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LUIZ ALMEIDA SAMPAIO FILHO

ON THE CHAOS IN THE FOREIGN EXCHANGE RATES AND CRYPTOCURRENCIES

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Dissertação apresentada ao Programa de Pós-Graduação em Economia – CAEN da Universidade Federal do Ceará – UFC, como requisito parcial para obtenção do título de Mestre em Economia.

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RESUMO

Estudou-se o comportamento dos mercados de câmbio e criptomoedas sob a perspectiva da teoria de sistemas dinâmicos. Utilizando o procedimento de reconstrução do espaço de fase sob a validade do teorema de Takens (1981). Investigou-se a presença de dependência serial por meio do teste BDS, a propriedade de sensibilidade às condições iniciais através do expoente máximo de Lyapunov e a distinção entre sinais determinísticos e estocásticos observando o comportamento da função E2(d) no método de Cao (1997). Avaliando 17 séries de log-retorno da taxa de câmbio, foram encontradas evidências de dependência serial, possivelmente não linear, em 11 delas. Quanto a sensibilidade às condições iniciais, nenhuma série mostrou resultados conclusivos sobre tal propriedade. Todas as séries apresentaram evidências de que seguem processos de natureza aleatória e incrementos não Gaussianos, do mesmo modo como as séries de log-retorno das criptomoedas. Das 10 séries de criptomoedas, a hipótese de IID foi rejeitada para 8 delas e nenhuma apresentou um resultado conclusivo quanto a um expoente de Lyapunov positivo. Como conclusão, não foram encontradas características consistentes de dinâmica caótica para os mercados de câmbio e moedas digitais no período analisado.

Palavras-chave: Caos; Criptomoedas; Mercado de Câmbio; Expoentes de Lyapunov.

ABSTRACT

The behavior of the foreign exchange and cryptocurrency markets was studied from the perspective of the theory of dynamical systems. Using the phase space reconstruction procedure under the validity of Takens' theorem (1981). The presence of serial dependence was investigated through the BDS test, the property of sensitivity to initial conditions through the Lyapunov maximum exponent and the distinction between deterministic and stochastic signals observing the behavior of the E2(d) function in Cao's method (1997). Evaluating 17 exchange rate log-return series, evidence of serial dependence, possibly non-linear, was found in 11 of them. As for sensitivity to initial conditions, no series has shown conclusive results on such a property. All series presented evidence that they follow processes of a random nature and non-Gaussian increments, in the same way as the cryptocurrency log-return series. Of the 10 series of cryptocurrencies, the IID hypothesis was rejected for 8 of them, and none presented a conclusive result regarding a positive Lyapunov exponent. As a conclusion, no consistent characteristics of chaotic dynamics were found for the foreign exchange and digital currency markets in the analyzed period.

Keywords: Chaos; Cryptocurrency; Foreign Exchange Market; Lyapunov Exponents.

SUMÁRIO

1 INTRODUCTION

In the last ten years, we have witnessed the rise of the digital coin market, popularly known as cryptocurrencies. These instruments emerged in an unstable economic environment with a high level of uncertainty and mistrust amidst the global financial crisis of 2008, with the proposal to act as an alternative form of payment to generate greater transparency, security, and reliability for investors, also, to increase speed and reduce transaction costs.

The idea was basically to create an electronic payment system based on cryptography to allow secure financial transactions between two parties, without the need for a regulatory institution to guarantee the validity of the transactions. The system would also serve as a solution to double-spending, the problem of spending the same currency twice.

Using a technology called *blockchain*, based on chained data blocks containing information on transactions performed and protected by encryption, in addition to a *peer-to-peer* network, Bitcoin was the first cryptocurrency to be created and today is the currency virtual with the highest market value (market capitalization around 157 billion dollars[1]). In January 2020, the capitalization of the cryptocurrency market already surpassed 238 billion dollars, highlighting Bitcoin, present in about 65.9% of transactions, according to data from the website *coinmarket.com*.

The use of the technology present in such a system corresponds perhaps its main difference in the classic coins. From now on, we will use the term "classical coins" to denote the group of traditional currencies - euro, dollar, real, among others and thus differentiate them from digital coins.

The international exchange market can be considered the largest financial market in the world by comparing the trading volume. According to the most recent triennial BIS survey - Bank for International Settlements - the daily average trading volume of foreign exchange markets was about US$ 6.6 trillion in April 2019. Meanwhile, the US dollar remained as currency dominant in the negotiations, being present in 88% of these, which corresponds to US$ 5.8 trillion traded per day in April 2019.

As for the cryptocurrency market, due to its characteristics of decentralization, large volume of trading, a certain degree of speculative content, high volatility and periods of rapid price and profitability variations, complexity in the technology of operation and consequently in the process of generating currencies (through what is known as "mining"), in addition to the

[1] Retrieved from https://coinmarketcap.com/ on January 20, 2020.

growing interest of the general public in this market, justify further study of the behavior of these instruments.

The purpose of this article is to characterize the dynamic behavior of the exchange rate log-return series as well as the cryptocurrencies. More specifically, we are interested in the classification of dynamics as stochastic or chaotic and also in the classification of series as being of a deterministic or stochastic nature.

We will use an operational definition of chaos generally accepted in the literature. A time series is considered chaotic if it presents characteristics of determinism, non-linearity, and dependence sensitive to the initial conditions.

For this, we used the behavior of the function E2(d) in the method of Cao (1997), the BDS test (BROOCK et al., 1996) and the calculation of the largest Lyapunov exponent, respectively, to identify such characteristics. We also tested the normality hypothesis for all series, according to the metric proposed in Jarque and Bera (1987).

We hope to contribute to a better understanding of the mechanisms responsible for the dynamics in such markets. We believe that this understanding allows for improvements in forecasting, as well as pricing modeling and risk management, elements that are of interest to investors.

In section 2, we present a brief review of the literature. Section 3 discusses the methodology used, and then we present a detailed description of the data in section 4. Section 5 summarizes the results. More details on the procedure for determining the minimum embedding dimension, as well as the choice of other parameters, are found in Appendix A and Appendix B.

2 LITERATURE REVIEW

Interest in the theory of chaos in economics and finance arose when stochastic models failed to provide reliable forecasts (BENSAÏDA; LITIMI, 2013).

Alves (2019) assesses the presence of chaos and randomness in price and return series from United States Brent Oil (BNO), Wipro Limited (WIT), Nasdaq, Inc. (NDAQ) and SPDR S&P 500 ETF (SPY), through a complex chaos quantifier, compared to the Lorenz's (1963) system. He concluded that the historical evolution of the prices and volatilities of these objects is chaotic and their series of returns are random.

BenSaïda and Litimi (2013) investigated the presence of noisy chaos in financial data for six stock indexes and six exchange rates. The hypothesis of chaotic dynamics was rejected for all data. Barkoulas (2008) investigated the presence of deterministic chaos in single-sum monetary aggregates in the USA. Using correlation dimension calculations, Lyapunov exponents and with the aid of recurrence graphs, he found that his study objects did not have a discernible chaotic structure.

Concerning studies on cryptocurrencies, we can highlight for example Yi, Xu and Wang (2018) who evaluated the volatility connectivity between eight typical digital currencies, finding a cyclical fluctuation in connectivity with a visible increase at the end of 2016. They also concluded through an analysis with 52 networked cryptocurrencies, that Bitcoin is not a dominant currency in which refers to the transmission of volatility.

Lahmiri and Bekiros (2018) investigated the presence of chaos, randomness and multi-scale time correlation for Bitcoin prices and returns divided into two periods: high and low, characterized by rapid and slow price increases, respectively. They concluded that only the price series showed a chaotic behavior, observed in both periods of analysis.

Matos, Benegas and Costa (2018) investigated characteristics of non-linearity and chaotic dynamics in some of the main indexes of the global banking sector. They found evidence of non-linearity and absence of chaotic behavior in such indexes, which according to the authors is a sign of efficiency.

Other works addressing some techniques presented here, among other approaches are, for example, Lahmiri and Bekiros (2019), Çoban and Büyüklü (2009), França (2015), Chu (2003), Hsieh (1993).

From the discussion presented above, it is clear that the use of such methodology applied to exchange rates and cryptocurrency is not exclusive of this work. However, we believe that the present analysis contributes to the literature due to the volume of data used and the way of comparing the analyzed markets.

3 METHODOLOGY

Our procedures are based on Takens's Theorem (1981) that allows the reconstruction of the phase space of a system through the lags of an observed time series, ensuring that the reconstructed attractor is topologically equivalent to the original attractor.

Topologically, in the sense that the invariant measures such as dimensions, entropies and Lyapunov exponents of the reconstructed attractor are the same as those of the original system underlying the data, which is unknown. This procedure allows us to recover all the information from the system underlying the data.

The reconstruction procedure consists of decomposing a scalar time series into a vector time series, using delay coordinates. Consider a time series with N observations, $x_1, \dots, x_N$. So, the reconstructed *i-th* vector of dimension m is: $y_i(m) = \left(x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\right)$, $i = 1, \dots, N - (m-1)\tau$, where τ is the time lag parameter and m is the embedding dimension. The idea is to transform each observation in the observable time series into an element of the phase space, therefore, a vector. From that, we need to determine two parameters: m and τ.

The τ parameter must be determined first. The estimation of m is explained in subsection 3.2, just below. In general, there are two main ways of determining it: through the autocorrelation function and the Mutual Information Method (FRASER; SWINNEY, 1986). Here we chose the latter because we are interested in the general dependence between the observations and not just the linear dependence.

The method consists of creating a histogram of the data distribution in the period covered by the sample. According to Kantz and Schreiber (2003), the Mutual Information for a τ time delay can be given by:

$$I_\varepsilon(\tau) = \sum_{i,j} p_{ij}(\tau)\ln(p_{ij}(\tau)) - 2\sum_i p_i ln(p_i) \tag{1}$$

where p_i is the probability that the signal takes on a value within the i-th compartment of the histogram and $p_{ij}(\tau)$ is the probability that x_t is in compartment i and $x_{t+\tau}$ is in compartment j. Therefore, $I_\varepsilon(\tau)$ is the amount by which a measure of x_t reduces the uncertainty about the knowledge of $x_{t+\tau}$ (FRASER; SWINNEY, 1986). Thus, a minimum in I means to demand the minimum of dependence between the observations in time.

We also use the space-time separation plots, introduced by Provenzale et al. (1992) to eliminate the effects of temporal separation on spatial separation, to guarantee that the proximity of the points (reconstructed vectors) in space comes from the attractor's geometry and not only from the correlation between them. Linear correlations often cause the reconstructed attractor to generate incorrect information about the topological features of the original attractor (SCHWARTZ; YOUSEFI, 2003).

From the above considerations, the methodology consists basically of three procedures: to verify the type of dependence of the series through the BDS Test, to calculate the minimum embedding dimension through the method proposed by Cao (1997) and finally, to use the algorithm presented in Rosenstein et al. (1993) for the calculation of the maximal Lyapunov exponent. The following will briefly describe the procedures mentioned above.

3.1 BDS Test

According to Broock et al. (1996), the method consists of a nonparametric test to verify the serial dependence and nonlinear structure of a time series. The null hypothesis is that the sample comes from an IID (independently and identically distributed) data generating process.

The test statistic has some advantages, such as being free of distribution, which means that we do not need to make initial assumptions about the distribution of the statistic for the application of the test. It is based on the calculation of the Correlation Integral, a measure introduced in Grassberger and Procaccia (1983).

The correlation integral for a given dimension m is given by:

$$C_{m,n}(\varepsilon) = \frac{1}{\binom{n}{2}} \sum_{1 \le s} \sum_{< t \le n} \chi_\varepsilon \left(\| y_s^m - y_t^m \| \right) \quad (2)$$

where n is the number of reconstructed vectors of dimension m, ε is a small distance and χ_ϵ is a Heavside function and,

$$C_m(\varepsilon) = \lim_{n \to \infty} C_{m,n}(\varepsilon) \quad (3)$$

Broock et al. (1996) demonstrated that under the null hypothesis of independence,

$$C_m(\varepsilon) = C_1(\varepsilon)^m \quad (4)$$

In addition, they proved that BDS statistics given by:

$$W_{m,n}(\varepsilon) = \sqrt{n} \frac{T_{m,n}(\varepsilon)}{V_{m,n}(\varepsilon)} \quad (5)$$

Converge in distribution to $N(0,1)$, where

$$T_{m,n}(\varepsilon) = C_m(\varepsilon) - C_1(\varepsilon)^m \quad (6)$$

and $V_{m,n}(\varepsilon)$ converge almost surely to V_m, obtained from the asymptotic variance

$$V_m^2 = 4[m(m-2)C^{2m-2}(K-C^2) + K^m - C^{2m} + 2\sum_{j=1}^{m-1}[C^{2j}\left(K^{m-j} - C^{2m-2j}\right) - mC^{2m-2}(K-C^2)] \quad (7)$$

where C and K can be consistently estimated by:

$$C_n(\varepsilon) = \frac{1}{n^2}\sum_{s=1}^{n}\sum_{t=1}^{n}\chi_\varepsilon(|u_s - u_t|) \quad (8)$$

$$K_n(\varepsilon) = \frac{1}{n^3}\sum_{r=1}^{n}\sum_{s=1}^{n}\sum_{t=1}^{n}\chi_\varepsilon(|u_r - u_s|)\chi_\varepsilon(|u_s - u_t|) \quad (9)$$

The rejection of the null hypothesis indicates that the data is not *iid*, indicating serial dependency.

3.2 Dimension Embedding

The method proposed by Cao (1997) presents some advantages over other methods for the calculation of the minimum embedding dimension based on Takens's Theorem (1981), for example, the absence of subjective parameters, the strong non-dependence of the quantity of available data points and the ability to distinguish clearly between deterministic and stochastic signals.

The test is based on the idea of the "nearest neighbors" method of Kennel, Brown and Abarbanel, (1992) using the measure given by:

$$a(i,d) = \frac{\left\|y_i(d+1) - y_{n(i,d)}(d+1)\right\|}{\left\|y_i(d) - y_{n(i,d)}(d)\right\|} \quad (10)$$

where $y_i(d) = \left(x_i, x_{i+\tau}, \ldots, x_{i+(d-1)\tau}\right)$, $i = 1, \ldots, N-(d-1)\tau$, is *iht* vector reconstructed with dimension d and τ is a time lag parameter. The distance function used is the maximum norm. The term $n(i,d)$ is an integer such that the distance between y_i both the numerator and the denominator are the smallest, that is, these terms are the smallest distance from each other in the reconstructed phase space of dimension d.

The false neighbor is identified by checking $a(i,d)$ is greater than a given threshold value. Due to the difficulty of finding threshold values that were independent of the dimension d and the specific point of the trajectory, Cao (1997) defined the following quantity:

$$E(d) = \frac{1}{N-d\tau}\sum_{i=1}^{N-d\tau} a(i,d) \quad (11)$$

For the determination of such a limit value, two functions, E1(d) and E2(d) are built where d is the dimension. The first is used to determine the minimum dimension while the second is used to distinguish between deterministic and stochastic signals. The function E1(d) is defined as:

$$E1(d) = \frac{E(d+1)}{E(d)} \tag{12}$$

E1(d) stops changing when d is greater than or equal to the minimum dimension.

The function E2(d) is defined as:

$$E2(d) = \frac{E^*(d+1)}{E^*(d)} \tag{13}$$

where $E^*(d)$ is defined as:

$$E^*(d) = \frac{1}{N-d\tau} \sum_{i=1}^{N-d\tau} \left| x_{i+d\tau} - x_{n(i,d)+d\tau} \right| \tag{14}$$

Note that E*(d) measures the average absolute variation of the distances between pairs of neighbors when the embedding dimension increases from d to d+1. For random data, since future values are independent of past values, E*(d) will be equal to one for any d. As deterministic signs show dependence between future and past values, E*(d) will be different from one for some d's.

3.3 Lyapunov Exponents

It is a quantifying measure of chaos in dynamical systems[2]. It measures the average rate of divergence between two near paths in state space. Through the procedure of reconstruction of the state space, the maximum Lyapunov exponent is calculated obtaining a measure of chaos and consequently complexity of the analyzed system. In this work, we follow the approach of Rosenstein et al. (1993), but it is worth noting that Kantz (1994) developed independently, essentially the same method, with some minor modifications.

Following the approach of Rosenstein et al. (1993), define:

$$D_j(0) = \min_{y_{\hat{j}}} \left\| y_j - y_{\hat{j}} \right\| \tag{15}$$

where $D_j(0)$ measures the starting distance between the *jth* point and its nearest neighbor, $y_{\hat{j}}$. The dot y_j is a reconstructed vector as $y_i(d)$ quoted above. Next, it is assumed that the distance from the *jth* point of its nearest neighbor after periods of time can be given by:

[2]Wolf et al. (1985) present excellent Lyapunov spectrum exposure.

$$D_j(i) \approx C_j e^{\lambda_1(i\Delta t)} \Longrightarrow \ln D_j(i) \approx \ln C_j + \lambda_1(i\Delta t) \tag{16}$$

where C_j is the initial separation and $i\Delta t$ is the representation of units of time, such as seconds for example.

Finally, a least squares fit is used for the regression line obtained from the equation below and estimated λ_1 as the growth rate of $D_j(i)$.

$$\Upsilon(i) = \frac{1}{\Delta t}\langle \ln D_j(i) \rangle \tag{17}$$

Where the $\langle . \rangle$ operator indicates we are averaging over all values of j.

4 DATA

The database is composed in total of 27 time series being 17 series of classic coins and 10 of cryptocurrency, with daily frequency and without seasonal adjustment. The classic coin series were drawn from the Federal Reserve Economic Data of St. Louis.

We extract data regarding cryptocurrencies from the *Investing.com* platform. It is a global financial portal that among other utilities, provides digital currency quotes as well as investment options. Although we take information from *coinmarket.com*, we prefer to use *Investing.com* data for the greater range of the series.

We chose to use the time series of the classic continuous currencies, updated and without seasonal adjustment. Data from other countries, such as Spain, Belgium, and Greece, for example, were also available, however, they were not used for the discontinuity of the time series and lack of updating of the same.

About the cryptocurrencies we decided to use only the most valued and with greater volume of negotiation, considering the relevance of the data. The ranking according to this criterion was given by the ranking available on the site *coinmarket.com*.

All quotes are in dollars, considering the importance and participation of the American currency in the total of negotiations. Concerning the exchange rate tables, the currency on the left is the base currency and the currency on the right is the quoted currency.

We evaluate the net income through the logarithmic difference in prices, as follows:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \quad (18)$$

This procedure is justified by the possibility of a direct comparison between currencies, as well as providing practical information to market agents.

5 RESULTS

For the calculation of measures in tables 2, and 5, the following packages were used in the software R: “DChaos”, “fNonlinear”, “nonlinearTseries” and “moments”. The values of Lyapunov's exponents are approximations to the fourth decimal place.

We emphasize an important observation regarding the estimation of the maximum Lyapunov exponent. The algorithms provided by both Rosenstein et al. (1993) and Kantz (1994) will always provide a numerical value. However, one should check if there is any linear region (at least approximately) on the graph, from which a slope can be extracted that will serve as an estimate for the Lyapunov exponent. If there is no such linear region, the estimates provided should be discarded.

From this, the positive exponents of the following series should not be taken into account: ETHEREUM (cryptocurrency); NZD/USD, USD/BRL, USD/HKD, USD/KRW and USD/MYR. We will keep the exponents estimates for these series in the tables for information purposes. The graphs with these analyzes can be seen in Appendices A and B, for cryptocurrencies and exchange rates, respectively.

From the analysis of tables 1 and 2, we can see that, except for the CARDANO and ETHEREUM series, all the daily log-return series for cryptocurrencies are leptokurtic, following heavy tails distributions. Besides, the hypothesis of normality was rejected for all series, as can be seen from the values of the Jarque-Bera statistic in Table 2.

Table 1 – Cryptocurrency Descriptive Statistics.

	Start	End	Size	Mean	Standard Deviation	Kurtosis	Skewness	Mín	Máx
BINANCE COIN	2017-11-10	2019-03-15	491	0.004	0.074	11.588	0.920	-0.402	0.487
BITCOIN	2010-07-19	2019-03-15	3161	0.003	0.067	34.436	0.016	-0.849	0.829
BITCOIN CASH	2017-08-04	2019-03-15	589	-0.001	0.089	5.989	0.498	-0.479	0.435
CARDANO	2018-01-01	2019-03-15	439	-0.006	0.069	2.882	0.364	-0.249	0.349
EOS	2017-07-03	2019-03-15	621	0.000	0.087	3.401	0.438	-0.357	0.356
ETHEREUM	2016-03-11	2019-03-15	1100	0.002	0.062	2.930	-0.022	-0.313	0.259
LITECOIN	2016-08-25	2019-03-15	933	0.003	0.067	13.242	1.840	-0.308	0.607
STELLAR	2017-02-23	2019-03-15	751	0.005	0.098	16.966	1.705	-0.343	0.728
TRON	2017-11-15	2019-03-15	486	0.005	0.105	20.049	2.525	-0.353	0.780
XRP RIPLE	2015-01-23	2019-03-15	1512	0.001	1.057	23.368	0.318	-0.997	0.937

Data Source: Self elaboration.

Table 2 – Analysis of Results for Cryptocurrencies.

	Minimum embedding dimension	Time lag	BDS Statistic	Maximum Lyapunov exponent	Jarque-Bera Statistic
BINANCE COIN	7	3	7.7065***	-0.0023	1578.1***
BITCOIN	8	2	-0.344	0.0000	130157***
BITCOIN CASH	9	2	6.5214***	-0.0014	913.76***
CARDANO	6	1	3.8774***	-0.0022	164.5***
EOS	7	2	5.2066***	-0.0013	322.88***
ETHEREUM	8	4	8.7484***	0.0002	396.51***
LITECOIN	8	2	6.9539***	0.0030	7380.6***
STELLAR	7	4	11.0405***	-0.0005	4372.3***
TRON	8	3	10.8256***	-0.0032	6402.5***
XRP RIPLE	8	1	-1.0005	0.0000	26162***

Data source: Self elaboration.
* p-value <0.1 ** p-value <0.05 *** p-value <0.01

The occurrence of leptokurtic distributions for financial assets is considered a stylized fact. Distributions with this characteristic show the occurrence of sudden and discontinuous variations in the price movements and returns of assets. It is associated with a greater probability of occurrence of extreme events such as crises and, consequently, large losses.

The *iid* process hypothesis was rejected for eight of the ten series, except BITCOIN and XRP RIPPLE. Only ETHEREUM and LITECOIN obtained positive Lyapunov exponents despite the low values.

It is worth mentioning that six series presented negative exponents and two, zero exponents. These results suggest a non-chaotic behavior in the digital currency market.

Next, we will evaluate the results for exchange rates.

Table 3 – Description of exchange rates and their countries.

Foreign Exchange Rates	Country/Region
AUD/USD	Australia
EUR/USD	Europe
GBP/USD	United Kingdom
NZD/USD	New Zealand
USD/BRL	Brazil
USD/CAD	Canada
USD/CHF	Switzerland
USD/DKK	Denmark
USD/HKD	Hong Kong
USD/INR	India
USD/JPY	Japan
USD/KRW	South Korea
USD/MYR	Malaysia
USD/NOK	Norway
USD/SEK	Sweden
USD/THB	Thailand
USD/ZAR	South Africa

Data Source: Self elaboration.

Considering the values up to three decimal places, all series showed zero means. Only EUR/USD showed no kurtosis greater than three. USD/HKD and USD/KRW stood out in this respect. As for the analysis of nonlinear dynamics, of the five series that showed positive exponents, the iid process hypothesis was not just rejected by USD/BRL.

We believe that such behavior is due to the incidence of noise in the series, as well as the small values of Lyapunov's exponents. This result is supported by evidence of randomness, which we will discuss later.

Table 4 – Descriptive Statistics for Exchange rates.

	Start	End	Size	Mean	Standard Deviation	Kurtosis	Skewness	Mín	Máx
AUD/USD	1971-01-04	2019-03-15	12084	0.000	0.007	79.830	-2.756	-0.193	0.107
EUR/USD	1999-01-04	2019-03-15	5070	0.000	0.006	2.269	0.111	-0.030	0.046
GBP/USD	1971-01-04	2019-03-15	12091	0.000	0.006	7.272	-0.396	-0.082	0.046
NZD/USD	1971-01-04	2019-03-15	12075	0.000	0.007	81.301	-3.052	-0.206	0.099
USD/BRL	1995-01-02	2019-03-15	6074	0.000	0.010	15.480	-0.097	-0.051	0.038
USD/CAD	1971-01-04	2019-03-15	12097	0.000	0.004	10.296	-0.097	-0.051	0.038
USD/CHF	1971-01-04	2019-03-15	12091	0.000	0.007	14.114	-0.399	-0.130	0.089
USD/DKK	1971-01-04	2019-03-15	12090	0.000	0.006	9.803	0.101	-0.078	0.081
USD/HKD	1981-01-02	2019-03-15	9591	0.000	0.002	224.450	4.949	-0.041	0.065
USD/INR	1973-01-02	2019-03-15	11583	0.000	0.005	81.760	2.943	-0.055	0.128
USD/JPY	1971-01-04	2019-03-15	12085	0.000	0.006	9.428	-0.659	-0.095	0.063
USD/KRW	1981-04-13	2019-03-15	9510	0.000	0.006	294.780	7.987	-0.074	0.208
USD/MYR	1971-01-04	2019-03-15	12069	0.000	0.004	72.447	-1.109	-0.092	0.072
USD/NOK	1971-01-04	2019-03-15	12090	0.000	0.007	7.080	0.227	-0.064	0.068
USD/SEK	1971-01-04	2019-03-15	12090	0.000	0.007	32.275	1.416	-0.054	0.151
USD/THB	1981-01-02	2019-03-15	9590	0.000	0.003	11.188	-0.479	-0.041	0.028
USD/ZAR	1980-01-02	2019-03-15	10227	0.000	0.010	15.667	-0.062	-0.144	0.103

Data Source: Self elaboration.

Table 5 – Analysis of Results for Exchange rates.

	Minimum embedding dimension	Time lag	BDS Statistic	Maximum Lyapunov exponent	Jarque-Bera Statistic
AUD/USD	9	2	7,2655***	0.0000	3225131***
EUR/USD	8	1	1,1083	0.0000	1099,9***
GBP/USD	9	4	1,9182*	-0.0003	26970***
NZD/USD	9	7	4,9844***	0.0003	3345471***
USD/BRL	9	2	-0,1046	0.0006	60860***
USD/CAD	9	2	21,6101***	-0.0002	53478***
USD/CHF	9	1	1,4005	-0.0002	100721***
USD/DKK	9	2	1,4961	0.0000	48454***
USD/HKD	9	3	7,995***	0.0005	20179078***
USD/INR	10	2	0,3318	-0.0014	3244074***
USD/JPY	9	3	2,5355**	-0.0004	45656***
USD/KRW	11	2	1,8545*	0.0009	34548065***
USD/MYR	10	6	-1,9533*	0.0005	2642755***
USD/NOK	9	2	4,6633***	0.0000	25364***
USD/SEK	10	2	5,0527***	0.0000	528985***
USD/THB	9	1	0,574	0.0000	9590***
USD/ZAR	10	5	3,1752***	0.0000	104651***

Data source: Federal Reserve Economic Data of St. Louis.

* p-value <0.1 ** p-value <0.05 *** p-value <0.01

Schwartz and Yousefi (2003) tested the existence of non-linear dependence using the BDS test at nine exchange rates. The authors found evidence of non-linear dependence between the observations. They also calculated Lyapunov's exponents for the series. The results were inconclusive and could not point in the direction of positive exponents.

Our results resemble those of Lahmiri and Bekiros (2018), who find evidence of chaotic behavior only for Bitcoin prices in two periods of analysis. However, they did not find such evidence for the returns.

To infer whether the data are deterministic or stochastic, we use the minimum dimension estimation graphs in Appendix A and look at the behavior of the E2(d) function in each graph. As explained in subsection 3.2, this function assumes a value equal to 1 for random signals and different from one for deterministic signals, since it captures the variation of function E2(d) about variations of d.

As for the determinism in the data, we used the graphs of the minimum embedding dimension estimates. Through the behavior of function E2(d), we can conclude that all series of both cryptocurrencies and foreign exchange, show evidence of a stochastic nature in the data.

As Kantz (1994) argues, the lack of a discernible (linear) scale region is related to the lack of determinism in the data. This last observation justifies our graphs regarding Lyapunov's exponents in Appendices A and B.

6 DISCUSSION

We investigated the presence of chaos, randomness and serial dependency in foreign exchange rates and cryptocurrencies. Despite positive Lyapunov exponents for five exchange rates, it is not possible to infer chaotic behavior based on these results due to the lack of discernible linear regions in the graphs. The same applies to the exponents for ETHEREUM and LITECOIN.

The results suggest that despite the complexity inherent in the foreign exchange and cryptocurrency markets, there is no conclusive evidence of chaotic behavior in such markets. Besides, all series appear to be stochastic in nature, indicating that irregular behaviors of returns must be caused by non-linear stochastic processes, given the rejection of the BDS statistic for most series.

We were also able to distinguish between random and deterministic signals in the data. We conclude that despite all the peculiar characteristics of the digital currency market discussed previously, there is no predominance of chaotic behavior in the data, at least about the log-return series.

Finally, we highlight that some improvements can be made to ensure the consistency of the results obtained. We can adjust ARMA or ARIMA models to filter the linear dependency and use the BDS test on the residuals of these models for non-linear dependence. A noise reduction methodology must also be applied, as described in Kantz and Schreiber (2003).

We believe that the results help to better understand the foreign exchange and digital currency markets. With this knowledge, more suitable models can be applied to increase the accuracy of forecasts in such markets, as well as models that explain the main factors of variations in these markets.

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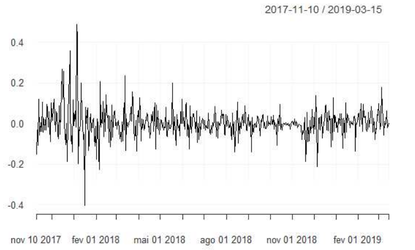
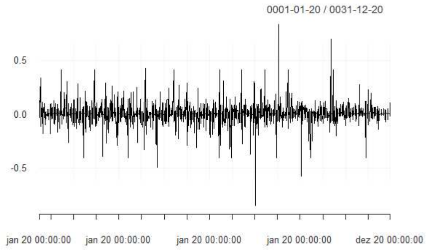
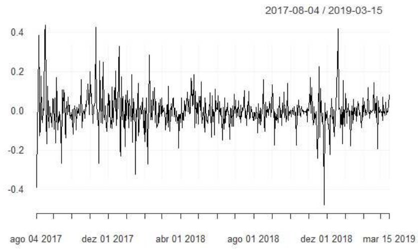
APPENDIX A – DETAILS ABOUT THE CRYPTOCURRENCIES PARAMETERS



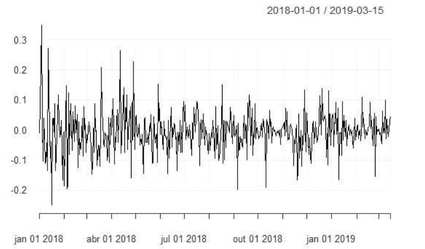
a. BINANCE COIN



b. BITCOIN



c. BITCOIN CASH



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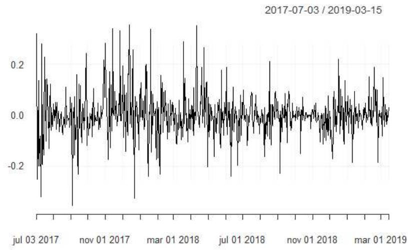

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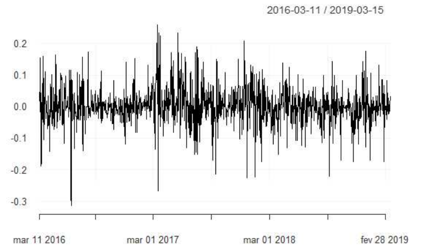
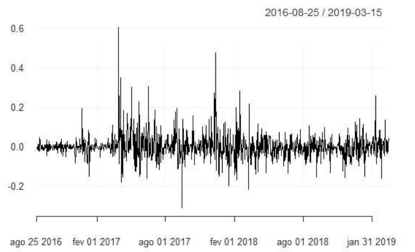


f. ETHEREUM



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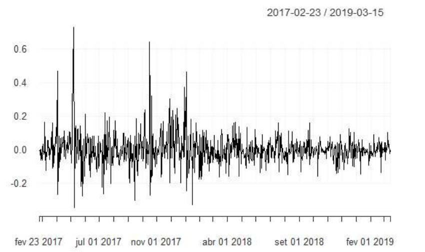

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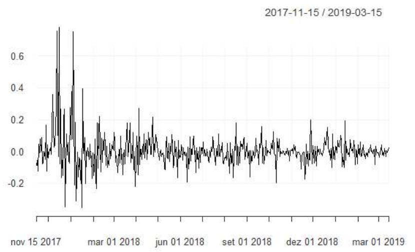

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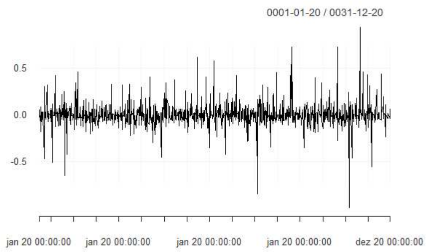
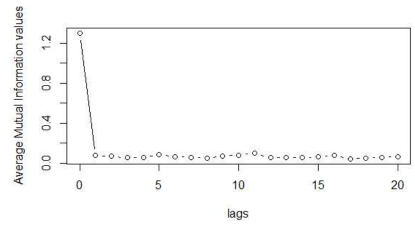


j. XRP RIPPLE

Figure 1. Cryptocurrency log-return series.



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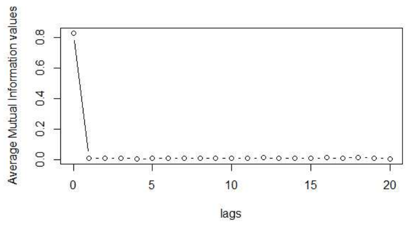

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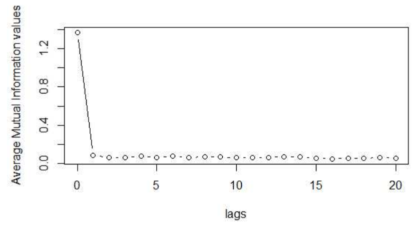

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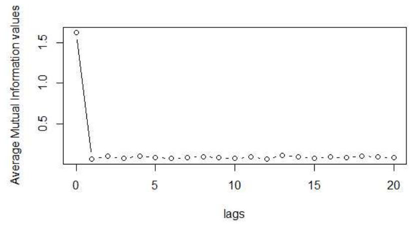

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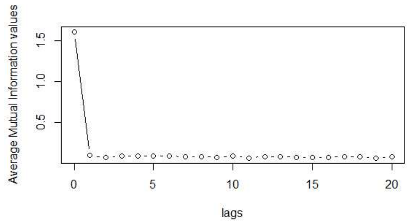
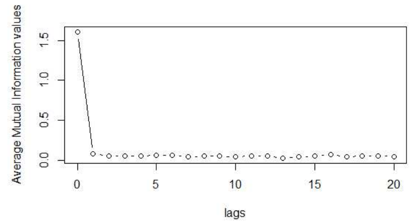
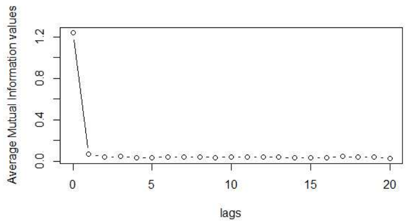


o. EOS



p. ETHEREUM



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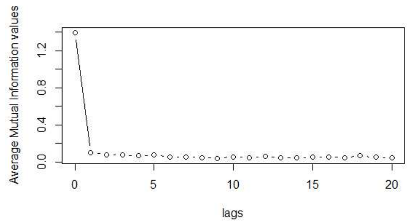
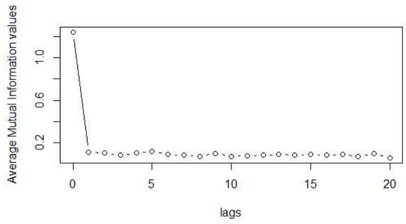


r. STELLAR



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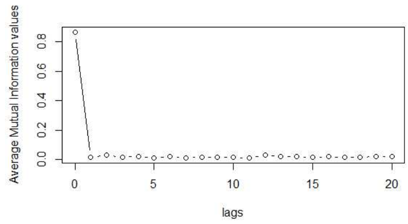


t. XRP RIPPLE

Figure 2. Average Mutual Information analysis for cryptocurrencies log-returns.

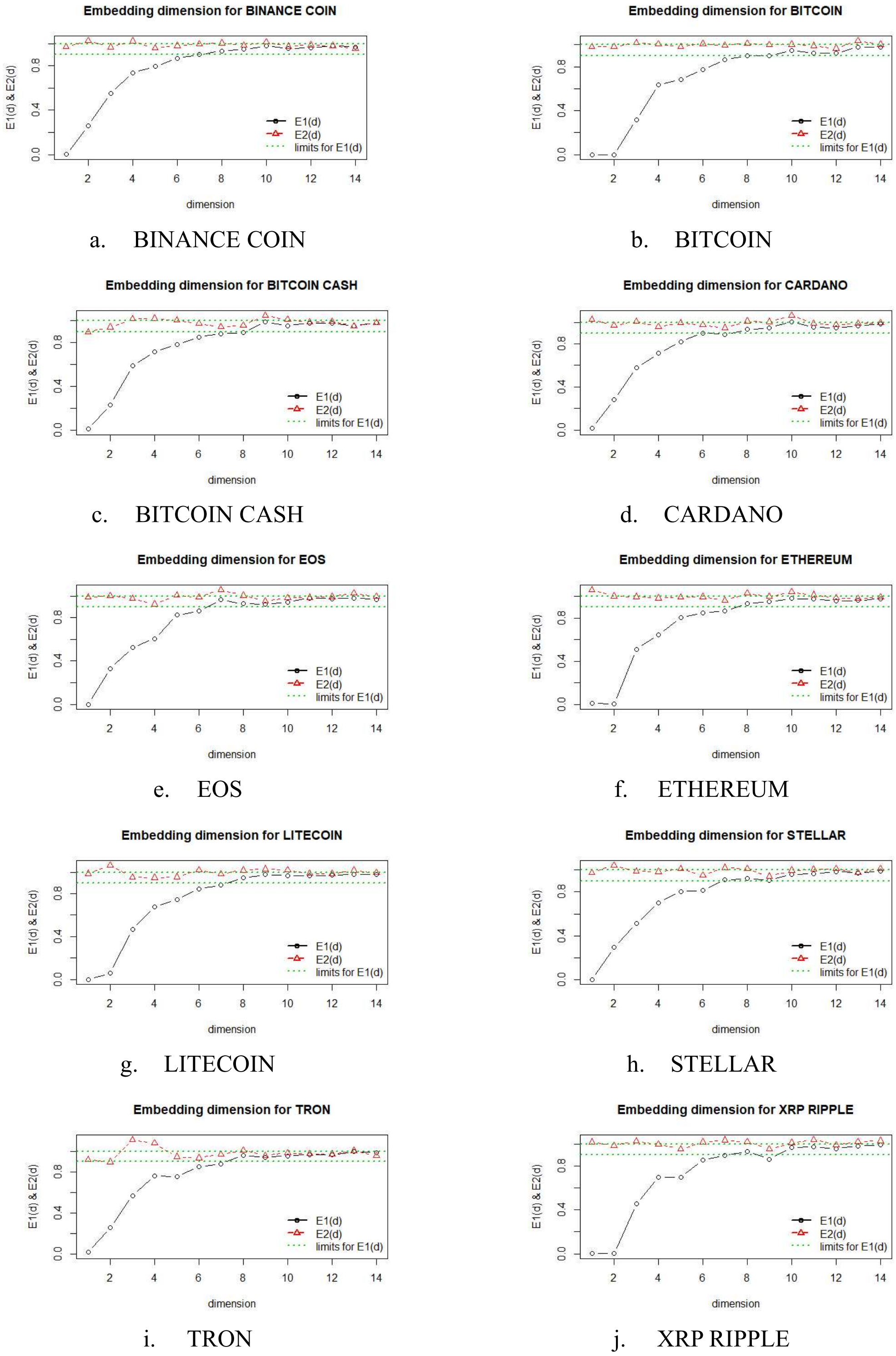
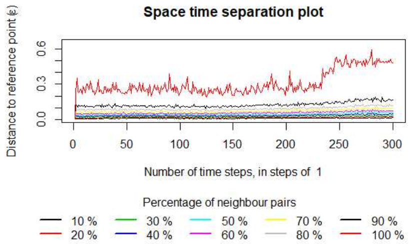


Figure 3. Embedding dimension analysis applied to cryptocurrencies log-returns.



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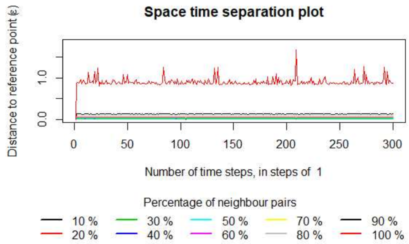

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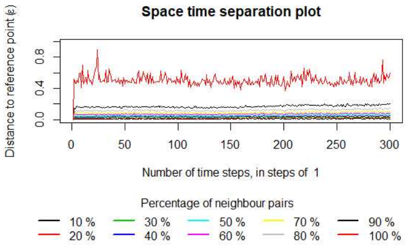
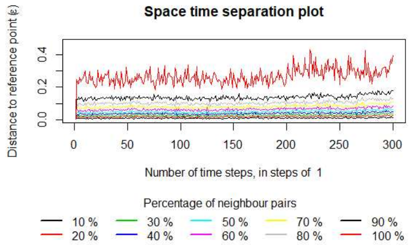
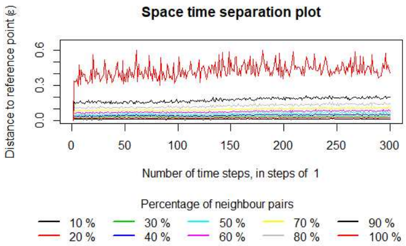


c. BITCOIN CASH



d. CARDANO



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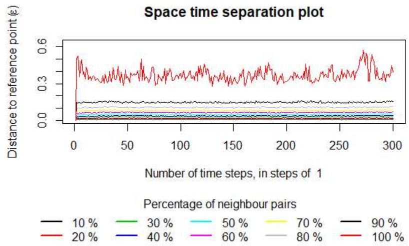
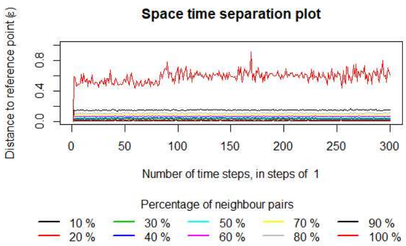


f. ETHEREUM



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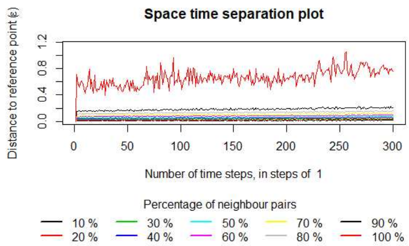

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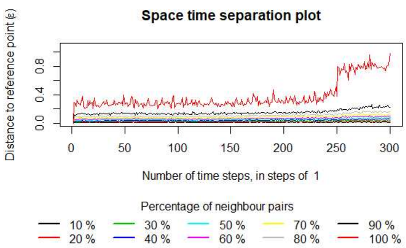
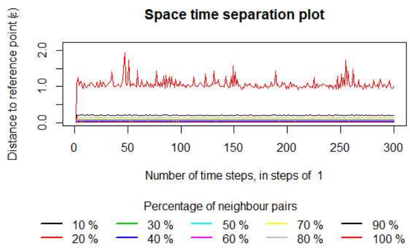


i. TRON



j. XRP RIPPLE

Figure 4. Space Time Separation plots for cryptocurrencies log-returns.

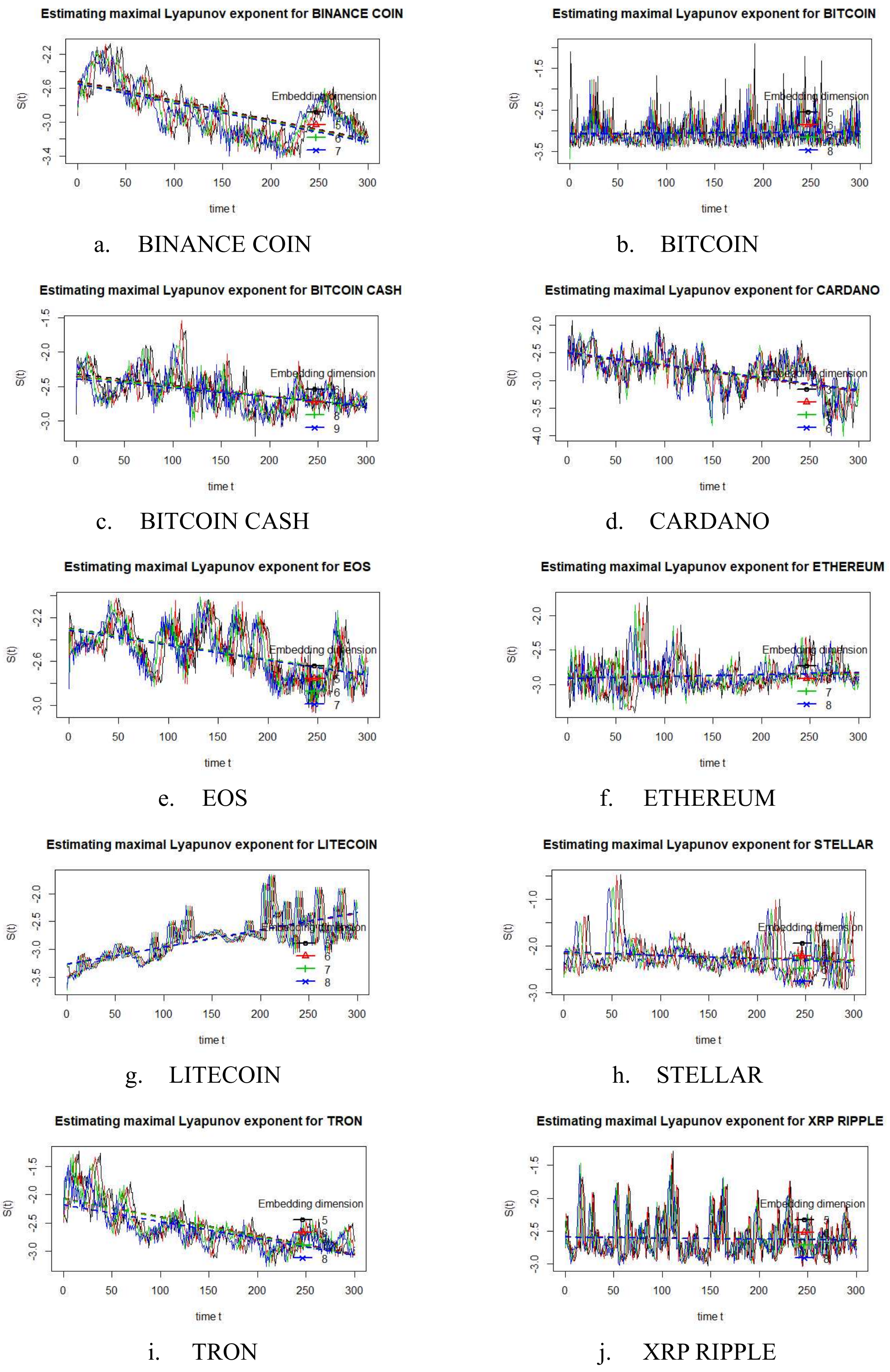
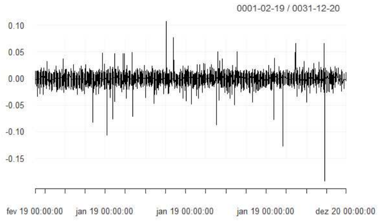
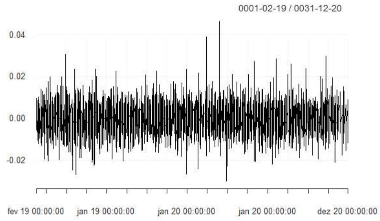


Figure 5. Estimation of Maximal Lyapunov Exponent for cryptocurrencies log-returns.

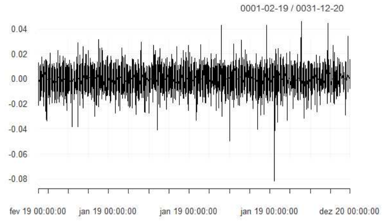
APPENDIX B – DETAILS ON THE CLASSIC COINS PARAMETERS



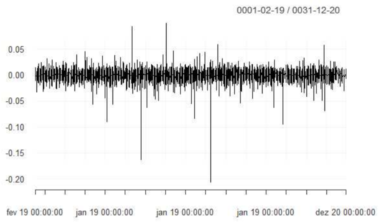
a. AUD/USD



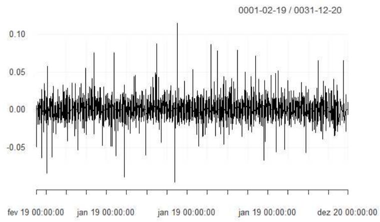
b. EUR/USD



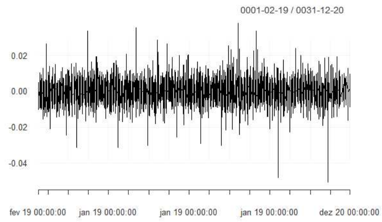
c. GBP/USD



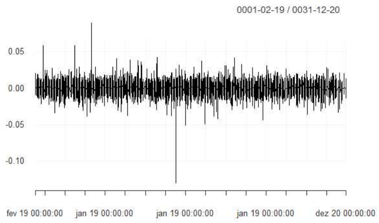
d. NZD/USD



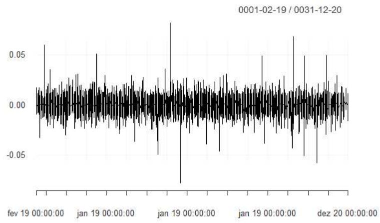
e. USD/BRL



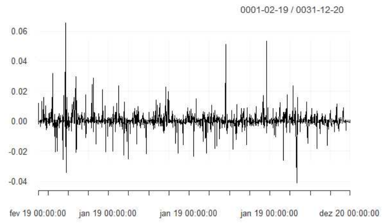
f. USD/CAD



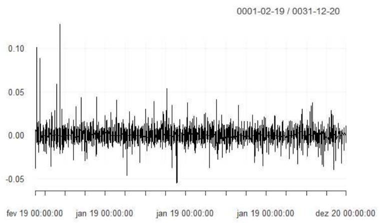
g. USD/CHF



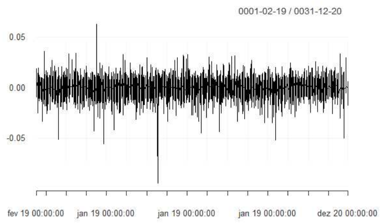
h. USD/DKK



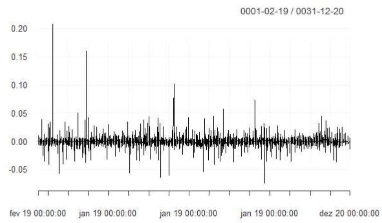
i. USD/HKD



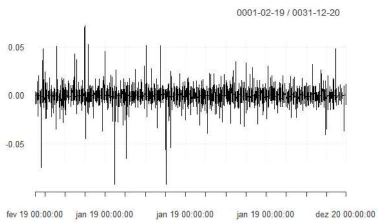
j. USD/INR



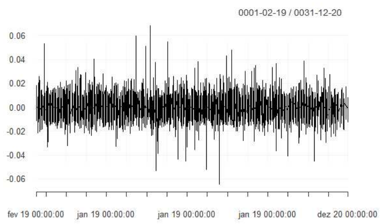
k. USD/JPY



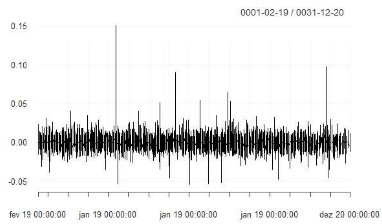
l. USD/KRW



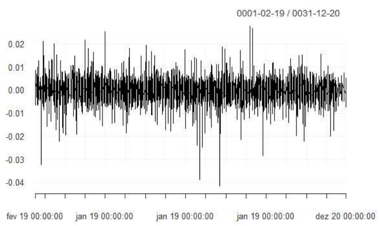
m. USD/MYR



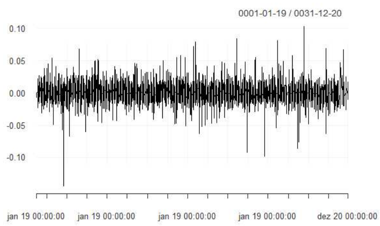
n. USD/NOK



o. USD/SEK

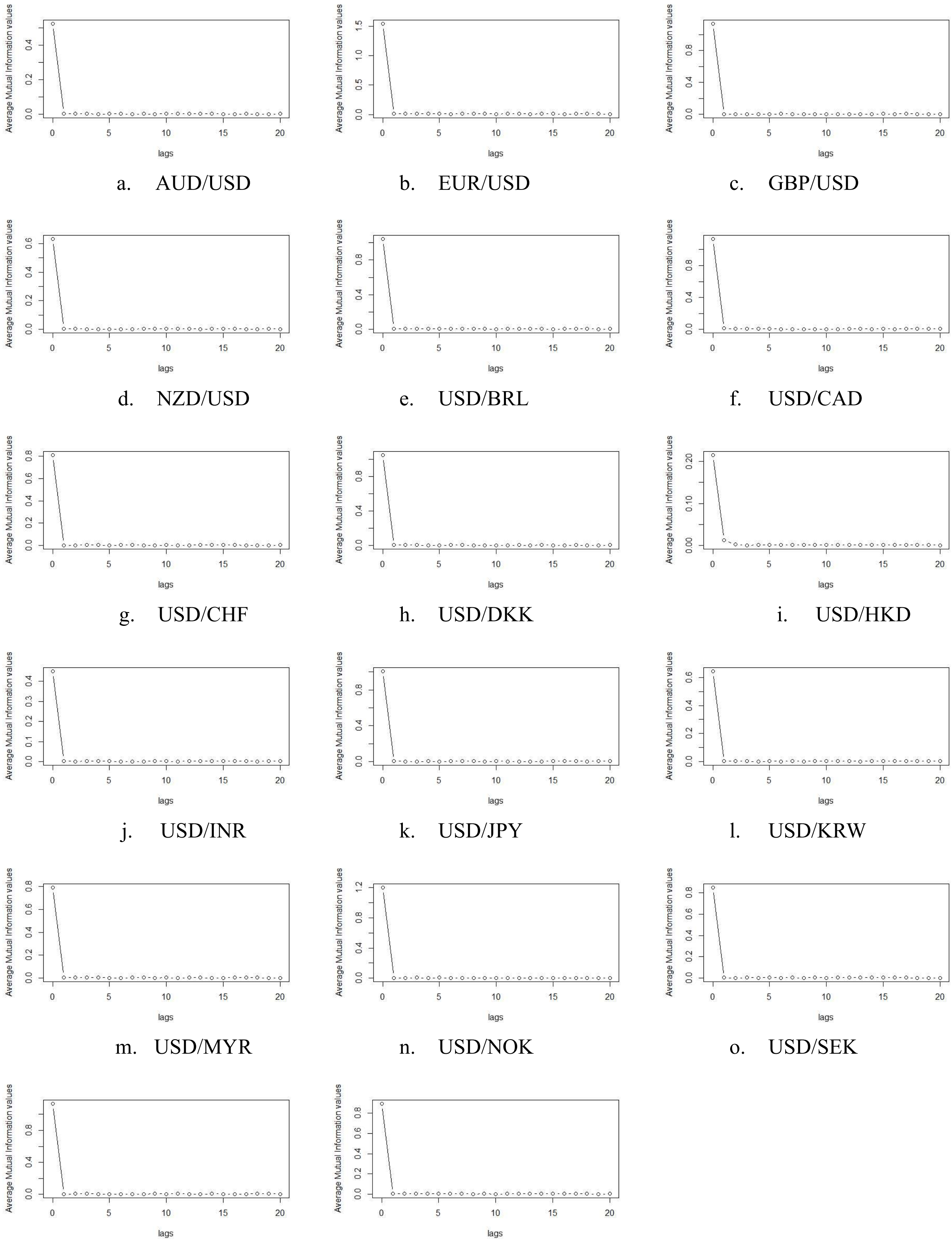


p. USD/THB



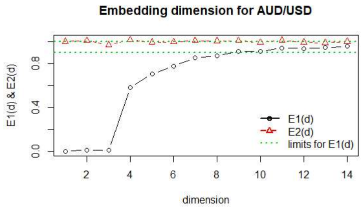
q. USD/ZAR

Figure 6. Exchange rate log-return series.

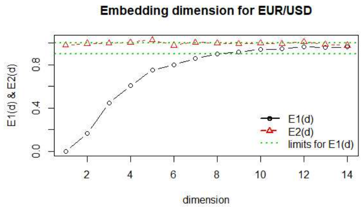


a. AUD/USD b. EUR/USD c. GBP/USD
d. NZD/USD e. USD/BRL f. USD/CAD
g. USD/CHF h. USD/DKK i. USD/HKD
j. USD/INR k. USD/JPY l. USD/KRW
m. USD/MYR n. USD/NOK o. USD/SEK
p. USD/THB q. USD/ZAR

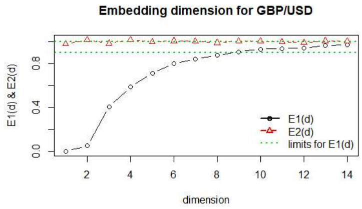
Figure 7. Average Mutual Information analysis for exchange rates log-returns.



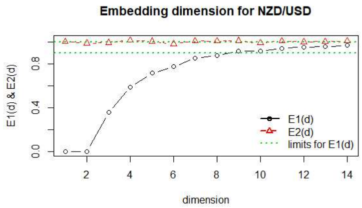
a. AUD/USD



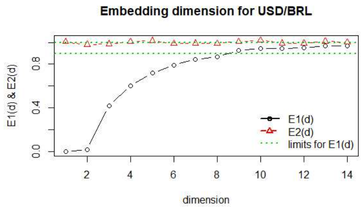
b. EUR/USD



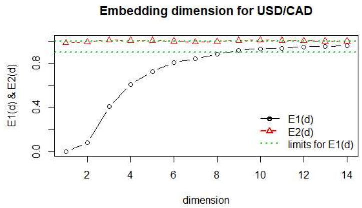
c. GBP/USD



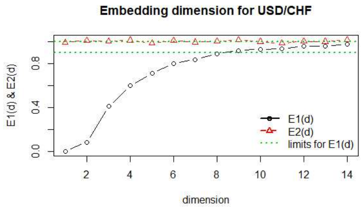
d. NZD/USD



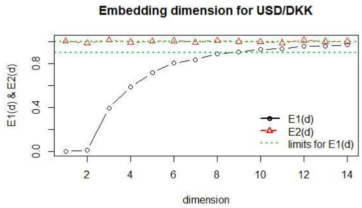
e. USD/BRL



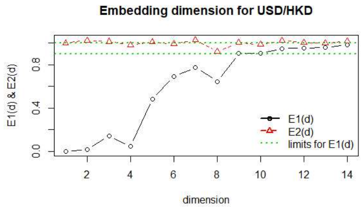
f. USD/CAD



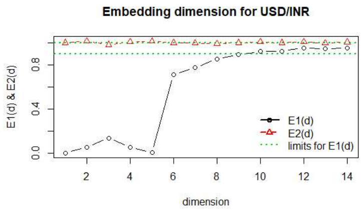
g. USD/CHF



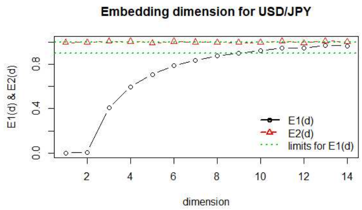
h. USD/DKK



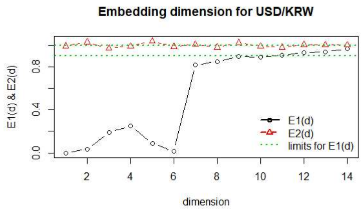
i. USD/HKD



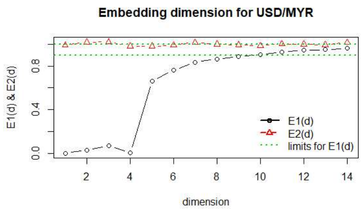
j. USD/INR



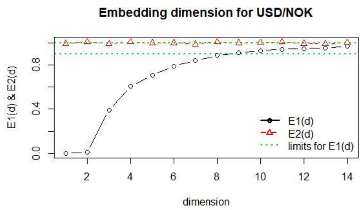
k. USD/JPY



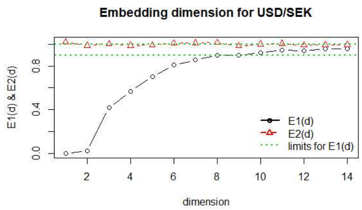
l. USD/KRW



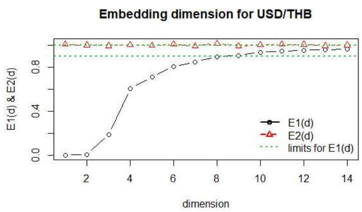
m. USD/MYR



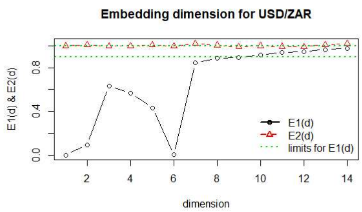
n. USD/NOK



o. USD/SEK



p. USD/THB



q. USD/ZAR

Figure 8. Embedding dimension analysis applied to exchange rates log-returns.

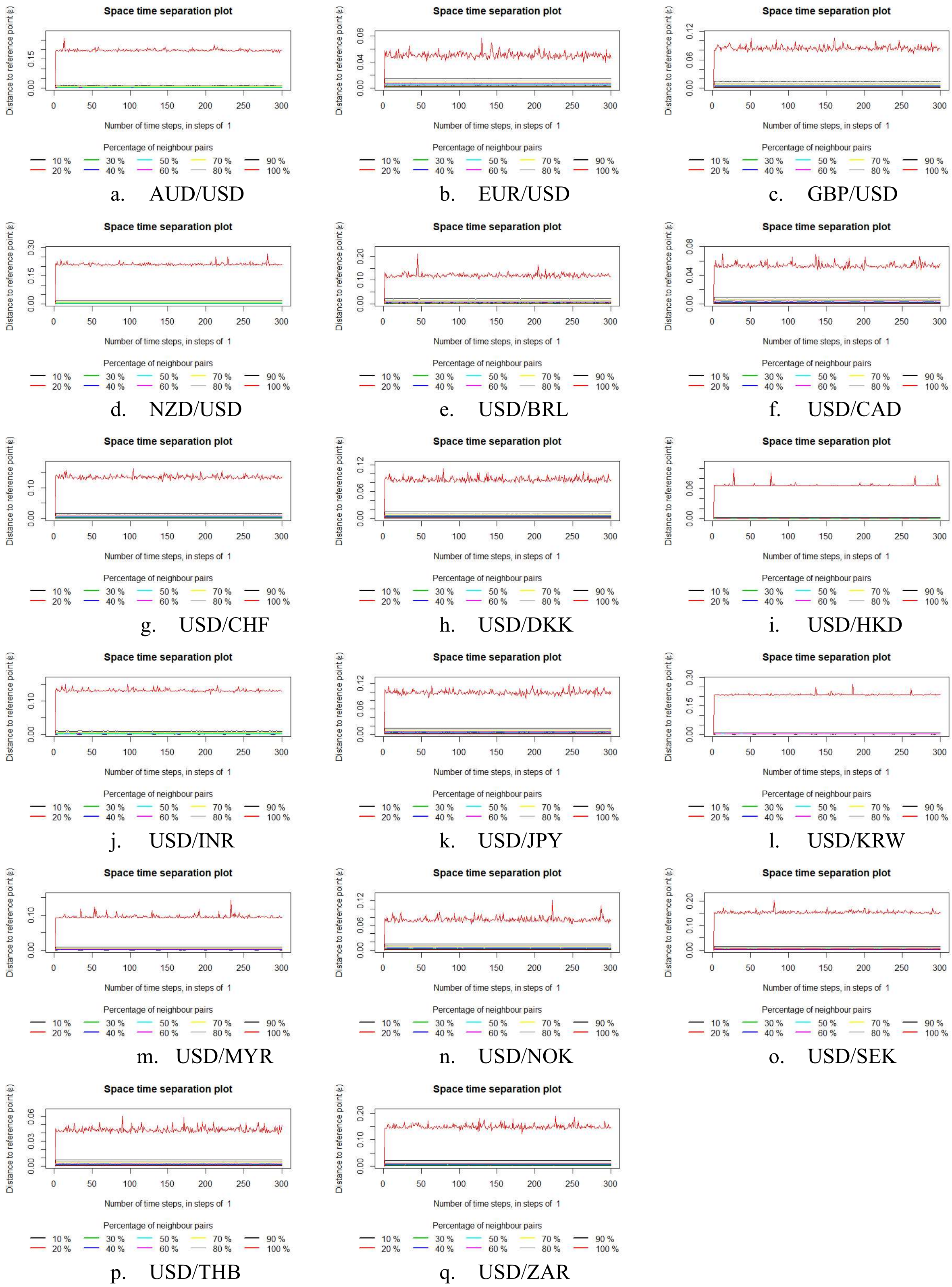
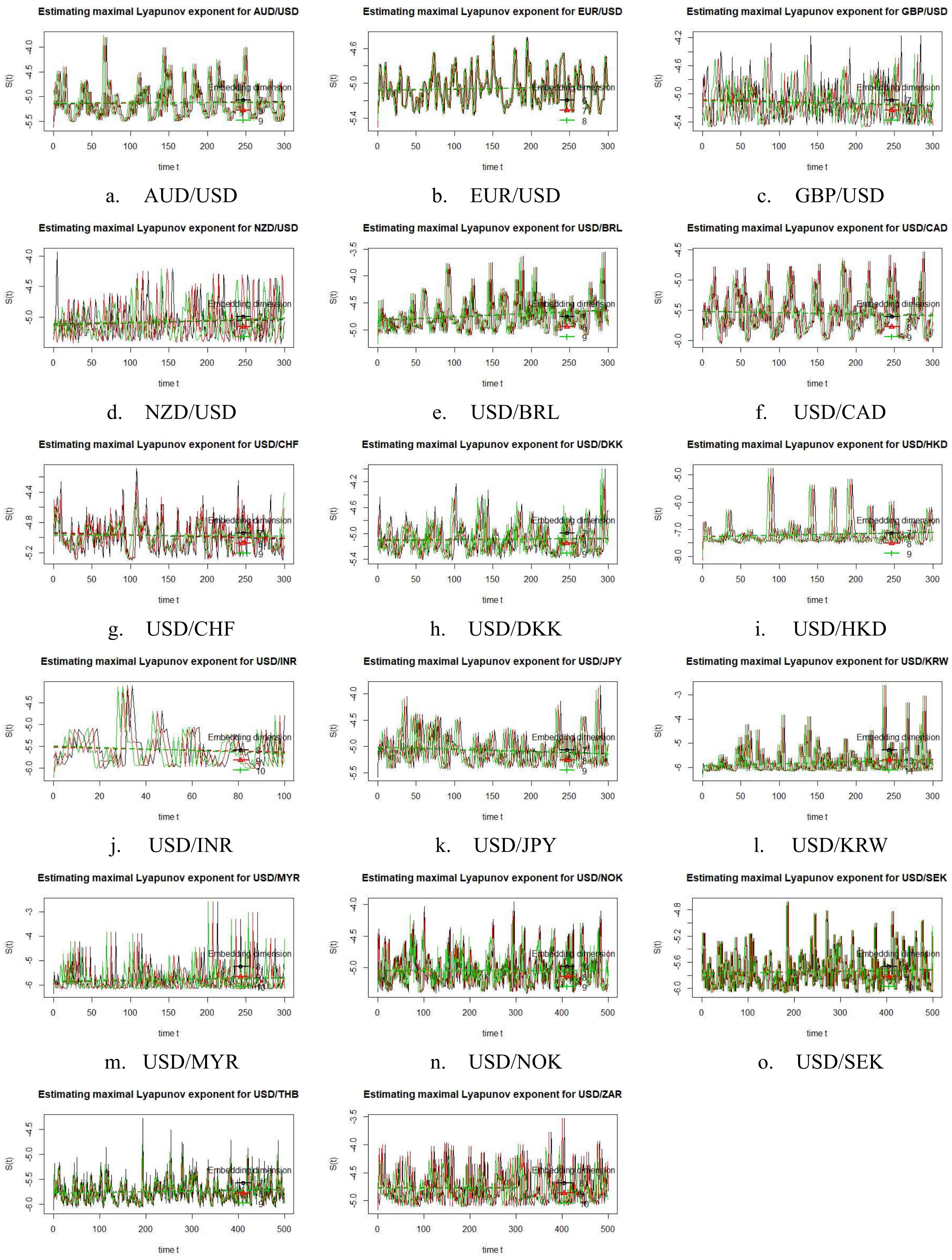


Figure 9. Space Time Separation plots for exchange rates log-returns.



a. AUD/USD
b. EUR/USD
c. GBP/USD
d. NZD/USD
e. USD/BRL
f. USD/CAD
g. USD/CHF
h. USD/DKK
i. USD/HKD
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k. USD/JPY
l. USD/KRW
m. USD/MYR
n. USD/NOK
o. USD/SEK
p. USD/THB
q. USD/ZAR

Figure 10. Estimation of Maximal Lyapunov Exponent for exchange rates log-returns.