An Integer Linear Programming Model for the Multiperiod Production Planning of Precast Concrete Beams

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Abstract: Unlike conventional beams, which are cast in the construction site, precast beams are cast in a production line in a beam factory. Their use may considerably reduce the completion time of construction projects, making them attractive to public housing. A key factor for the success of their use is the ability of efficiently producing them, which depends on the quality of the production planning. The objective of this paper is to propose an integer linear programming model for the precast concrete beams production problem. It is first shown that this problem is isomorphic to the well-known multiperiod cutting stock problem. The objective function is the minimization of the production loss of a production order, subject to the available capacity of forms used to cast the beams. A case study is presented with data of a real scale instance, so as to demonstrate the applicability of the model in industrial settings. The proposed model was implemented and ran on the CPLEX solver, which reached an optimal solution within an acceptable running time. The results indicate that significant gains may be achieved in terms of reduction of planning time through the application of the proposed model. **DOI: 10.1061/(ASCE)CO.1943-7862.0000991.** *© 2015 American Society of Civil Engineers.*

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Introduction

The filigree wide-slab method is a procedure currently used in civil construction, usually for the execution of slabs in a building (Elliott 2002; Bachmann and Steinle 2011; Elliot and Jolly 2013). This procedure is generally applied to low-cost buildings, particularly to public housing. The aforementioned method is based on the assembly of prefabricated pieces, such as bricks and beams, which are produced in a factory.

Precast beams are of great importance for the success of a filigree wide-slab method. In factories, concrete beams are molded informs, which have a fixed capacity bounded by their lengths. A loss can be defined as the unfilled empty space in a form after receiving a set of beams to be concreted. Depending on the demand faced by a factory, it can be very difficult to meet the deadlines required by customers. The production orders should be scheduled so as to maximize the utilization of the forms, or equivalently to minimize the losses in the forms.

With regard to precast concrete beams, several papers have been reported in the literature. Structural optimization techniques, information technology approaches, and management methodologies are presented by Sacks et al. (2004), Castilho et al. (2007), Ergen et al. (2007), Jeong et al. (2009), Yin et al. (2009), Castilho and Lima (2012), Chen et al. (2010a, b), Albuquerque et al. (2012), Demiralp et al. (2012), Senaratne and Ekanayake (2012), Fernandez-Ceniceros et al. (2013), Hong et al. (2014), and Jailon and Poon (2014). However, studies which approach the optimization of precast beam production with the objective of minimizing the losses and the total production time are scarce in the literature.

The main objective of this paper is to propose a model based on integer linear programming in order to optimize the production of precast beams by seeking a solution that minimizes the losses at minimal time. This paper is structured as follows: "Problem Description" states in detail the problem under study; "Proposed Model" presents the formulation of the proposed model; "Case Study" presents a case study that describes real data; and "Conclusions" draws some concluding remarks and suggestions for future research.

Problem Description

In order to describe the precast beams production problem, we must first recognize its similarity to the well-known cutting stock problem. The cutting stock problem, first proposed by Kantarovich (1960), is a widely studied combinatorial optimization problem with several industrial applications. Due to its practical relevance, this problem has received great attention from researchers over the years. We refer to Haessler and Sweeney (1991) for a thorough review of the problem.

The simplest case of the cutting stock problem is in a one-dimensional setting. It is often illustrated by an application in the paper industry, in which one has a stock of rolls of paper with fixed widths for the fulfillment of orders made by customers. The orders are typically composed of different quantities of rolls with varying widths. The problem faced by a production planner is: How to cut the rolls according to the orders demanded by customers so as to minimize waste?

In order to treat the production of precast beams as a cutting stock problem, we make the following analogy: the forms represent the rolls and the beams represent the demand of the customers. In this case, we do not have real waste, but a loss of production

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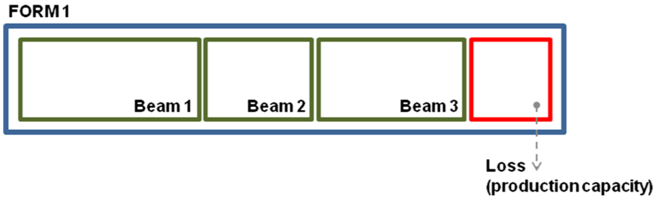

Fig. 1. Example of a beam scheduling in a form

Table 1. Size and Demand of the Precast Beams according to Their Types

Type	Size (m)	Demand
1	1.22	24
2	1.45	60
3	2.35	56
4	2.50	72
5	2.65	16
6	2.95	17
7	3.30	12

capacity due to unfilled spaces in the forms. In practice, an order by a customer often requires several time periods to be fulfilled, so that we have to consider a multiperiod approach to the problem. Fig. 1 illustrates the application of the cutting stock problem in the precast beam production.

Proposed Model

The cutting stock problem is traditionally approached by integer programming techniques. Let us introduce some notation. Parameters: m = number of forms; n = number of beam types; c_i = capacity of form i; l_j = length of a beam of type j; and d_j = demand for a beam of type j. Integer decision variables: x_{ij} = quantity of beams of type j precast in form i; t = number of time periods required for the completion of the production orders; and x_{ijk} = quantity of beams of type j precast in form i in time period k.

The objective is to schedule the production of the beams so as to minimize the empty spaces in the forms, considering a planning horizon of t time periods. The problem can be formulated as follows:

$$\text{minimize} \sum_{i=1}^{m}\sum_{k=1}^{t}\left(c_{ik}-\sum_{j=1}^{n} l_j x_{ijk}\right) \tag{1}$$

subject to:

$$\sum_{j=1}^{n} l_j x_{ijk} \leq c_i \qquad i=1,\ldots,m; \qquad k=1,\ldots,t \tag{2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{t} x_{ijk} = d_j \qquad j=1,\ldots,n \tag{3}$$

$$x_{ijk} \geq 0 \quad \text{and integer } i=1,\ldots,m; \qquad j=1,\ldots,n; \qquad k=1,\ldots,t \tag{4}$$

The objective function Eq. (1) is the sum of empty spaces in all forms for all time periods. Constraints in Eq. (2) assure that the total length of the beams is less than or equal to the capacity of the forms. Constraints in Eq. (3) assure that total quantity of the beams is equal to the demand. Finally, constraints in Eq. (4) define the scope of the model variables.

This model has $n \times m \times t$ integer decision variables. Even for small problems, with few forms, beams and time periods, the number of possible solutions is high, making it difficult for planners to elaborate optimal schedules on an empirical basis. This justifies the use of an optimization algorithm.

Case Study

We used data from a real case in order to demonstrate the application of the proposed model, which consists in the execution of a plate for the construction of a building, for which 257 precast beams were required for the assembly of a filigree system. The cross sections of all the beams are the same, so that the problem is similar to a one-dimensional cutting stock problem. The beams can be classified in seven types according to their lengths. The total length of all demanded beams is equal to 560.03 m. Table 1 presents the data.

The contractor outsources the precast beams to a specialized manufacturer, who builds them according to a make-to-order production strategy. The manufacturer avoids carrying inventory—this is a major constraint of the production process. Another important constraint of the production process is that the required time for drying the concrete is 1 day.

Reference to the production capacity, the manufacturer has seven forms, whose total length is 78 m. However, each form needs a residual space for the concreting of the beams, which results in an effective production capacity of 77.65 m (Table 2).

After receiving the customer's order, the production requirement is calculated by subtracting the beams already in stock from the demand amounts. Next, the planners calculate the quantity of materials (gravel 3/8, cement, river sand, water, and truss iron) required to produce the beams. Then, they elaborate a production schedule by trial and error with the aid of the spreadsheet application Microsoft Excel. In summary, the current practice of the manufacturer is to set empirically how many beams of each type will be produced in each form so that the empty spaces are acceptable and all the constraints are satisfied.

In order to demonstrate the modeling and solving of the problem by mathematical programming, we used the software application IBMCPLEX optimizer version 12.6.0.0. The computational experiment was performed on a PC with processor Intel Core Duo 3GHz and 4GB of memory.

Since demand cannot be fulfilled in a single time period, which in our case is 1 day, the problem is in a multiperiod setting. First, we need to determine the required production time, by dividing the total demand of precast beams by the total daily capacity of all the forms. This value should be rounded up to the lowest integer. In the present case, we have a total demand of 560.03 m of precast beams and a total daily production capacity of 77.65 m, which results in a planning horizon of 8 days.

Table 2. Production Capacity of the Forms

Type	Capacity (m)
1	11.95
2	11.95
3	11.95
4	11.95
5	11.95
6	11.95
7	5.95

Table 3. Losses for the Optimal Solution

Forms	Periods (m)							
	1	2	3	4	5	6	7	8
1	0.10	0.20	0.20	0.05	0.00	0.10	1.95	1.95
2	0.00	0.05	0.20	1.05	0.75	0.65	1.95	9.45
3	0.05	0.13	0.05	0.00	1.35	0.73	0.05	0.20
4	0.05	0.08	0.13	0.03	0.00	0.94	1.95	11.95
5	0.01	0.48	0.25	0.05	0.00	0.00	1.95	11.95
6	0.05	0.25	0.08	0.03	0.00	0.73	0.05	4.70
7	0.95	0.00	0.30	0.00	0.95	0.15	0.95	0.95

Using $t = 8$, we ran the model given by Eqs. (1)–(4) and found the optimal solution with total length of unfilled spaces of 61.17 m. The computation time was only 4.07 s, which indicates the feasibility of the application of the model to practical cases.

It is a known fact in the literature that the cutting stock usually has many optimal solutions (Haessler and Sweeney 1991), among which one may be more preferable according to some criterion. For example, the manufacturer may prefer that the unfilled spaces are placed in the last days in the schedule, so that they may more likely be used in case a new production order arrives during the execution of the current order.

Adding the set of constraints given by Eq. (5), one can generate a schedule with a maximum admissible loss ε_k in the time period k. The values of ε_k must be set by the planner and are input parameters of the model. Several scenarios can be generated, varying the values of ε_k

$$\sum_{i=1}^{m}\left(c_{ik} - \sum_{j=1}^{n} l_j x_{ijk}\right) \leq \varepsilon_k \qquad k = 1, \ldots, t \tag{5}$$

We generated some scenarios, varying the maximal admissible loss per day. According to the experience of the authors, the parameter vector [1.22, 1.22, 1.22, 1.22, 3.3, 3.3, 11.95, 77.17] generated a good solution from a practical standpoint. We used the information about the beams and the forms to generate this vector. An optimal solution of 61.17 m was found in 9.20 s.

It should be highlighted that the set of constraints (5) tends to increase the computational times. Reducing the admissible losses in the first days, it may take several minutes or even hours to find an optimal solution. Depending on the set of values for the admissible losses, the RAM memory may fail and the CPLEX stop (out of memory error).

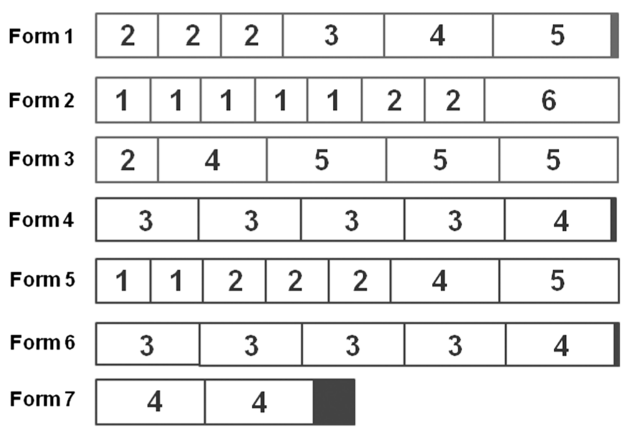

Fig. 2. Graphical visualization of the form scheduling for the first period

Table 3 presents the losses per day and per form. Among the first six planning days, on only two production days the schedule exhibited losses greater than 1 m. Fig. 2 illustrates the form scheduling for the first time period. The numbers represent the beam types and the gray spaces represent the losses. The losses are small in comparison with the capacities of the forms.

Conclusions

The model described in this paper is a promising technique for scheduling precast beams in forms, considering the constraints inherent to the forms (capacity) and to the beams (lengths and demands).To the best of the authors' knowledge, there is no other formulation in the literature for the production planning of precast beams based on the multiperiod cutting stock problem.

The approach proposed in this paper is a useful tool to support decision making in precast beam production, which can also be applied to other types of precast concrete parts. The model allows the identification of minimum loss solutions in an automated process, contributing to the reduction of planning times in industry. The computational time required by the solver to obtain the optimal solution was less than 10 s in the case study. This represents a considerable gain from a practical point of view, as there is a substantial decrease in the effort required by the planning process. As the optimal solutions were found in acceptable computational times, we think this is a promising approach in real world applications.

We also presented an extension of the model by adding new constraints, which allow the control of maximum admissible losses in specific time periods of the planning horizon.

As a further development of the work presented in this paper, the authors are currently working on a multiobjective extension of the model, so as to take into account other critical issues, which arise in practice, as for instance the carrying of inventory over the planning horizon. The model can also easily tackle the case of multiple customers since it is irrelevant if the beams are from different customers. In addition, if the objective function was the minimization of lead time, the model should be extended to consider time constraints related to the customers.

Finally, we do not compare the obtained results with the results from the current planning practices in the case study because the real scheduling was done under somewhat different conditions from the ones assumed in our model. For example, the current planning practices enable inventory formation and we plan to extend our model in order to incorporate this feature in future studies.

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