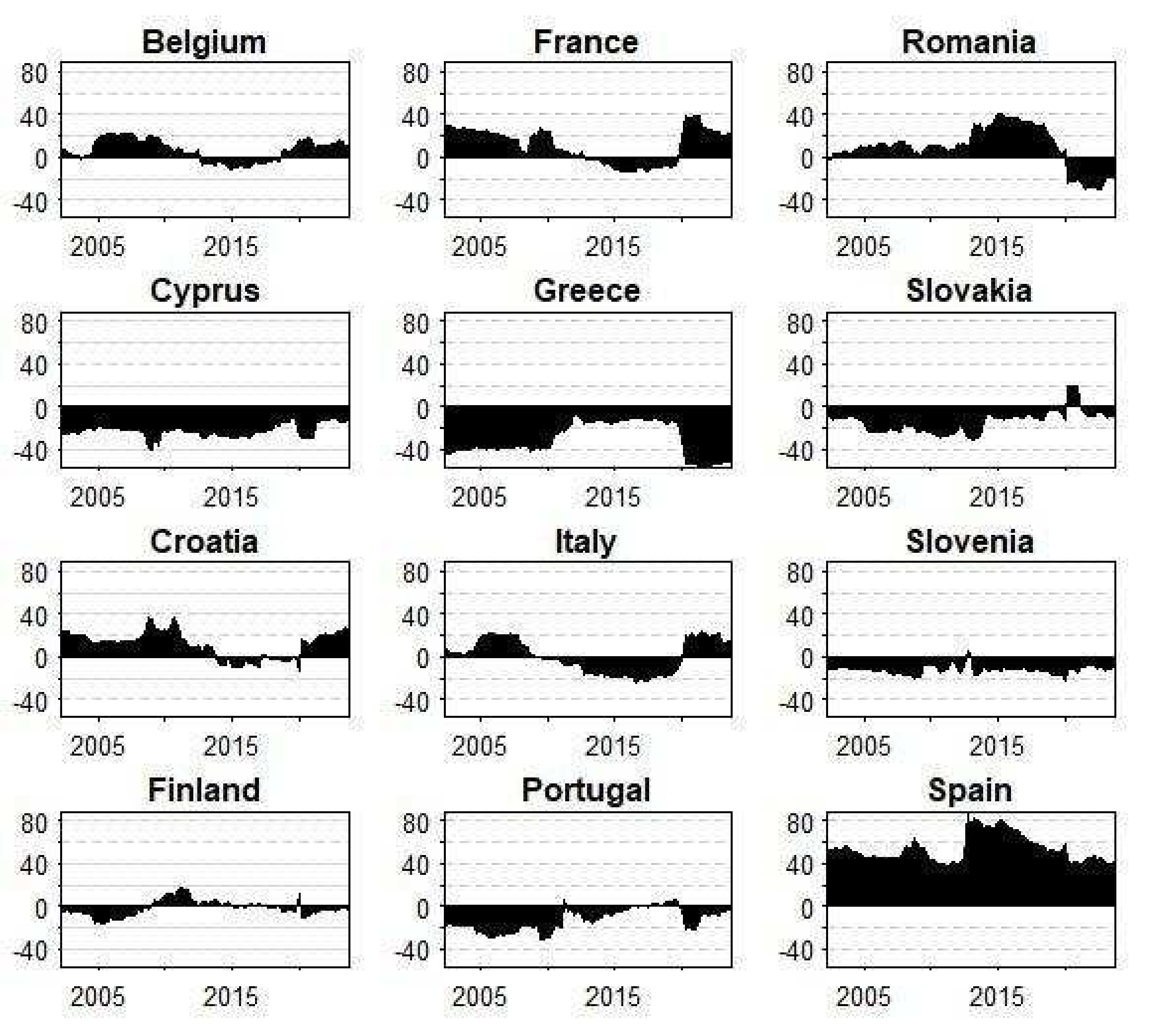
Source: Own elaboration.

Below is the chart of Net Total Connectivity for each country. This is the same information as the "NET" row in the connectivity table. Belgium, Croatia, France, Italy (weakly), Romania and Spain are net transmitters of connectivity. It is interesting to note some patterns. A first highlight is Spain, which throughout the sample is a strong net transmitter of shocks reaching peaks in 2008, 2012, and 2015, and a decreasing trend from then until 2020. France is the second main transmitter of shocks. Romania transitions from net transmitter to net receiver in 2020. Cyprus and Greece are net receivers throughout the sample. Croatia is a prominent net transmitter during the sovereign debt crisis.

Ando, Greenwood-Nimmo, Shin (2022), studying credit risk connectivity between sovereign governments and their respective financial sectors, also found strong spillovers from the Belgian financial sector, which, according to the same authors, highlights the important role played by Belgium in the sovereign debt crisis.

The fact that Greece is identified as a net receiver of spillovers may seem strange given its role in the sovereign debt crisis. However, several studies in the contagion literature, which is closely related to the connectivity approach, find no evidence of contagion from the Greek crisis (MINK; DE HANN, 2012; PHILIPPAS; SIRIOPOULOS, 2013; SAMITAS; TSAKALOS, 2013; PRAGIDIS et al., 2015). Aguiar-Conraria and Soares (2011), studying the synchronization of business cycles, found that France and not Germany, as is often assumed, leads the European cycle. The results presented here show that, in addition to business cycles, France also plays a leading role in the spillover of debt shocks in the European Union.

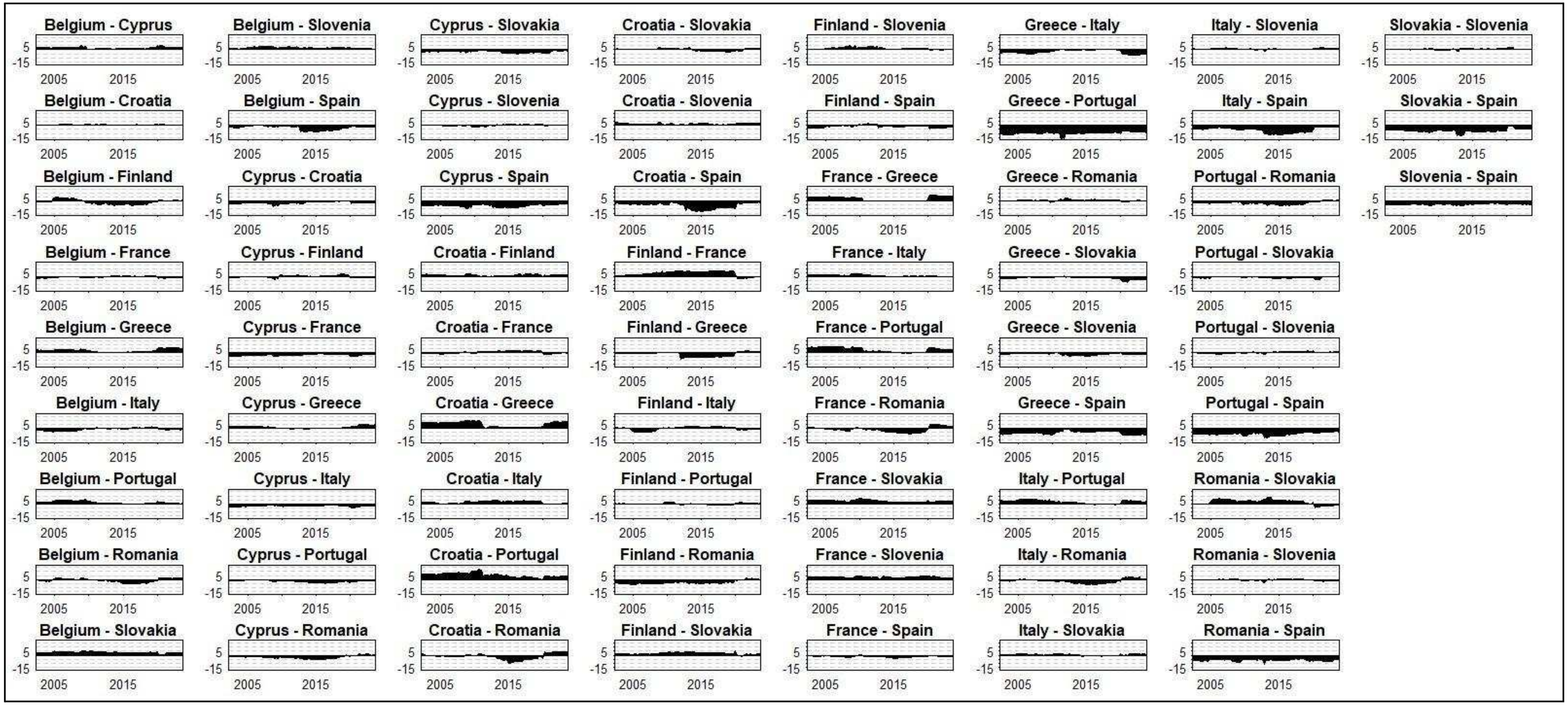
Figure 24 – Net Total Connectivity for each country.



Source: Own elaboration.

Analysis of the Net Pairwise Dynamic Connectedness as shown in Figure 25, reveals interesting patterns. First, Finland's strong dominance over two of the largest liquid transmitters, France and Belgium. Also surprising is the large transmission of shocks from Portugal to Greece, especially during the sovereign debt crisis and from Spain to Italy. It is worth noting the transmission of shocks from Croatia to Portugal throughout the sample, an unexpected transmission channel. Lastly, there is also a strong influence of Cyprus and Croatia on Greece, particularly during the pandemic.

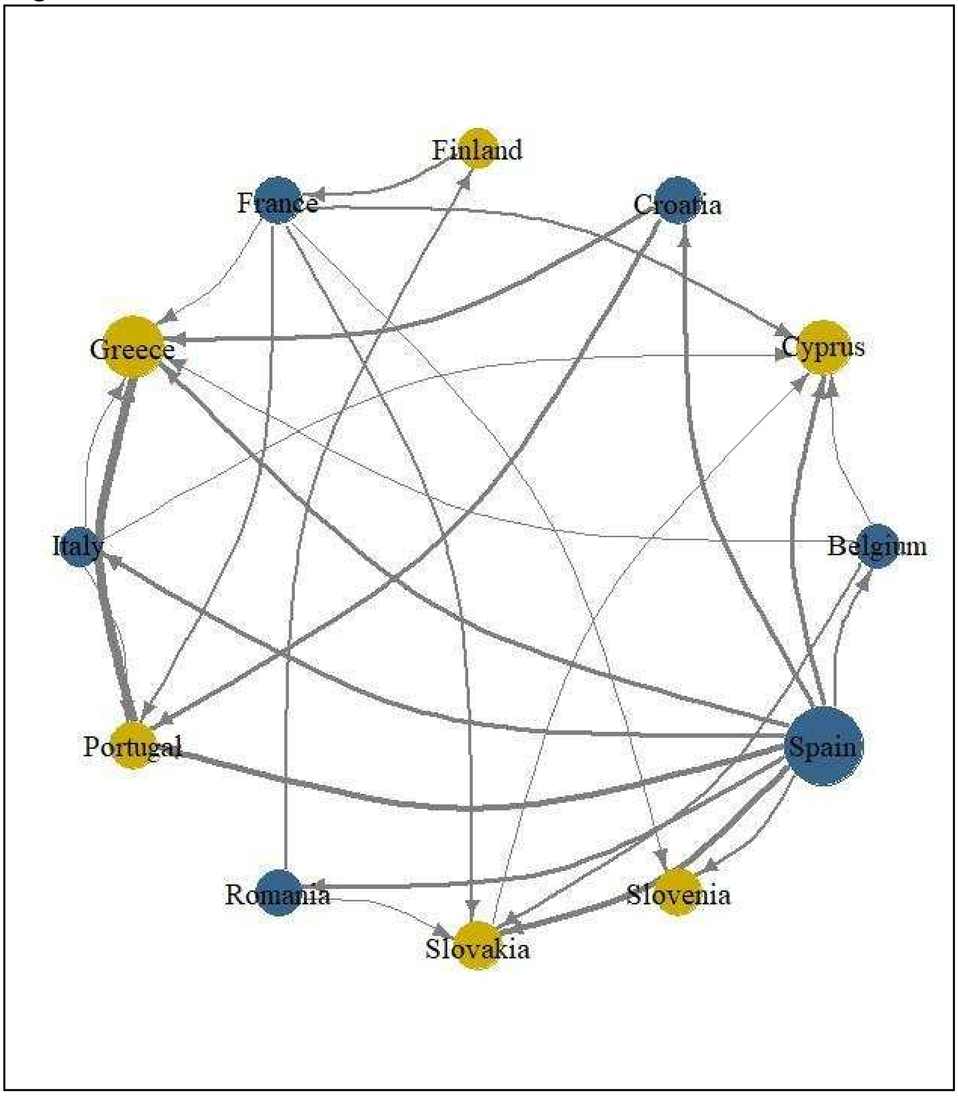
Figure 25 – Net Pairwise Dynamic Connectedness (NPDC).



Source: Own elaboration.

Finally, we have the network chart. All the charts are based on the connectivity table. That is, they represent the same information in different ways. The blue circles represent the net transmitters (positive NET value in the table), and the yellow circles refer to the net receivers (negative NET value in the table). The size of the circles indicates the magnitude of the corresponding values from the table. For example, Greece is the country that receives the most shocks (with the most negative NET value in the table). Therefore, it has the largest yellow circle.

Figure 26 – Network Connectedness.



Source: Own elaboration.

6. CONCLUSION

The literature on European public debt is extensive and controversial. For this reason, this study used a systematic approach to overcome the main difficulties found in the literature.

Firstly, the issue of classifying countries into groups according to their debt behavior was addressed. Using cluster techniques, it was initially possible to distinguish three groups with a good level of precision. Next, in an attempt to consider specific characteristics of the debt trajectory, with an emphasis on long-term behavior, in order to detect possible convergence clubs, the convergence testing methodology proposed by Phillips and Sul (2007, 2009) and the algorithms proposed by von Lyncker and Thoennessen (2017) were applied. The results confirm the existence of two debt convergence clubs in the European Union. Taking the intersection of the cluster and convergence results, a sub-sample of twelve countries was defined for which a solvency analysis was applied. The sub-sample consists of the following countries: Italy, France, Spain, Portugal, Greece, Belgium, Cyprus, Finland, Slovenia, Croatia, Slovakia, and Romania.

Continuous Wavelet Transform (CWT) was then used to assess the solvency/sustainability of the public debt of the twelve countries in the sub-sample. This technique makes it possible to assess the behavior of the debt in different frequency ranges, enabling a separation between liquidity and solvency risks, facilitating the interpretation of the results as well as the analysis of the effectiveness of the policies adopted in the period. The results point to evidence of insolvent behavior in eight of the twelve countries analyzed. The results for Croatia, Cyprus and Spain show that insolvent behavior was more associated with liquidity risks. In addition, there is evidence that the policies adopted by these countries in times of crisis were effective.

Finally, in order to analyze the transmission of debt shocks between these countries, the connectivity spillover approach proposed by Diebold and Yilmaz (2009, 2012, 2014) was applied in a VAR model structure with time-varying parameters (TVP-VAR) according to Antonakakis, Chatziantoniou, and Gabauer (2020), in order to capture the dynamic effect of spillovers. The results show that 63.91% of the total connectivity of the network formed by the twelve countries in the sub-sample comes from spillovers, in this case from debt shocks between these countries. Spillovers increase during periods of crisis, especially during the COVID-19 pandemic. In addition, Belgium, Croatia, France, Italy (weakly), Romania and

Spain are identified as net transmitters of spillovers, with particular emphasis on Spain, while Greece plays a special role among net receivers, being the country most affected by the others throughout the sample period together with Cyprus. Croatia proves to be an influential transmitter of shocks during the sovereign debt crisis and Spain appears as a leading economy in the sense of transmitting debt shocks within the group, dominating all the other countries in a pairwise analysis.

The results of this study are useful in that they provide a better understanding of debt behavior within the European Union. The existence of debt clubs made it possible to identify the most vulnerable countries. Solvency analysis made it possible to characterize periods of illiquidity and insolvency and to verify the effectiveness of the policies adopted. The analysis of spillovers made it possible to identify which countries exert the greatest influence on the transmission of debt shocks and which are most affected. These conclusions are particularly important for policymakers. Particularly with regard to evaluating fiscal policy and mapping the transmission of shocks, which makes it possible to adopt preventive actions aimed at reducing the negative impacts of a possible crisis, especially for a region with an unfavorable fiscal history and threatened by recent turbulence such as the European Union.

REFERENCES

AFONSO, António; JALLES, João Tovar. Euro area time‐varying fiscal sustainability. **International Journal of Finance & Economics**, [*s.l*], v. 22, n. 3, p. 244-254, jul. 2017.

AFONSO, António; VERDIAL, Nuno. **Sovereign debt crisis in Portugal and Spain**. EconPol Working Paper, 2020.

AGUIAR-CONRARIA, Luís; AZEVEDO, Nuno; SOARES, Maria Joana. Using wavelets to decompose the time–frequency effects of monetary policy. **Physica A: Statistical mechanics and its Applications**, [*s.l*], v. 387, n. 12, p. 2863-2878, may 2008.

AGUIAR-CONRARIA, Luis; MARTINS, Manuel MF; SOARES, Maria Joana. Estimating the Taylor rule in the time-frequency domain. **Journal of Macroeconomics**, [*s.l*], v. 57, p. 122-137, sep. 2018.

AGUIAR-CONRARIA, Luís; SOARES, Maria Joana. Business cycle synchronization and the Euro: A wavelet analysis. **Journal of Macroeconomics**, [*s.l*], v. 33, n. 3, p. 477-489, sep. 2011.

AGUIAR‐CONRARIA, Luís; SOARES, Maria Joana. The continuous wavelet transform: Moving beyond uni‐and bivariate analysis. **Journal of economic surveys**, [*s.l*], v. 28, n. 2, p. 344-375, 2014.

AGUIAR-CONRARIA, Luís; SOARES, Maria Joana; SOUSA, Rita. California's carbon market and energy prices: a wavelet analysis. **Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences**, [*s.l*], v. 376, n. 2126, p. 20170256, aug. 2018.

AHLBORN, Markus; WORTMANN, Marcus. The core–periphery pattern of European business cycles: A fuzzy clustering approach. **Journal of Macroeconomics**, [*s.l*], v. 55, p. 12-27, mar. 2018.

AHMED, Shaghil; ROGERS, John H. Government budget deficits and trade deficits Are present value constraints satisfied in long-term data? **Journal of Monetary Economics**, [*s.l*], v. 36, n. 2, p. 351-374, nov. 1995.

AKHTARUZZAMAN, Md; BOUBAKER, Sabri; SENSOY, Ahmet. Financial contagion during COVID–19 crisis. **Finance Research Letters**, [*s.l*], v. 38, p. 101604, jan. 2021.

ALLOZA, Mario et al. Public debt dynamics: a stochastic approach applied to Spain. **Banco de Espana Occasional Paper**, n. 2420, 2024.

ANDO, Tomohiro; GREENWOOD-NIMMO, Matthew; SHIN, Yongcheol. Quantile connectedness: modeling tail behavior in the topology of financial networks. **Management Science**, [*s.l*], v. 68, n. 4, p. 2401-2431, apr. 2022.

ANDRÉ, Mathias et al. French public finances through the financial crisis: it's a long way to recovery. **Fiscal Studies**, [*s.l*], v. 36, n. 4, p. 431-452, 2015.

ANDREWS, Donald WK. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. **Econometrica: Journal of the Econometric Society**, v. 59, n. 3, p. 817-858, may 1991.

ANTONAKAKIS, Nikolaos et al. Dynamic connectedness of uncertainty across developed economies: A time-varying approach. **Economics Letters**, [*s.l*], v. 166, p. 63-75, may 2018.

ANTONAKAKIS, Nikolaos; CHATZIANTONIOU, Ioannis; GABAUER, David. Refined measures of dynamic connectedness based on time-varying parameter vector autoregressions. **Journal of Risk and Financial Management**, [*s.l*], v. 13, n. 4, p. 84, 2020.

ANTONAKAKIS, Nikolaos; GABAUER, David; GUPTA, Rangan. Greek economic policy uncertainty: Does it matter for Europe? Evidence from a dynamic connectedness decomposition approach. **Physica A: Statistical Mechanics and Its Applications**, [*s.l*], v. 535, p. 122280, dec. 2019.

APERGIS, Nicholas; PANOPOULOU, Ekaterini; TSOUMAS, Chris. Old wine in a new bottle: Growth convergence dynamics in the EU. **Atlantic Economic Journal**, [*s.l*], v. 38, p. 169-181, mar. 2010.

ARESTIS, Philip; PHELPS, Peter. Endogeneity analysis of output synchronization in the current and prospective EMU. **Journal of Common Market Studies**, [*s.l*], v. 54, n. 3, p. 525-543, 2016.

ARTIS, Michael J.; ZHANG, Wenda. Core and periphery in EMU: A cluster analysis. **Economic Issues**, [*s.l*], vol. 6, n. 2, p. 39-59, sep. 2001.

ARTIS, Michael J.; ZHANG, Wenda. Membership of EMU: A fuzzy clustering analysis of alternative criteria. **Journal of Economic Integration**, [*s.l*], v. 17, n. 1, p. 54-79, mar. 2002.

BADINGER, Harald; MÜLLER, Werner; TONDL, Gabriele. Regional convergence in the European Union, 1985-1999: A spatial dynamic panel analysis. **Regional Studies**, [*s.l*], v. 38, n. 3, p. 241-253, 2004.

BAJO-RUBIO, Oscar; DÍAZ-ROLDÁN, Carmen; ESTEVE, Vicente. Is the budget deficit sustainable when fiscal policy is non-linear? The case of Spain. **Journal of Macroeconomics**, [*s.l*], v. 28, n. 3, p. 596-608, sep. 2006.

BALCILAR, Mehmet; GABAUER, David; UMAR, Zaghum. Crude Oil futures contracts and commodity markets: New evidence from a TVP-VAR extended joint connectedness approach. **Resources Policy**, [*s.l*], v. 73, p. 102219, oct. 2021.

BALDI, Guido; STAEHR, Karsten. The European debt crisis and fiscal reactions in Europe 2000–2014. **International Economics and Economic Policy**, [*s.l*], v. 13, p. 297-317, jan. 2015.

BARRO, Robert J.; SALA-I-MARTIN, Xavier. Convergence across states and regions. **Brookings papers on economic activity**, [*s.l*], v. 1991, n. 1, p. 107-182, 1991.

BATTISTI, Michele; DE VAIO, Gianfranco. A spatially filtered mixture of β-convergence regressions for EU regions, 1980–2002. **Empirical Economics**, [*s.l*], v. 34, p. 105-121, 2008.

BELKE, Ansgar; DOMNICK, Clemens; GROS, Daniel. Business cycle synchronization in the EMU: Core vs. periphery. **Open Economies Review**, [*s.l*], v. 28, p. 863-892, sep. 2017.

BEZDEK, James C. **Pattern recognition with fuzzy objective function algorithms**. 1 ed. New York, Springer Science & Business Media, 2013.

BOHN, Henning. Are stationarity and cointegration restrictions really necessary for the intertemporal budget constraint? **Journal of Monetary Economics**, [*s.l*], v. 54, n. 7, p. 1837-1847, oct. 2007.

BOHN, Henning. The behavior of US public debt and deficits. **The Quarterly Journal of Economics**, [*s.l*], v. 113, n. 3, p. 949-963, aug. 1998.

BOHN, Henning. The sustainability of budget deficits in a stochastic economy. **Journal of Money, credit and banking**, [*s.l*], v. 27, n. 1, p. 257-271, feb. 1995.

BOREIKO, Dmitri. EMU and accession countries: Fuzzy cluster analysis of membership. **International Journal of Finance & Economics**, [*s.l*], v. 8, n. 4, p. 309-325, sep. 2003.

BRKIĆ, Mislav. Greek sovereign debt crisis: Causes, fiscal adjustment programs and lessons for Croatia. **Croatian Economic Survey**, [*s.l*], v. 18, n. 1, p. 71-99, 2016.

BUBÁK, Vít; KOČENDA, Evžen; ŽIKEŠ, Filip. Volatility transmission in emerging European foreign exchange markets. **Journal of Banking & Finance**, [*s.l*], v. 35, n. 11, p. 2829-2841, nov. 2011.

CAMACHO, Maximo; PEREZ-QUIROS, Gabriel; SAIZ, Lorena. Are European business cycles close enough to be just one?. **Journal of Economic Dynamics and Control**, [*s.l*], v. 30, n. 9-10, p. 1687-1706, sep.-oct. 2006.

CASCIO, Iolanda Lo. A wavelet analysis of US fiscal sustainability. **Economic Modelling**, [*s.l*], v. 51, p. 33-37, dec. 2015.

CHATZIANTONIOU, Ioannis; GABAUER, David; STENFORS, Alexis. Interest rate swaps and the transmission mechanism of monetary policy: A quantile connectedness approach. **Economics Letters**, [*s.l*], v. 204, p. 109891, jul. 2021.

COLLIGNON, Stefan. Fiscal policy rules and the sustainability of public debt in Europe. **International economic review**, [*s.l*], v. 53, n. 2, p. 539-567, may 2012.

COSTA, Antonio; MATOS, Paulo; DA SILVA, Cristiano. Sectoral connectedness: New evidence from US stock market during COVID-19 pandemics. **Finance Research Letters**, [*s.l*], v. 45, p. 102124, mar. 2022.

DIEBOLD, Francis X.; YILMAZ, Kamil. Better to give than to receive: Predictive directional measurement of volatility spillovers. **International Journal of forecasting**, [*s.l*], v. 28, n. 1, p. 57-66, jan.-mar.2012.

DIEBOLD, Francis X.; YILMAZ, Kamil. Measuring financial asset return and volatility spillovers, with application to global equity markets. **The Economic Journal**, [*s.l*], v. 119, n. 534, p. 158-171, jan. 2009.

DIEBOLD, Francis X.; YILMAZ, Kamil. On the network topology of variance decompositions: Measuring the connectedness of financial firms. **Journal of Econometrics**, [*s.l*], v. 182, n. 1, p. 119-134, sep. 2014.

ENGLE, Roter F. Interpreting spectral analyses in terms of time-domain models. **Annals of Economic and Social Measurement**, [*s.l*], v. 5, n. 1, 1976.

EUROSTAT. **Glossary: European Free Trade Association (EFTA)**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:European_Free_Trade_Association_(EFTA). Access in: 05 march 2024.

EUROSTAT. **Glossary: General government sector**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:General_government_sector. Access in: 05 march 2024.

EUROSTAT. **Glossary: Government revenue and expenditure**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Government_revenue_and_expenditure. Access in: 05 march 2024.

EUROSTAT. **Glossary: Stability and growth pact (SGP)**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Stability_and_growth_pact_(SGP). Access in: 05 march 2024.

EUROSTAT. **Glossary:Government debt**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Government_debt. Access in: 05 march 2024.

EUROSTAT. **Glossary:Treaty on European Union**. Luxembourg, 2024. Available at: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Treaty_on_European_Union. Access in: 05 march 2024.

FIDRMUC, J. et al. Slovakia: A Catching Up Euro Area Member. In and Out of the Crisis (IZA Policy Paper No. 55). **Bonn: Institute for the Study of Labor**, 2013.

FINCKE, Bettina; GREINER, Alfred. How to assess debt sustainability? Some theory and empirical evidence for selected euro area countries. **Applied Economics**, [*s.l*], v. 44, n. 28, p. 3717-3724, 2012.

FISCHER, Manfred M.; STIRBÖCK, Claudia. Pan-European regional income growth and club-convergence: insights from a spatial econometric perspective. **The Annals of Regional Science**, [*s.l*], v. 40, p. 693-721, aug. 2006.

GABAUER, David; GUPTA, Rangan. On the transmission mechanism of country-specific and international economic uncertainty spillovers: Evidence from a TVP-VAR connectedness decomposition approach. **Economics Letters**, [*s.l*], v. 171, p. 63-71, oct. 2018.

GADEA-RIVAS, M. Dolores; GÓMEZ-LOSCOS, Ana; BANDRÉS, Eduardo. Clustering regional business cycles. **Economics Letters**, [*s.l*], v. 162, p. 171-176, jan. 2018.

GE, Zhongfu. Significance tests for the wavelet cross spectrum and wavelet linear coherence. In: **Annales Geophysicae**. Göttingen, Germany: Copernicus Publications, 2008. p. 3819-3829.

GENÇAY, Ramazan; SELÇUK, Faruk; WHITCHER, Brandon. Scaling properties of foreign exchange volatility. **Physica A: Statistical mechanics and its applications**, [*s.l*], v. 289, n. 1-2, p. 249-266, jan. 2001.

GORDO, Luis; DE COS, Pablo Hernández; PÉREZ, Javier J. Developments in Spanish public debt since the start of the crisis. **Economic Bulletin**, p. 19-36, 2013.

GOURINCHAS, Pierre-Olivier; MARTIN, Philippe; MESSER, Todd. **The Economics of Sovereign Debt, Bailouts, and the Eurozone Crisis**. International Monetary Fund, 2023.

HAKKIO, Craig S.; RUSH, Mark. Is the budget deficit “too large?”. **Economic inquiry**, [*s.l*], v. 29, n. 3, p. 429-445, jul. 1991.

HAMILTON, James D.; FLAVIN, Marjorie A. On the Limitations of Government Borrowing: A Framework for Empirical Testing. **The American Economic Review**, [*s.l*], v. 76, n. 4, p. 808-819, sep. 1986.

HAUG, Alfred A. Cointegration and government borrowing constraints: Evidence for the United States. **Journal of Business & Economic Statistics**, [*s.l*], v. 9, n. 1, p. 97-101, 1991.

HODRICK, Robert J.; PRESCOTT, Edward C. Postwar US business cycles: an empirical investigation. **Journal of Money, credit, and Banking**, [*s.l*], v. 29, n. 1, p. 1-16, feb. 1997.

INTERNATIONAL MONETARY FUND STRATEGY, POLICY, AND REVIEW DEPARTMENT. **Review of the debt sustainability framework for market access countries**. International Monetary Fund, 2021.

JAGRIČ, Timotej; OVIN, Rasto. Method of analyzing business cycles in a transition economy: The case of Slovenia. **The Developing Economies**, [*s.l*], v. 42, n. 1, p. 42-62, mar. 2004.

KAUFMAN, Leonard; ROUSSEEUW, Peter J. **Finding groups in data: an introduction to cluster analysis**. 1 ed. [*s.l*], John Wiley & Sons, 1990.

KOOP, Gary; KOROBILIS, Dimitris. A new index of financial conditions. **European Economic Review**, [*s.l*], v. 71, p. 101-116, oct. 2014.

KOOP, Gary; PESARAN, M. Hashem; POTTER, Simon M. Impulse response analysis in nonlinear multivariate models. **Journal of econometrics**, [*s.l*], v. 74, n. 1, p. 119-147, sep. 1996.

LILLY, Jonathan M.; OLHEDE, Sofia C. Higher-order properties of analytic wavelets. **IEEE Transactions on Signal Processing**, [*s.l*], v. 57, n. 1, p. 146-160, 2009.

LIN, Fu-Lai et al. Stock and bond return relations and stock market uncertainty: Evidence from wavelet analysis. **International Review of Economics & Finance**, [*s.l*], v. 55, p. 285-294, may 2018.

MAHARAJ, Elizabeth Ann; D'URSO, Pierpaolo; CAIADO, Jorge. **Time series clustering and classification**. 1 ed. [*s.l*], Chapman and Hall/CRC, 2019.

MANDLER, Martin; SCHARNAGL, Michael. Money growth and consumer price inflation in the euro area: a wavelet analysis. **Deutsche Bundesbank**, 2014.

MANKIW, N. Gregory; ROMER, David; WEIL, David N. A contribution to the empirics of economic growth. **The quarterly journal of economics**, [*s.l*], v. 107, n. 2, p. 407-437, may 1992.

MATOS, Paulo et al. Credit, default, financial system and development. **The Quarterly Review of Economics and Finance**, [*s.l*], v. 79, p. 281-289, feb. 2021.

MATOS, Paulo; COSTA, Antonio; DA SILVA, Cristiano. COVID-19, stock market and sectoral contagion in US: a time-frequency analysis. **Research in International Business and Finance**, [*s.l*], v. 57, p. 101400, oct. 2021.

MATOS, Paulo; MONTEIRO, Valdeir. A Note on the public investment-debt-cash linkages: a Brazilian cross-state analysis. **Economics Bulletin**, [*s.l*], v. 43, n. 2, p. 1027-1035, 2023.

MINK, Mark; DE HAAN, Jakob. Contagion during the Greek sovereign debt crisis. **Journal of International Money and Finance**, [*s.l*], v. 34, p. 102-113, apr. 2013.

MONFORT, Mercedes; CUESTAS, Juan Carlos; ORDOÑEZ, Javier. Real convergence in Europe: A cluster analysis. **Economic Modelling**, [*s.l*], v. 33, p. 689-694, jul. 2013.

MOŻDZIERZ, Anna. Strengthening the post-crisis fiscal rules–the case of Spain, Slovakia and Sweden. **Equilibrium. Quarterly Journal of Economics and Economic Policy**, [*s.l*], v. 10, n. 2, p. 31-52, 2015.

NECK, Reinhard; STURM, Jan-Egbert. Sustainability of public debt: introduction and overview. *In:* NECK, Reinhard; STURM, Jan-Egbert (ed.). **Sustainability of Public Debt.** Cambridge: MIT PRESS, 2008. p. 1-13

NETO, David. Tracking fiscal discipline. Looking for a PIIGS on the wing. **International Economics**, [*s.l*], v. 163, p. 147-154, oct. 2020.

OECD. **OECD Economic Surveys: Belgium 2013**. OECD Publications Centre, 2013.

OECD. **OECD Economic Surveys: Belgium 2015**. OECD Publications Centre, 2015.

OECD. **OECD Economic Surveys: Finland 2006**. OECD Publications Centre, 2006.

OECD. **OECD Economic Surveys: Finland 2008**. OECD Publications Centre, 2008.

OECD. **OECD Economic Surveys: France 2019**. OECD Publications Centre, 2019.

OECD. **OECD Economic Surveys: Italy 2015**. OECD Publications Centre, 2015.

OECD. **OECD Economic Surveys: Slovak Republic 2024**. OECD Publications Centre, 2024.

PAL, Nikhil R.; BEZDEK, James C. On cluster validity for the fuzzy c-means model. **IEEE Transactions on Fuzzy systems**, [*s.l*], v. 3, n. 3, p. 370-379, aug. 1995.

PANIAGUA, Jordi; SAPENA, Juan; TAMARIT, Cecilio. Fiscal sustainability in EMU countries: A continued fiscal commitment? **Journal of International Financial Markets, Institutions and Money**, [*s.l*], v. 50, n. 50, p. 85-97, sep. 2017.

PENTECÔTE, Jean-Sébastien; HUCHET-BOURDON, Marilyne. Revisiting the core-periphery view of EMU. **Economic Modelling**, [*s.l*], v. 29, n. 6, p. 2382-2391, nov. 2012.

PESARAN, H. Hashem; SHIN, Yongcheol. Generalized impulse response analysis in linear multivariate models. **Economics letters**, [*s.l*], v. 58, n. 1, p. 17-29, jan. 1998.

PHILIPPAS, Dionisis; SIRIOPOULOS, Costas. Putting the "C" into crisis: Contagion, correlations and copulas on EMU bond markets. **Journal of International Financial Markets, Institutions and Money**, [*s.l*], v. 27, p. 161-176, 2013.

PHILLIPS, Peter CB; SUL, Donggyu. Economic transition and growth. **Journal of applied econometrics**, [*s.l*], v. 24, n. 7, p. 1153-1185, oct. 2009.

PHILLIPS, Peter CB; SUL, Donggyu. Transition modeling and econometric convergence tests. **Econometrica**, [*s.l*], v. 75, n. 6, p. 1771-1855, oct. 2007.

PORTUGAL freed from debt mechanism, as Spain threatens Greece. **Euro News**. Lyon, 16 jun. 2017. Europe News. Available at: https://www.euronews.com/my-europe/2017/06/16/portugal-freed-from-debt-mechanism-as-spain-threatens-greece. Access in: 11 october 2024.

PRAGIDIS, I. C. et al. Contagion effects during financial crisis: Evidence from the Greek sovereign bonds market. **Journal of Financial Stability**, [*s.l*], v. 18, p. 127-138, 2015.

QUAH, Chee-Heong. Revisiting business cycles in the Eurozone: A fuzzy clustering and discriminant approach. **Acta Oeconomica**, [*s.l*], v. 64, n. 2, p. 161-180, jul. 2014.

RAMSEY, James B.; LAMPART, Camille. The decomposition of economic relationships by time scale using wavelets: expenditure and income. **Studies in Nonlinear Dynamics & Econometrics**, [*s.l*], v. 3, n. 1, apr. 1998.

RAVN, Morten O.; UHLIG, Harald. On adjusting the Hodrick-Prescott filter for the frequency of observations. **Review of economics and statistics**, v. 84, n. 2, p. 371-376, 2002.

ROUSSEEUW, Peter J. Silhouettes: a graphical aid to the interpretation and validation of cluster analysis. **Journal of computational and applied mathematics**, [*s.l*], v. 20, p. 53-65, nov. 1987.

SAMITAS, Aristeidis; TSAKALOS, Ioannis. How can a small country affect the European economy? The Greek contagion phenomenon. **Journal of International Financial Markets, Institutions and Money**, [*s.l*], v. 25, p. 18-32, 2013.

ŠIMOVIĆ, Hrvoje. Impact of public debt sustainability on fiscal policy in Croatia. **Acta oeconomica**, [*s.l*], v. 68, n. 2, p. 231-244, 2018.

SOLOW, Robert M. A contribution to the theory of economic growth. **The quarterly journal of economics**, [*s.l*], v. 70, n. 1, p. 65-94, feb. 1956.

TORRENCE, Christopher; COMPO, Gilbert P. A practical guide to wavelet analysis. **Bulletin of the American Meteorological society**, [*s.l*], v. 79, n. 1, p. 61-78, 1998.

TREHAN, Bharat; WALSH, Carl E. Testing intertemporal budget constraints: Theory and applications to US federal budget and current account deficits. **Journal of Money, Credit and Banking**, [*s.l*], v. 23, n. 2, p. 206-223, may 1991.

VON LYNCKER, Konrad; THOENNESSEN, Rasmus. Regional club convergence in the EU: evidence from a panel data analysis. **Empirical Economics**, [*s.l*], v. 52, p. 525-553, 2017.

WILCOX, David W. The sustainability of government deficits: Implications of the present-value borrowing constraint. **Journal of Money, credit and Banking**, [*s.l*], v. 21, n. 3, p. 291-306, aug. 1989.

WONG, H. et al. Modelling and forecasting by wavelets, and the application to exchange rates. **Journal of Applied Statistics**, [*s.l*], v. 30, n. 5, p. 537-553, 2003.

ZENIOS, Stavros A. The Cyprus debt: perfect crisis and a way forward. **Cyprus Economic Policy Review**, [*s.l*], v. 7, n. 1, p. 3-45, 2013.

APPENDIX A – CLUSTER ROBUSTNESS

Below are the results of the cluster analysis concerning the robustness tests by varying the values of the *fuzziness* parameter, m.

For both m = 1.5 and m = 2.5, the composition of the groups is exactly the same as for m = 2 in all cases.

A.1 Results for m=1.5

Table A1 – Cluster Results for 3 Groups and m=1.5.

Country	Cluster 1	Cluster 2	Cluster 3	Assigned Cluster
Belgium	**0.61**	0.10	0.29	1
Estonia	**0.92**	0.02	0.07	1
Greece	**0.99**	0.00	0.01	1
Spain	**1.00**	0.00	0.00	1
France	**0.99**	0.00	0.01	1
Croatia	**0.98**	0.00	0.02	1
Italy	**1.00**	0.00	0.00	1
Cyprus	**0.93**	0.01	0.05	1
Latvia	**0.60**	0.01	0.40	1
Lithuania	**0.94**	0.00	0.06	1
Luxembourg	**0.95**	0.00	0.05	1
Portugal	**0.86**	0.00	0.13	1
Romania	**0.99**	0.00	0.01	1
Slovenia	**0.99**	0.00	0.01	1
Slovakia	**0.95**	0.01	0.04	1
Finland	**0.99**	0.00	0.00	1
Czechia	**0.52**	0.01	0.48	1
Bulgaria	0.04	**0.93**	0.03	2
Denmark	0.04	**0.86**	0.10	2
Malta	0.03	**0.87**	0.10	2
Sweden	0.01	**0.98**	0.01	2
Norway	0.15	**0.66**	0.19	2
Germany	0.03	0.03	**0.94**	3
Ireland	0.04	0.01	**0.95**	3
Hungary	0.08	0.01	**0.91**	3
Netherlands	0.05	0.03	**0.92**	3
Austria	0.16	0.01	**0.84**	3
Poland	0.11	0.01	**0.88**	3
	Sample Average Silhouette		**0.53**	
	Dunn Index		**0.83**	
	Normalized		**0.75**	

Source: Own elaboration.

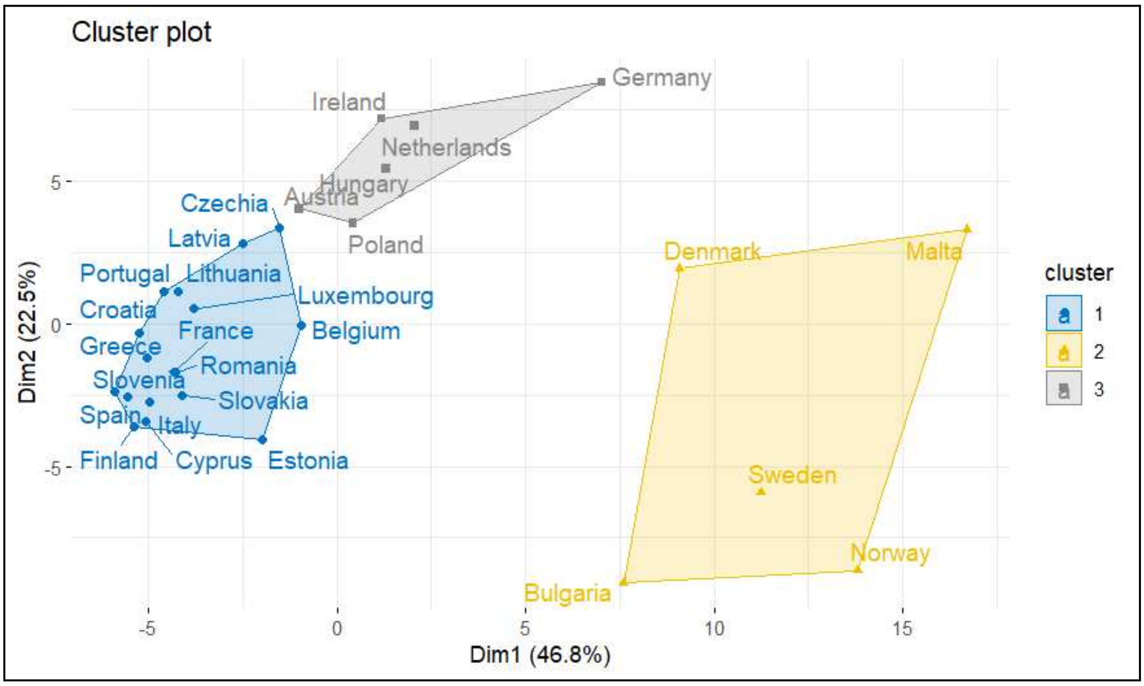
Table A2 – Silhouette Plot Information for 3 groups and m=1.5.

Country	Cluster	Neighbor	Silhouette Width
Spain	1	3	0.81
Finland	1	3	0.80
Italy	1	3	0.79
Romania	1	3	0.77
Slovenia	1	3	0.75
France	1	3	0.75
Greece	1	3	0.72
Slovakia	1	3	0.71
Croatia	1	3	0.68
Estonia	1	3	0.66
Cyprus	1	3	0.66
Luxembourg	1	3	0.64
Lithuania	1	3	0.62
Portugal	1	3	0.49
Belgium	1	3	0.36
Latvia	1	3	0.29
Czechia	1	3	0.23
Sweden	2	3	0.59
Bulgaria	2	1	0.42
Norway	2	3	0.22
Malta	2	3	0.22
Denmark	2	3	0.10
Germany	3	1	0.66
Netherlands	3	1	0.54
Ireland	3	1	0.52
Hungary	3	1	0.39
Poland	3	1	0.30
Austria	3	1	0.12

Cluster	Number of Members	Size average Silhouette Width
1	17	0.63
2	5	0.31
3	6	0.42

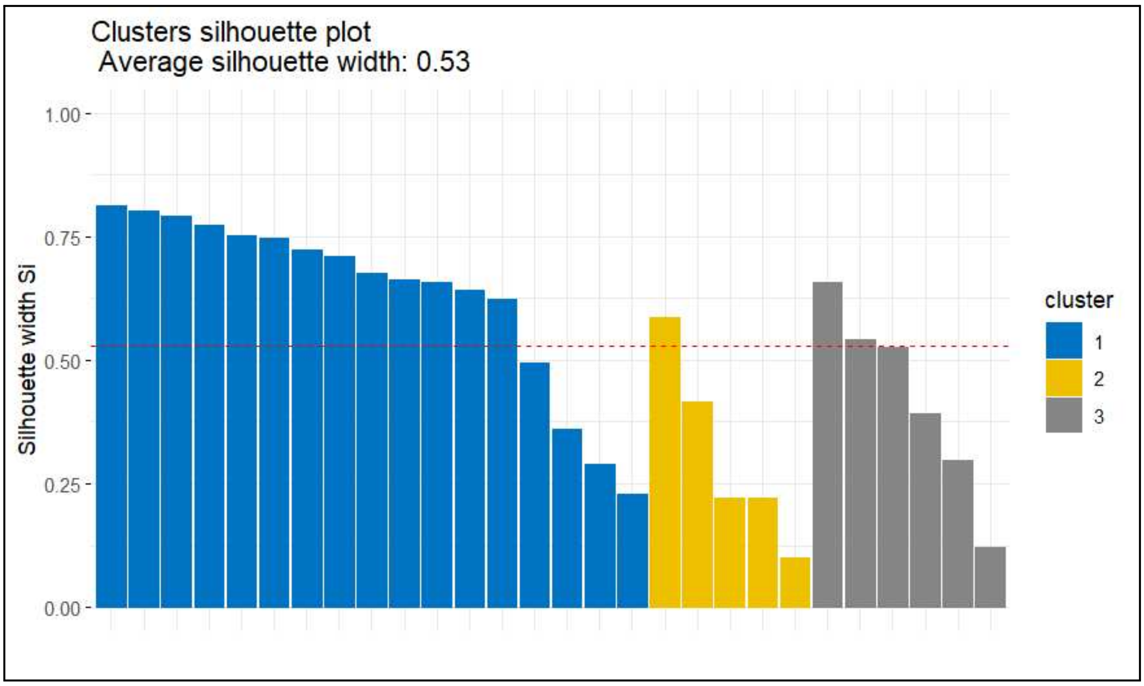
Source: Own elaboration.

Figure A1 – Cluster Results for 3 Groups and m=1.5.



Source: Own elaboration.

Figure A2 – Silhouette plot 3 clusters and m=1.5.



Source: Own elaboration.

Table A3 – Cluster Results for 4 Groups and m=1.5.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Assigned Cluster
Belgium	**0.59**	0.11	0.28	0.02	1
Czechia	**0.50**	0.01	0.49	0.00	1
Estonia	**0.91**	0.01	0.07	0.01	1
Greece	**0.99**	0.00	0.01	0.00	1
Spain	**1.00**	0.00	0.00	0.00	1
France	**0.99**	0.00	0.01	0.00	1
Croatia	**0.98**	0.00	0.02	0.00	1
Italy	**1.00**	0.00	0.00	0.00	1
Cyprus	**0.93**	0.01	0.05	0.00	1
Latvia	**0.59**	0.01	0.40	0.00	1
Lithuania	**0.94**	0.00	0.06	0.00	1
Luxembourg	**0.95**	0.00	0.05	0.00	1
Portugal	**0.86**	0.00	0.13	0.00	1
Romania	**0.99**	0.00	0.01	0.00	1
Slovenia	**0.99**	0.00	0.01	0.00	1
Slovakia	**0.95**	0.01	0.04	0.00	1
Finland	**0.99**	0.00	0.00	0.00	1
Bulgaria	0.03	**0.92**	0.02	0.03	2
Denmark	0.02	**0.91**	0.05	0.01	2
Malta	0.04	**0.76**	0.11	0.10	2
Sweden	0.01	**0.98**	0.01	0.01	2
Germany	0.03	0.02	**0.93**	0.01	3
Ireland	0.04	0.01	**0.95**	0.00	3
Hungary	0.08	0.01	**0.91**	0.01	3
Netherlands	0.05	0.03	**0.92**	0.01	3
Austria	0.15	0.01	**0.84**	0.00	3
Poland	0.11	0.01	**0.88**	0.01	3
Norway	0.00	0.00	0.00	**1.00**	4
	Sample Average Silhouette			**0.54**	
	Dunn Index			**0.84**	
	Normalized			**0.79**	

Source: Own elaboration.

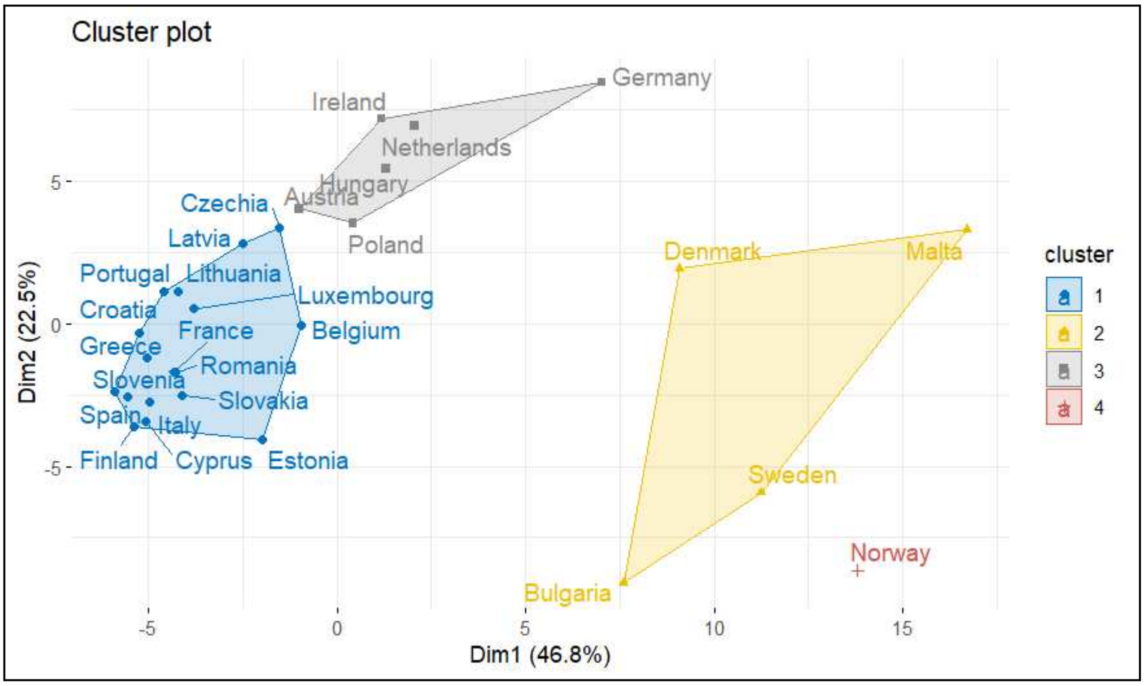
Table A4 – Silhouette Plot Information for 4 groups and m=1.5.

Country	Cluster	Neighbor	Silhouette Width
Spain	1	3	0.81
Finland	1	3	0.80
Italy	1	3	0.79
Romania	1	3	0.77
Slovenia	1	3	0.75
France	1	3	0.75
Greece	1	3	0.72
Slovakia	1	3	0.71
Croatia	1	3	0.68
Estonia	1	3	0.66
Cyprus	1	3	0.66
Luxembourg	1	3	0.64
Lithuania	1	3	0.62
Portugal	1	3	0.49
Belgium	1	3	0.36
Latvia	1	3	0.29
Czechia	1	3	0.23
Sweden	2	4	0.58
Bulgaria	2	4	0.53
Denmark	2	3	0.39
Malta	2	3	0.31
Germany	3	1	0.66
Netherlands	3	1	0.54
Ireland	3	1	0.52
Hungary	3	1	0.39
Poland	3	1	0.30
Austria	3	1	0.12
Norway	4	2	0.00

Cluster	Number of Members	Size average Silhouette Width
1	17	0.63
2	4	0.45
3	6	0.42
4	1	0.00

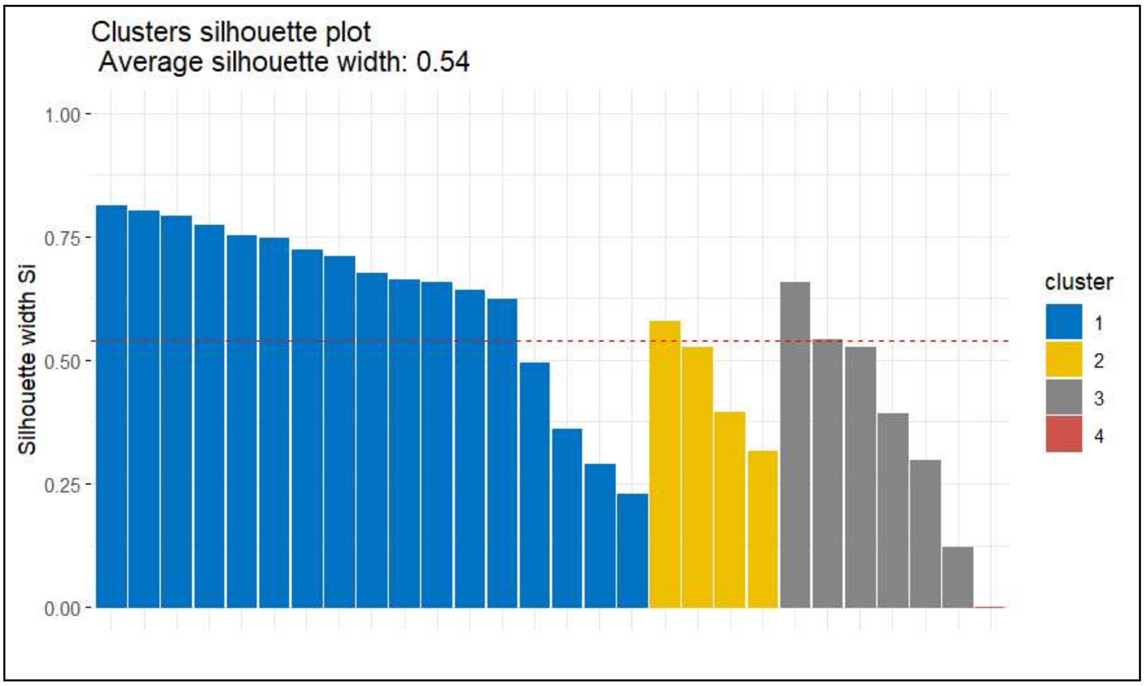
Source: Own elaboration.

Figure A3 – Cluster Results for 4 Groups and m=1.5.



Source: Own elaboration.

Figure A4 – Silhouette plot 4 clusters and m=1.5.



Source: Own elaboration.

Table A5 – Cluster Results for 5 Groups and m=1.5.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Assigned Cluster
Belgium	**0.42**	0.08	0.36	0.13	0.01	1
Estonia	**0.75**	0.01	0.21	0.02	0.01	1
Greece	**0.87**	0.00	0.12	0.01	0.00	1
Spain	**1.00**	0.00	0.00	0.00	0.00	1
France	**0.85**	0.00	0.15	0.00	0.00	1
Croatia	**0.82**	0.00	0.17	0.01	0.00	1
Italy	**0.98**	0.00	0.01	0.00	0.00	1
Cyprus	**0.87**	0.01	0.10	0.02	0.00	1
Romania	**0.90**	0.00	0.10	0.01	0.00	1
Slovenia	**0.96**	0.00	0.04	0.00	0.00	1
Slovakia	**0.85**	0.01	0.12	0.02	0.00	1
Finland	**0.98**	0.00	0.01	0.00	0.00	1
Bulgaria	0.02	**0.92**	0.02	0.02	0.02	2
Denmark	0.02	**0.84**	0.03	0.09	0.01	2
Malta	0.03	**0.63**	0.06	0.18	0.10	2
Sweden	0.00	**0.98**	0.00	0.01	0.01	2
Czechia	0.11	0.00	**0.85**	0.04	0.00	3
Latvia	0.08	0.00	**0.90**	0.02	0.00	3
Lithuania	0.31	0.00	**0.67**	0.01	0.00	3
Luxembourg	0.42	0.00	**0.57**	0.01	0.00	3
Hungary	0.08	0.01	**0.60**	0.31	0.01	3
Austria	0.08	0.00	**0.80**	0.11	0.00	3
Poland	0.09	0.01	**0.63**	0.26	0.01	3
Portugal	0.41	0.00	**0.55**	0.03	0.00	3
Germany	0.01	0.01	0.04	**0.94**	0.00	4
Ireland	0.01	0.00	0.04	**0.94**	0.00	4
Netherlands	0.01	0.01	0.03	**0.95**	0.00	4
Norway	0.00	0.00	0.00	0.00	**1.00**	5
	Sample Average Silhouette			**0.36**		
	Dunn Index			**0.74**		
	Normalized			**0.67**		

Source: Own elaboration.

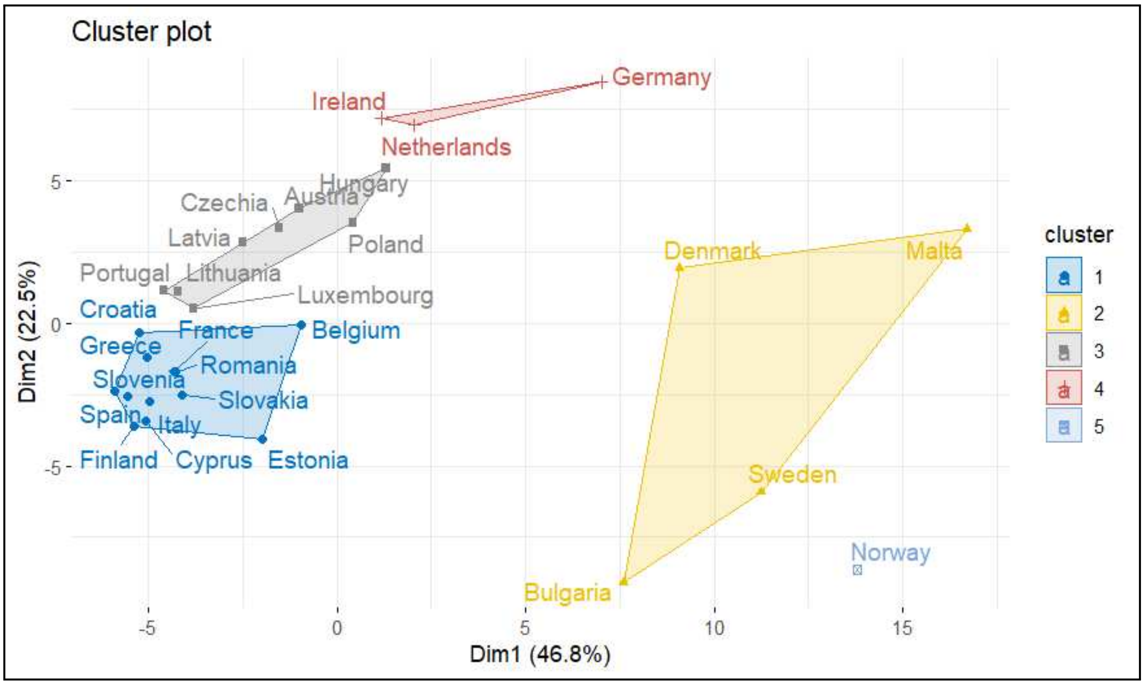
Table A6 – Silhouette Plot Information for 5 groups and m=1.5.

Country	Cluster	Neighbor	Silhouette Width
Spain	1	3	0.65
Finland	1	3	0.64
Italy	1	3	0.59
Slovakia	1	3	0.56
Slovenia	1	3	0.54
Cyprus	1	3	0.53
Romania	1	3	0.51
Estonia	1	3	0.42
Greece	1	3	0.31
France	1	3	0.29
Croatia	1	3	0.26
Belgium	1	3	0.21
Sweden	2	5	0.58
Bulgaria	2	1	0.51
Denmark	2	4	0.24
Malta	2	4	0.15
Latvia	3	1	0.41
Austria	3	4	0.40
Czechia	3	1	0.31
Poland	3	4	0.25
Hungary	3	4	0.22
Portugal	3	1	0.02
Luxembourg	3	1	-0.04
Lithuania	3	1	-0.11
Netherlands	4	3	0.58
Germany	4	3	0.54
Ireland	4	3	0.48
Norway	5	2	0.00

Cluster	Number of Members	Size average Silhouette Width
1	12	0.46
2	4	0.37
3	8	0.18
4	3	0.53
5	1	0.00

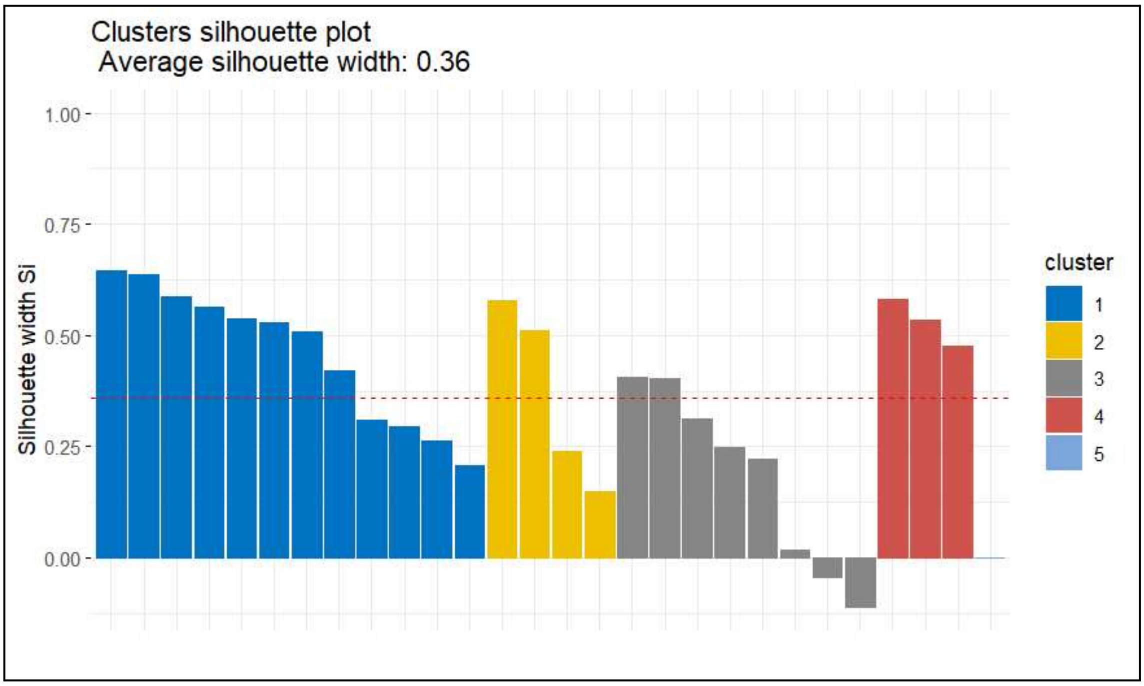
Source: Own elaboration.

Figure A5 – Cluster Results for 5 Groups and m=1.5.



Source: Own elaboration.

Figure A6 – Silhouette plot 5 clusters and m=1.5.



Source: Own elaboration.

A.2 Resultados para m = 2.5

Table A7 – Cluster Results for 3 Groups and m=2.5.

Country	Cluster 1	Cluster 2	Cluster 3	Assigned Cluster
Belgium	**0.39**	0.24	0.37	1
Estonia	**0.55**	0.15	0.30	1
Greece	**0.70**	0.07	0.23	1
Spain	**0.82**	0.05	0.13	1
France	**0.70**	0.07	0.22	1
Croatia	**0.67**	0.07	0.26	1
Italy	**0.78**	0.06	0.16	1
Cyprus	**0.58**	0.14	0.28	1
Latvia	**0.56**	0.08	0.35	1
Lithuania	**0.60**	0.08	0.32	1
Luxembourg	**0.53**	0.09	0.38	1
Portugal	**0.68**	0.09	0.23	1
Romania	**0.72**	0.08	0.20	1
Slovenia	**0.58**	0.14	0.28	1
Slovakia	**0.75**	0.07	0.18	1
Finland	0.21	**0.59**	0.20	2
Bulgaria	0.19	**0.57**	0.24	2
Denmark	0.19	**0.55**	0.26	2
Malta	0.14	**0.70**	0.16	2
Sweden	0.28	**0.42**	0.30	2
Norway	0.38	**0.11**	0.51	3
Czechia	0.25	0.24	**0.51**	3
Germany	0.26	0.17	**0.57**	3
Ireland	0.41	0.10	**0.49**	3
Hungary	0.29	0.14	**0.57**	3
Netherlands	0.26	0.23	**0.51**	3
Austria	0.29	0.12	**0.59**	3
Poland	0.28	0.14	**0.57**	3
	Sample Average Silhouette		**0.49**	
	Dunn Index		**0.47**	
	Normalized		**0.20**	

Source: Own elaboration.

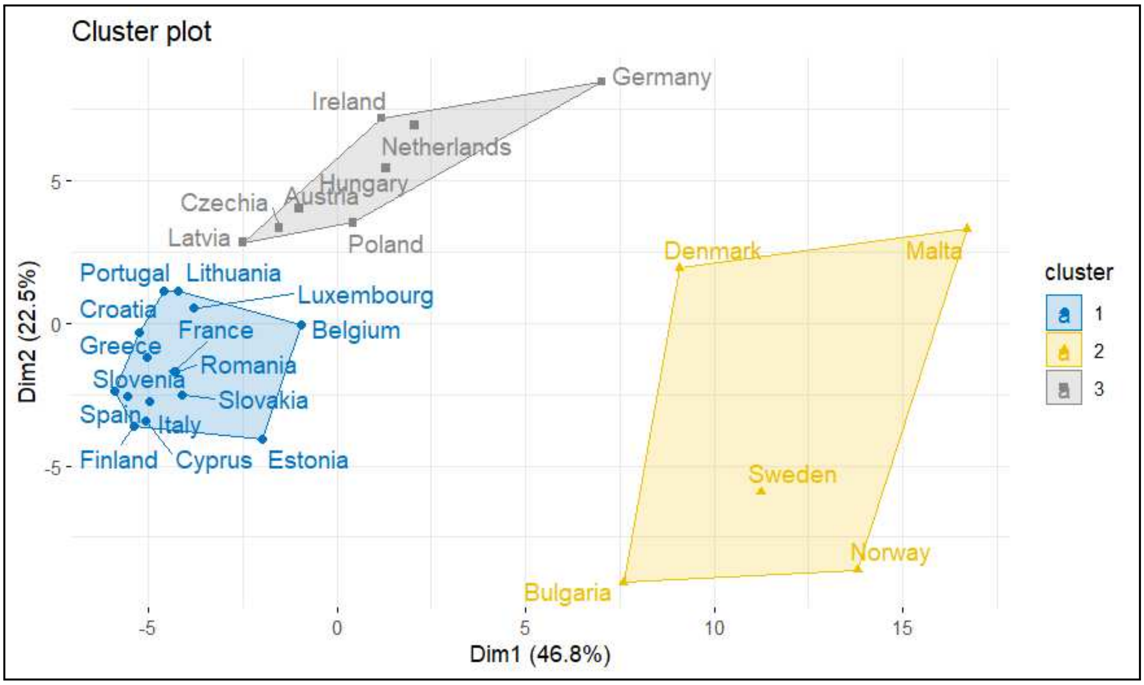
Table A8 – Silhouette Plot Information for 3 groups and m=2.5.

Country	Cluster	Neighbor	Silhouette Width
Spain	1	3	0.81
Finland	1	3	0.79
Italy	1	3	0.79
Slovenia	1	3	0.75
Romania	1	3	0.73
France	1	3	0.70
Greece	1	3	0.69
Slovakia	1	3	0.69
Cyprus	1	3	0.67
Croatia	1	3	0.65
Estonia	1	3	0.62
Luxembourg	1	3	0.54
Lithuania	1	3	0.53
Portugal	1	3	0.42
Belgium	1	3	0.32
Sweden	2	3	0.61
Bulgaria	2	1	0.40
Malta	2	3	0.29
Norway	2	3	0.24
Denmark	2	3	0.15
Germany	3	1	0.61
Ireland	3	1	0.53
Netherlands	3	1	0.49
Hungary	3	1	0.46
Poland	3	1	0.35
Austria	3	1	0.25
Czechia	3	1	-0.15
Latvia	3	1	-0.22

Cluster	Number of Members	Size average Silhouette Width
1	15	0.65
2	5	0.34
3	8	0.29

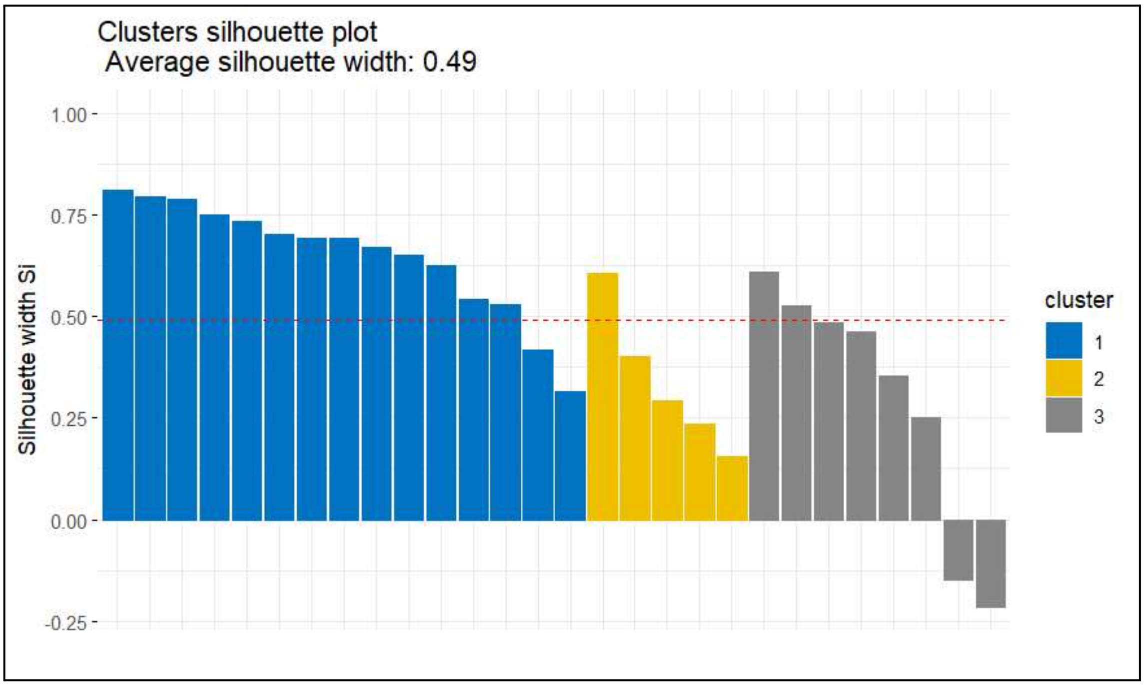
Source: Own elaboration.

Figure A7 – Cluster Results for 3 Groups and m=2.5.



Source: Own elaboration.

Figure A8 – Silhouette plot 3 clusters and m=2.5.



Source: Own elaboration.

Table A9 – Cluster Results for 4 Groups and m=2.5.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Assigned Cluster
Belgium	**0.31**	0.16	0.24	0.29	1
Czechia	**0.39**	0.07	0.25	0.29	1
Latvia	**0.42**	0.07	0.22	0.29	1
Lithuania	**0.49**	0.05	0.13	0.34	1
Luxembourg	**0.44**	0.05	0.13	0.38	1
Austria	**0.42**	0.06	0.17	0.36	1
Portugal	0.15	**0.54**	0.15	0.15	2
Bulgaria	0.18	**0.40**	0.25	0.17	2
Denmark	0.18	**0.39**	0.27	0.16	2
Malta	0.09	**0.73**	0.10	0.08	2
Sweden	0.22	**0.32**	0.24	0.22	2
Norway	0.19	0.13	**0.53**	0.16	3
Germany	0.21	0.09	**0.52**	0.17	3
Ireland	0.29	0.09	**0.39**	0.23	3
Hungary	0.20	0.12	**0.51**	0.16	3
Netherlands	0.33	0.09	**0.34**	0.24	3
Poland	0.29	0.10	**0.38**	0.23	3
Estonia	0.34	0.10	0.16	**0.40**	4
Greece	0.38	0.04	0.10	**0.47**	4
Spain	0.26	0.03	0.07	**0.64**	4
France	0.37	0.05	0.10	**0.48**	4
Croatia	0.39	0.05	0.12	**0.45**	4
Italy	0.30	0.04	0.08	**0.59**	4
Cyprus	0.32	0.09	0.16	**0.43**	4
Romania	0.37	0.06	0.11	**0.47**	4
Slovenia	0.32	0.05	0.10	**0.53**	4
Slovakia	0.34	0.09	0.15	**0.42**	4
Finland	0.30	0.05	0.09	**0.57**	4
	Sample Average Silhouette			**0.28**	
	Dunn Index			**0.35**	
	Normalized			**0.13**	

Source: Own elaboration.

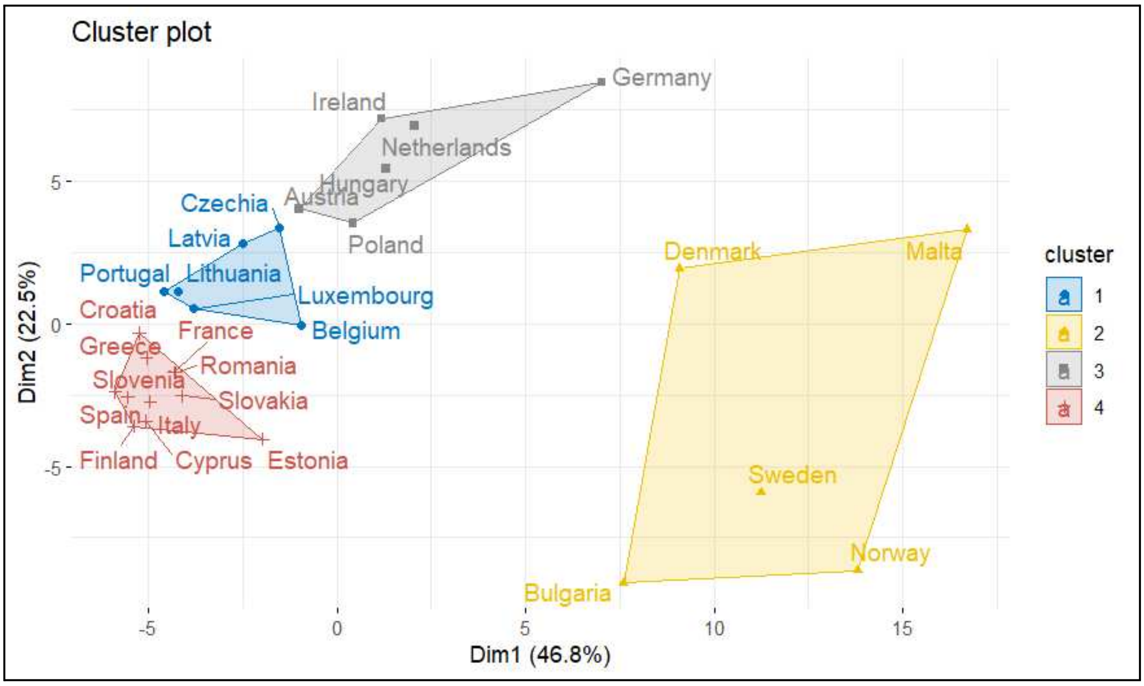
Table A10 – Silhouette Plot Information for 4 groups and m=2.5.

Country	Cluster	Neighbor	Silhouette Width
Latvia	1	4	0.28
Czechia	1	4	0.19
Lithuania	1	4	0.01
Belgium	1	4	-0.11
Luxembourg	1	4	-0.19
Portugal	1	4	-0.21
Sweden	2	3	0.59
Bulgaria	2	4	0.40
Norway	2	3	0.22
Malta	2	3	0.22
Denmark	2	3	0.10
Germany	3	1	0.55
Netherlands	3	1	0.41
Ireland	3	1	0.33
Hungary	3	1	0.19
Poland	3	1	0.09
Austria	3	1	-0.14
Spain	4	1	0.64
Finland	4	1	0.63
Italy	4	1	0.61
Slovenia	4	1	0.55
Cyprus	4	1	0.48
France	4	1	0.39
Greece	4	1	0.37
Estonia	4	1	0.36
Romania	4	1	0.36
Slovakia	4	1	0.34
Croatia	4	1	0.31

Cluster	Number of Members	Size average Silhouette Width
1	6	-0.01
2	5	0.30
3	6	0.24
4	11	0.46

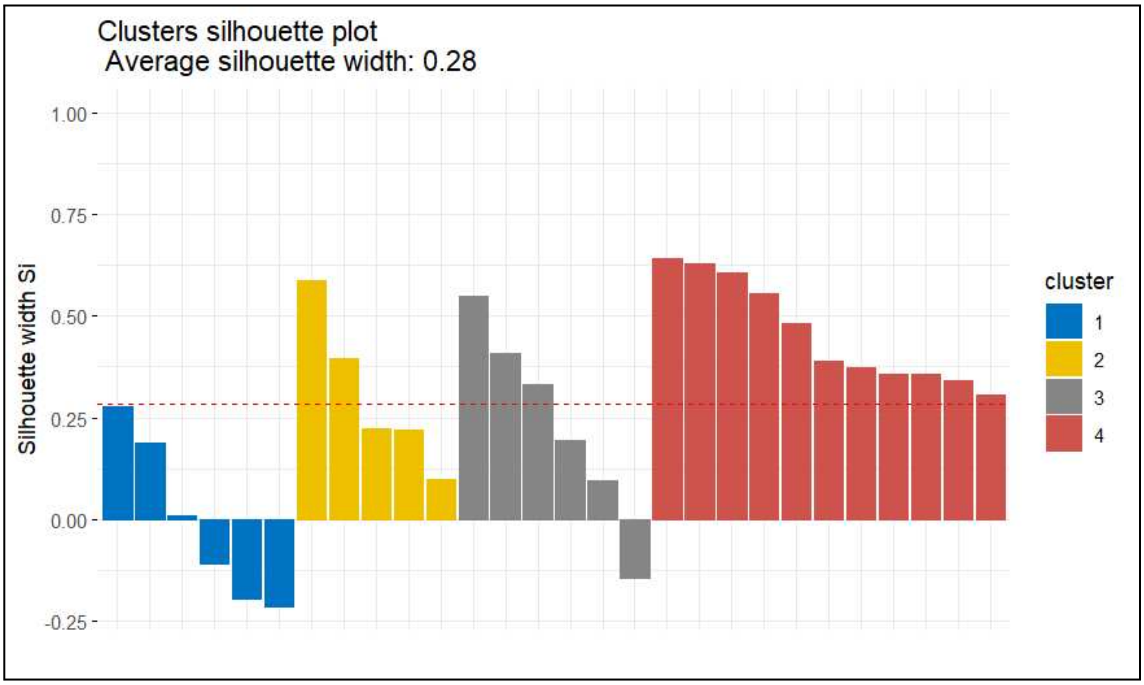
Source: Own elaboration.

Figure A9 – Cluster Results for 4 Groups and m=2.5.



Source: Own elaboration.

Figure A10 – Silhouette plot 4 clusters and m=2.5.



Source: Own elaboration.

Table A11 – Cluster Results for 5 Groups and m=2.5.

Country	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Assigned Cluster
Belgium	**0.24**	0.11	0.18	0.23	0.24	1
Czechia	**0.34**	0.05	0.16	0.22	0.24	1
Latvia	**0.33**	0.05	0.14	0.23	0.26	1
Lithuania	**0.31**	0.07	0.26	0.18	0.19	1
Luxembourg	**0.36**	0.06	0.21	0.18	0.20	1
Austria	**0.32**	0.07	0.25	0.18	0.19	1
Portugal	0.13	**0.47**	0.13	0.13	0.13	2
Bulgaria	0.18	**0.30**	0.23	0.14	0.15	2
Denmark	0.18	**0.30**	0.25	0.14	0.14	2
Malta	0.05	**0.79**	0.06	0.05	0.05	2
Sweden	0.19	**0.25**	0.20	0.18	0.18	2
Germany	0.20	0.09	**0.46**	0.12	0.13	3
Ireland	0.23	0.07	**0.42**	0.14	0.15	3
Hungary	0.20	0.09	**0.45**	0.13	0.14	3
Netherlands	0.21	0.07	0.11	**0.31**	0.30	4
Poland	0.16	0.03	0.07	**0.37**	0.37	4
Estonia	0.10	0.03	0.05	**0.48**	0.35	4
Greece	0.16	0.03	0.07	**0.37**	0.36	4
Spain	0.12	0.03	0.06	**0.44**	0.36	4
France	0.18	0.07	0.11	**0.33**	0.31	4
Croatia	0.17	0.04	0.08	**0.37**	0.35	4
Italy	0.14	0.04	0.07	**0.40**	0.35	4
Cyprus	0.19	0.07	0.11	**0.33**	0.31	4
Romania	0.13	0.04	0.06	**0.43**	0.34	4
Slovenia	0.17	0.03	0.08	0.35	**0.36**	5
Slovakia	0.25	0.04	0.09	0.29	**0.33**	5
Finland	0.22	0.04	0.09	0.31	**0.34**	5
Norway	0.24	0.04	0.12	0.29	**0.31**	5
	Sample Average Silhouette			**0.16**		
	Dunn Index			**0.28**		
	Normalized			**0.10**		

Source: Own elaboration.

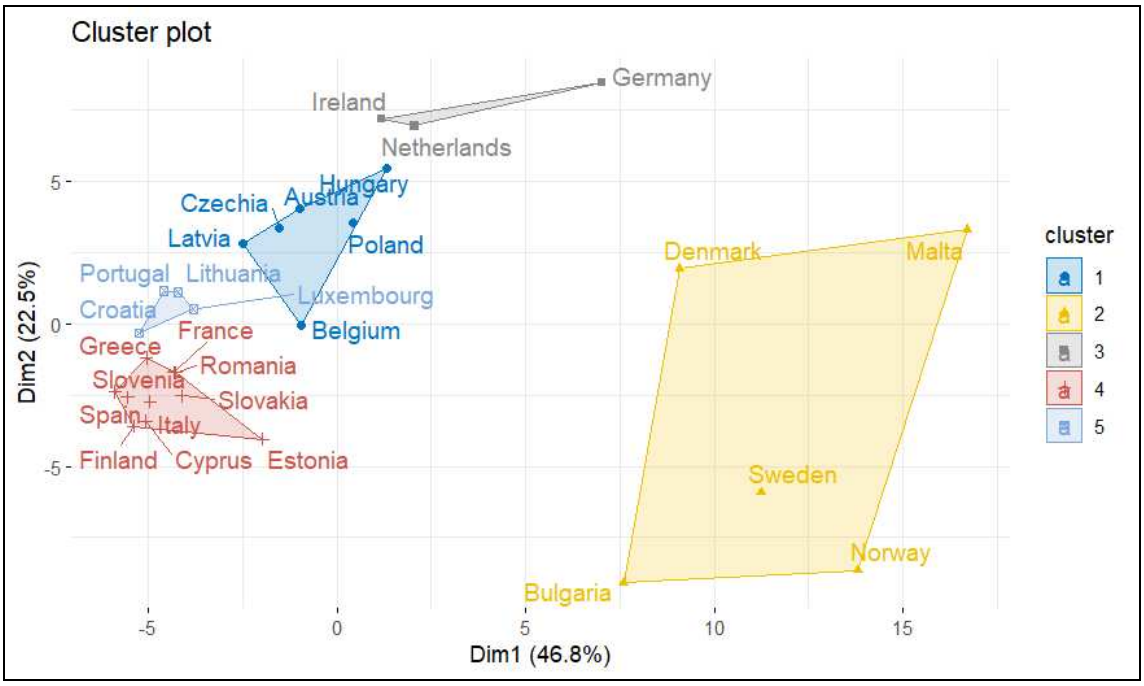
Table A12 – Silhouette Plot Information for 5 groups and m=2.5.

Country	Cluster	Neighbor	Silhouette Width
Poland	1	5	0.00
Hungary	1	5	0.00
Austria	1	5	0.00
Belgium	1	4	-0.26
Czechia	1	5	-0.28
Latvia	1	5	-0.52
Sweden	2	3	0.54
Bulgaria	2	4	0.39
Norway	2	1	0.24
Malta	2	3	0.03
Denmark	2	3	-0.11
Netherlands	3	1	0.57
Ireland	3	1	0.52
Germany	3	1	0.51
Finland	4	5	0.40
Estonia	4	5	0.35
Slovakia	4	5	0.30
Spain	4	5	0.28
Romania	4	5	0.25
Italy	4	5	0.24
Cyprus	4	5	0.10
Slovenia	4	5	-0.04
France	4	5	-0.12
Greece	4	5	-0.45
Portugal	5	4	0.57
Croatia	5	4	0.37
Lithuania	5	4	0.36
Luxembourg	5	4	0.29

Cluster	Number of Members	Size average Silhouette Width
1	6	-0.17
2	5	0.22
3	6	0.53
4	10	0.13
5	4	0.40

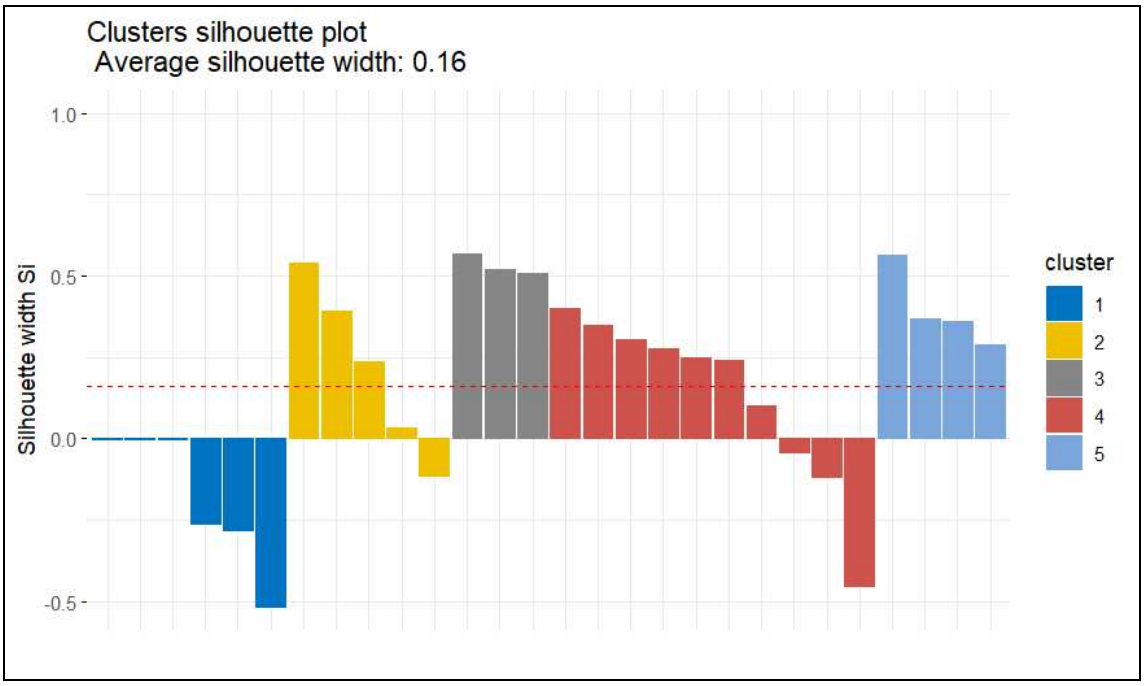
Source: Own elaboration.

Figure A11 – Cluster Results for 5 Groups and m=2.5.



Source: Own elaboration.

Figure A12 – Silhouette plot 5 clusters and m=2.5.



Source: Own elaboration.