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**DEVELOPMENT OF A NON-DESTRUCTIVE TEST FOR VISCOELASTIC
CHARACTERIZATION OF ASPHALT MIXTURES**

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CHARACTERIZATION OF ASPHALT MIXTURES

M.Sc. Thesis presented to the Graduate Program in Transportation Engineering, as a partial fulfillment of the requirements for the Master's degree in Transportation Engineering at Federal University of Ceará. Area within the Graduate Program: Transportation infrastructure

Advisor: Prof. Dr. Jorge Barbosa Soares.
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A Deus.

Aos meus pais, Ana e Valtenio.

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RESUMO

Tradicionalmente, a obtenção do módulo complexo de misturas asfálticas é feita por um ensaio cíclico e quase estático, que demanda bastante tempo e exige mão de obra especializada. Assim, justifica-se a importância do desenvolvimento de métodos alternativos mais econômicos, simples e com possibilidade de aplicação em campo. Isso é particularmente importante considerando a emergência de métodos de dimensionamento e de controle de campo cada vez mais mecânicos. Neste contexto, o objetivo desse trabalho foi desenvolver um ensaio não destrutivo, baseado no princípio de propagação de ondas e ressonância por impacto, para a determinação da rigidez de misturas asfálticas. Para isso, montou-se uma primeira versão do equipamento de ensaio de ressonância por impacto (*impact resonance test* – IRT), validado com testes em materiais elásticos. Os resultados mostraram que o IRT proposto produz respostas de ressonância adequadas e similares a testes clássicos (quase-estático, estático e ultrassônico). Após esta etapa e com uma segunda versão do equipamento de IRT, realizou-se o ensaio com medida de Função de Resposta de Frequência (FRF) em um corpo de prova de mistura asfáltica em 3 temperaturas (-10, 4 e 20°C) com 2 equipamentos de IRT, sendo um o desenvolvido na Universidade Federal do Ceará (UFC) e o outro o existente na *École de Technologie Supérieure* (ÉTS) da Universidade do Québec. Os resultados das FRF experimentais obtidas com os dois testes mostraram boa similaridade, com menos de 1% de diferença percentual entre os picos das frequências ressonantes. Finalmente, para análise dos resultados de IRT e obtenção de curvas mestras, uma análise inversa foi proposta, baseada em simulação com *software* de Elementos Finitos (FEM). Essa metodologia se mostrou adequada, pois as curvas mestras globais obtidas com o teste tradicional cíclico e os dinâmicos (IRT UFC e IRT ÉTS) apresentam boa correlação. As diferenças se devem a dispersões experimentais e a eventuais efeitos de não linearidade do material viscoelástico (possível efeito do nível de carga no impacto). Portanto, esse trabalho de mestrado trouxe o desenvolvimento pioneiro no Brasil de um teste de IRT com medida de FRF e a proposição de um método de análise inversa para obtenção de curvas mestras de misturas asfálticas. Contribuiu ainda diretamente para a aproximação entre os laboratórios da UFC-Brasil e da ÉTS-Canadá, além da *Ecole Nationale des Travaux Publics de l'Etat* (ENTPE) da Universidade de Lyon-França, onde equipamentos de mesma natureza haviam sido pioneiramente desenvolvidos.

Palavras-Chaves: viscoelasticidade linear; propagação de onda; função de resposta de frequência; misturas asfálticas; análise inversa.

ABSTRACT

Traditionally, the complex modulus of asphalt mixtures is obtained by a cyclic and quasi-static test, which demands more time and requires specialized personnel. This justifies the importance of developing alternative methods that are more economical, simple and applicable in the field. This is particularly important considering the emergence of increasingly mechanistic pavement design methods and field control. The objective of this research was to develop a non-destructive test, based on the principle of wave propagation and impact resonance for determining the stiffness of asphalt mixtures. For this purpose, the first version of the (*Impact Resonance Testing* - IRT), pioneer in Brazil, equipment was developed and validated with elastic materials. The results indicated that the proposed IRT produces adequate resonance responses similar to classical tests (quasi-static, static and ultrasonic). After this step and with the second version of the IRT equipment, the test was performed with the Frequency Response Function (FRF) measurement on an asphalt mixture specimen at 3 temperatures (-10, 4 and 20°C) with 2 IRT equipment, one developed at the Federal University of Ceará (UFC) and another already existing at the *École de Technologie Supérieure* (ÉTS) of the University of Quebec. The results of the experimental FRF obtained with the two tests for the same specimen of asphalt mixture presented less than 1% difference between the peaks of the resonant frequencies. Finally, to analyze the IRT test results and obtain the master curve, an inverse analysis was proposed, based on simulation with a Finite Element Software and divided into 4 main steps. This methodology showed to be adequate, and when comparing the global master curves obtained with the traditional cyclic test and the dynamic ones (IRT UFC and IRT ÉTS), they presented a very close correlation. The existing differences are due to experimental dispersions and nonlinearity effects of the viscoelastic material (possible effect of load level on impact). Therefore, this master's thesis brought up the development in Brazil of an IRT test with FRF measurement and proposed an inverse analysis to obtain asphalt mixture master curves. It also contributed directly to the approximation between research laboratories of UFC-Brazil and ÉTS-Canada, in addition to the *Ecole Nationale des Travaux Publics de l'Etat* (ENTPE) of the University of Lyon-France, where equipment of the same nature had been pioneered.

Keywords: linear viscoelasticity; wave propagation; frequency response function; asphalt mixtures; inverse analysis.

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1 INTRODUCTION

The complex modulus ($|E^*|$) is a parameter that contains the information about the viscoelastic behavior of materials such as asphalt mixtures used in pavement surface courses, providing stiffness values at different temperatures and frequencies. This is a fundamental input for asphalt pavements design (CARRET *et al.*, 2018), since mechanistic-empirical methods involve the structural analysis of the layer system and require the stiffnesses of the various layers as input.

This parameter is also used in asphalt mixtures performance equations in countries such as France since 1960 and the United States since 1990 (HUET, 1963; SAYEGH, 1965; ODÉON; CAROFF, 1997). In Brazil, currently the $|E^*|$ is obtained in the laboratory by cyclic and quasi-static tests standardized by DNIT-ME 416 (2019). This lab test requires more resources and time than others in the paving industry, besides requiring a skilled labor force. It is also worth mentioning that it is not adapted to the field. This set of factors discourages its use, despite the fact that it is considered more effective for material characterization than others, in public agencies, consulting firms and construction companies. Therefore, there is demand for developing alternative methods which are more economical, simple to perform and that can be applied for stiffness measurements in the field, serving for technological control during construction.

To circumvent the above mentioned difficulties, several stiffness prediction models have been developed in recent decades with the aim of estimating the complex modulus curve (CHRISTENSEN *et al.*, 2003; BIRGISSON *et al.*, 2004; BARI, 2005; KIM; KING; MOMEN, 2005; YU; SHEN, 2012; YANG *et al.*, 2014; GEORGOULI; LOIZOS; PLATI, 2016; GOUVEIA, 2016). However, these models need to be calibrated for validation through laboratory testing, and even if calibrated, they will always be subject to inter-material variability and estimation errors. Researchers have also presented non-destructive methodologies based on wave propagation and impact resonance for stiffness determination in the laboratory and in the field, which is the object of the study presented herein.

Ultrasonic tests for the determination of $|E^*|$ have been performed by measuring the time-of-flight (ToF) (DI BENEDETTO; SAUZÉAT; SOHM, 2009; MOUNIER *et al.*, 2012). The main limitation of these tests is that the $|E^*|$ results at a given temperature are determined for a single frequency, which requires running the test at different temperatures to obtain part of the master curve for high frequencies.

Based on this difficulty, Gudmarsson (2014) and Carret (2018) developed a method based on a test that involves wave propagation and impact resonance with the analysis being based on Frequency Response Functions (FRF, which are ratios of the integral transforms of the output vibration intensities and of the input excitations, in frequency domain), allowing to characterize viscoelastic materials over a wide range of frequencies (REN; ATALLA; GHINET, 2011). Measurements with FRF have shown promising results and allow a direct comparison between the master curves obtained from the traditional cyclic test and the alternative impact resonance and wave propagation tests (GUDMARSSON; RYDEN; BIRGISSON, 2012a).

Given the importance of $|E^*|$, the possibility of innovation in the design and quality control of pavements, and also considering the lack of national research (COSTA; ALBUQUERQUE; FREITAS, 2017) on this topic, it is justified to investigate how the alternative methods (non-destructive and based on wave propagation and impact resonance) are useful to provide the linear viscoelastic (LVE) properties of asphalt mixtures. This Master's dissertation intends to contribute to the continuous improvement of asphalt mixture LVE characterization processes, based on the implementation of new non-destructive testing technologies. The idea is to make it possible to obtain stiffness data compatible with the behavior of the material, considering more productive methodologies with respect to labor, materials and equipment, and obtaining faster results and lower maintenance costs.

1.1 Objectives

The main objective of this research was develop a non-destructive testing prototype, based on the principle of wave propagation and impact resonance, for determining the stiffness of asphalt mixtures, investigating its ability to determine accurate LVE properties. To achieve the main objective, the following specific objectives were defined:

- To validate the test through the characterization of materials considered elastic, comparing with results from other types of classical tests (static, quasi-static and ultrasonic);
- To compare the operation and the results of the prototype developed at Federal University of Ceará (UFC) with international prototypes through a partnership with other institutions;

- To develop an inverse analysis tool for viscoelastic characterization of asphalt mixtures at different temperatures, obtaining global master curves, and compare the results with the traditional complex modulus test.

1.2 Manuscript structure

The work was divided into 6 chapters, where chapters 3, 4 and 5 are papers produced from the research conducted, corresponding to each of the individual specific objectives listed before. The overall structure is as follows:

- Chapter 1, in which a contextualization of the subject, the justification and objectives of the research were presented;
- Chapter 2, where the main issues related to linear viscoelasticity and wave propagation in asphalt mixtures are addressed in a brief Literature Review;
- Chapter 3, which corresponds to a paper published in a national journal (*Transportes*) with the objective to present the non-destructive impact resonance test (IRT) developed in the Pavement Mechanics Laboratory (LMP) of Federal University of Ceará (UFC);
- Chapter 4, which corresponds to a paper accepted and presented in an international conference (International Symposium of Asphalt Pavements - ISAP 2022 in Costa Rica), which aimed to present the results of the first stage of validation of the development of the IRT test in elastic materials;
- Chapter 5, which corresponds to a paper to be submitted to a journal aiming to propose an inverse analysis methodology to obtain viscoelastic properties of asphalt mixtures from the IRT test results;
- Chapter 6, contains the final considerations and consolidates the contributions of the research presented as part of the Master's work developed.

2 LITERATURE REVIEW

This chapter aims to briefly present the main issues that contribute significantly to the contextualization and understanding of the concepts related to the research object. The focus of this literature review was on the following items: linear viscoelastic behavior, to understand the properties that govern this behavior and what are the main continuous spectrum models for asphalt mixtures; wave propagation and its vibration modes in linear viscoelastic materials; advantages and limitations of dynamic tests used to determine stiffness; and modeling and numerical simulations of dynamic tests, to understand the parametric analysis and inverse analysis methods. Some topics of particular interest to the results and discussion sections (Chapters 3, 4 and 5) are more detailed in each of the corresponding chapters.

2.1 Linear Viscoelasticity (LVE)

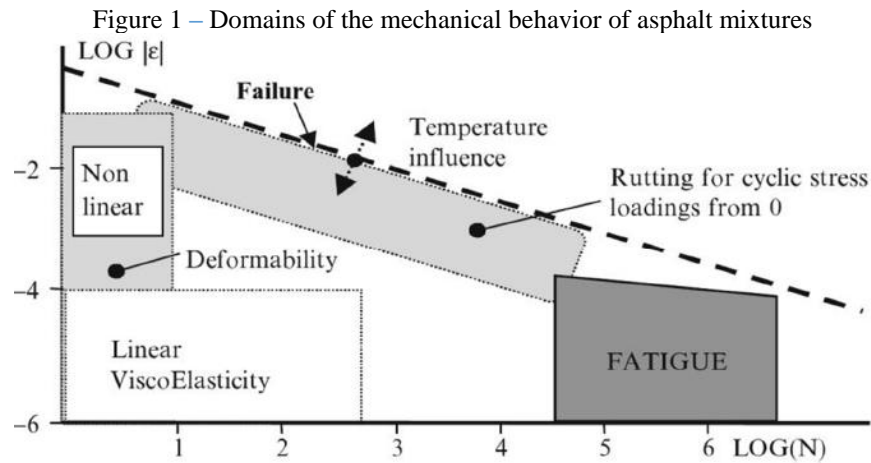
Materials present different mechanical behavior in their stress-strain relationships. For example, Portland cement concrete and Newtonian liquids are represented by linear constitutive relations, Hooke's and Newton's Laws, respectively. In the case of the first one, there is a constant of proportionality between stresses and strains, where the path of loading and unloading is equal.

Asphalt materials, on the other hand, are dependent on time (loading frequency), temperature, and loading intensity. They are therefore viscoelastic, i.e., exhibit both elastic and viscous properties. At low temperatures and when subjected to high frequencies (higher velocity of load application) they resemble a solid, and at high temperatures and low frequencies they take the form of a viscous liquid.

This viscoelastic behavior can be linear or nonlinear, depending on the strain level, but in both cases, there is a time dependence in the stress-strain relationship. At small strain levels, linear behavior is expected, and at high strain levels, nonlinear behavior occurs. For the application of linear viscoelasticity (LVE), it is necessary to observe Boltzmann's principles of homogeneity and superposition. Viscoelastic materials are also dependent on the loading history.

Within the linear regime, elastic behavior is also expected at very low temperatures, and viscoelastic behavior is expected at high temperatures (BABADOPULOS, 2017).

However, in asphalt mixtures the granular skeleton is responsible for a portion of elastic behavior at high temperatures. Figure 1 shows the behavior regimes discussed above.



Source: Adapted Di Benedetto *et al.* (2013).

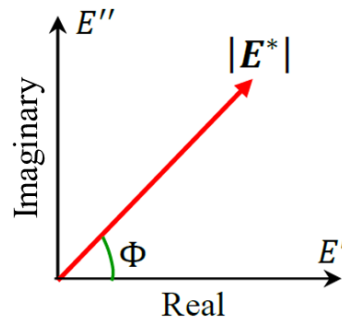
In the characterization of viscoelastic materials, as is the case of asphalt cement refined from petroleum, several properties are used, including the modulus (E) and Poisson's ratio (ν). Asphalt binders are tested by applying an oscillatory load, obtaining the parameters dynamic shear modulus ($|G^*|$) and phase angle (Φ). In asphalt mixtures, oscillating axial loads are applied to the specimens. The parameters of complex modulus ($|E^*|$) and phase angle (Φ) are obtained from the stress and corresponding axial strain.

When having harmonic sinusoidal loading, the relationship between stress and strain is called complex modulus ($|E^*|$), shown in Equation 1 (PRITZ, 1998). $|E^*|$ characterizes the stiffness of asphalt mixtures, a property highly dependent on temperature and applied loading (KIM, 2009). It is a complex number that contains a phase angle (Φ) that relates its imaginary part and its real part (Figure 2) (OLIVEIRA, 2019). This lag happens between the moment when the stress is applied and the moment when the material responds to that stress. Purely elastic materials have Φ of 0° , whereas purely viscous materials have Φ of 90° , and viscoelastic materials within a range of $0^\circ < \Phi < 90^\circ$.

$$E^* = \frac{\sigma}{\varepsilon} = \frac{\sigma_0 e^{i\omega t}}{\varepsilon_0 e^{i(\omega t - \Phi)}} = \frac{\sigma_0}{\varepsilon_0} \cos \Phi + i \frac{\sigma_0}{\varepsilon_0} \sin \Phi \quad (1)$$

where σ_0 is the loading stress amplitude; ε_0 is the recoverable strain amplitude; ω is the angular frequency; t is the time; and Φ is the phase angle.

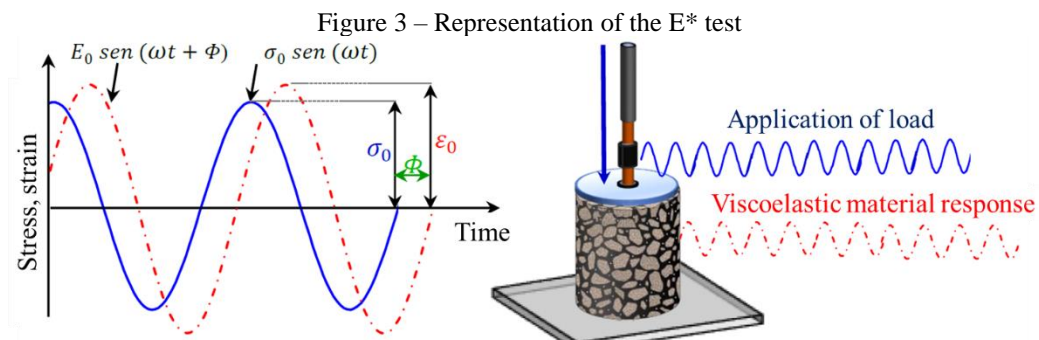
Figure 2 – Complex modulus representation



Source: Adapted Kim (2009).

The real part is also known as storage modulus (E') and is related to the elastic response of the material. The imaginary part or loss modulus (E'') has to do with the viscous response. The relationship between stress and strain amplitudes is the absolute value of the complex modulus (Equation 2 and Figure 3) (PAPAZIAN, 1962).

$$|E^*| = \frac{\sigma_0}{\varepsilon_0} \quad (2)$$



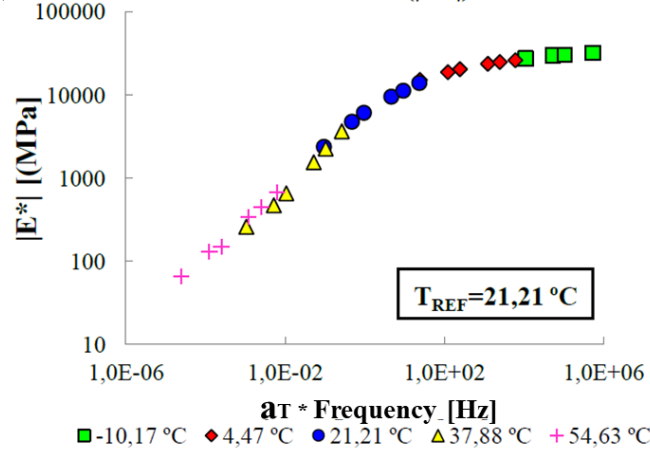
Source: Adapted Lucas Júnior (2018).

2.1.1 Time-Temperature Superposition Principle (TTSP)

In order to enable the determination of the complex modulus of asphalt mixtures, the Time-Temperature Superposition Principle (TTSP) is used, since the analysis of linear viscoelastic behavior must be done over a wide range of time (frequency) (SILVA 2009; OLIVEIRA, 2019). This would not be experimentally feasible, as it would require about 3 years and time intervals of the order of 10^{-5} seconds (time scale at the level of molecular vibrations) (SILVA, 2009). To circumvent this fact, the TTSP equivalence between time and temperature is used, where an increase in temperature is equivalent to a reduction in the frequency (increase in time) of loading application (GOUVEIA, 2016). When the TTSP is valid, the material is

considered thermoreologically simple and it is possible to superimpose the experimental viscoelasticity data obtained at different temperatures and frequencies into a single smooth and continuous curve, known as the master curve (Figure 4) (FERRY 1980; GOUVEIA, 2016).

Figure 4 – Example of a general dynamic modulus master curve



Source: Adapted Lucas Júnior (2018).

For the construction of a master curve, a horizontal time-temperature shift factor (aT) is determined, which represents the shift time related to the temperature change (FERRY, 1980; DI BENEDETTO *et al.*, 2009; BABADOPULOS, 2017), at a given chosen reference temperature (Tr). The reduced frequency is also determined using Equation 3, which shows the ratio between the test frequency (f) and the displacement factor (aT) for the respective temperature.

$$aT = \frac{f}{f_r} > \hat{f} = \frac{f}{aT} > \log(\hat{f}) = \log(f) - \log(aT) \quad (3)$$

The following Williams-Landel-Ferry (WLF) Equation 4 is commonly used in the literature to obtain the displacement factor.

$$\text{Log}(aT) = \frac{-C_1(T-Tr)}{C_2 + T-Tr} \quad (4)$$

Where aT is the shift factor at temperature T ; T is the temperature at which the data were obtained; Tr is the reference temperature for curve construction; and C_1 and C_2 are empirical curve-fitting constants, found in the literature.

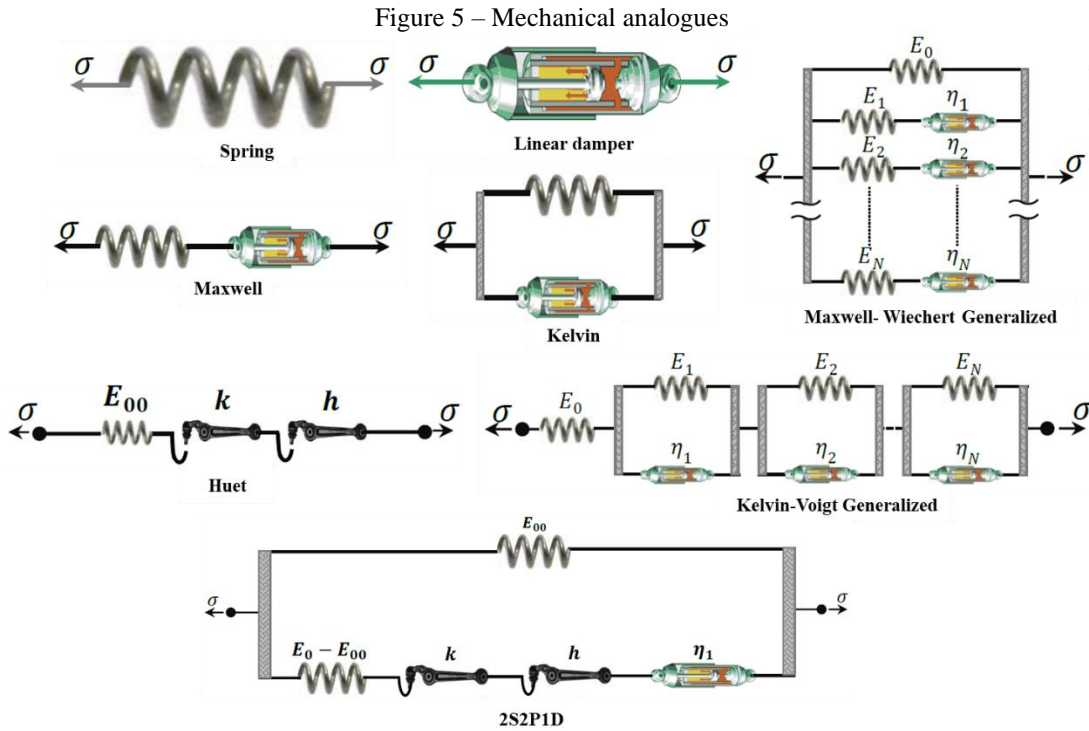
Linear viscoelastic behavior can also be represented with other types of curves. In *Cole-Cole* curves, the imaginary part of the complex modulus is plotted as a function of its real part. In the Black Space curves, the complex modulus is plotted versus the phase angle, being more accurate for low modulus values when compared to the *Cole-Cole*. In isotherm curves, the complex modulus is plotted as a function of frequency, in different curves at fixed temperature. It is worth remembering that at low frequencies it is not possible to perform such a wide frequency sweep, due to equipment limitations and the time required to perform the tests. Thus, the application of TTSP on the isotherms for the construction of the Master Curve is an alternative tool for analyzing the behavior of the viscoelastic material (OLIVEIRA, 2019).

2.1.2 Constitutive models of LVE behavior

For predicting the LVE behavior of asphalt mixtures there are several constitutive models in the literature, with a wide variety of complexity (PARK; SCHAPERLY, 1999; OLARD; DI BENEDETTO, 2003; KIM, 2009; MAHMOUDI *et al.*, 2010; TIOUAJNI *et al.*, 2011; LAMOTHE *et al.*, 2014). Each model represents the various mechanical behaviors that these materials can assume. They are called mechanical analogues, and consist of springs, linear dampers (dashpot), and parabolic dampers, and facilitate the understanding of the materials' behavior. The series or parallel association of these elements gives rise to the various existing mechanical models (Figure 5): Maxwell, Kelvin, Generalized Maxwell-Wiechert, Generalized Kelvin-Voigt, Huet and 2S2P1D (two springs, two parabolic elements and one dashpot).

The models can be represented by the relationships between stresses and strains through differential equations, which can be converted to an integral form. With the master curve, they can be fitted to experimental data.

The 2S2P1D is the closest representation of the actual material behavior (BABADOPULOS, 2017), and can be applied to binders and asphalt mixtures (OLARD; DI BENEDETTO, 2003; DI BENEDETTO, *et al.*, 2004). In addition, it offers more flexibility to model the material behavior because of its constants (CARRET, 2018). The implementation of the model and its analytical investigation is facilitated by its small number of parameters and its continuous relaxation spectrum (CAO; LACROIX; KIM, 2022). It consists of two springs in parallel and two parabolic dampers and a linear damper associated in series.



Source: Adapted Lucas Júnior (2018).

The model has seven physical parameters (Equation 5), plus two WLF equation constants (nine total for 1D modeling), since, asphalt mixtures obey the TTSP and the WLF can be used.

$$E^*(\omega) = E_{00} + \frac{E_0 - E_{00}}{1 + \delta(i\omega\tau) - k + (i\omega\tau) - h + ((i\omega\beta\tau) - 1)} \quad (5)$$

where E_0 is the value of the modulus when the frequency tends to infinity; E_{00} is the value of the modulus when the frequency tends to zero; k and h are the dimensionless constants of the two parabolic elements; δ is the dimensionless constant of the form factor; β is the constant related to viscosity; ω is the angular frequency; and τ is the characteristic time.

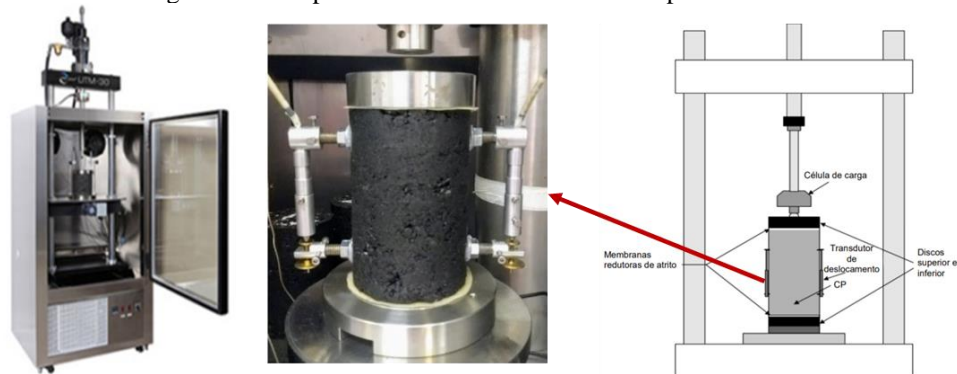
In addition to these parameters and for the three-dimensional modeling, three other parameters are added for the description of the Poisson's ratio (Equation 6). Where ν_0 and ν_{00} are the values of the asymptotic Poisson's ratio at low and high temperatures, respectively, and $\tau\nu$ is a factor responsible for a characteristic time of the Poisson's ratio, directly linked to the characteristic time of the complex modulus ($\tau\nu = \tau / \gamma$, where γ is a dimensionless constant). This makes a total of 12 parameters.

$$\nu(\omega) = \nu_{00} + \frac{\nu_0 - \nu_{00}}{1 + \delta(i\omega\tau\nu) - k + (i\omega\tau\nu) - h + ((i\omega\beta\tau\nu) - 1)} \quad (6)$$

2.2 Tests to determine LVE properties

As previously mentioned, the test for asphalt mixtures stiffness determination is standardized in Brazil by DNIT-ME 416 (2019). It is performed in a robust press through a sinusoidal request, at 5 different temperatures (-10, 4, 20, 40 and 54°C) and frequencies (25, 10, 5, 1, 0.5 and 0.1Hz), with the application of small stress-strain amplitudes. The strain in the specimen is measured using LVDTs. Figure 6 shows the schematic representation of the complex modulus test.

Figure 6 – Complex Modulus Test Schematic Representation



Source: Author (2023).

Through the loading parameters, it is possible to obtain the modulus as a function of the loading frequency at each temperature. At the end of the test, the master curve (described in item 2.1.1) is constructed by TTSP and with a reference temperature. The modulus of asphalt mixtures can vary theoretically from approximately 40MPa (high temperature and low speed) to 40,000MPa (low temperature and high speed), depending on its equilibrium modulus, temperature and loading frequency (SANTOS, 2020).

Without taking into account the viscoelastic properties of asphalt mixtures, Whitmoyer and Kim (1994) applied an impact resonance method based on the recommendations of the American standard applied to Portland cement concrete, and concluded that the test had good repeatability and reproducibility. Kweon and Kim (2006) applied an impact resonance test, a dynamic modulus test originally developed for concrete, and the traditional cyclic stiffness method. The authors noted that all three tests showed promising results, but that the complex modulus was only obtained for a resonance frequency that provided only one modulus per measurement temperature. In these early impact resonance tests,

the sample was externally excited by a mechanical force applied manually by an operator using a steel ball.

Ryden (2011) used three-dimensional number calculations to estimate the complex modulus from various resonance frequencies. The specimen was manually excited by a miniature impact hammer. However, measurements were made only on cylindrical disk specimens and limited to the longitudinal vibration mode.

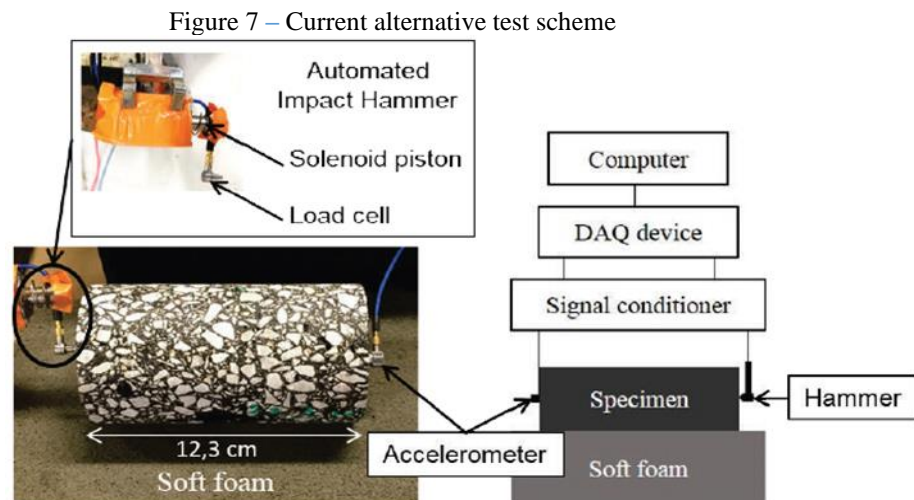
Mounier *et al.* (2012) applied ultrasonic testing with compression P-waves on several asphalt mixtures. Two methods of measurements that allow complex modulus of mixtures to be obtained were performed and compared, direct time-of-flight measurement and IRT. The calculated moduli at different temperatures showed a good fit compared to both classical stress-compression complex modulus tests. However, the results of $|E^*|$ at a given temperature were determined for a single frequency.

In ultrasonic testing, the parameters that characterize the propagation of the wave in the material studied are velocity and attenuation (weakening of the wave by dissipative effects in the material). The velocity of propagation is directly related to the modulus of elasticity, Poisson's ratio (ν) and the specific mass of the material. In asphalt mixtures, the modulus can be affected by factors such as pulse frequency and test temperature (MOUNIER *et al.*, 2012). $|E^*|$ can be determined by calculating the compression wave velocity (P-wave), this velocity being obtained by directly measuring the time-of-flight of the wave (DI BENEDETTO; SAUZÉAT; SOHM, 2009; MOUNIER *et al.*, 2012). In this method, the results of $|E^*|$ at a given temperature are determined for a single frequency, which is characteristic of the emitting equipment. Obtaining measurements at different temperatures allows plotting part of the master curve for high frequencies (around 10kHz). The complete characterization of the master curve is developed with the help of the rheological model 2S2P1D, which allows the adjustment of the curve for low frequencies (DI BENEDETTO; SAUZÉAT; SOHM, 2009; MOUNIER *et al.*, 2012).

Testing based on vibration analysis consists of measuring the natural (resonant) frequencies of the material, which vary with the mode of vibration, geometry, density, and material properties. And in the case of viscoelastic materials, they also vary with the complex modulus and Poisson's ratio (CARRET *et al.*, 2018). The general limitation in adapting Resonant Ultrasonic Spectroscopy (RUS) and Resonant Acoustic Spectroscopy (RAS) methods to asphalt mixtures has been to define $|E^*|$ at various frequencies at each test temperature. Moreover, the evaluation of the results is based on mathematical models, dependent on the geometry and limited vibration modes for the definition of the resonant frequencies, and

consequently, limited values of $|E^*|$. The limitation in the construction of the master curve is also due to the experimental obtaining of the transfer adjustment factors of the conventional test curve (GUDMARSSON, 2014).

Based on these difficulties, Gudmarsson (2014) and Carret *et al.* (2018) developed an IRT using the principle of Frequency Response Functions (FRF), the ratio between the output signals (acceleration) and the input signals (force), calculated from the signals in the frequency domain recorded by the system. The experimental results of FRF allow theoretical analysis by three-dimensional modeling of the vibration of the solid in finite element software, where the parameters of a linear viscoelastic model determine the vibration of that solid over a wide range of frequencies. The main improvements of the test developed by Carret *et al.* (2018) (Figure 7) over that of Gudmarsson (2014) were as follows: use of doubled sampling rate, which ensures a more detailed description of the recorded signals in the time domain; and the sample was excited by an automatic impact hammer that allows better suitability for different geometries and measurement positions, as well as higher quality and repeatability of the test.



Source: Carret (2018).

The most recent innovation for IRT based on the wave propagation principle was developed by Bekele *et al.* (2019). The authors investigated the effects of cooling and heating cycles on 3 dense asphalt mixture specimens making use of an automated non-contact resonance frequency measurement method, where the excitation of the specimen is done through a loudspeaker.

2.3 Wave propagation

The first musical instruments aroused interest in vibration. Pythagoras, a Greek philosopher and mathematician, conducted an experiment with a vibrating string and is considered to be the pioneer in research into musical sounds. Many human activities involve vibration, for example, hearing works because the eardrums vibrate, breathing is a result of the vibration of the lungs, speech and the movement of the legs and hands are associated with an oscillatory motion (RAO, 2008).

In engineering, the design of machines, foundations, structures, engines, turbines, and control systems, among others, have been motivated by the applications of vibration. Recent studies are using the principle of wave propagation in nonlinear viscoelastic solids (BERJAMIN *et al.*, 2018), viscoelastic composites (HU; OSKAY, 2018), linearly elastic and inhomogeneous geological materials (ITURRIOZ; RIERA, 2021), anisotropic (SELVI; ANITHA, 2021) and isotropic materials (MOHABUTH *et al.*, 2019), porous layer mounted on a foundation with sandy elastic stratum (KUMAR *et al.*, 2021), among others. Moreover, some authors have devoted the study of wave propagation in tests for determining the stiffness of Portland cement and asphalt concrete (Section 2.3).

Vibration or oscillation is any motion that repeats after a time interval and is the study of oscillatory movements of bodies and the forces associated with them (RAO, 2008). Free vibration is one in which a system, after an initial perturbation, continues to vibrate on its own. Forced vibration, on the other hand, is when a system is subject to an external force.

In solid materials, forced vibration causes a disturbance around its equilibrium state, causing small deformation. This perturbation is called a mechanical wave, which progresses through the solid medium by particle motions without any mass transport (CARRET, 2018). In continuum mechanics of solids, waves propagating in a solid are known as body waves and can be of two types, P waves and S waves. (MANDEL, 1966; CARRET, 2018; ITURRIOZ; RIERA, 2021). In P waves, also called longitudinal or primary waves, the displacement direction is the same as the wave propagation direction, and the waves are associated with displacements of relatively small particles (CARRET, 2018). Whereas S waves, also called shear or secondary waves, always propagate in the direction perpendicular to that of the wave propagation, and are slower than P waves (CARRET, 2018).

Iturrioz and Riera (2021) report that these waves are characterized by propagation velocities V_p and V_s , respectively, which are functions of the elastic modulus (E) and the

specific mass of the material (ρ), as indicated by Equation 7 and Equation 8, valid for homogeneous and isotropic materials:

$$Vp = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \quad (7)$$

$$Vs = \sqrt{\frac{E}{2\rho(1+\nu)}} \quad (8)$$

For the wave propagation velocity of LVE materials the frequency of $|E^*|$, the complex Poisson's ratio (ν) at the same frequency, and the phase angle (Φ), must be taken into account. Equation 9 and Equation 10 show the wave velocities P and S, respectively:

$$Vp = \frac{1}{\cos\left(\frac{\Phi}{2}\right)} \sqrt{\frac{|E^*|(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \quad (9)$$

$$Vs = \frac{1}{\cos\left(\frac{\Phi}{2}\right)} \sqrt{\frac{|E^*|}{2\rho(1+\nu)}} \quad (10)$$

These mechanical wave velocities have a relationship with material properties. The estimation of the modulus of bituminous materials from the measurement of the time-of-flight over a known distance has been investigated and reported in the literature (DI BENEDETTO; SAUZÉAT; SOHM, 2009). An interesting alternative to record the time-of-flight of body waves is to exploit the resonance phenomenon and apply the Resonant Ultrasonic Spectroscopy (RUS) and Resonant Acoustic Spectroscopy (RAS) methods (RYDEN, 2011; GUDMARSSON; RYDEN; BIRGISSON, 2012a).

When the natural frequency (associated with free vibration) of a material coincides with its external excitation frequency (associated with forced vibration), the resonance phenomenon occurs (RAO, 2008; CARRET 2018). When a material is forced to vibrate at its resonance frequency, it will suffer dangerously large oscillations because the response to the excitation will be of large amplitude. This fact is what often leads to the failure of structures such as buildings, bridges, turbines, and aircraft wings. Despite this damaging effect, resonance vibration is used in various applications, such as in vibratory testing of materials, vibratory processes of electronic finishing and circuits in filtering out unwanted frequencies, improving

machining, casting and welding processes, earthquake simulations in geological surveys and in the design of nuclear reactors (RAO 2008).

2.3.1 Frequency Response Function (FRF)

The Frequency Response Function (FRF) has been used in studies with asphalt mixtures (GUDMARSSON; RYDEN; BIRGISSON, 2012a; GUDMARSSON *et al.*, 2014; GUDMARSSON *et al.*, 2015; CARRET *et al.*, 2018). It is a tool that identifies the resonant frequencies and the damping properties, linked to the peak amplitudes, of LVE materials (CARRET, 2018). Also, according to Carret (2018), they are complex numbers that contain an amplitude and a phase, but only the amplitude is needed for material characterization and it does not depend on the phase. Thus, it can be easily compared at different temperatures to highlight changes in material properties, a main advantage of this tool. By definition, the FRF is the ratio of the output spectrum (Y) to the input spectrum (X), being signals in the frequency domain (BERJAMIN *et al.*, 2018). The output can be a displacement or acceleration and the input can be a force (CARRET, 2018). Equation 11 shows the expression for calculating the FRF of a material:

$$H(f) = \frac{S_{xy}(f)}{S_{xx}(f)} = \frac{X^*(f)Y(f)}{X^*(f)X(f)} \quad (11)$$

where $H(f)$ is the FRF in the frequency domain; S_{xy} is the cross power spectrum; S_{xx} is the automatic input of the power spectrum; $X(f)$ and $X^*(f)$ is the input in the frequency domain and its complex conjugate; and $Y(f)$ is the output in the frequency domain.

In order to evaluate the quality of the measurements, coherence functions are used, correlating the input and output signals at each frequency (HALVORSEN; BROWN, 1977). A value of 1 indicates that the output is fully explained by the input, and decreasing values mean that there is noise in the system that interrupted the test (CARRET, 2018). Equation 12 shows the expression for calculating the Coherence Function, $CF(f)$, of a material:

$$CF(f) = \frac{S_{xy}^2}{S_{xx} S_{yy}} \quad (12)$$

where S_{xy} is the cross power spectrum; S_{xx} the automatic power spectrum input; S_{yy} equals $Y^*(f) Y(f)$ (automatic power spectrum output); and $Y(f)$ and $Y^*(f)$ the output in the frequency domain and its complex conjugate.

2.4 Modeling with finite element methods and numerical simulations

The study of wave propagation in solids began with the so-called analytical approach, which leads to useful results in situations where the material properties can be assumed uniformly distributed and the system is characterized by simple boundary conditions (ITURRIOZ; RIERA, 2021). To analyze more complex problems, engineering attention quickly turned to numerical methods, such as the Finite Element Method (FEM). FEM aims to model complex problems with a finite number of simpler elements (discretization) by solving differential equations that explain the problem, where the elements are connected by nodes (points where the response will be calculated). The elements can assume several shapes and patterns and can be one-dimensional, two-dimensional and three-dimensional, with a variety of standard shapes and with distinct numbers of nodal points on their sides and faces (SORIANO, 2009). There are several commercial software for this type of analysis, e.g., ANSYS®, COMSOL® and ABAQUS®.

After defining the geometry, mesh, material behavior, boundary conditions, and other model parameters, the theoretical FRFs must be calculated three-dimensionally, in addition to the experimentally measured FRFs, and calculated according to Equation 11 (section 2.3.1). The following wave motion equation (Equation 13) is used for each specified and applicable frequency:

$$-\rho\omega^2\mathbf{u} - \Delta \sigma = 0 \quad (13)$$

where ρ is the bulk density; ω the angular frequency; \mathbf{u} the displacement vector; Δ the gradient tensor operator; and σ the Cauchy stress tensor.

The calculation results are processed to obtain the amplitude of the acceleration at the position and direction of vibration of the output sensor during the experimental test. Different values of complex modulus, Poisson's ratio and variables of the adopted models can be assumed to calculate the FRF that can be compared to the measured FRF. After that, the complex modulus and complex Poisson's ratio are iteratively adjusted until the calculated FRFs match the measured ones (inverse analysis problem). This procedure requires the use of a model

(section 2.1.2) capable of accounting for the complex (viscoelastic) behavior along with temperature and the dependence that asphalt concrete behavior has on frequency.

Carret (2018) performed a parametric analysis, where first the complex modulus and complex Poisson's ratio were considered independent of frequency and temperature and the influence of the norm and the phase angle of the complex modulus and complex Poisson's ratio were studied. Then, the complex modulus and complex Poisson ratio were considered temperature and frequency dependent, and modeled with the three-dimensional version of the 2S2P1D model. In this parametric analysis, the influence of the constants of the 2S2P1D model was investigated.

With the inverse analysis process, it is possible to determine the LVE properties of a material from measured FRF, and different methods based on FEM calculations can be used. Carret (2018) used five methods, four of which provided access to the complex modulus only, and another method in which it is possible to determine the complex Poisson's ratio. However, two of the methods are not recommended for application in asphalt mixtures. Gudmarsson (2014) and Carret (2018) used COMSOL software for modeling and a MATLAB function for the optimization process of the FRF.

3 DEVELOPMENT OF IMPACT RESONANCE TEST TO DETERMINE THE STIFFNESS OF DIFERENT MATERIALS

(Content of the paper published in the Brazilian journal *TRANSPORTES*, 2022. DOI:10.14295/transportes.v30i3.2757)

ABSTRACT

Non-destructive tests have been used for viscoelastic characterization of asphalt mixtures. An impact resonance test was developed at the Federal University of Ceará, with the intention to apply it primarily to bituminous materials. This paper presents the assembly and first results of the test. As a step in the validation, experiments with linear elastic materials were performed, being 1 steel and 6 mortar specimens. The materials were submitted to classical quasi-static stiffness tests and to the developed dynamic impact resonance test. The results indicated a small difference between the modulus values of the two tests, 2.15% for steel and between 4-13% for mortar. This indicates that the developed impact resonance test produces interpretable resonance results and has the potential to be used to determine the viscoelastic properties of bituminous materials.

Keywords: non-destructive testing; impact resonance; bituminous materials; stiffness.

3.1 Introduction

The stiffness of a material is the property that indicates its capacity against deformability, and is determined by the relations between applied stresses and strains. In linear elastic materials it is given directly by the proportionality ratio between stress and strain. This property is important in structural design and for technological control in the field in several areas of knowledge. In engineering, stiffness properties are necessary to predict mechanical behavior in terms of stresses and strains, a phenomenon linked to the appearance of stresses such as sinking and cracking, which functionally and structurally degrade constructions. Stiffness can be obtained through quasi-static mechanical tests, which generate stress-strain diagrams, or by measuring some indirect quantity through dynamic tests, as is the case of measuring the time-of-flight of ultrasonic waves (ultrasound tests) or measuring resonance frequencies (impact resonance tests).

In the case of viscoelastic materials, as observed in asphalt materials used in paving, the stiffness is translated in terms of the complex modulus, which has an absolute value, equivalent to the ratio between stress and strain amplitude in the permanent regime, and a phase angle. This value is known in the national asphalt materials literature as the complex modulus, despite the fact that there are no inertial effects or wave propagation in the materials during the tests, which are quasi-static. The magnitude of the complex modulus is the property required to calculate stresses and strains in structures containing viscoelastic material, and it is able to

translate the time and temperature dependence of its mechanical behavior. It has been recommended for the most modern methods of pavement design.

The literature reports that in construction, destructive and non-destructive methods have been used in determining the stiffness of cementitious materials (CARRASCO *et al.*, 2017; ARAGON *et al.*, 2019; MAKOOND; PELA; MOLINS, 2020). There are different non-destructive techniques involving inertial effects and wave propagation, where dynamic moduli are determined. Among these techniques, ultrasonic and IRT have been gaining notoriety in recent years. Carrasco *et al.* (2017) concluded that it is possible to use acoustic impact wave tests to estimate the dynamic modulus of iron ore tailings mortars, rather than traditional ultrasonic techniques, which rely on the use of expensive equipment. Wang and Gupta (2021) analyzed the modulus of elasticity properties of cementitious repair material from an IRT, obtaining a strong linear correlation (87%) with static modulus of elasticity values.

For asphalt materials, traditionally, stiffness determination is done with a laboratory cyclic loading test in a hydraulic press or by means of prediction models, which are not as accurate as real tests (GUDMARSSON; RYDEN; BIRGISSON, 2014). The results are essential for mechanistic pavement design methods. The conventional test for obtaining the stiffness of asphalt mixtures is governed by the DNIT-ME 416 (2019) standard (2019), and is performed in a robust press using a sinusoidal request at five temperatures (-10, 4, 20, 40, and 54°C) and six frequencies (25, 10, 5, 1, 0.5 and 0.1 Hz), with application of small amplitudes of stress and strain (on the order of 50 to 75 μ m/m peak-to-peak amplitude), in order to ensure assumptions of linearity. Strain in the specimen is commonly measured using Linear Variable Differential Transformer (LVDT) type sensors. Meanwhile, despite the importance of stiffness properties of asphalt materials, field control in Brazil is still restricted to the testing of volumetric and composition properties, which may not ensure the expected properties of the materials, including stiffness.

The use of more economical, precise techniques that open the possibility of a future non-destructive quality control of pavement materials has great potential. Such techniques are not yet widespread and few works are found, even in the international literature. Ultrasonic tests with compression wave propagation have been performed on cylindrical samples of asphalt mixtures (MOUNIER; DI BENEDETTO; SAUZÉAT, 2012). Although useful for an evaluation in terms of stiffness control at high frequencies (characteristics of ultrasonic testing), only one complex modulus value can be determined per test temperature, always at high frequencies.

In order to remedy this limitation of ultrasonic testing applied to asphalt mixtures, methods based on vibrational and impact resonance principles capable of determining resonant

frequencies have been performed. The application of resonant acoustic spectroscopy to frequency-dependent asphalt concrete has enabled the determination of complex modulus at more than one resonant frequency (OSTROVSKY *et al.*, 2001; GUDMARSSON; RYDEN; BIRGISSON, 2012a), which shows potential for characterizing these viscoelastic materials. However, to determine master curves based solely on mechanical wave measurements, analyses should not be limited to discrete resonance frequencies. The Frequency Response Function (FRF), obtained by dividing the acceleration by the applied force in the frequency domain, provides information of the material properties over a wide frequency range, including resonance peaks and valleys in a frequency response curve (GUDMARSSON *et al.*, 2014).

Gudmarsson *et al.* (2015) and Carret *et al.* (2018) developed an IRT using FRF. The experimental results enabled theoretical analysis by three-dimensional modeling of the vibration of the solid in finite element software, where the parameters of a linear viscoelastic model determine the vibration of that solid over a wide frequency range. The main improvements of the test developed by Carret *et al.* (2018) compared to that of Gudmarsson *et al.* (2014) were the use of doubled sampling rate, which ensures a more detailed description of the recorded signals in the dominium of time, and the sample was excited by an automatic impact hammer that allows better suitability to different geometries and measurement positions, as well as higher quality and repeatability of the test.

Nationally in Brazil, there is an even greater lack of research on this topic. Costa, Albuquerque and Freitas (2017) used an IRT to terminate the natural frequencies and, based on the concepts of vibration mechanics (bandwidth method), the Poisson's ratio, the complex modulus, and the phase angle were calculated for each asphalt mixture sample. When compared to the traditional axial compression test, both showed similar modulus and frequency values in the frequency range of 0.01 to 25Hz, at temperatures of 4, 20 and 40°C, and similar behavior in the development of the curves.

Given this context, and linked to the possibility of pavement control in the field and the lack of research on non-destructive IRT, this paper aims to present the assembly and development of a non-destructive test to determine the stiffness parameter of different materials, with an intermediate validation step for elastic materials detailed in this article, and a future target in asphalt materials (viscoelastic).

The prototype was developed at the Laboratory of Pavement Mechanics (LMP) of the Federal University of Ceará (UFC). Previous research were performed and they did not return patent protections, only generic results, without specific patent protections for the developed prototype. Thus, this new test developed on this reserarch, pioneer in the national

scenario, contributes to the understanding of the stiffness property, opening the way for further research with similar tools and with the potential to be useful in the short term for designers and field engineers.

3.2 Materials and methods

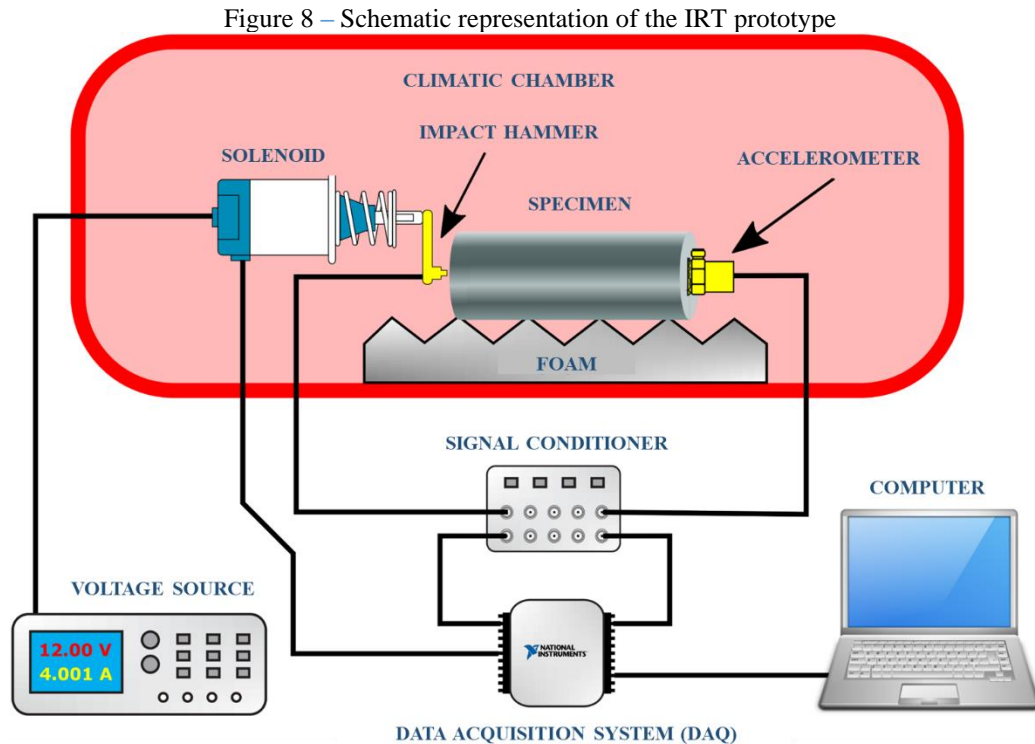
Based on the equipment developed by Carret *et al.* (2018), the appropriate components for assembling the prototype were defined. In parallel with the definition of the components, the code responsible for receiving and recording the signals during the impact tests was developed. The first tests were performed on samples of materials classically considered elastic. With the tests were obtained results of the output signals in the time and frequency domain, obtained by the accelerometer and necessary for determining the stiffness.

3.2.1 Test prototype

The test prototype was built following the schematic representation presented in Figure 8. The experiment setup consists of a specimen, a miniature impact hammer equipped with a 222N load cell (PCB model 086E80) coupled to a solenoid (SOLOTEC model 040) and an accelerometer (PCB model 353B15) bonded to the specimen. The solenoid, hammer, accelerometer, and specimen are inside a climate chamber that uses T-type thermocouple sensors for temperature control. A voltage supply was also used to power the solenoid, a signal conditioner (PCB model 482C15) that amplifies and conditions the signals measured in the hammer and accelerometer, a Data Acquisition Equipment (DAQ model NI USB 6002) to convert analog data/signals to digital at a rate of 50 thousand samples per second (kS/s), and a computer with the software for processing the collected signals.

The impact hammer is used to excite the sample externally and generate stationary waves inside, which are measured by the accelerometer. With the hammer a force signal is generated, measured at its tip (the point where the load is applied), and captured by the signal conditioner. The impact hammer and the accelerometer are connected to the signal conditioner, which prepares the signals for conversion from analog to digital. This signal conditioner is connected to a data acquisition system. The signal generated in the test is in the time domain. After acquisition a Fast Fourier Transform (FFT) is performed and the signal is transformed to the frequency domain. It is worth noting that there are different vibration modes, which are distinguished according to the position of the accelerometer and hammer and the way the impact

is applied. Figure 8 illustrates the longitudinal vibration mode, where the accelerometer is positioned in the opposite direction to the impact. However, it is also possible to perform the test with the accelerometer and the impact in the flexural direction, where the accelerometer and the hammer are positioned on the same surface of the specimen, something that is not the focus of this article.



Source: Author (2023).

The specimen should be placed on top of a foam for simulation of free boundary conditions. Contact with the surroundings should not interfere with the stationary waves generated by the impact. For samples of viscoelastic materials, the test should be performed inside a climatic chamber and executed at different temperatures, monitored with the thermocouples. The hammer impact is produced five times for each test temperature, and the vibrations captured by the accelerometer are sent to the signal conditioner and then to a data acquisition system. These signals are received and recorded by the computer using a signal analysis software that was developed by this research group in LabVIEW language.

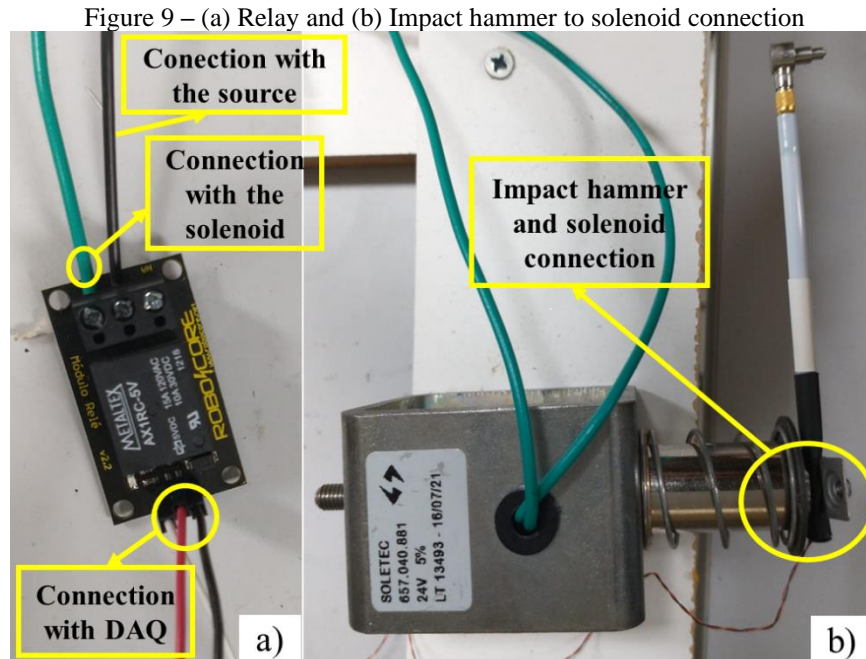
3.2.2 LabVIEW code

LabVIEW, an environment based on the graphical programming language "G" (MITRA; JAGTAP; KACHARE, 2018), was used to develop a software for acquiring and processing the signals experienced in the time dominium during the execution of the test, which are converted to the frequency dominium (LADIPO; MUTHALIF, 2012). In this software, a function was developed to control the speed and duration of the impact of the hammer on the tested material. In previous work, in order to automate the impact (CARRET *et al.*, 2018), the hammer was connected to a microcontroller (Arduino Uno R3), programmed with the Arduino interface. In the present research, the automation of the hammer-solenoid system was fully programmed in LabVIEW.

LabVIEW was chosen because no programming experience is required to use the language because it makes use of terminology, icons, and ideas familiar to technicians, scientists, and engineers, and because it relies on graphical symbols rather than a textual language to describe program actions (MITRA; JAGTAP; KACHARE, 2018). Through its use to generate prototypes, test, and implement instruments, it is possible to reduce development time and increase productivity. All LabVIEW programs have a front panel and a block diagram (MITRA; JAGTAP; KACHARE, 2018). The front panel is the graphical user interface of the virtual instrument and collects user inputs and displays the program's output. The block diagram contains the graphical source code of the virtual instrument, which is capable of controlling and performing functions on the inputs and output created on the front panel.

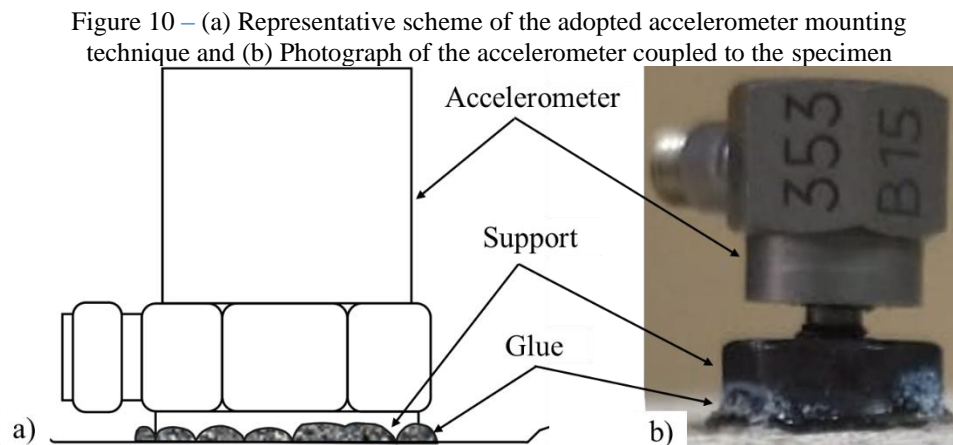
3.2.3 Test set-up

The first step of the assembly was to test the functionality of the components. The first was the accelerometer, which was connected to the signal conditioner and consequently to the DAQ. The first impacts were made with a hammer and the signals captured by the accelerometer and obtained from the DAQ were observed. With the coupling of the solenoid, it was possible to use the miniature impact hammer (PCB model 086E80) and test it in an automated way. For the operation of the solenoid, it was necessary to use a relay (Figure 9a), because the DAQ does not have the voltage required by the solenoid, which is 24V. Thus, the solenoid was connected to a source and activated by a pulse generated by the DAQ and sent to the relay. In order to secure the solenoid and to prevent it from moving during the tests, a simple wooden stand was constructed, on which the solenoid was screwed. To fix the impact hammer on the solenoid, a screw and a steel piece were used (Figure 9b).



Source: Author (2023).

The accelerometer was fixed to the sample using a fast-acting glue (*cinoacrilato* superglue). The accelerometer used is the adhesive mount type (Figure 10), indicated for temporary use or when the surface of the test object cannot be adequately prepared for the assembly of accelerometers with pins. Since different specimens are tested in this research and the test is nondestructive, this type was found to be more suitable. This accelerometer has a mounting base (glued to the specimen) where the accelerometer is threaded.



Source: Author (2023).

3.2.4 Materials

One cylindrical steel specimen (named A1) and six mortar specimens (M1, M2, M3, M4, M5, and M6) were used to investigate the ability of the developed equipment to generate reliable data. Specimen A1 is 0.153 m long and 0.10m in diameter. Specimens M1, M2, M3, M4, M5 and M6 are 0.10m in diameter and 0.116, 0.120 and 0.120m, 0.207, 0.203m and 0.206m in length, respectively. All specimens were submitted to stiffness tests after 28 days of curing.

The mortar mixture has proportions corresponding to 50kg of cement, to 20kg of lime, to 299kg of natural sand, the mass trace being 1:0.4:5.98 (cement:lime:sand). It was used Portland Composite Cement, CP II-F-32, with specific gravity of 3.17 g/cm³. The hydrated lime used was type I, CH-I, with a specific gravity of 2.54 g/cm³. The water/binder factor of 1.08 was used to obtain the consistency index established by NBR 13276 (2016) for the consistency table test, of 260 ± 5mm. The steel investigated is an austenitic 316L stainless steel alloy, and a standard sample was used for calibration of the hydraulic presses in the UFC laboratory. For this paper, elastic materials were chosen in the development of the prototype over bituminous materials because their behaviors are not frequency dependent and are poorly sensible to temperature changes.

3.2.5 Performing impact resonance test

In the IRT developed, initially the samples were placed on the foam, the accelerometer base was connected to them with fast-acting glue and the impact hammer was positioned to send mechanical waves in the longitudinal direction of the samples, toward the accelerometer. Then, the test was started and after 5 blows of approximately 300ms each, the results of the output signal accelerometer were obtained in the dominium of time and frequency. With the results, Young's modulus was determined using the resonant frequency. With the resonant frequency, which corresponds to the peak maximum amplitude of vibration, and the mass and dimension values of the samples, the dynamic modulus (E) was calculated with Equation 1 (ASTM C215, 2019).

$$E = DM(n')^2 \quad (14)$$

Where D equals $5.093 \cdot (L/d^2)$ for cylinders, L is the length, d the diameter, M the mass, and n' the resonant frequency.

3.2.6 Execution of static modulus of elasticity test

In Brazil, the test to determine the modulus of deformation and particularly the static modulus of elasticity of concretes is standardized by NBR 8522-1 (2021). This test was applied to the six mortar specimens (cf. Figure 17). As recommended by the standard, a testing machine is required for the application of the loads, a load cell for measuring the loads and subsequent calculation of the stresses, two strain gauges for measuring the deformations. At least 5 cylindrical specimens should be used to perform the test (2 for determining compressive strength and 3 for modulus). For the two samples tested in compressive strength, the average of the results was 5.4MPa, a value considered in the stiffness test. This test is divided into 4 sequential steps.

In the first step, the samples are loaded at a constant force corresponding to 30% of the average stress found in the previous step for 60 seconds, and unloaded at a contact load (the force is close to zero and used only to ensure that contact is not lost). In the second step, the samples were loaded for 60 seconds at a force corresponding to a strain of 0.5MPa, then loaded for 60 seconds at a force corresponding to the initial strain from step 1, and then unloaded at a force close to zero. The third step is a repeat of the second step. Step 4 consists of loading the samples at a force equivalent to a 0.5MPa stress for 60 seconds and reading the strains for the next 30 seconds. Next, the specimens are loaded at the same initial force as in step 1 for 60 seconds, and strain readings are taken for the next 30 seconds. After the strain readings, the samples are loaded at the same speed (0.083MPa/s) as before, until failure occurs, where the actual strength of the tested sample is. If the effective strength is not more than 20% different from the compressive strength found initially, the results are considered reliable and the modulus of elasticity is calculated.

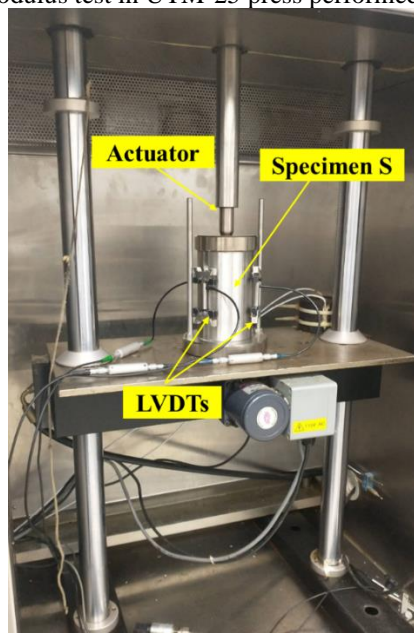
It is important to note that the definition of Young's modulus of elasticity only applies to idealized perfectly linear elastic materials, which is not the case for real materials. For example, in the article, the moduli of elasticity (linear zone) would certainly occur for very small strains, but not for those applied in static or quasi-static tests, which although small and held to less than 30% of the yield stress of the material, would not yield the same results as if smaller stresses had been used. For this reason, in this article the result produced by the test described in this topic is referred to as the static modulus of elasticity.

3.2.7 Quasi-static modulus testing

The steel specimen was subjected to the conventional complex modulus test typically used in asphalt mixtures (DNIT-ME 416, 2019/ASTM 3497-79) with quasi-static sinusoidal loading, whose setup is shown in Figure 11. Loading amplitude of 2000 kPa was used. Only the frequencies of 0.1, 0.5, 1 and 5Hz and the ambient temperature were selected for the analysis. The corresponding strain observed was also sinusoidal, as expected. With loading, the load and displacement amplitude values measured at the specimen were obtained. The ratio between these amplitudes is the value of the magnitude of the complex modulus, which describes the stiffness of the material. The test parameters and the modulus results obtained are presented in Table 1. The process allowed confirming the negligible effect of the loading frequency on the measured modulus.

It is worth noting that in this paper the authors adopted the nomenclature of complex modulus for the property obtained in this quasi-static test, a property represented by a complex number characterized by its magnitude (absolute value of the complex modulus, commonly called in the literature dynamic modulus despite there is no dynamic effect in the measurement) and its lag (phase angle) (HUANG; DI BENEDETTO, 2015). There is dynamic modulus testing and it corresponds to modulus measurements taken with wave propagation of waves (item 2.5), which is not the case in quasi-static complex modulus tests (GROSS; HENNEAUX; SEVRIN, 1953; HUET, 1963; MANDEL, 1966; FERRY, 1980).

Figure 11 – Conventional modulus test in UTM-25 press performed on cylindrical steel specimen



Source: Author (2023).

Table 1 - Traditional quasi-static test parameters

Name	Preconditioning	Scan 1	Scan 2	Scan 3	Scan 4
Frequency (Hz)	5	5	1	0.5	0.1
Cycles	10	10	10	5	5
Modulus (MPa)	195.149	190.919	192.497	192.632	192.482

Source: Author (2023).

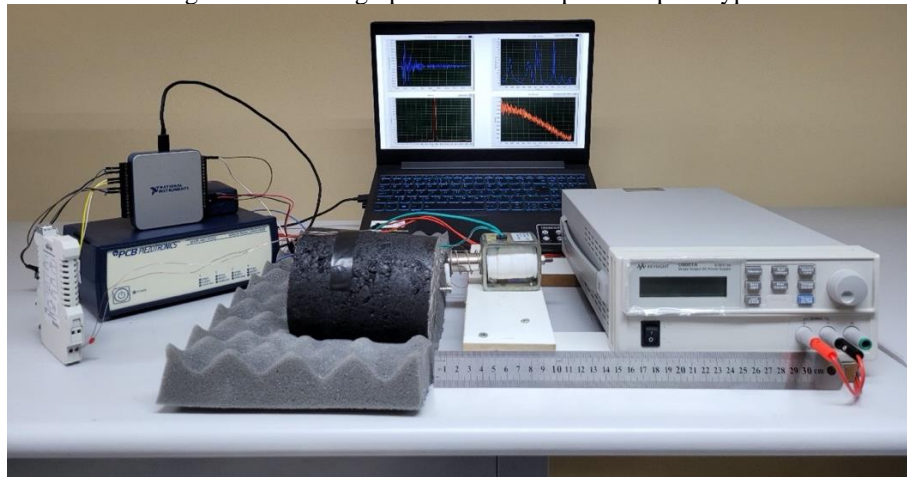
3.3 Results and Discussion

3.3.1 Development of the acquisition system

A relevant result of this research is shown in Figure 12, which shows the test equipment assembled in the stage in which it was used for the validation presented here. It is not yet inside the climatic chamber, as planned for future versions and presented in Figure 8. Temperature control will be indispensable for thermo-viscoelastic materials such as asphalt mixtures, but has little effect for materials such as mortar and concrete. Figure 13a shows an example of a front panel and Figure 13b the block diagram constructed for this research in LabVIEW.

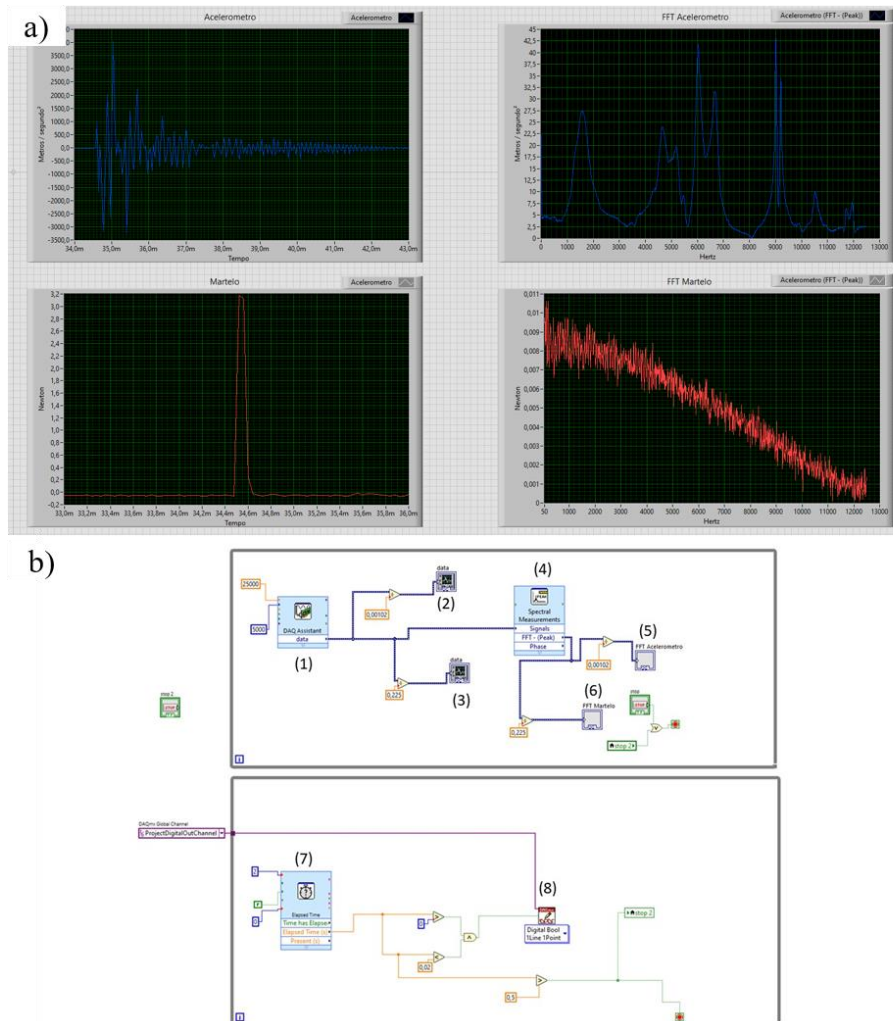
The front panel is where the graphs of the sensor measurements in the time and frequency domain are plotted. It is possible to view and save images of the graphs obtained and export the data to spreadsheets. Due to limitations of the DAQ used, simultaneously capturing the accelerometer and hammer signals halved the amount of samples obtained from each sensor, limiting the maximum measurement frequency to 12.5KHz. Therefore, only the data from the signals obtained by the accelerometer were possible to be exported to Excel, results that are sufficient to calculate the modulus of elastic materials. In Figure 13a, the graphs on the left side are the responses of the accelerometer (top) and the hammer (bottom) in the dominium of time. The graphs on the right side are the responses of the accelerometer (top) and the hammer (bottom) in the dominium of frequency. It is on the front panel that the test run is triggered.

Figure 12 – Photograph of the developed IRT prototype



Source: Author (2023).

Figure 13 – (a) Front panel showing accelerometer (top) and hammer (bottom) responses in time domain (left side) and frequency domain (right side) and (b) Block diagram



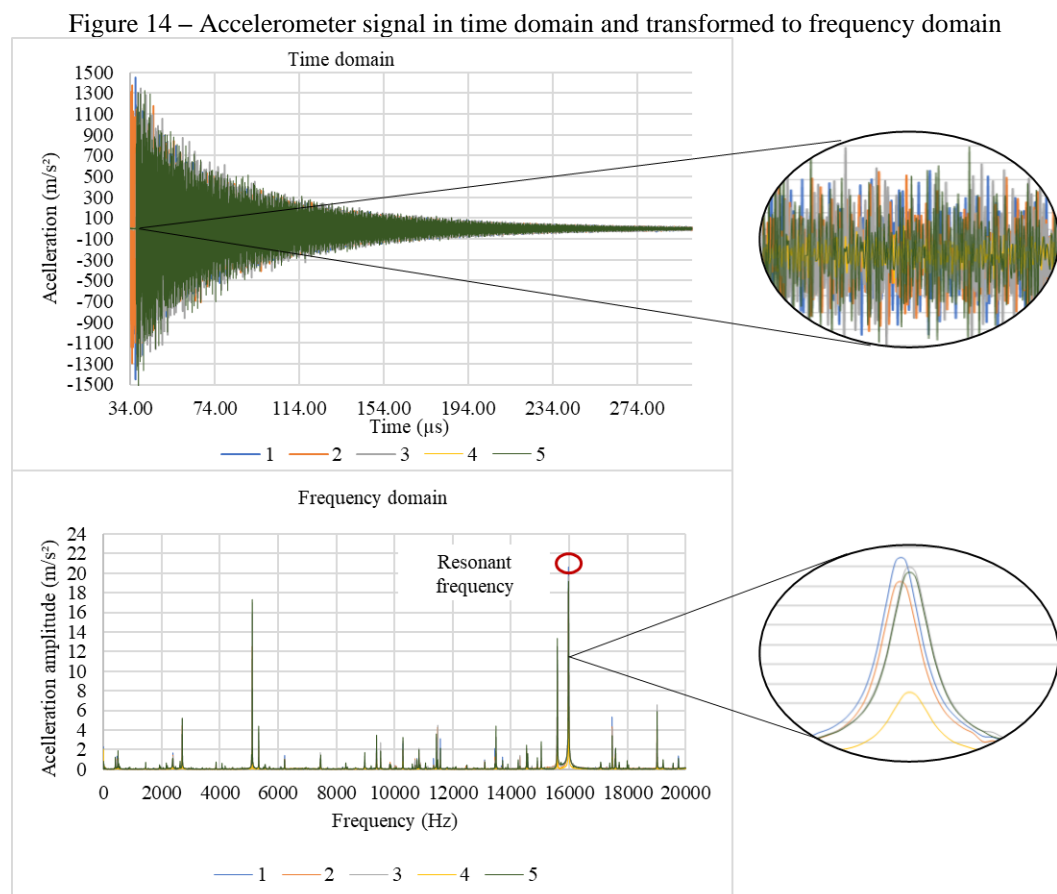
Source: Author (2023).

The block diagram in Figure 13b contains a module "(1)" that does the signal acquisition from the hammer and accelerometer, which sends the signals to the graphs (2 and

3), making the conversion of voltage to the usual units of measure, being the Newton (N), the load signal measured in the hammer and meter/second² (m/s²) of the accelerometer. This signal is sent to the "(4)" module, which converts the signals in the time to frequency domain, resulting in two graphs (5 and 6). Module "(7)", on the other hand, concerns the operation of the solenoid, which automates the impact of the hammer. In it, a controlled pulse is generated from one of the DAQ's output ports (8) to the relay, triggering the solenoid.

3.3.2 Testing with steel specimen

The results presented in this section correspond to measurements performed on the same steel cylinder. The accelerometer output signals in the time and frequency domains of the 5 impacts are shown in Figure 14.



Source: Author (2023).

With Figure 14 it is observed that the signals in the time and frequency domains of the 5 impacts are practically overlapped, indicating the good repeatability of the developed test, evidenced ahead with the analysis of the results. With the resonance frequency (cf. Figure 14),

which corresponds to the peak of maximum amplitude of vibration, the value of the modulus of elasticity of the steel specimen was calculated with Equation 1, obtaining a value of 188,587.07MPa, corresponding to the average value of resonance frequency equal to 15,970.7Hz. The individual values for each of the 5 impacts were 15,973.33Hz, 15,970.00Hz, 15,970.00Hz, 15,970.00Hz and 15,970.00Hz, which correspond to the modulus values of 188,650.05MPa, 188,571.32MPa, 188,571.32MPa, 188,571.32MPa and 188,571.32MPa, shown in Table 2. The coefficient of variation (CV) observed was 0.009%.

Table 2 – Statistical analysis of the IRT for A1

Impacts	F. Resonance (Hz)	E (MPa)
1	15,973.33	188,650.05
2	15,970.00	188,571.32
3	15,970.00	188,571.32
4	15,970.00	188,571.32
5	15,970.00	188,571.32
Average	15,970.67	188,571.32
Standard deviation	1,49	35,21

Source: Author (2023).

It was observed that, in fact, the effect of the loading frequency of the quasi-static test is very low for the steel specimen, which is expected for linear elastic behavior. By averaging the modulus values found for the 5 frequencies tested in the traditional test, a value of 192,735.2MPa was arrived at (cf. Section 3.2.7). The difference between the values found in the two tests was only 2.15%, within the variability of results observed between the different tests. Demonstrating that the IRT is able to accurately and reliably estimate the elastic modulus. Moreover, it is observed that the result is compatible with results available in the literature for 316L steel (SINGH; MAUSAM; SHARMA, 2021), having already observed measurements on the order of 193,000MPa.

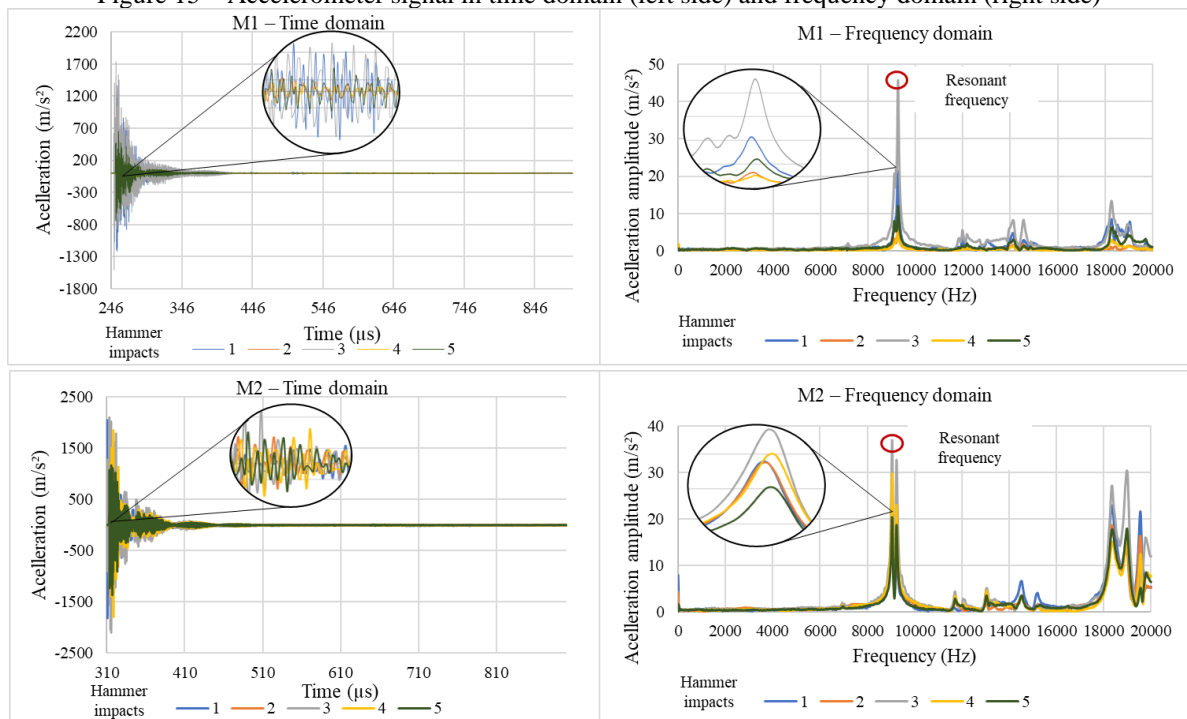
3.3.3 Testing mortar specimens

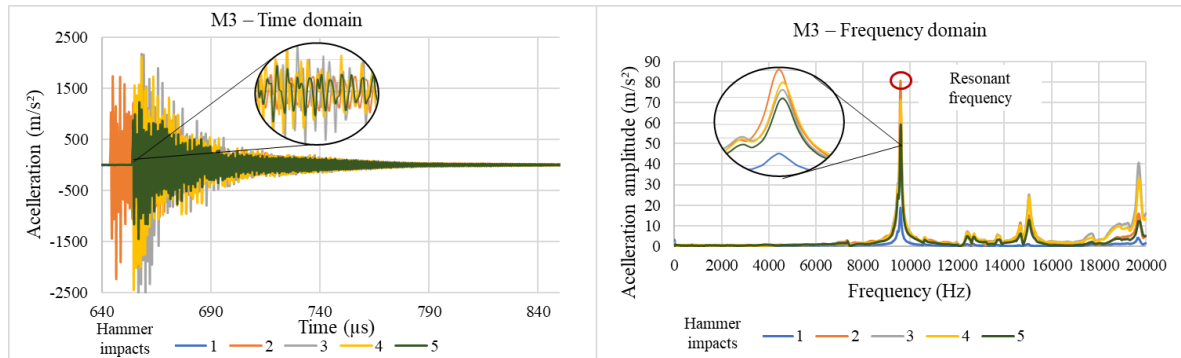
In order to verify the repeatability of the IRT, the curves regarding the response received by the accelerometer in the dominions of time and frequency were plotted. Figure 15 presents these results for the three mortar specimens (M1, M2, and M3) tested considering the five hammer impacts. It can be seen that the graphs of the 5 impacts are overlapping in all specimens and that the maximum amplitude of the vibration peak occurred at the same

frequency range. The maximum peak was at a frequency of approximately 9KHz (highlighted in Figure 15) for all 3 specimens.

The average results of the static modulus of elasticity tests and those obtained by impact resonance performed on the mortar specimens are shown in Figure 16. The variability observed, in terms of standard error. The elastic modulus data obtained are in agreement with the literature, in relation to the range of expected values. The E values obtained with all the methodologies used in this study are relatively similar to each other. When comparing the results of all the specimens, there is a similar trend of results, where the values of dynamic modulus (obtained by vibrational testing) are higher than those of static modulus of elasticity, as reported in the literature (ARAGON *et al.*, 2019; MARQUES *et al.*, 2020). The dynamic modulus of elasticity (obtained with wave propagation tests) of concrete is commonly observed to have superior values than Young's modulus of elasticity determined with static or quasi-static tests (MEHTA; MONTEIRO, 2014), due to small physical nonlinearity effects of the stress-strain behavior (the modulus is slightly lower for larger strain levels). This reinforces that the dynamic nondestructive method is reliable and of good quality, even requesting the material with strains more suitable for the linear zone of the material than traditional tests. The percentage difference between the values found in the two tests for specimens M1, M2, M3, M4, M5, and M6 were 4.81, 7.79, and 12.83, 11.25, 23.49, and 18.56%, respectively.

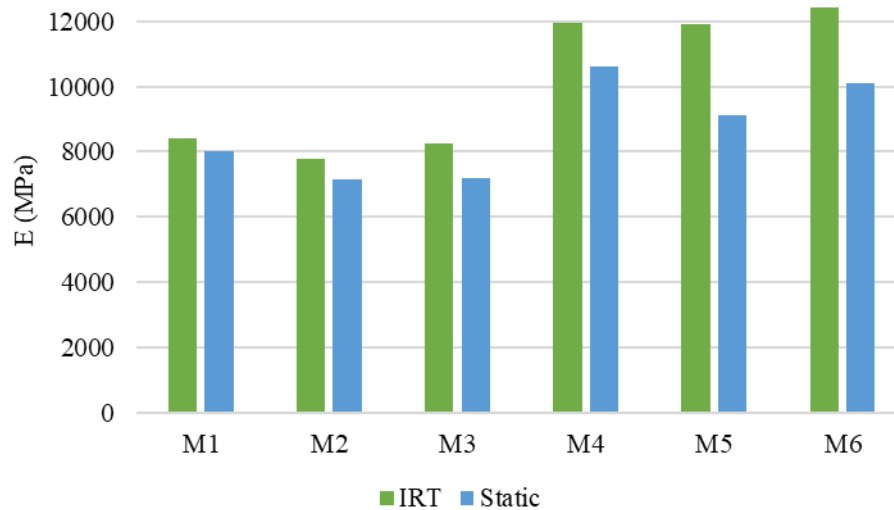
Figure 15 – Accelerometer signal in time domain (left side) and frequency domain (right side)





Source: Author (2023).

Figure 16 – Modulus of elasticity of mortar specimens tested with different methods



Source: Author (2023).

The results of the IRT were statistically analyzed through the mean, standard deviation and CV, shown in Table 3 and Table 4. Only specimen M2 presented higher values of standard deviation (reaching 174.38MPa, which corresponds to 2.24% CV), maybe due to some damage during the transportation process to perform the tests. The other results had less than 10MPa of standard deviation (corresponding to less than 0.2% CV), indicating a great repeatability of the test. All specimens presented CVs close to zero, indicating that the data are homogeneous, reinforcing the good repeatability of the IRT developed. Besides that, specimens M4, M5 and M6 presented the same resonance frequency in all 5 impacts of the hammer, which implied standard deviation and CV values equal to zero.

Table 3 – Statistical analysis of the IRT

Impacts	M1		M2		M3	
	F. Resonance (Hz)	E (MPa)	F. Resonance (Hz)	E (MPa)	F. Resonance (Hz)	E (MPa)
1	9255.56	8421.51	9200.00	7966.20	9044.44	8279.02
2	9255.56	8421.51	9014.29	7647.83	9044.44	8279.02
3	9255.56	8421.51	9014.29	7647.83	9033.33	8258.69
4	9255.56	8421.51	9014.29	7647.83	9044.44	8279.02
5	9266.67	8441.74	9200.00	7966.20	9033.33	8258.69
Average	9257.78	8425.56	9088.57	7775.18	9040.00	8270.89
Standard deviation	4.97	9.05	101.72	174.38	6.09	11.13
Coefficient of variation (%)						
	0.05	0.12	1.12	2.24	0.07	0.13

Source: Author (2023).

Table 4 – Statistical analysis of the IRT

Impacts	M4		M5		M6	
	F. Resonance (Hz)	E (MPa)	F. Resonance (Hz)	E (MPa)	F. Resonance (Hz)	E (MPa)
1	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78
2	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78
3	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78
4	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78
5	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78
Average	6029.41	11965.93	6176.47	11920.13	6176.47	12418.78

Source: Author (2023).

3.4 Conclusions

This paper aims to present the development and assembly of the test equipment for determining the modulus of asphalt mixtures through wave propagation that has been implemented in the UFC, and to validate it for elastic materials in comparison with classic quasi-static loading tests. As a validation step, 1 steel and 3 mortar specimens were tested with traditional methods (quasi-static tests of sinusoidal loading and static modulus of elasticity) and with the equipment object of the research. The following conclusions could be obtained:

- The resonance by impact test developed showed satisfactory results, since the percentage difference between the steel modulus values found in the two tests was only 2.15% and for the results with mortar between 4 and 13%;
- The dynamic resonance impact test has the advantage of being fast, inexpensive and non-destructive, and is potentially useful for determining the viscoelastic behavior properties of bituminous materials. For this, the frequency mix, and not

just the maximum resonance frequency as studied in this paper, will become important;

- Due to the limitations of the DAQ used for the development of this article, it was not possible to obtain the data from the impact hammer signals with the necessary precision in the time domain, which made it impossible to calculate the FRF;
- The next step is to experimentally obtain FRF and characterize bituminous and other viscoelastic materials. In addition to contributing to the dissemination of a new test, more practical and efficient, and with the determination of the stiffness property of asphalt mixtures, it is expected to provide more tools to pavement designers and engineers who monitor road works, enabling more assertive designs that can have their stiffness assumptions controlled in the field.

ACKNOWLEDGMENT

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4 APPLICATION OF NON-DESTRUCTIVE TESTING FOR THE DETERMINATION OF THE STIFFNESS OF DIFFERENT MATERIALS

(Content of the paper presented at the INTERNATIONAL SYMPOSIUM ON ASPHALT PAVEMENTS, ISAP, 2022)

ABSTRACT

Due to the need for innovation in design methods and quality assessment and quality control of pavements in the field, non-destructive tests based on the principles of wave propagation and impact resonance have been used for viscoelastic characterization of asphalt mixtures. An impact resonance test, intended to be applied in bituminous materials, is being developed at the Federal University of Ceará (UFC). As a first step in the validation of the test, materials with simpler mechanical behaviour are under investigation. In the work herein, 3 specimens of mortar and 1 of steel were subjected to static (classical quasi-static stiffness tests) and dynamic tests (ultrasonic and impact resonance) to determine their stiffnesses. The results indicated some differences (20%-40%) between the static and ultrasonic methods, with the impact resonance test returning results similar to the static tests. Since dynamic testing is performed at much lower strain levels than traditional static testing, these results were expected and agree with previous research on the material dependence of the strain level. In the case of the ultrasonic pulse velocity method, this could be due to inadequate Poisson's ratio. The research has shown that dynamic testing, with emphasis on impact resonance testing has the potential to be used to determine the viscoelastic properties of bituminous materials in laboratory and in the field. It may be an alternative characterization tool with the great advantage of being rapid, inexpensive, non-destructive, and more easily adaptable to the field.

Keywords: Non-destructive testing, asphalt mixes, concrete, dynamic modulus.

4.1 Introduction

The stiffness characterization of asphalt mixtures is of paramount importance for pavement design. The values of this viscoelastic characterization are inputs to mechanistic-empirical design methods. Traditionally in pavement engineering, the complex modulus is determined by a standardized laboratory test performed in a robust press. This requires skilled labour-intensive work and is time consuming. In addition, it is hardly adaptable for in situ testing. While for materials with instantly recoverable mechanical behavior (elastic), uniaxial monotonic testing at only one temperature is usually enough for stiffness testing, for viscoelastic materials quasi-static sinusoidal loading tests and many temperatures and frequencies are usually required (DI BENEDETTO *et al.*, 2001).

Aragon *et al.* (2019) indicate that the static modulus of elasticity of quasi elastic materials such as mortars can be obtained from uniaxial tests, using the stress-strain diagram. The measurement is obtained from the slope of an ideal straight line between 5 and 33% of the yield stress, ignoring the tested results at strains less than 5% (due to accommodation and

signals noise). The results obtained are called secant modulus of elasticity, and for perfectly elastic materials it is interpreted as the modulus of elasticity. In contrast, materials tend to behave nearer linearity for small strain. When dynamic tests are used to determine this modulus, the vibrational waves are of low amplitude, so the numerical value of the elastic modulus corresponds to small strains near the origin of the stress-strain curve. This fact is one of the reasons that drives the use of dynamic tests based on the principle of wave propagation and impact resonance for viscoelastic characterization of bituminous materials. Experimental results on such materials show that linear viscoelastic behavior can be considered with good accuracy in the small strain domain (up to 10^{-4} m/m) (MOUNIER; DI BENEDETTO; SAUZÉAT, 2012), but there is evidence of non-linearity for much lower strains (MANGIAFICO *et al.*, 2018).

Due to the need for innovation in design methods and in pavement quality assessment and quality control in the field, non-destructive tests based on the principle of wave propagation and impact resonance have been used. Dynamic ultrasonic tests that make direct measurements of the time-of-flight (ToF) have been performed on asphalt mixtures (DI BENEDETTO, SAUZÉAT; SOHM, 2009; NORAMBUENA-CONTRERAS *et al.*, 2010). However, only one modulus value is determined for each test at a given temperature and imposed frequency (given by the used emitter). In the meantime, resonant acoustic spectroscopy makes it possible to impose a large number of different frequencies simultaneously for which the complex modulus can be calculated (OSTROVSKY *et al.*, 2001). Together with spectroscopy, the Frequency Response Function (FRF) has been used by Gudmarsson *et al.* (2015) and Carret *et al.* (2018) for viscoelastic characterization of bituminous materials over a wide frequency range. Laboratory measurements of FRF for asphalt mixtures have shown promising results and allow a direct comparison between master curves obtained from uniaxial compressive tests and dynamic tests. Gudmarsson *et al.* (2014;2015) showed that the combination of conventional measurements and dynamic tests is useful to improve the characterization of asphalt mixtures. The authors showed that small differences could be obtained between the FRF method and classical quasi-static testing due to the known nonlinearity of asphalt concrete (NGUYEN *et al.*, 2015; MANGIAFICO *et al.*, 2018).

Carret *et al.* (2018) used basically the same approach as Gudmarsson *et al.* (2015) for characterization of asphalt mixtures, but with some improvements in the IRT. The two main improvements were: (i) the doubled sampling rate, which ensures a more detailed description of the recorded signals in the time domain; (ii) the sample was excited by an automatic impact

hammer that allows better suitability to different geometries and measurement positions, as well as higher quality and repeatability of the test.

The same kind of test is being developed at Federal University of Ceará (UFC), aiming to be applied in bituminous materials. As a first validation step of the IRT, this work intends to investigate the behaviour of materials classically considered as elastic. For that, 3 specimens of mortar and 1 of steel were subjected to static monotonic tests and to dynamic tests (both ultrasonic ToF testing and impact resonance testing) to determine their stiffness.

4.2 Experimental plan

4.2.1 Materials

The mortar mixture has proportions correspondent to 50kg of cement, 20kg of lime, and 299kg of natural sand, measured using a volume approach (5 x 40-L batches of sand were used for each 50kg bag of cement). This way, the mass composition was 1:0.4:5.98 (cement to lime to sand ratio). The amount of water was chosen to reach the consistency index established by standard, 260 ± 5 mm measured in the standard consistency table, obtaining a water/binder ratio of 1.08.

The investigated steel is an austenitic stainless-steel alloy 316L, being a standard specimen used for calibration of the hydraulic presses in UFC's lab, used for mechanical tests. Table 5 shows the dimensions and masses of the specimens. The steel specimen was named S and the mortar specimens were named M1, M2 and M3.

Table 5 – Characteristics of the specimens tested for stiffness properties

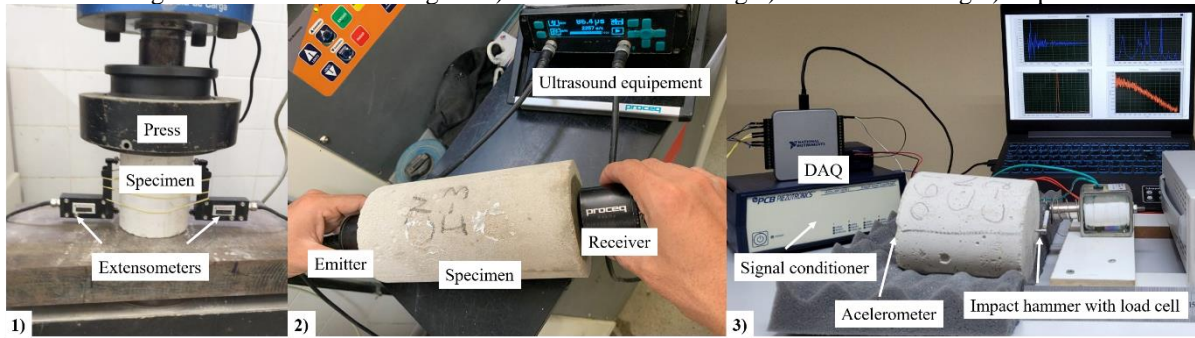
Specimens	Length (m)	Diameter (m)	Specific Gravity (kg/m ³)
M1	0.116	0.10	1827.37
M2	0.120	0.10	1634.82
M3	0.120	0.10	1757.96
S	0.153	0.10	7884.77

Source: Author (2023).

4.2.2 Methods

Three test methods were performed on the 4 specimens. The first one was static modulus (Figure 17.a), then the ultrasonic (Figure 17.b), and then the impact resonance (Figure 17.c).

Figure 17 – Test methodologies: 1) Static modulus testing 2) Ultrasonic testing 3) Impact



Source: Author (2023).

4.2.2.1 Static modulus test

In Brazil, the test for determining the static modulus is standardized by NBR 8522-1 (2021). As recommended by this standard, a testing machine, extensometers (2 in this study), and at least 5 cylindrical specimens (2 for determining compressive strength and 3 for determining modulus) are required to perform the test. For the two tested specimens in compressive strength, the average of the results was 0.054MPa considered for the design of the stiffness test.

The test for determining the static modulus of elasticity was divided into 4 sequential steps. In the first step, the specimens are loaded at a force corresponding to 30% of the average stress found (0.054MPa) in the initial compressive strength test., held at this load for 60 seconds, and unloaded to a contact load (force is close to zero and used just to ensure contact is not loss). In the second step, specimens were loaded for 60 seconds to a force corresponding to 0.5MPa stress then loaded again for 60 seconds to a force corresponding to the initial strain of step 1, and then unloaded to near zero force. The third step is a repetition of the second step. Step 4 consists of loading the specimens again to a force equivalent to 0.5MPa stress for 60 seconds, and then reading the strains over the next 30 seconds. Next, the specimens are loaded to the same initial force as in step 1 for 60 seconds, and strain readings are taken for the next 30 seconds. After reading the strains, the specimens are loaded at the same speed rate (0.083MPa/s) as before until failure, where the actual strength of the tested specimen is found. If the effective strength is not more than 20% different from the compressive strength found initially, then the results are said to be reliable and the modulus of elasticity can be calculated.

4.2.2.2 Ultrasonic test

The ultrasonic test was performed with a Portable Ultrasonic Non-Destructive Digital Indicating Test (PUNDIT Lab) equipment from PROCEQ. The test consists in inducing the ultrasonic pulse in one surface of the cylindrical specimen and measuring the arrival of the wave in the other. Flat surface transducers with 50mm diameter were used, working at a nominal frequency of 54kHz. They emit and receive compression waves, called P waves. At the end, the test provides the time-of-flight (ToF) and, according to NBR 15630 (2009), the velocity of wave propagation and the modulus of elasticity of the samples are determined, provided that the Poisson's ratio is known or assumed. As suggested by the standard, Poisson's ratio was assumed as 0.2.

4.2.2.3 Impact Resonance test

This dynamic test was built based on Carret *et al.* (2018). The specimens are externally excited 5 times by an automatic impact hammer, generating waves that are registered by an accelerometer. The hammer and the accelerometer are connected to a signal conditioner, which in turn is connected to a data acquisition system linked to a computer. The positions of the impact point and the accelerometer depend on the chosen geometry of the specimen and the intended vibration mode. In this study, only the longitudinal mode of vibration of the cylinders was considered. Thus, the impact happened at the center of one side of the cylinder while the acceleration was measured at the center of the opposite side. The signals are received and recorded with a software developed in LabVIEW. In this same environment, the time domain signal measured by the accelerometer was transformed to the frequency domain with a Fast Fourier Transform (FFT). With the obtained frequency domain response, the methodology for calculating the dynamic modulus for elastic materials recommended by ASTM C215 (2019) was adopted. With the resonant frequency, i.e., the frequency that corresponds to the peak of maximum amplitude, and the values of mass and dimensions of the specimens, the dynamic modulus (E) was calculated with Equation 15.

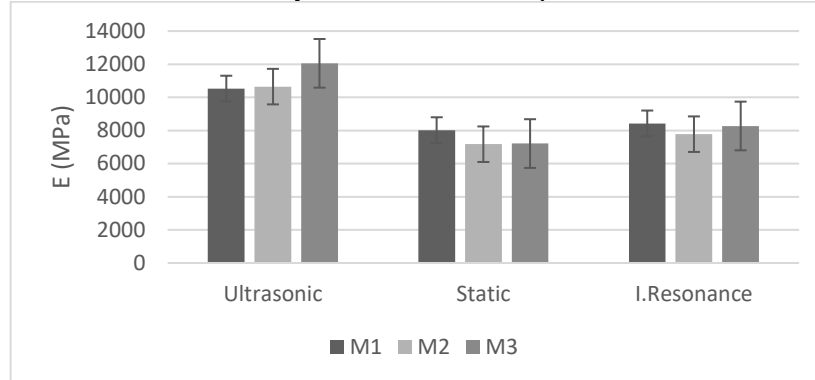
$$E = DM(n') \quad (15)$$

Where D equals $5.093N.s^2$ ($kg.m^2$) (result of L/d^2 for the used geometry) for cylinders, L is the length, d the diameter, M the mass, and n' the resonant frequency.

4.3 Results

The results of the 3 tests described above for the mortar specimens are presented in Figure 18. The modulus of elasticity data is in agreement with the literature, in relation to the range of expected values (MARQUES *et al.*, 2020). The E values obtained with all methodologies used in this study are relatively similar to each other. When comparing specimens M1 and M2, it can be seen that the 3 tests present the same trend, where the values for M1 are higher than M2. This reinforces that the dynamic non-destructive methods are reliable and of good quality.

Figure 18 – Modulus of elasticity of the tested mortar specimens with different methods



Source: Author (2023).

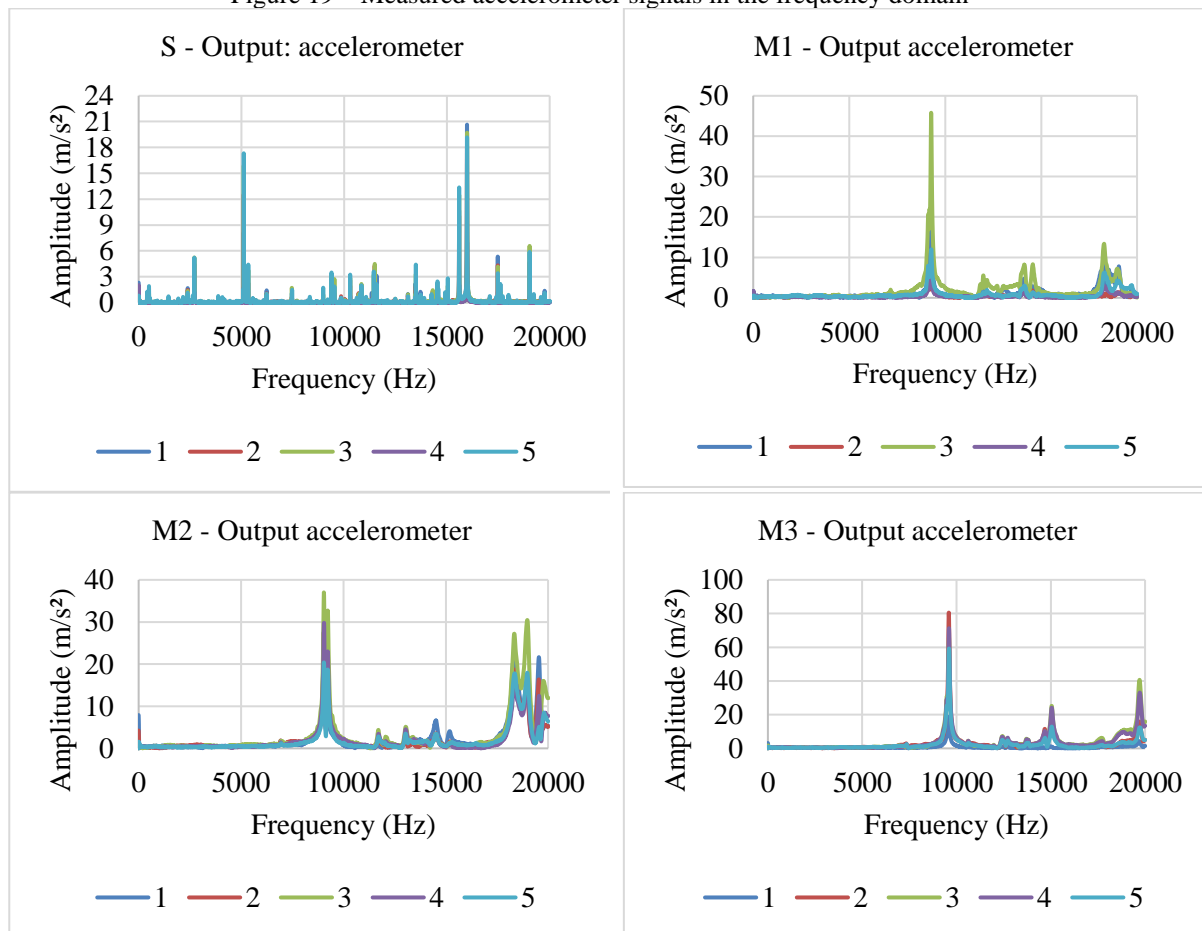
Comparing static and dynamic E values (ultrasonic and I.Resonant), the dynamic ones are higher than the static ones. The dynamic modulus of elasticity of concrete is higher than the static modulus of elasticity (MEHTA; MONTEIRO, 2014). It is possible to observe that the values of the IRT are closer to those of static modulus, but are still higher than them. This fact also happened with the results found by (MEHTA; MONTEIRO, 2014; CARRASCO *et al.*, 2017, ARAGON *et al.*, 2019;). As described in Makoond, Pela and Molins (2019), dynamic modulus of elasticity obtained by vibration impulse impact are more reliable and representative of the modulus of elasticity of the entire specimen when compared to ultrasonic pulse velocity estimates. However, it is worth noting that there is generally good agreement between the two types of dynamic estimates for isotropic materials (CARRASCO *et al.*, 2017). One important thing to consider is that Poisson's ratio was assumed as 0.2, which is not necessarily true for the tested specimen. This is also a source of difference in results. If a value of Poisson's ratio of 0,3 was used, the results would show a difference of approximately 18%.

The results of the steel specimen's modulus were 192,735.2MPa with the quasi-static test. Since no clear effect of frequency was observed on the measured ratio between stress

and strain amplitudes (also known as dynamic modulus), an average was calculated with the results for all frequencies (5, 1, 0.5, and 0.1Hz). With the Impact Resonance equipment, the value of the steel specimen's modulus was 188,587.07MPa. The difference between the two results is only 2.15%, which shows that the IRT is able to obtain adequate modulus values, when compared to quasi-static tests. Both tests were carried out at room temperature. The 316L steel used by Singh, Mausam and Sharma (2021) had a modulus of 193,000MPa.

In order to verify the repeatability of the IRT, the curves referring to the response received by the accelerometer in the frequency domain were plotted. Figure 19 presents these results for the 4 specimens tested in this work and considering the 5 impacts of the hammer. It can be observed that the graphs of the 5 impacts are overlapping in all specimens and that the maximum peak amplitude occurred in the same frequency range. For the steel specimens the maximum peak was at a frequency of approximately 16kHz and for the 3 mortar specimens it was close to 9kHz.

Figure 19 – Measured accelerometer signals in the frequency domain



Source: Author (2023).

4.4 Conclusions

In the work presented, 3 test methods that have been applied to characterize the stiffness of elastic materials and the viscoelastic behaviour of asphalt mixtures were used and compared. As a first validation step of the test that has been developed at UFC, 4 specimens of different materials usually considered as elastic, 3 of mortar and 1 of steel, were tested with all 3 methods and the results were analysed. It is shown that the conventional approach provides lower modulus values, possibly due to small non-linearities. In the case of the ultrasonic pulse speed method, this could be due to inadequate Poisson's ratio. The differences observed between static and dynamic methods had already been highlighted in previous work reported in the literature. Since dynamic tests are performed at a much lower strain level than traditional tests, these results were expected and agree with former research on the dependence of the strain level of materials. Therefore, the research presented herein shows that dynamic testing, with emphasis on IRT, which has the great advantage of being fast, inexpensive, and non-destructive, has the potential to be used to determine the viscoelastic behaviour properties of bituminous materials.

5 INVERSE ANALYSIS METHODOLOGY PROPOSAL FOR VISCOELASTIC CHARACTERIZATION OF ASPHALT MIXTURES FROM RESONANCE IMPACT TEST (IRT) RESULTS

(The content of this Chapter will compose a third paper related to this Master's work and will be submitted to a scientific journal).

ABSTRACT

The analysis of impact resonance test (IRT) results for the characterization of viscoelastic materials, such as asphalt mixtures, requires an elaborate approach that requires performing an inverse analysis process. Thus, the main objective of this paper is to propose a new inverse analysis methodology for viscoelastic characterization of asphalt mixtures using IRT results with frequency response functions (FRF) measurements. To verify the applicability of the proposed method, IRTs were performed using two different equipment prototypes, one developed at UFC and another at ÉTS. In order to validate the proposed method, traditional complex modulus test results were used to compare with those obtained from IRTs inverse analysis. The proposed inverse analysis method is divided into 4 main steps: (i) determination of the set of possible initial input values for the parameters E_0 and τ_e of the 2S2P1D model (initial guess of rheological model parameters, total of 81 parameter sets) for a given temperature; (ii) calculation of the FRF with finite element methods (FEM), called FRFcalc; (iii) calculation of the error between the experimental FRF and the 81 calculated functions FRFcalc; (iv) choice of the best set (least error and whose corresponding FRFcalc is called FRFopt) of rheological parameters amongst the 81 tested, and determination of the material's modulus master curve for the analyzed temperature and frequency range around the resonance peak; (v) repetition of the process for all other tested temperatures to obtain the experimental IRT isotherms; (vi) determination of the global master curve, with consequent rheological modeling of the material using the test results for the whole set of investigated temperatures. The results showed a good similarity, with a percentage difference of less than 1% in the three test temperatures, between the experimental FRF's of the IRT test of the UFC and ÉTS (considering their respective resonant frequencies), and also between the FRFexp and FRFopt. With the optimization results, global master curves were constructed that presented good approximation with the results of a traditional stress-compression test, with a percentage difference of less than 6% for the complex modulus of the UFC and ÉTS. The differences observed between the traditional and the proposed dynamic test are due to experimental dispersion and nonlinearity factors of the asphalt mixture. The new inverse analysis approach proposed is simpler than previous methods in the literature and reduces considerably the computational time, offering great potential to calculate the complex modulus of asphalt mixtures from FRF measurements using only the first resonance frequency peak.

Keywords: impact resonance test; asphalt concrete; inverse analyses; modulus complex; frequency response function; finite element method.

5.1 Introduction

The viscoelastic characterization of asphalt mixtures has already been carried out using non-destructive technologies, with ultrasound tests to measure the time-of-flight (ToF) (DI BENEDETTO; SAUZÉAT; SOHM, 2009; NORAMBUENA-CONTRERAS *et al.*, 2010; MOUNIER) or impact resonance (IRT), either with consideration of the fundamental resonance frequency (WHITMOYER; KIM, 1994; KWEON; KIM, 2006; LACROIX; KIM; FAR, 2009) or by measuring a FRF (REN; ATALLA; GHINET, 2011; RUPITSCH *et al.*, 2011; GUDMARSSON; RYDEN; BIRGISSON, 2012b; CARRET *et al.*, 2018; CARRET; DI BENEDETTO; SAUZÉAT, 2018b). However, the results of these dynamic tests do not allow the direct determination of the properties that characterize the linear viscoelastic (LVE) behavior of asphalt mixtures. An inverse analysis process is required (CARRET, 2018). In the paving materials field, such kind of inverse analysis process combined with the finite element method (FEM) is consolidated for example for the falling weight deflectometer (FWD) test, in which the response of the pavement under various dynamic loads is simulated with FEM in order to determine the stiffness of the individual paving layers (BESKOU; HATZIGEORGIOU; THEODORAKOPOULOS, 2016; LI *et al.*, 2017; LIU; WANG; OESER, 2017). In the case of the LVE behavior of asphalt mixtures, the complexity comes from the fact that the material has modulus dependent of loading frequency, and an infinite number of loading frequencies is excited in IRT.

For materials considered elastic (cementitious materials, steel, etc.) the analysis of the IRT test results can be done in a more simplified way, with the direct application of Equation 14, which relates analytically the resonant frequency and geometric parameters with the stiffness of the materials, as shown in detail in Bezerra *et al.* (2022). For viscoelastic materials, such as asphalt mixtures, a more complicated process is necessary, requiring the aid of modeling to obtain the optimized FRF from the variation of LVE model parameters of the material (CARRET, 2018). In other words, from an iterative process, the parameters are determined that characterize the stiffness behavior of the viscoelastic material for any frequency and temperature. The iteration process uses the calculated value of the error between the experimental and optimized response, and then determines the values of the variables of the viscoelastic model that represents the material so that the error between modeled and experimental results is minimized.

Carret (2018) proposed 5 inverse analysis methods, finding adequate and satisfactory results with 3 of them. In general, the calculation methodology of those methods is divided into

two main stages. In the first step, an optimization process is repeated at each tested temperature to obtain the LVE parameters at the corresponding temperature, giving a part of the LVE property master curve (normally the absolute value of complex modulus). In the second step, the global LVE behavior of the material is determined using the master curve results obtained at each temperature in the first step. Differences between the methods consist of which variables of the viscoelastic model are fixed and which are optimized and whether only one peak or all peaks of resonant frequency are used. The step-by-step procedure of the methods and their differences are described in Carret (2018).

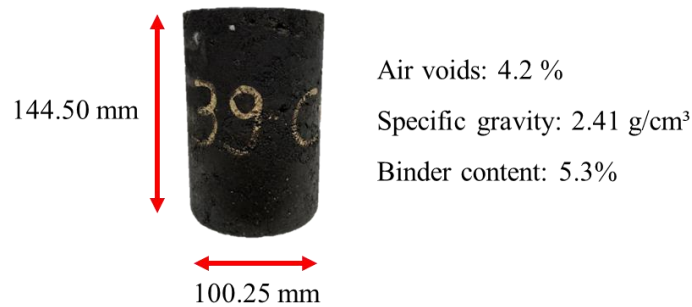
Given that context, it is clear that the accurate characterization of the LVE behavior of asphalt mixtures using FRF obtained in the IRT is a challenge and that simplified approaches such as the existent for elastic materials (ASTM 215, 2019) will not give LVE characterization. Then, a more elaborated approach is required, and a proposition is given in this paper to avoid long calculation. Therefore, the objective of this research is to propose an inverse analysis methodology (also known in the literature as back-calculation) for viscoelastic characterization of asphalt mixtures with IRT test results with FRF's measurements that may rapidly produce sufficiently accurate experimental results. In addition, the work compares the operation of two IRT prototypes, one from Federal University of Ceará (UFC), Brazil, and one from the *École de Technologie Supérieure* (ÉTS), Canada.

5.2 Materials and methods

An asphalt mixture composed of a penetration-graded binder CAP 50/70 (PG 64S-XX) from Lubnor refinery in Fortaleza, and granite aggregate with an addition of 5% fly ash in relation to the total mass of the mix. The design process followed the Superpave method (AASHTO R 35, 2017), aiming to obtain mixtures with a Nominal Maximum Aggregate Size (NMAS) of 12.5mm and 4.0% air voids. The main characteristics of the cylindrical specimen (called herein AM1) necessary for this investigation are presented in Figure 20.

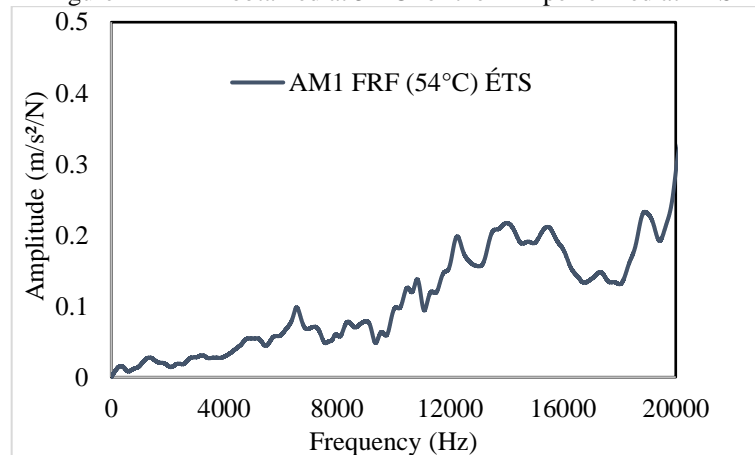
Specimen AM1 was tested in two different IRT tests, in the UFC equipment and in the ÉTS equipment. The tests were carried out at 3 temperatures (-10, 4 and 20°C). Higher test temperatures were not considered because they affect the material viscosity, impacting the signals and bringing noise that hinders further analysis of the results to obtain the stiffness parameters. Figure 21 shows an example of the FRF of the IRT at 54°C. In order to compare the IRT results, 3 specimens of the same mixture (AM1) were tested with the traditional dynamic modulus test standardized by DNIT-ME 416 (2019).

Figure 20 – Characteristics of the specimen AM1



Source: Author (2023).

Figure 21 – FRF obtained at 54°C for the IRT performed at ÉTS

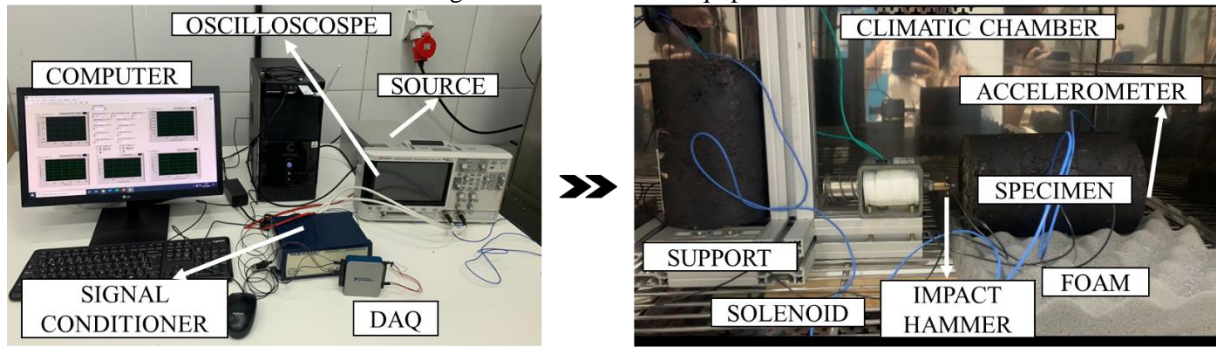


Source: Author (2023).

5.3 Measurement of the FRF with non-destructive impact resonance test

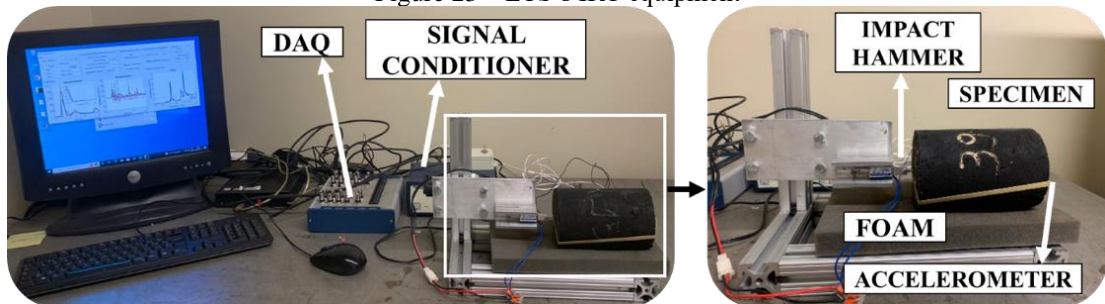
The impact resonance test (IRT) was performed on specimen AM1 as previously explained (cf Section 2.5. and 2.5.3). However, the hardware of the test apparatus was modified to obtain the experimental FRF (FRF_{exp}) in the IRT test, since, as mentioned in section 3.4, due to DAQ limitations, it was not possible to obtain this result. Thus, an oscilloscope (with the same DAQ function) was added to obtain data from the impact hammer signals with the necessary precision in the time domain for calculating the FRF_{exp}. In addition, the test was set up and carried out inside a climatic chamber at 3 temperatures (-10, 4 and 20°C). Figure 22 shows the UFC IRT test in its current version, with oscilloscope and climatic chamber. Figure 23 shows the ÉTS IRT equipment.

Figure 22 – UFC’s IRT equipment



Source: Author (2023).

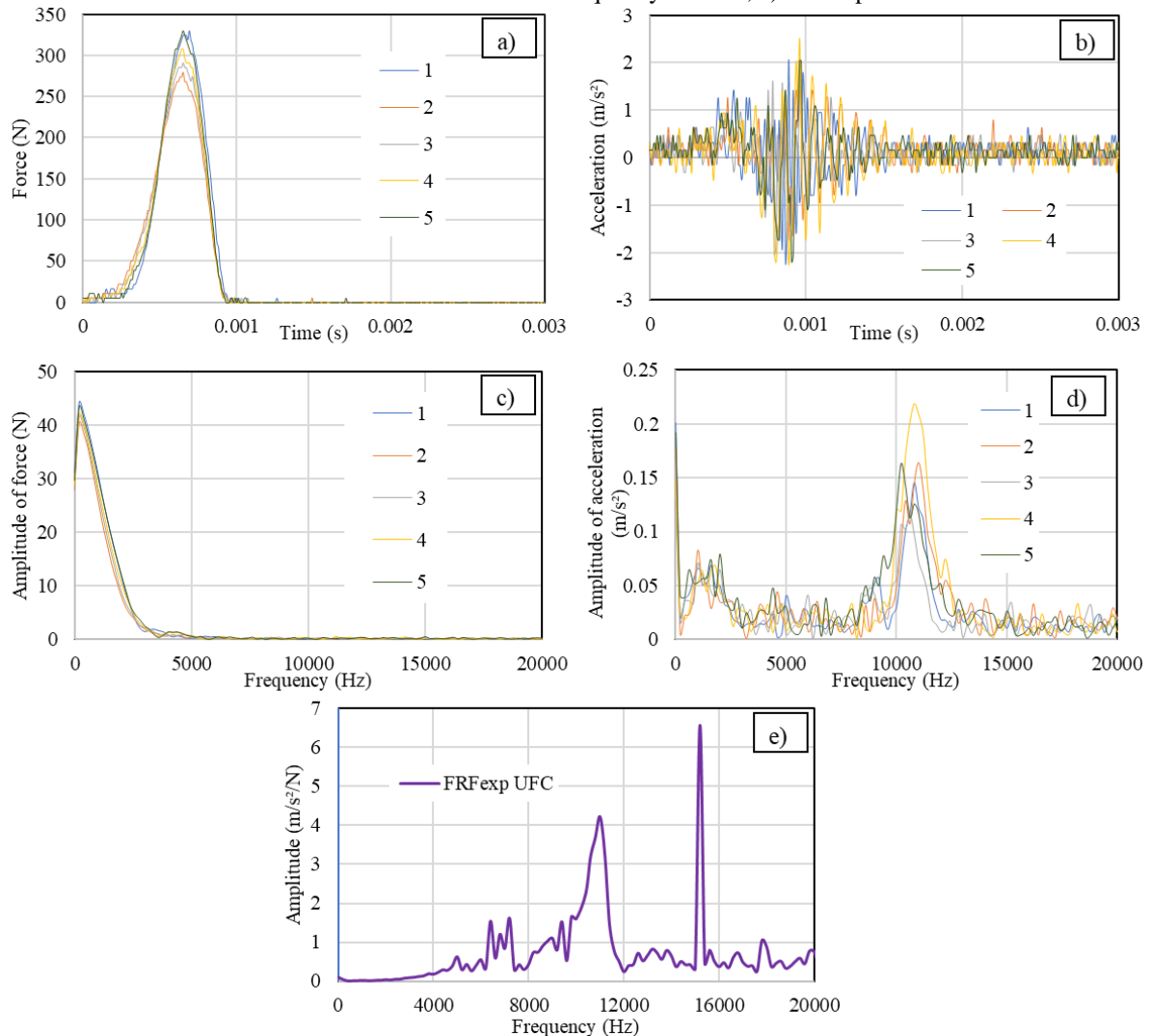
Figure 23 – ÉTS’s IRT equipment



Source: Author (2023).

In order to recall the test procedure, a brief description is given here. More details can be found in Section 3.2.1 and/or in Bezerra et al. (2022). The impact hammer and accelerometer were positioned in the longitudinal direction (opposite faces) on specimen AM1, placed under a foam. Once the test started, the specimen was excited 5 times and results were obtained in both time (acceleration and force vs. time) and frequency domains (acceleration output over load force input, both after fast Fourier transforms to the frequency domain). With the signals in the frequency domain, a simple ratio (cf. Equation 11) provides the experimentally measured FRF_{exp} . Figure 24 shows an example of results in the time domain and frequency domain for five impacts and the respective FRF_{exp} for the temperature 20°C in the UFC IRT test.

Figure 24 – Example of results for the impact resonance test on AM1: a) impact hammer load cell results in time domain; b) accelerometer results in time domain; c) impact hammer load cell results in frequency domain; d) accelerometer results in frequency domain; e) FRFexp

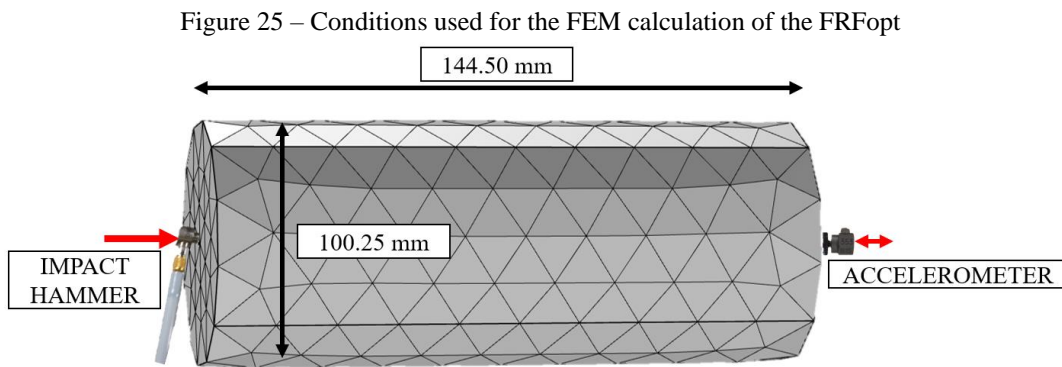


Source: Author (2023).

5.4 Calculation of FRF with FEM

In order to obtain the optimized FRF (FRFcalc), a numerical modeling of the asphalt mixture was performed with the FEM software COMSOL. The rheological modeling considered an isotropic linear viscoelastic model (cf. Equation 5 and Equation 6) for material behavior. However, in the FEM software, only one value of modulus is considered for dynamics equations solution and calculation of FRF, considering that temperature is constant and that the frequency range around the considered peak of resonant frequency is not wide enough to require more refined viscoelastic analysis. After the geometry, mesh (maximum element size of 2cm), material behavior and free boundary conditions (except at the impact position, where a unitary

load is applied, as a function of time) have been defined, the numerical FRF can be calculated by solving in COMSOL with wave motion equation (cf. Equation 13) for each specified and applicable frequency. Figure 25 illustrates the conditions used for calculating the FRFcalc with FEM. The frequencies for which the calculation was performed were chosen to match the frequency range accessible with experimental testing. Thus, the frequency resolution was set to 20 Hz to limit computational time.



Source: Author (2023).

5.5 Inverse analysis for determination of the LVE properties from IRT

An inverse analysis method is proposed as part of the research. With the FRFexp (cf. Section 5.3) and the FRFcalc (cf. Section 5.4) the error (Equation 16) between them is calculated, considering only 10 points around the peak of the first resonance frequency. The FRFcalc is used as a characteristic of the vibration of the solid in the 3D modeling in FEM together with the parameters of an isotropic linear viscoelastic model, which will characterize the asphalt mixture. This way, a process of inverse analysis is possible to determine these parameters by optimization of the 3D model consisting of an interactive process of $|E^*|$ values calculated with the 2S2P1D model. Unlike in Carret *et al.* (2018), here a code was developed in Matlab for calculating this error separately and requiring the FRFcalc results. Carret *et al.* (2018) used the `fminsearch` algorithm in Matlab to automatically and objectively minimize the difference between the calculated and the measured FRF.

$$Error = \sum_{j=1}^{Npeaks} \sum_{i=1}^{10} \left(\frac{\left| |H_{exp_{ji}}| - |H_{calc_{ji}}| \right|}{|H_{exp_{ji}}|} \right) \quad (16)$$

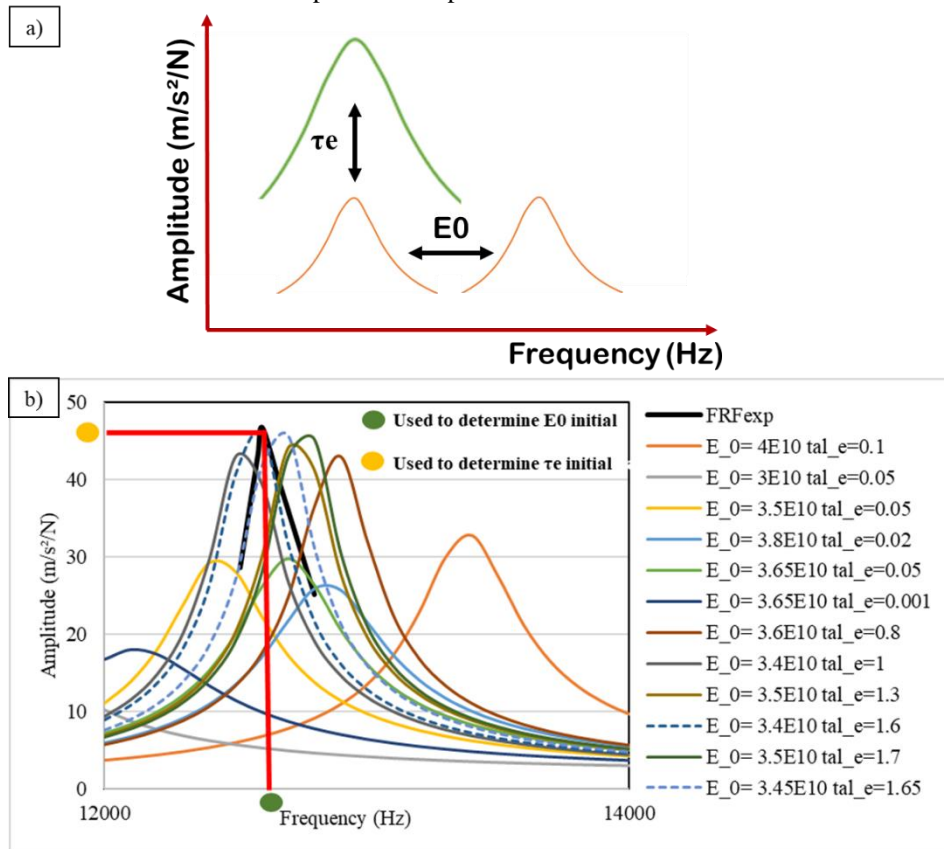
where H_{exp} is the experimental FRF, H_{calc} is the calculated FRF, N_{peaks} is the number of resonance peaks, j is the index of the peak and i is the index of the frequencies.

For the proposed inverse analysis, only 4 parameters (E_0 , k , δ and τ_e , cf. Equation 5) were optimized and 8 parameters were fixed. The values of the fixed parameters for the modeling were adopted from the reference material used by Carret, Di Benedetto and Sauzéat (2020). This decision was made from the parametric analysis conducted by Carret *et al.* (2018), who intended to identify the constants (2S2P1D model) that have a negligible influence and that can be fixed for the inverse analysis. The referred authors observed that 4 constants have the greatest influence on the FRF: E_0 has important effect on the resonant frequency and k , δ and τ_e have influence on both resonant frequencies and amplitudes.

For the determination of the FRF_{calc} in COMSOL, 3 possible values are inserted for each 4 parameters (E_0 , k , δ and τ_e), and each iteration another set of values is used. Considering that FRF are calculated for the 10 frequency points of the first resonant peak, a matrix with 81 possible combinations is generated. These 10 points are chosen from the resonant frequency of the peak, considering a distance of 200Hz between them. With those combinations and the result of the FRF_{exp} , the code in Matlab returns which combination produces the smallest error (cf. Equation 16) between the FRF_{calc} and the FRF_{exp} among the tested sets of parameters values. Carret *et al.* (2018) explained that this limitation of the number of constants to be determined is of great interest because limiting the number of optimized parameters reduces considerably the computational time and the risk to obtain a wrong solution corresponding to a local minimum.

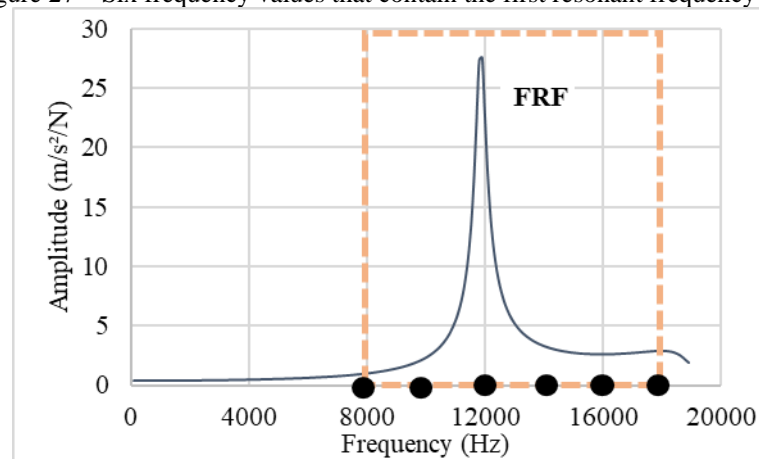
However, before this step, the initial inputs (3 values) for the parameters E_0 and τ_e are defined through a manual process in COMSOL. It is known that E_0 has a direct correlation with the resonant frequency and τ_e with the amplitude of the FRF. Thus, manually and individually (by entering only one value), it is found which initial values of these two parameters make the FRF_{calc} closest to the FRF_{exp} . Figure 26a shows a schematic representation of the process needed to choose the E_0 and τ_e inputs. When the value of E_0 changes, the curve moves to the right and left, altering the resonance frequency. When the value of τ_e changes, the curve moves up or down, altering the amplitude of the FRF. Figure 26b shows an example of this process for specimen AM1 at -10°C . It is seen that the blue dashed curves are closer to the FRF_{exp} (black continuous curve). For the parameters k and δ , 3 values were used as initial inputs, based on the literature.

Figure 26 – Process for finding the initial inputs for the parameters: a) schematic representation; b) example of this process for specimen AM1 at -10°C



After this step and with the error value and consequently the 4 variable parameters determined, 6 frequency values are selected for each temperature (shown in Figure 27) in a range containing all the resonance frequencies around the first peak. The 6 frequency values are determined from the resonant frequency, and 2000Hz is added or subtracted until the 6 values are complete. These frequencies together with the values determined in the inverse analysis are used in the modeling in an Excel spreadsheet until they are adjusted to the 2S2P1D model (called herein "FRF 2S2P1D") to obtain the global master curve.

Figure 27 – Six frequency values that contain the first resonant frequency peak



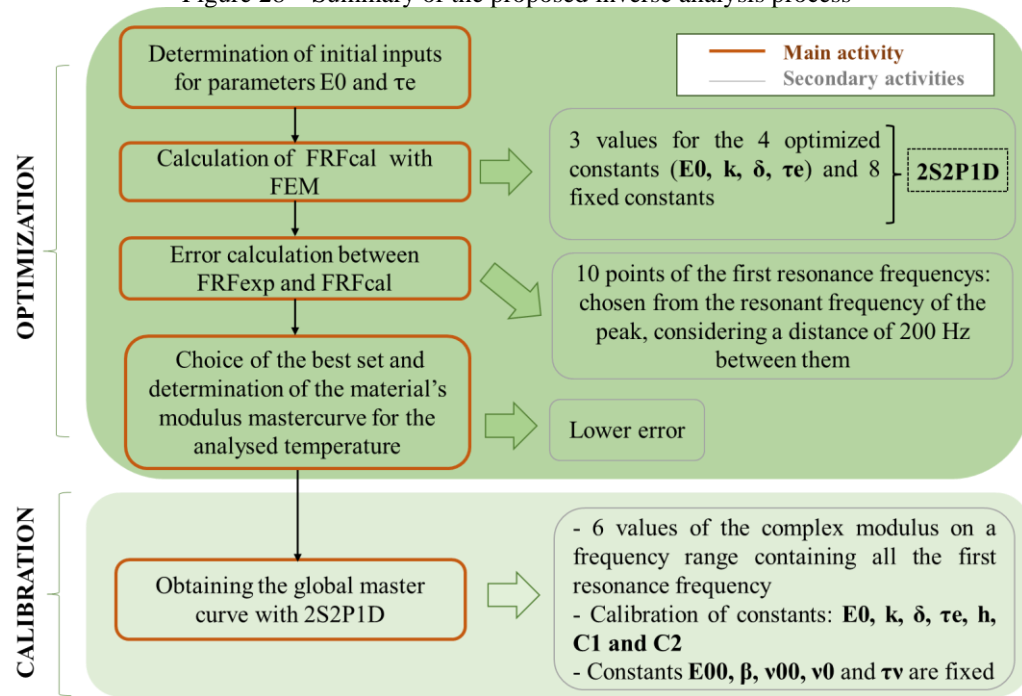
Source: Author (2023).

In summary, the inverse analysis proposed is divided into the following 4 steps:

- (i) determination of the initial inputs for the parameters E_0 and τ_e of the 2S2P1D model (initial guess of rheological model parameters) for a given temperature;
- (ii) calculation of the FRFcalc with FEM (cf. Section 5. 4);
- (iii) calculation of the error between FRFexp and 81 calculated functions FRFcalc;
- (iv) choice of the best set (least error and whose corresponding FRFcalc is called FRFOpt) of rheological parameters amongst the 81 tested, and determination of the material's modulus master curve for the analyzed temperature and frequency range around the resonance peak;
- (v) repetition of the process for all other tested temperatures in order to obtain other isotherms from IRT;
- (vi) determination the master curve with the calibration of the 2S2P1D model and constants of the WLF equation simulating the global behavior of LVE considering the results of step (ii) at all temperatures, 6 frequency values of the FRF's, and the values of the LVE model parameters.

Figure 28 presents graphically the optimization process of the inverse analysis proposed.

Figure 28 – Summary of the proposed inverse analysis process



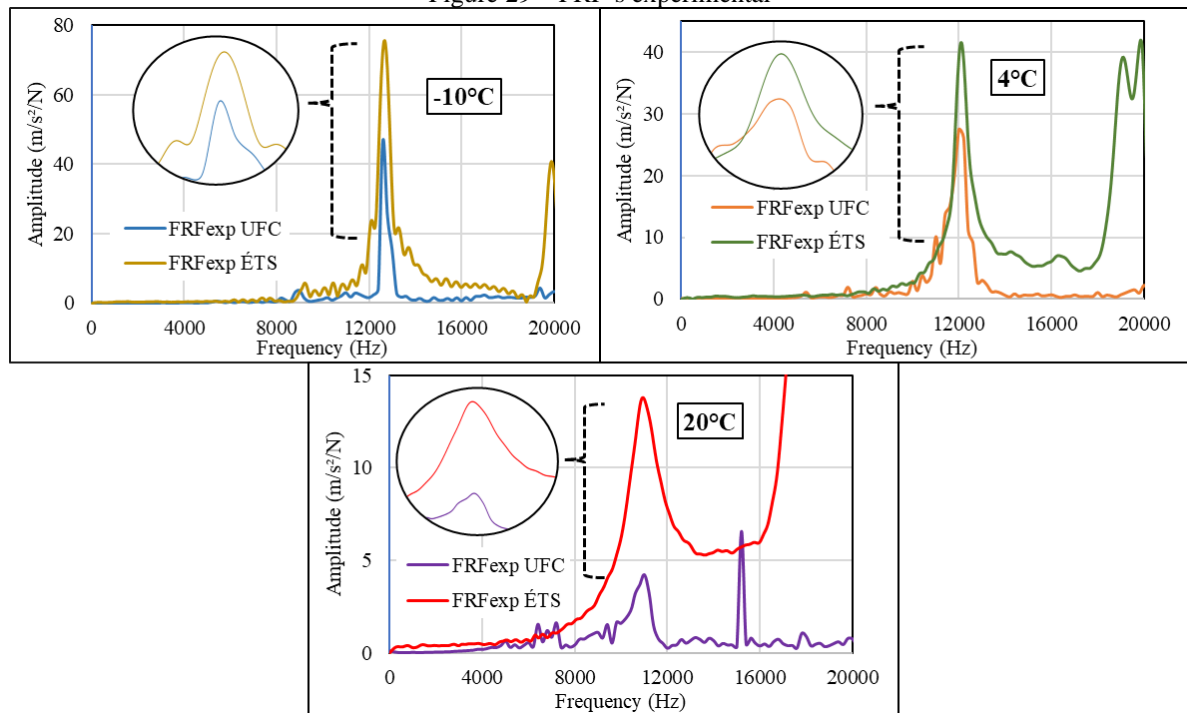
Source: Author (2023).

5.6 Results

5.6.1 FRF experimental

Figure 29 presents the results of the experimental FRF's for specimen AM1 at the 3 test temperatures, considering the results obtained in the IRT tests performed with ÉTS and UFC prototypes. At all temperatures the results are quite similar with respect to the peak of the first resonance frequency. For -10°C the resonance frequency values were 12600Hz and 12660Hz, for 4°C 12000Hz and 12100Hz, and for 20°C 11000Hz and 10940Hz for UFC and ÉTS, respectively. The difference observed in the y-axis (of amplitude in $\text{m/s}^2/\text{N}$) is credited to the different impact applied. It is believed that the speed, intensity and inclination of the impacts are not adequate and are causing energy loss outside of the vibration of the material. If executed correctly, the FRFs should be the same independent of the impact force. This is the case because the FRF's is the ratio of acceleration to measured force, and it is a unit load applied at each frequency.

Figure 29 – FRF's experimental



Source: Author (2023).

The results show that the resonance frequency decreased and the amplitude increased with increasing temperature, showing changes in the material properties due to the viscoelastic behavior of asphalt mixtures, as also found by Carret *et al.* (2018) and Bekele *et al.* (2019).

5.6.2 Initial parameter inputs of the LVE model adopted for inverse analysis

Table 6 presents the initial input values determined in the first step of the optimization procedure (cf. Section 5.5 and Figure 26) of the inverse analysis for the 2S2P1D model parameters E_0 and τ_e . For k and δ fixed ranges of values were used for all temperatures, based on the values adopted by Carret *et al.* (2018). For k , 0.1, 0.15 and 0.2 were used as input values, and, for δ , 1.5, 1.75 and 2.

An observation to be highlighted is that the variability existing between the values found in the first step for the variables E_0 and τ_e at each test temperature is common in this type of proposed inverse calculation. Because the model parameters 2S2P1D needed to calculate modulus master curves are not the final parameter values to be considered, but only necessary to obtain “experimental” isotherms from IRT for the tested temperature and frequency range, which are in small range of frequency compared to the whole master curve. It

is only after having various parts of the master curve (frequency-shifted isotherms at different temperatures) that a global 2S2PID model can be fitted to the results.

Table 6 – Initial input values for E_0 and τ_e in the FRF optimization procedure

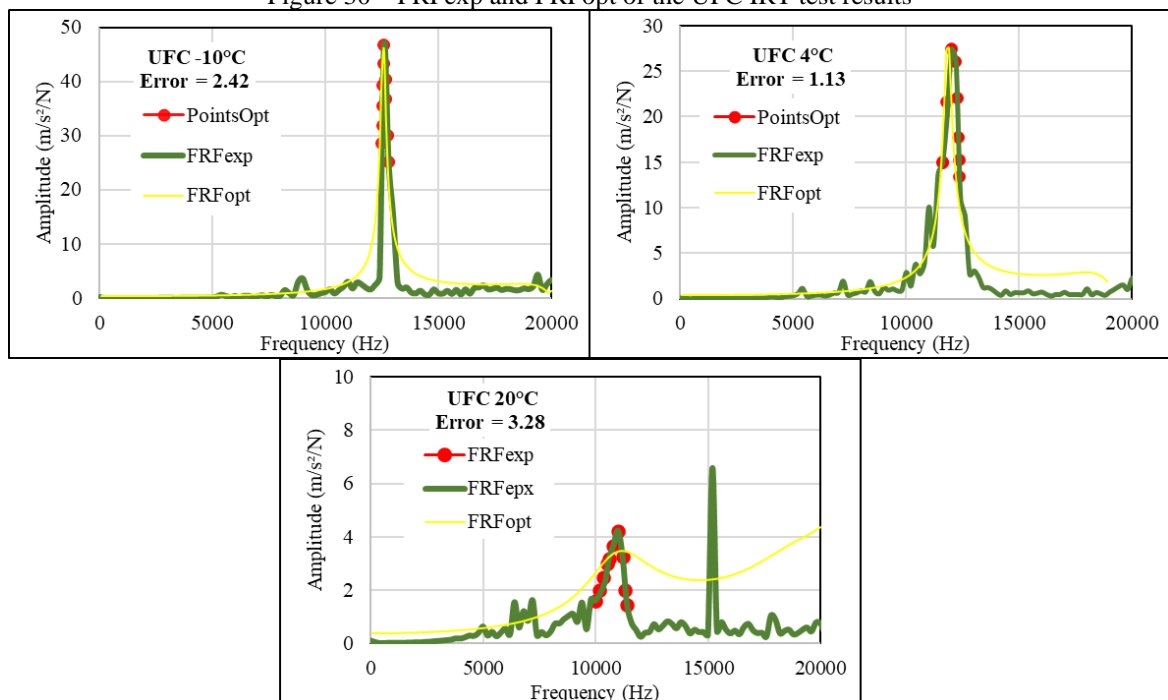
	-10°C		4°C		20°C	
	IRT UFC	IRT ÉTS	IRT UFC	IRT ÉTS	IRT UFC	IRT ÉTS
E_0(MPa)	3.40E10;	3.30E10;	3.20E10;	3.00E10;	6.0E10;	2.90E10;
	3.45E10;	3.35E10;	3.30E10;	3.20E10;	6.5E10;	2.95E10;
	3.50E10	3.40E10	3.40E10	3.40E10	7.0E10	3.00E10
τ_e (s)	1.60;	50;	11;	210;	0.001;	3.5;
	1.65;	55;	12;	220;	0.002;	4.0;
	1.70	60	13	230	0.003	4.5

Source: Author (2023).

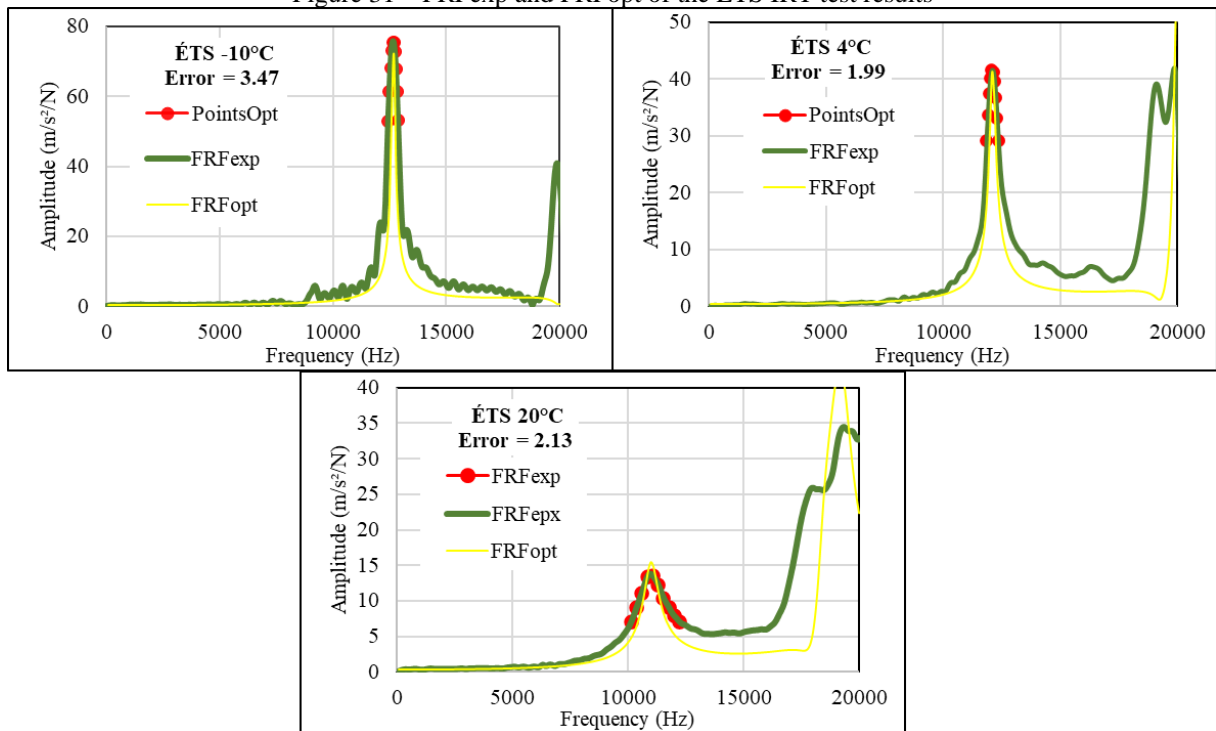
5.6.3 Inverse analyses

The FRF_{exp}, the FRF_{opt} and the 10 points used in the optimization process (for the error calculation) for specimen AM1 for the UFC and ÉTS IRT test are presented in Figure 30 and Figure 31, respectively. In addition, the error values between the experimental and the optimized FRF's are presented. A good approximation between the FRF_{exp} and FRF_{opt} can be observed, with the exception of the 20°C UFC temperature. The results demonstrate that the inverse analysis method worked properly in this verification step.

Figure 30 – FRF_{exp} and FRF_{opt} of the UFC IRT test results



Source: Author (2023).

Figure 31 – FRF_{exp} and FRF_{opt} of the ÉTS IRT test results

Source: Author (2023).

Table 7 shows the parameter values of the 2S2P1D model (FRF 2S2P1D) optimized after the inverse analysis process. Note that only the value of E_0 is different between the UFC and the ÉTS IRT test. It should be noted that the values of the constants E_{00} , h and β are assumed fixed because they have little effect on the complex modulus values in the frequency range involved during the dynamic tests.

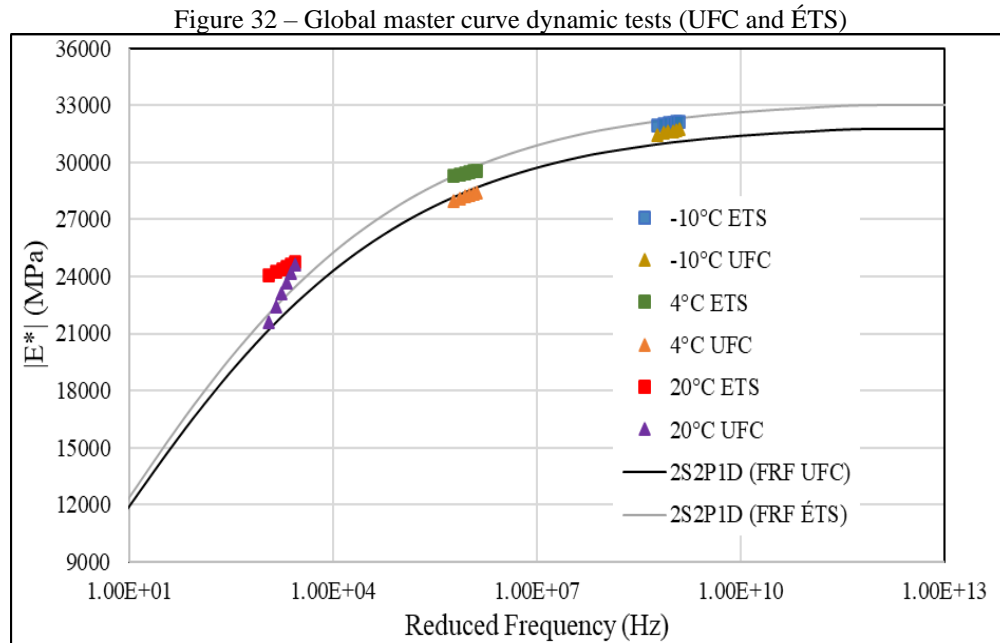
Table 7 – Parameter values of the 2S2P1D model after the inverse analysis process

	E_{00} (MPa)	E_0 (MPa)	δ	k	h	β	$\tau_{e15^\circ\text{C}}$ (s)	$\tau_{v15^\circ\text{C}}$ (s)	ν_0	ν_{00}	C1	C2
UFC	100	34000	1.5	0.2	0.53	250	0.005	4.43	0.19	0.45	23.3	150.9
ÉTS	100	33300	1.5	0.2	0.53	250	0.005	4.43	0.19	0.45	23.3	150.9

Source: Author (2023).

With the results of the inverse analysis, the parts of the master curves were obtained, and, then, a global master curve could be built with a global 2S2P1D parameter fitting. Figure 32 presents this result, considering the IRT equipment of both UFC and ÉTS. The master curve of the norm of the complex modulus at 15°C was obtained considering the validity of the TTSP. A good fit of the 2S2P1D curves can be observed for both tests, which provide acceptably similar results (only 2.06% difference in global E_0 explains all the experimentally observed difference in the master curves), with only one parameter (E_0) being affected by the test equipment. Even with this existing variability between the values found in the first step for the variables E_0 and τ_e (cf. Section 5.1.2), the values of the complex modulus recalculated at each

temperature produced a good fit to a global 2S2P1D model, which provided the global LVE behavior of sample AM1, as shown in Figure 32.

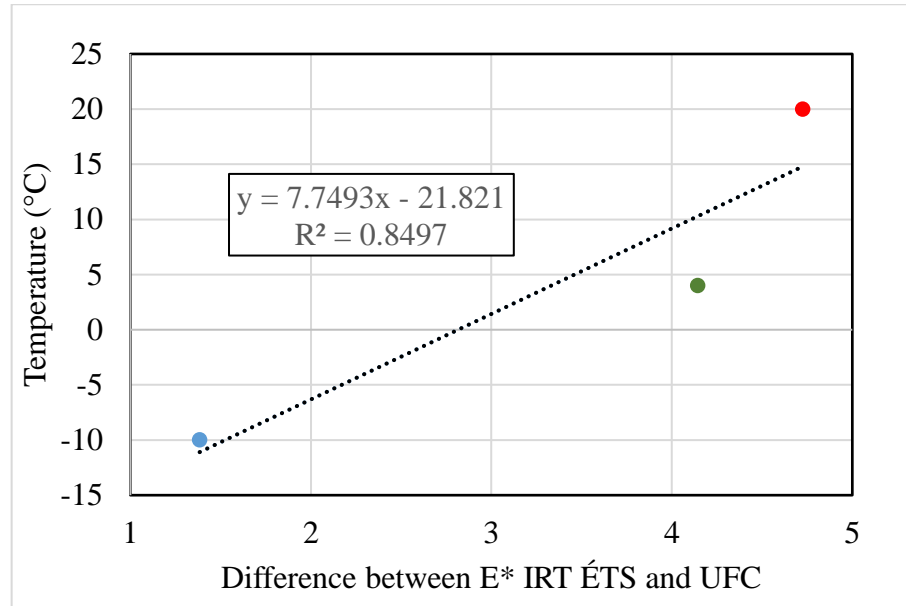


Source: Author (2023).

The complex modulus values obtained with the ÉTS test were slightly higher than those of the UFC, with percent differences of 1.38, 4.14 and 4.73% for temperatures of -10, 4 and 20°C, respectively. Figure 33 shows the correlation between temperature and the difference between the modulus values determined with IRT UFC and IRT ÉTS. There is a correlation of 85% between the modulus increase measured by the ÉTS equipment to that of the UFC, considering the temperature. That is, the higher the temperature, there is a tendency that the tendency for the ÉTS equipment to obtain higher modulus than the UFC. It is believed that this difference is associated with the execution of the test, in relation to the speed and intensity of the impacts, which can cause different vibrations of the same material, causing different responses.

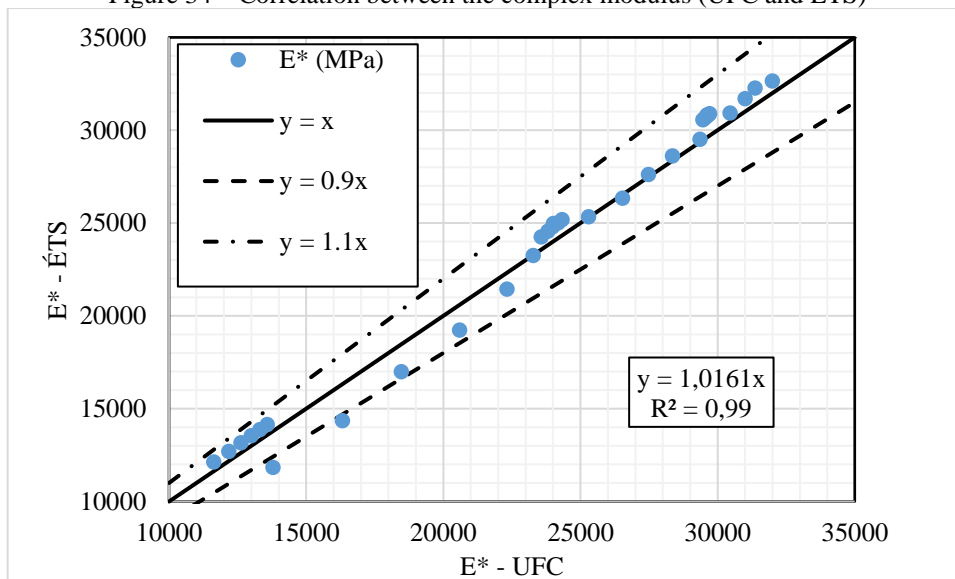
The good correlation between the complex modulus values found with the IRT of UFC and ÉTS are shown in Figure 34. Considering the 2S2P1D model, shown in Figure 31, complex modulus values were calculated for 30 frequency values, between 1.0E+03 and 1.0+E10. It can be seen that there is a high coefficient of determination, with R^2 of 0.99. On average, the modulus of the two tests were only 1.016% different, with few observations reaching 10% difference. The lines ($y=0.9x$ and $y=1.1x$) have been inserted in a way that shows that the data is dispersed and mostly in the error zone of +/-10%.

Figure 33 – Correlation between temperature and the difference between the modulus values determined with IRT UFC and IRT ÉTS



Source: Author (2023).

Figure 34 – Correlation between the complex modulus (UFC and ÉTS)



Source: Author (2023).

5.6.4 Comparison between Cyclic and Dynamic Tests Results

Table 8 presents the results of the 2S2P1D modeling for the Traditional Compressive (TC) stress test for three specimens of the same mixture (materials) as AM1.

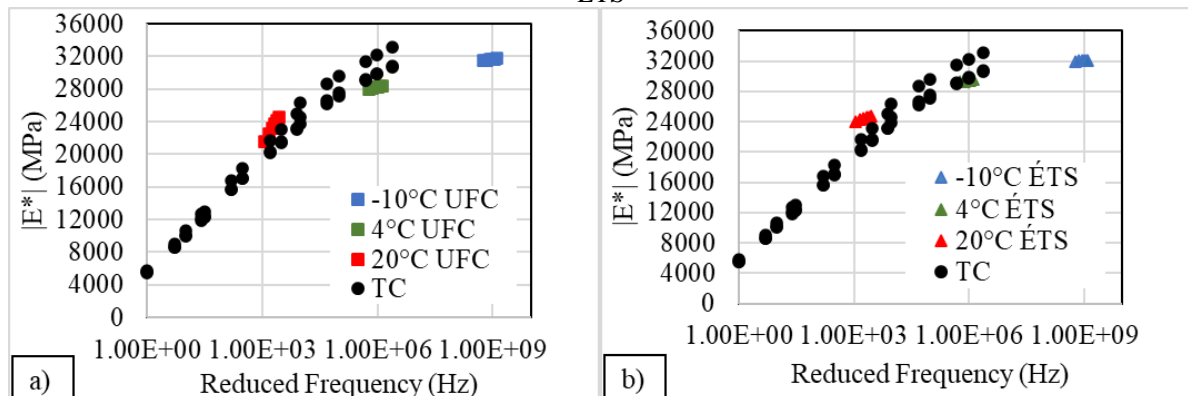
Table 8 – Parameter values of the 2S2P1D model for traditional complex modulus test

	E00 (MPa)	E0 (MPa)	δ	k	h	β	$\tau_{15^\circ\text{C}}$ (s)	$\tau_{15^\circ\text{C}}$ (s)	ν_0	ν_{00}	C1	C2
AM1-A	100	33500	1.6	0.2	0.53	250	0.005	4.43	0.19	0.45	16.98	135.45
AM1-B	100	33500	1.6	0.2	0.53	250	0.005	4.43	0.19	0.45	16.98	135.45
AM1-C	100	33500	1.6	0.2	0.53	250	0.005	4.43	0.19	0.45	16.98	135.45

Source: Author (2023).

In order to compare the results found between the traditional cyclic test and the IRT test results of the UFC and ÉTS, the master curve of Figure 35 was constructed. The observed differences can be justified due to the different frequency ranges between the traditional and dynamic test. The percentage difference in the complex modulus is greater for greater reduced frequencies (or lower temperatures). When comparing the traditional result (averages of the three specimens of the same mixture) with the UFC IRT (Figure 35a) test the difference is 3.06, 5.87 and 0.07% for the temperatures of -10, 4 and 20°C. The results of this difference for the ÉTS (Figure 35b) test are 4.40, 1.48 and 0.12%. These results are in agreement with previous studies using FRF (GUDMARSSON; RYDEN; BIRGISSON, 2012b; CARRET *et al.*, 2018). It is important to underline that, in order to calculate those percentage differences, it was necessary to frequency-shift the traditional cyclic test results to the corresponding frequencies at the temperature adopted for the comparison (-10°C) to obtain complex modulus values in the same frequency range as found in the dynamic IRT tests. For such frequency-shift, the reference temperature was changed and consequently new shift factor values were altered. For -10°C the shift factor was from 5.04 to 1.19E+08, for 4°C from 2.44 to 2.92E+05 and for 20°C from 0 to 1.49E+03.

Figure 35 – Master curve comparison between traditional and IRT: a) traditional and IRT; b) traditional and IRT ÉTS



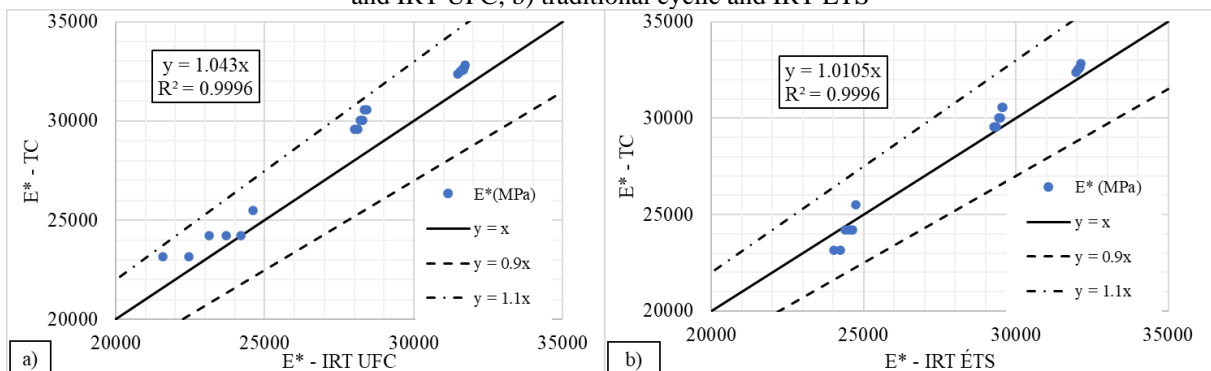
Source: Author (2023).

One factor that may influence this difference is the different stress levels applied in the cyclic and dynamic tests. Due to the nonlinear behavior of asphalt mixtures (NGUYEN;

BENEDETTO; SAUZÉAT, 2015; MANGIAFICO *et al.*, 2018) even at small strain levels, the case of the dynamic tests, the increase in complex modulus occurs with decreasing strain amplitude. In addition, aging of the materials also justifies these differences, since the dynamic tests were performed years (about 2 years) after the cyclic test. As a means for comparison, the expected effect of nonlinearity (which is usually a neglected effect for LVE characterization) is about around 5 to 10% when changing the strain amplitudes from $0\mu\text{m/m}$ to $50\mu\text{m/m}$ (MANGIAFICO *et al.*, 2018), depending on temperature. Thus, the difference of about 6% observed between IRT are on the same order as that expected to nonlinearity. However, it is to be noticed that it is expected that higher strain lead to lower modulus, which was inversed in the comparison between IRT and the traditional quasi-static test.

Figure 36 shows the correlation between the complex modulus obtained in the traditional cyclic test and the dynamic tests (IRT UFC and IRT ÉTS). Considering the 2S2P1D model, complex modulus values were calculated for a frequency range between $1.5\text{E}+03$ and $4.8\text{E}+09$. A high correlation is observed, with R^2 of 0.99 for IRT UFC (cf. Figure 36a) and IRT ÉTS (cf. Figure 36b). Also, with 1.04 and 1.01% difference between the complex moduli of IRT UFC and IRT ÉTS, respectively. This shows that there was a satisfactory correlation throughout the frequency range analyzed.

Figure 36 – Correlation between the complex modulus traditional cyclic and dynamic tests: a) traditional cyclic and IRT UFC; b) traditional cyclic and IRT ÉTS



Source: Author (2023).

5.7 Conclusions

An inverse analysis method for viscoelastic characterization of asphalt mixtures from results of FRF of a dynamic test considering different temperatures and frequencies was proposed. For verification, results from two IRT prototypes from two different universities, UFC in Brazil and ÉTS in Canada, were used. To validate the proposed method, results from

the traditional complex modulus test were used to compare with those obtained from the IRT tests. The main conclusions from this investigation were:

- The experimental FRF's of both UFC and ÉTS presented similar behavior, where the resonance frequency values were very close, with a percentage difference of less than 1% in the three test temperatures. However, in relation to the amplitude of the FRF's, the values presented a greater difference. It is believed that this fact is due to the different impact applied to the two pieces of equipment;
- The first step of the proposed inverse analysis process (determination of the initial inputs for the parameters E_0 and τ_e) facilitated the calculation and reduces the computational effort, since it allows the determination of values that produce FRF_{cal} close to FRF_{exp} ;
- Except for the 20°C temperature with the IRT test of the UFC, a good convergence of the FRF_{opt} towards the FRF_{exp} was observed. This demonstrates the efficiency of the process for optimizing FRF;
- The overall LVE behavior determined after the optimization process showed a good similarity between the results of both IRT tests (UFC and ÉTS) performed. The percentage difference for the complex modulus was less than 6%, and this difference increased with the test temperature;
- It is possible that the differences obtained between the traditional stress-compression test and the dynamic tests could be due to other effects, such as dispersion experimental when measuring the FRF_{exp} or nonlinearity of the asphalt mixture. Even so, there was a satisfactory correlation over the entire frequency range between the dynamic tests and the tension-compression tests;
- It is believed that the combination of the two test methods (traditional and dynamic) is useful for improving the characterization of the LVE behavior of asphalt mixtures over a wider frequency range, because the dynamic tests provide access to the material behavior on higher frequencies;
- The proposed inverse analysis approach is more simplified than other existing ones and considerably reduces computational time (about 1 minute to run the 81 sets in the FEM). Thus, the analysis offers great potential to calculate the complex modulus of asphalt mixtures from FRF measurements using only the first resonance frequency peak.

6 CONCLUSIONS FROM THIS Master's RESEARCH AND RECOMMENDATIONS

6.1 Initial considerations

With the present master's work, a resonance impact test was set up, based on the principle of wave propagation and resonance by impact. In addition, a methodology for the analysis of its results was developed to characterize the linear viscoelastic behavior of asphalt mixtures, by determining the material's modulus master curve. The main contribution of the work was have pioneered the development in Brazil of an IRT test with FRF measurement and the proposal of an inverse analysis method to obtain asphalt mixture master curves.

6.2 Summary of conclusions

- The validation step of the first version of the IRT test involved tests on elastic materials (mortar and steel) and comparison with the results of classical tests for determining modulus (quasi-static, static and ultrasonic tests). The results of that step with the developed IRT were satisfactory, since the percentage difference between the steel modulus values in both tests (IRT and quasi-static) was only 2.15% and for the tests (IRT and static) with mortar between 4% and 13%. These results fulfill what was outlined in the first specific objective of this work;
- After the validation step and with a second version of the prototype of the IRT apparatus, tests were performed with a specimen of asphalt mixture and FRF measurements were obtained for three temperatures (-10°C, 4°C and 20°C). This same specimen was tested with the ÉTS IRT test at the same temperatures to verify the repeatability and reproductibility of the UFC IRT. The results of the experimental FRF presented a good similarity between the two test prototypes (UFC IRT and ÉTS IRT), with resonant frequency of the first peak close (less than 1% of percent difference in 3 tests in 1 specimen). Regarding the amplitude of the FRF, the difference was greater and this is due to the different speed and intensity of the impact produced in the tests, which may be related to some material nonlinearity (effect of loading amplitudes on LVE properties). These results were related to the second specific objective outlined;
- With the FRF results, global master curves of the tested asphalt mixture were obtained through the proposed inverse analysis methodology (cf. Section 5). This analysis was observed to be appropriate for asphalt mixtures characterization, since the global master curves of the dynamic tests (both using UFC IRT and ÉTS IRT)

presented good correlation with that of the traditional cyclic test. The differences observed between them are due to possible experimental dispersion of the IRT and to material nonlinearity. These conclusions show that the third specific objective has been achieved.

Based on the above-mentioned conclusions, it is believed that this work presented a relevant and pioneer development in Brazil of an IRT with FRF measurement, a non-destructive technique, of low cost, simple execution and with the potential to contribute in supplying more tools to pavement designers and engineers that manage road construction. It was also important for the development of a new inverse analysis method with the potential to calculate the complex modulus of asphalt mixtures over a wide range of frequencies from FRF measurements, using only the first resonance frequency peak.

The research has also contributed for internationalization of Brazilian research since it constituted a collaboration of laboratories at UFC-Brazil, University of Lyon-France and ÉTS-Canada. The cooperation has been another step towards improving the processes of characterization of asphalt mixtures by implementing a new technology and methodology for non-destructive testing based on the principle of wave propagation and impact resonance. It is believed that it will help in the development of a future standard for dynamic testing.

6.2 Recommendations and suggestions for future research

This master's research was useful in identifying the need for the development of further investigations to improve the understanding of viscoelastic characterization of asphalt mixtures with nondestructive dynamic testing. Suggestions for future research are given below:

- To improve the test setup to have better, shorter and less intense impacts with less energy loss out of material vibration. This can improve the quality of the test setup by shortening the impact time interval and can reach temperatures of 30°C or 40°C;
- To perform a sensitivity analysis to determine which value of impact force causes adequate vibration in the material for IRT;
- To perform IRT with a larger amount of asphalt mixtures specimens (possibly mastics and fine aggregate mixes was well, at sufficiently low temperatures to get appropriate vibration for testing) using different types of materials, in to validate the repeatability and the IRT. In addition to performing tests with these specimens in the IRT ÉTS equipment to validate the reproducibility;

- To investigate other testing temperatures for IRT, including a wider range, starting at -20°C and increasing every 10°C , and evaluating the effect on the determined LVE master curves;
- To perform IRT in different vibration modes (transverse and torsional) in addition to the longitudinal mode, to determine LVE properties from enhanced inverse analysis;
- To perform ultrasonic tests comparing with the master curves obtained with the IRT and the traditional tension-compression or compression tests;
- To perform an ultrasonic test with S-wave measurement, in order to determine the Poisson's ratio, evaluating the impact this property has on the final modulus value;
- To investigate the effect of using the FRF measurements to analyze the IRT results in materials considered classically elastic in order to detect small phase angle variations and small modulus variations;
- To investigate inverse analysis methods considering simpler rheological models to determine the absolute value of the complex modulus and its phase angle for each temperature.

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