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A Branch-and-Bound Algorithm for Optimal Pump Scheduling in Water Distribution Networks

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Abstract Nowadays water distribution operation systems are accomplished with the aid of qualified professionals who use their experience in order to achieve a satisfactory performance of the several hydromechanical devices, which are part of the system, such as boosters and valves. In general, these operational rules are empirical and the main goal is to assure the availability of water for the population, with no special concerns about saving energy used in pumping systems. Besides, these empirical rules often disregard hours of lower energy rates. There are several research works concerning the developments of operational rules optimization applied to specific water distribution systems. However, in this work, a general optimization routine integrated with EPANET is presented, which allows the determination of strategic optimal rules of operation for any type of water distribution system. Moreover, a Branch-and-Bound algorithm is also used, where finding the global optimal solution is guaranteed, in admissible computational times. The water distribution system used in this work corresponds to a hypothetical network proposed in the specialized literature.

Keywords Water supply networks · Energy efficiency · Combinatorial optimization · Branch-and-bound

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1 Introduction

Nowadays, many countries have been investing in technological innovations aiming at the energy efficiency of the several production sectors. This practice has also been applied in water supply companies. The consumption of electric energy, due to the pumping systems, represents the biggest parcel of the energy expenses in the water sector. Among the practical solutions which can enable the reduction of energy consumption, the change in the pumping operational procedures shows to be very effective, since it does not need any additional investment and, besides, the economy with the reduction of the energy cost occurs in a short term.

However, the task of the operators of the water supply networks (WSN) is very complex. Several distinct goals are involved in this process. In order to determine, among an extensive set of possibilities, operational rules that watch out for the quality of the service and that also provide economy with the cost of electric energy, the utilization of models which take into consideration all the elements involved is necessary. The technological advances in the computational area enabled, in the last years, the magnification of the quality of the scientific works related to the optimization of models, as well as the reduction of the energy cost in the operation of WSN. Nevertheless, most of the models developed were applied to specific cases.

For the determination of operational policies with reduced costs, several approaches have been reported in the literature. Models and algorithms aiming to minimize the operational costs in a WSN are presented by Brion and Mays (1991); Jowitt and Germanopoulos (1992); Ormsbee and Reddy (1995); Goldman and Mays (1999); Martinez et al. (2007); Rao and Salomons (2007); Salomons et al. (2007); Shamir and Salomons (2008); Vieira and Ramos (2008); Ramos and Ramos (2009); Cohen et al. (2009); Costa et al. (2010); Ramos et al. (2012); Kougias and Theodossiou (2013); Fayzul et al. (2014).

The optimization of pump scheduling in WSN is a hard combinatorial optimization problem, with discrete decision variables. Let n be the number of pumps in a WSN, assuming a planning horizon of 24 h, there are 2^{24n} possible solutions. In addition, the abovementioned problem has a set of hard operational constraints, related to the hydraulic features of the system, with implications in the complexity of the problem. The limitation of the models based on Operational Research approaches in WSN's concerns the complexity of the equations which guarantee the hydraulic equilibrium in the network and the difficulty to generalize these models for any WSN.

Traditionally, heuristic techniques such as Genetic Algorithms (GA's) are used for the determination of pump schedules with optimized operational costs. However, according to the experience of the authors, a GA has a great difficulty in finding high quality solutions. The random nature of the initial population, as well as the crossover and the mutation operations, does not guarantee the finding of feasible solutions. Therefore, the computational cost of this approach is too high. Aiming to reduce the computational times, Rao and Alvarruiz (2007) have introduced the concept of Artificial Neural Networks (ANN) for the substitution of the hydraulic simulations performed by EPANET, having expressive gains, resulting in the model proposed by Rao and Salomons (2007), which integrates ANN - GA.

Regarding Costa et al. (2010), the correction of infeasible solutions, in a Hybrid Genetic Algorithm (HGA) perspective, improves the quality of generated solutions and decreases the computational times substantially. However, these procedures imply high computational costs. Furthermore, the use of heuristic algorithms, such as a GA or HGA, does not guarantee the finding of global optimal solutions, as well as the gap between the best solution in the search domain. In the exact methods, an algorithm can prove the optimality of a given solution.

Nevertheless, the computational costs are usually higher than heuristic methods, for problems with several discrete decision variables. One can observe in WSN the number of pumps, in several practical situations, is not too high. Therefore, the application of exact methods based on the enumeration of the feasible solutions can lead to optimality, since the number of combinations is computationally tractable.

A Branch-and-Bound (B&B) algorithm is a divide-and-conquer strategy for problem solving. It divides the problem into small subproblems, by the decision variable fixing. The B&B algorithm was proposed by Lang and Doig (1960), for resolution of pure and mixed integer programming problems. Since the complete enumeration is impossible for problems with dozens of discrete variables, B&B uses the information about the feasibility of a given solution and the computation of lower bounds (in a minimization problem).

Other works that apply B&B in WSN are quite limited with relation to the other optimization methods (linear programming and heuristics). Costa et al. (2001) presents a B&B algorithm to the optimal design of water distribution networks. The objective function represents the global network cost. In the branching scheme, the nodes represent pipe diameters (discrete decision variables). This work did not consider the pump scheduling.

Ghaddar et al. (2014) proposes a Lagrangian decomposition to the pump scheduling problem, aiming the reduction of the search space, decoupling the problem into smaller subproblems. In each subproblem are generated solutions (feasible and infeasible) through a Lagrangian relaxation. For the infeasible solutions of each subproblem, a ILDS algorithm (Korf 1996) is applied for the correction of infeasibility. The above mentioned method finds high quality solutions, although without optimality guarantee. The constraints related to the energy conservation restrict the model to the wide-spread networks. In addition, Lagrangian relaxation requires a large number of parameters.

This paper aims at presenting an innovative Branch-and-Bound approach for the pump scheduling problem. To the best of the authors' knowledge, the proposed approach is the first to apply the B&B as an exact method for finding global optimal solution in the pump scheduling problem. In addition, the proposed algorithm can be used to any type of network (wide-spread or meshed) modeled on EPANET and it is parameter-free.

The remaining parts of this paper are structured as follow: (i) in the second section, the problem statement is presented; (ii) in the third section, the proposed algorithm is exposed; (iii) in the fourth section, the algorithm is applied and the results are discussed; (iv) at the end, the paper presents some conclusions and recommendations for future research.

2 Problem Statement

The search for the optimal control settings of pumps in a real WSN is seen as a problem of high complexity, due to the fact that it involves a high number of decision variables and several constraints, particular to each system.

The decision variables are the operational states of the pumps $\times t$ ($\times 1\, t$, $\times 2\, t$, …, $\times Nt$), where N represents the number of pumps and t is the time-step throughout the operational time.

To represent the states of the decision variables in each time-step, the binary notation was utilized. The configuration of the pump is represented by a bit where 0 and 1 represent the stated turned off and turned on, respectively.

The main goal of the model is to find the scheduling of pumps which proceed in the lowest energy cost possible scenario, in the operational time duration. However, in order to calculate this cost, several intermediate variables must be considered, in each time-step, for example, the variation of consumption, energy tariff pattern and the status of the pumps.

The objective function is the sum of energy generated by the pumps, in every operational time, due to the power originated from the service at the consumption points and storage of the tanks. It can be expressed according to Eq. (1).

In what follows, some notation is introduced to model a WSN with an Integer Programming Problem.

Sets

N	number of pumps
T	number of time steps
I	number of nodes in the WSN

The objective is to find the configuration of pumps and valves that proceeds at the possible lowest energy cost during the operational situation.

Parameters

E_{nt}	consumed energy (kWh) of the n^{th} pump in the t^{th} period.
C_{nt}	energy tariff cost of the n^{th} pump in the t^{th} period.
P_{it}	pressure on i^{th} node in the t^{th} period.
$P_{min\ i}$	minimum pressure required for i^{th} node.
$P_{max\ i}$	maximum pressure required for i^{th} node.
S_{jt}	level of j^{th} tank in the t^{th} period.
$S_{min\ j}$	minimum level of the j^{th} tank.
$S_{max\ j}$	maximum level of the j^{th} tank.
NA_k	number of actuation of k^{th} pump.
$NA_{max\ k}$	maximum amount allowed for k^{th} pump.

Binary Decision Variables

$$X_{nt} = \begin{cases} 1, & \text{if the } n^{\text{th}} \text{ pump is turned on in the } t^{\text{th}} \text{ period} \\ 0, & \text{otherwise} \end{cases}$$

The WSN problem can now be formulated as follows:

$$minimize\ z = \sum_{n=1}^{N} \sum_{t=1}^{T} C_{nt} E_{nt} X_{nt} \tag{1}$$

subject to:

$$P_{min\ i} \le P_{it} \le P_{max\ i} \quad \forall i^*, \forall t \tag{2}$$

$$S_{min\ i} \leq S_{it} \leq S_{max\ i} \quad \forall i, \forall t \tag{3}$$

$$S_{j(24h)} \geq S_{j(0h)} \quad \forall j \tag{4}$$

$$NA_k \leq NA_{\max\ k} \quad \forall k \tag{5}$$

$$X_{nt} \in \{0, 1\} \quad \forall n, \forall t \tag{6}$$

The objective function (1) minimizes the sum of energy generated by the pumps, in every operational time, due to the power originated from the service at the consumption points and storage of the tanks. The constraints (2) are related to pressures in the network nodes. For each time-step of operational time the pressures in some critical nodes must be between the minimum and maximum limits. The constraints (3) and (4) are related to the level of the storage tanks. The levels of the storage tanks must be between the minimum and maximum limits for each time-step. Besides, at the end of the operational time duration, they must be superior to the levels at the beginning of the time duration. This last constraint assures that the levels of the tanks do not lessen with the repetitions of the operational cycles. The constraints (5) are related to the actuation of the pumps. The number of pumping actions in the operational strategy must be inferior to a pre-established limit. This constraint, presented by Lansey and Awumah (1994), influences the maintenance of each pump, since the more it is put into action in a same operational cycle, the greater will be its wear. Finally the constraints (6) define the scope of the model variables.

3 Branch-and-Bound Algorithm

3.1 Background

A Branch-and-Bound is an implicit enumeration algorithm in which a large number of possible solutions are discarded. Dividing the search space in subsets, some of them can be eliminated from the search process, because they cannot lead to the optimal solution. Information about the quality or the feasibility of an incumbent solution can be used for the reduction of search space.

In a B&B algorithm, a bound is an optimistic estimative of the quality of a given solution when it becomes a complete solution. In a minimization problem, a lower bound is a solution which is less than or equal to every feasible solution in the search space.

Let *candidate_set* be the set of solutions which will be evaluated, *best_value* be the best evaluated value for objetctive function, *best_solution* be the best solution founded and *lb* be a lower bound for the problem, a standard B&B algorithm for a minimization problem is presented on Fig. 1.

In a B&B algorithm, for a minimization problem, a solution can be discarded from the search in two situations: (i) if the solution is infeasible; and (ii) if the lower bound of this solution is greater than the upper bound of some node of the search tree.

Fig. 1 A standard B&B algorithm

```
begin
while candidate_set is not empty do
      choose a branching node from candidate_set
      remove node k from active_set
      generate a solution from k and a lbi
      for i=1 to nk do
            if lbi worse than best_value then
                  eliminate solution;
            else if solution is a complete solution then
                  best_value • lbi
                  best_solution • solution i
            else add solution i to candidate_set
      end-of-for
end-of-while
end
```

There are two different strategies to transverse an enumeration tree: Depth-First-Search (DFS) and Breadth-First-Search (BDF). In the first strategy, the search is transversal in the graph because the algorithm explores as far as possible at each branch before proceeding a backtracking. In the second strategy, the search occurs in layers, because the algorithm inspects all the neighboring nodes.

3.2 Proposed Algorithm

The first stage of the process is characterized by the generation of a solution by fixing the decision variables (i.e. amount of turned on pumps). Next, these variables are converted to the binary format and used by EPANET (the hydraulic simulator), which calculates the hydraulic variables of the system, all of them being necessary to the evaluation of the solution. If the partial solution is feasible, the branching procedure occurs, expanding the partial solution in new $(n+1)$ partial solutions. This procedure is repeated until the 24th hour. In the enumeration tree, a path represents a partial solution. However, the hydraulic simulation is performed hourly. In the 24th hour, a path represents a complete solution. A given node in the proposed B&B represents the quantity of turned on pumps. After the determination of all feasible and complete solutions, the final stage is characterized by the calculation of the objective function, which is obtained from the total energy cost. The optimal solution is the feasible solution that has the minimal operational cost. The B&B for the WSN is described as follows. In this case, we have a pumping station with three pumps and an operation of 24 h per day. A schematic view of the proposed B&B is illustrated on Fig. 2. The beginning of the algorithm has an empty solution. In the first hour, one can have four possibilities: (i) not starting any of the pumps; (ii) starting one pump; (iii) starting two pumps; and (iv) starting three pumps. A given node is the amount of turned pumps.

For each generated solution, all the mentioned hydraulic variables are calculated by EPANET. If a given node consists of an infeasible solution after the EPANET hydraulic simulation, this node is pruned of the search, reducing substantially the implicit enumeration tree.

This type of pruning is the sole used in the proposed B&B, because the achievement of lower bounds for the problem in issue is not an easy task. Taking into consideration that constraints (2), (3) and (4) depend on the decision variable values, being calculated dynamically by EPANET, one cannot find lower bounds by the classic methods, like the linear relaxation.

In the B&B tree, the solutions are evaluated in a Breadth-First-Search strategy. In a same hour (level of the search), all the nodes are tested and, if necessary, pruned. There are not backtracking moves, such as in a Depth-First-Search strategy.

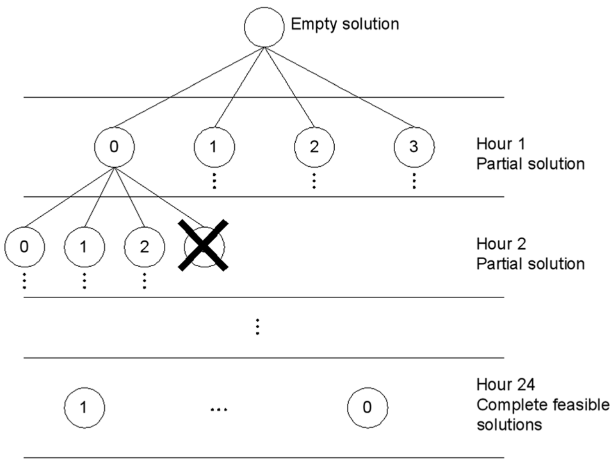


Fig. 2 A partial view of the implicit enumeration tree

In the next hour, for each feasible node, the hydraulic simulation is performed by EPANET and the feasibility of the solutions is determined. Infeasible solutions are pruned of the search. This procedure is repeated until the 24th hour, the final stage of the planning. At the end of the algorithm, the process determines all the feasible solutions for the problem. Depending on the hydraulic variables of the problem, the node (quantity of pumps) can be bounded. This occurs due to the existence of a hydraulic infeasibility in a given operational hour which is not corrected in the next hours.

Let us take the example illustrated on Fig. 2. While there was not any turned pump in the first operation hour, turning on three pumps in the second hour (partial solution 03) implied in the occurrence of a hydraulic infeasibility, interrupting the search of the partial enumeration tree in that branch. The remaining partial solutions in the second hour (00, 01 and 02) are feasible solutions, enabling the continuity of the branching process until obtaining a partial feasible solution or, in the 24th hour, a complete feasible solution. However, the solution vector, which is generated by the proposed B&B, has a size varying from 1 to 24, being defined for the hour (level of the search) that is processed. Each element of the vector represents the quantity of turned on pumps, as illustrated in Fig. 3.

In the coding scheme used by the proposed B&B, as shown in Fig. 2, it can be used, in a given operation hour, 0, 1, 2, or with 3 pumps. Before the solution evaluation, a procedure for the conversion of the codification is performed as follows: an amount of turned on pumps is converted to a binary vector of size N (number of pumps). In the case of one pump, initially, one can consider the first turned on (100). When this pump reaches the maximal number of actions, one can turn on the second pump and turn off the first pump (010), being considered one actuation. When the second pump reaches the actuation limit, the third pump is turned on and the second pump is turned off (001). Therefore, according to the constraint of the maximal number of actions reached, the

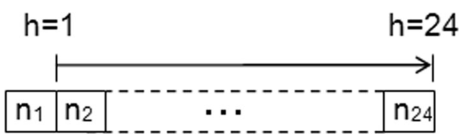


Fig. 3 Coding scheme used by B&B

configuration of the pumps is changed. For one turned on pump, the scheduling is as follows: (100), (010), (001). For the case with two turned on pumps, the configurations are changed as follows: (110), (101) and (011). For the case with three turned on pumps, the configuration is the one (111). This procedure decreases the amount of enumerated solutions substantially, because the search is disciplined for the number of actions in a sequential way. One can observe that this strategy is valid for the case of pumping stations with equal pumps.

In the case of a pump station with different pumps, the amount of evaluated solutions tends to increase. For example, with three identical pumps and one turned on pump, the solutions (100), (010) and (001) could represent the same solution and a single solution would be analyzed, considering the maximal number of actions. In the case of three different pumps, the above mentioned solutions would be distinct, increasing the amount of branched solutions.

The proposed algorithm is performed in 24 stages. Each stage represents the time step of 1 h. The procedure begins in the first hour, in which $n+1$ partial solutions are generated (n is the quantity of pumps). These solutions are stored in a set of partial solutions for the first hour (SPS_1). Then, all these solutions are evaluated, and, if the feasibility is confirmed, the branching procedure is performed, generating, for each feasible partial solution, $n+1$ new partial solutions, which will be stored in SPS_2.

From 2 to 23 h, for each feasible partial solution founded in the SPSh, $n+1$ solutions are created (branching procedure), which are stored in the SPS$h+1$. In the twenty-fourth hour, the branching procedure does not occur. The feasible complete solutions in the SPS24 are stored in a set of feasible complete solutions and the algorithm is ended with the calculation of the energy cost for all these solutions, performed by EPANET.

From Fig. 4, the hydraulic simulations are performed in the duration of 1 h (from h-1 to h). For each solution, the initial levels of the storage tanks, on step h, are the final levels in the time step h-1. These levels are stored in the course of the procedure, together with the partial solutions. Since the set constraints of type (5) is not dependent on the hydraulic simulation results, it is verified initially. The set of constraints of type (4) is verified only on time step 24. The other solutions are evaluated in all time steps.

When the end of the algorithm is reached, besides the definition of the global optimal solution, all the existent feasible solutions are also determined.

4 Case Study: a Hypothetical Network

Aiming at the evaluation of the proposed algorithm, the network Any Town (modified), also called AT(M) network, used by Rao and Salomons (2007) is used in this research. This new version is an expansion of the net created by Walski et al (1987). The AT(M) network is composed by a source of supply, three pumps and three storage tanks (Fig. 5).

The limitations of the tanks levels, of the nodes pressure and also of the amount of pump actions compose the constraints of the problem. The maximum, minimum and initial tanks' levels are 71.53, 66.53 and 66.93 m, respectively. The minimum pressures for the nodes are 51 m (node 90), 42 m (node 50) and 30 m (node 170). The time-step assumed is 1 h for an operational time of 24 h.

The demand patterns, the constraints and the tariff pattern applied at the network are the same utilized by Rao and Salomons (2007). In the proposed B&B algorithm, two special features were embedded: (i) in the peak hours of tariff (18 to 21 h), the actuation of all the three pumps was forbidden; and (ii) the maximal number of pump actions is three.

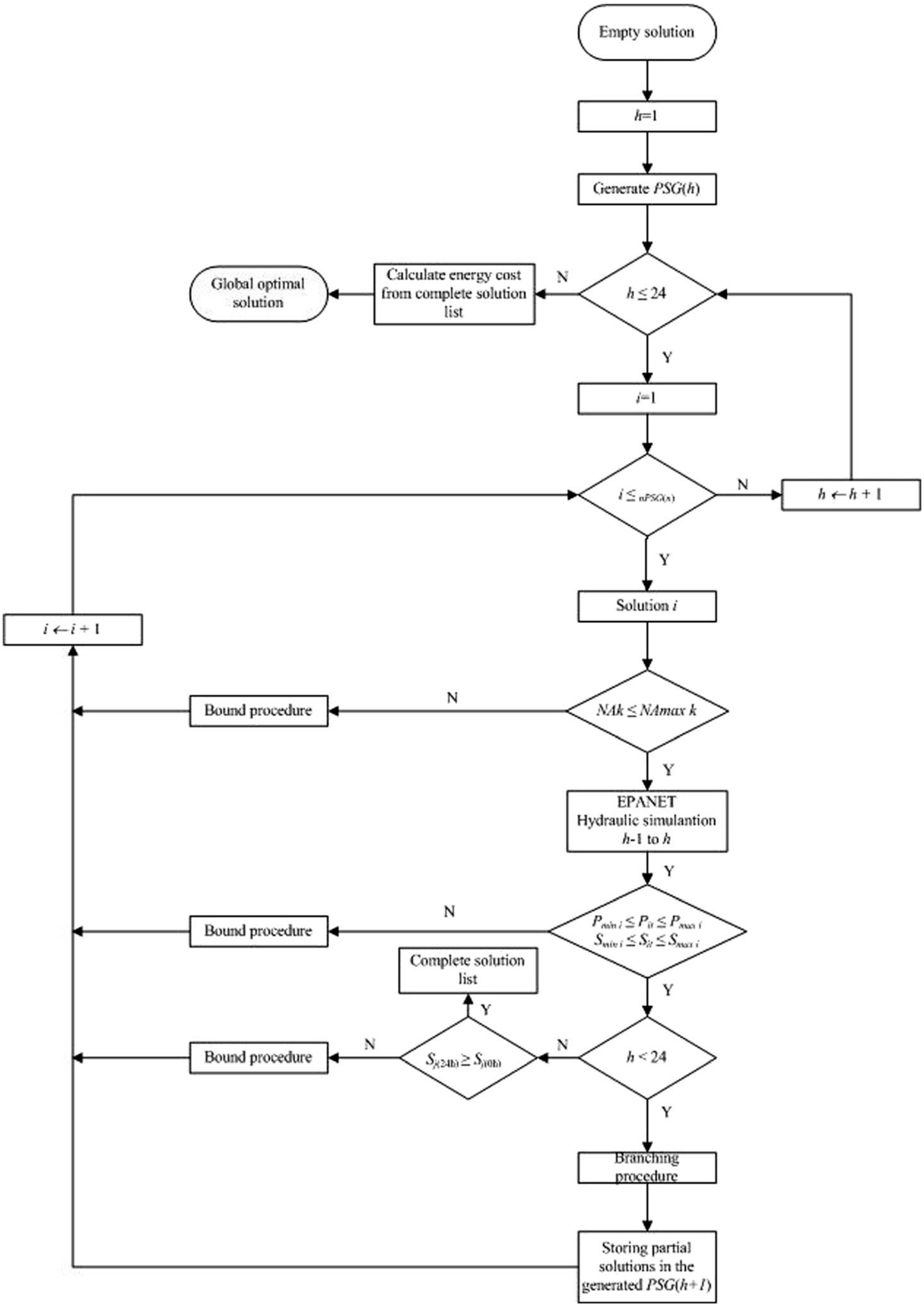


Fig. 4 Proposed algorithm

The tariff used by Rao and Salomons (2007) presents the higher values between hours 18 and 21, as illustrated in Fig. 7. Therefore, it was adopted in this period a strategy with the aim of does not turn on the three pumps. One can observe that turning on the three pumps in the most expensive period, certainly leads to more expensive solutions. Furthermore, taking into account

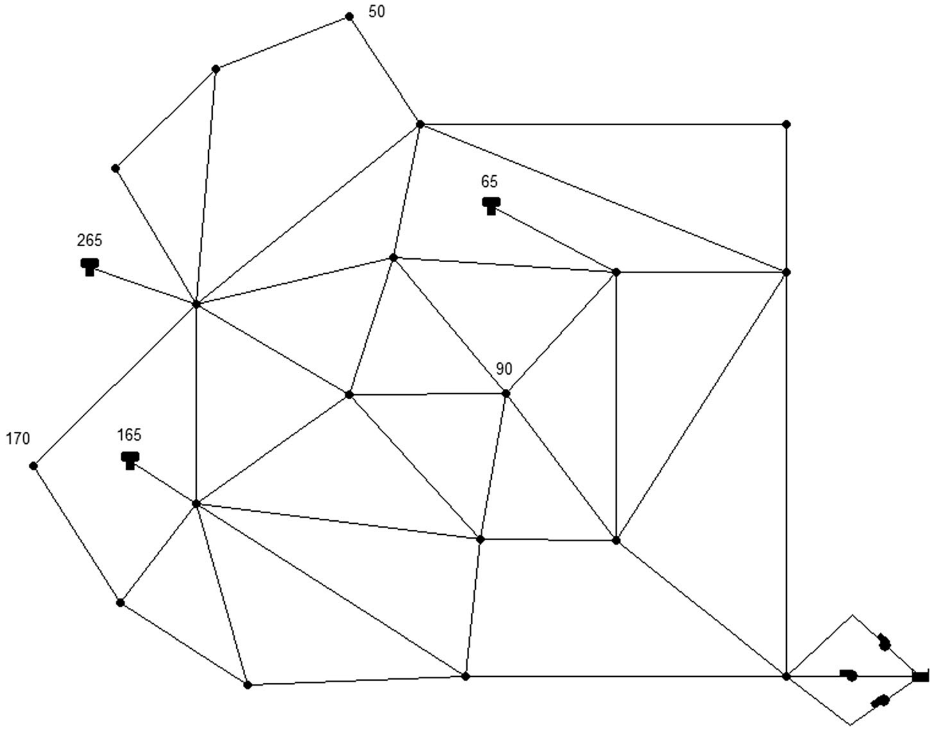


Fig. 5 AT (M) network, Font (Rao and Salomons 2007)

this strategy, one can reduce substantially the search space, because the branching process will have three branches (0 1 2) instead of four branches (0 1 2 3). In this period (hours between 18 and 21), for each feasible partial solution generated, in the hour *h*, will be generated three new partial solutions in the following hour, instead of four branches. This strategy has the objective to remove expensive solutions of the search space, improving the algorithm performance.

The reduction of the number of pump actions also contributes for the reduction of maintenance costs. According to Lansey and Awumah (1994), increasing the number of actions per operational cycle proportionally increases the wear of the pumps. Therefore, besides enhancing the algorithm performance, the proposed strategy involves the cost reductions in the long term.

5 Results and Discussion

The abovementioned algorithm was implemented in the Lazarus integrated development environment (http://www.lazarus.freepascal.org/). The computational experience was performed on a PC with Intel Core i7-4771 CPU 3.50GHz and 32GB memory. The computational results are on Table 1.

In the first hour, considering three pumps, there were created four partial solutions: 0, 1, 2 and 3. After the hydraulic simulations, it was verified that 3 solutions are feasible and 1 infeasible, as illustrated in Table 1. The partial solution 0 is infeasible because turning off all the pumps in the first hour, the level of the tanks will be less than the minimum, infringing the

Table 1 Computational results

Hour	Feasible solutions	Infeasible solutions	Total solutions	Hour	Feasible solutions	Infeasible solutions	Total solutions
1	3	1	4	13	194,904	71,499	266,403
2	10	2	12	14	412,249	172,463	584,712
3	27	13	40	15	910,649	326,098	1,236,747
4	75	33	108	16	2,111,104	620,843	2,731,947
5	197	103	300	17	4,414,277	1,919,035	6,333,312
6	511	277	788	18	6,817,056	2,011,498	8,828,554
7	1436	608	2044	19	10,522,678	3,111,434	13,634,112
8	3639	669	4308	20	13,714,154	396,367	14,110,521
9	7872	3045	10,917	21	19,648,694	7,779,614	27,428,308
10	18,989	4627	23,616	22	37,009,232	41,585,544	78,594,776
11	40,991	15,976	56,967	23	73,182,809	74,854,119	148,036,928
12	88,801	34,172	122,973	24	27,843,662	264,887,574	292,731,236

constraints of type (3). The cut of this infeasible solution will result in the exclusion of all the solutions that begins with 0, reducing the search space in 25 %.

After the feasibility of the solutions 1, 2 and 3 has been confirmed, the branching process is performed for each of these solutions, generating new solutions as follows: 10, 11, 12 and 13 (pump 1); 20, 21, 22 and 23 (pump 2); 30, 31, 32 and 33 (pump 3), being generated 12 (3×4) new partial solutions for the second hour, as illustrated in the Table 1. In this set of 12 partial solutions, it was verified that 10 solutions are feasible (11, 12, 13, 20, 21, 22, 23, 30, 31 and 32) and 2 solutions area infeasible (10 and 33). In the two infeasible solutions, it was performed the cut procedure. In the ten feasible solutions, it was run the branch procedure, generating 40 (10x4) new partial solutions for the hour 3, of which 27 solutions are feasible and 13 solutions area infeasible. In the 24th hour, after this procedure, it was founded 27,843,662 feasible solutions.

Analyzing the enumeration procedure illustrated in Fig. 6, in the cut of the partial solution 0, in the first hour, all the solutions in the search space, in which in the first hour there is no turned on pump, what are infeasible are excluded of the search process. These excluded solutions represents 25 % of the search space. In the second hour, the same situation occurs with the cuts of the partial solutions 10 and 33, resulting, for each of these solutions, in the exclusion of 6,25 % of the solutions in the search space. Therefore, it can be stated that the higher the amount of infeasible partial solutions generated, the more efficient will be the proposed algorithm. With the increase of h, each cut eliminates fewer solutions in the search space; however, the amount of cuts increases considerably during the process. Constraints of type (5) support this behavior of the algorithm, because the amount of actions increases during the day. According to the Table 1, it can be verified a substantial reduction of the generated feasible partial solutions in the 23rd hour and the complete feasible solutions in the 24th hour. This is due to the set of constraints of type (4).

The amount of possible solutions is extremely high; however, the majority of these solutions was not evaluated. This happens because the B&B prunes the partial solutions when any infeasibility occurs. The proposed algorithm obtained a set of 27,843,662 complete feasible solutions, in a computational time of 81.12 h. Although the computational time is

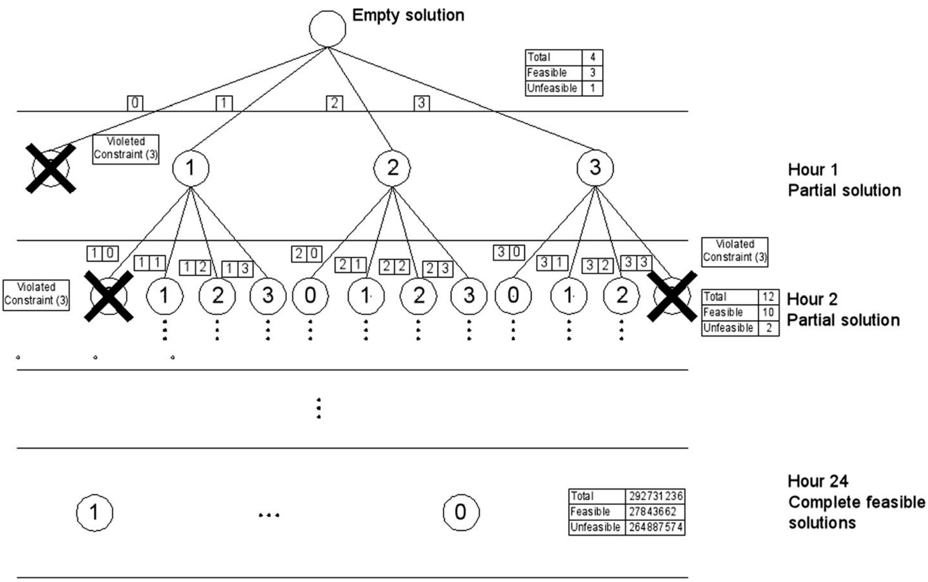


Fig. 6 Illustration of the branch and bound procedures in the case study

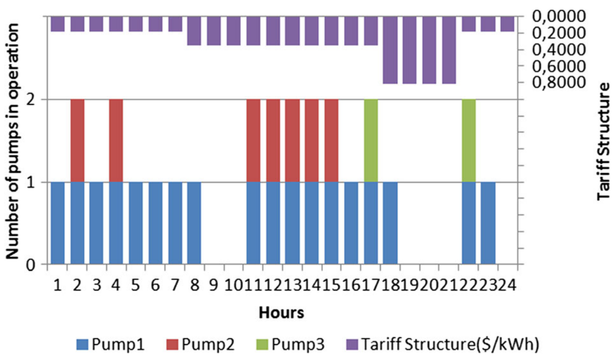
longer than the literature results, which are usually based in metaheuristcs, the guarantee of obtaining the global optimal solution is a powerful result.

In all feasible solutions, one global optimal solution with a cost of \$ 3578.66 is obtained. The global optimal solution is 121211110022222121000210. In this string with 24 elements, each component is the amount of turned on pumps in a given hour. The optimal solution is presented on Fig. 7.

For a B&B algorithm, in contrast to what happens with metaheuristics, the more constrained the problem is, the faster the search process is. In this paper, two sets of constraints were considered, (4) and (5), which were not considered by Rao and Salomons (2007). The achieved solution by the abovementioned authors had a cost of \$3612.84, more expensive than the proposed solution.

Figure 7 shows that in the hours on which the tariff cost is higher (hours 18 until 21), only one pump was turned on (at the 18 h) and the three pumps do not violate the constraint of the maximum number of pump actions.

Fig. 7 Global optimal control pumping strategy for a 24th planning period



On Fig. 8, the level variation of the three reservoirs is presented. The storage levels are between the minimum and maximum allowed values along the day, according to constraints of type (3), and the tanks' level at the end of the day are higher than the levels at the beginning of the day, according to constraints of type (4).

Regarding the pressures, according to constraints of type (2), the most critical situations occurred for nodes 55, 90 and 170, respectively with 42.61 m (hour 10), 51.52 m (hour 10) and 30.11 m (hour 21).

In this case study, the property of the hydraulic time step adopted in the EPANET was 30 min. When using a hydraulic time step of 60 min, a substantial reduction of the feasible solutions founded (251,542 solutions) was verified, as well as the computational time (less than 1 h). However, since the simulation results are more precise with a hydraulic time step of 30 min, this value was used.

To the best of the authors' knowledge, a metaheuristic based on a standard Genetic Algorithm would have difficulties in finding a feasible solution for the pump scheduling problem with harder constraints, as a minor number of pump actions. In contrast to what usually happens with the metaheuristics for the abovementioned problem, for the proposed B&B, the more constrained the problem is, the more efficient the algorithm is.

Aiming to evaluate the performance of the proposed approach in other operational contexts, two other scenarios were generated, with two and one pump actions. The results are on Table 2, as follows:

In the scenario with a maximum number of 2 pump actions, we have 1,447,133 complete feasible solutions, a global optimal solution with a cost of $ 3618.59 and a computational time of 36,914 s. In the scenario with a maximum number of 1 pump action, we have 313,340 complete feasible solutions, a global optimal solution with a cost of $ 3916.98 and a computational time of 425 s.

These results illustrate the robustness of the proposed approach. The use of harder constraints leads to faster computational times, as well as high quality solutions, in terms of

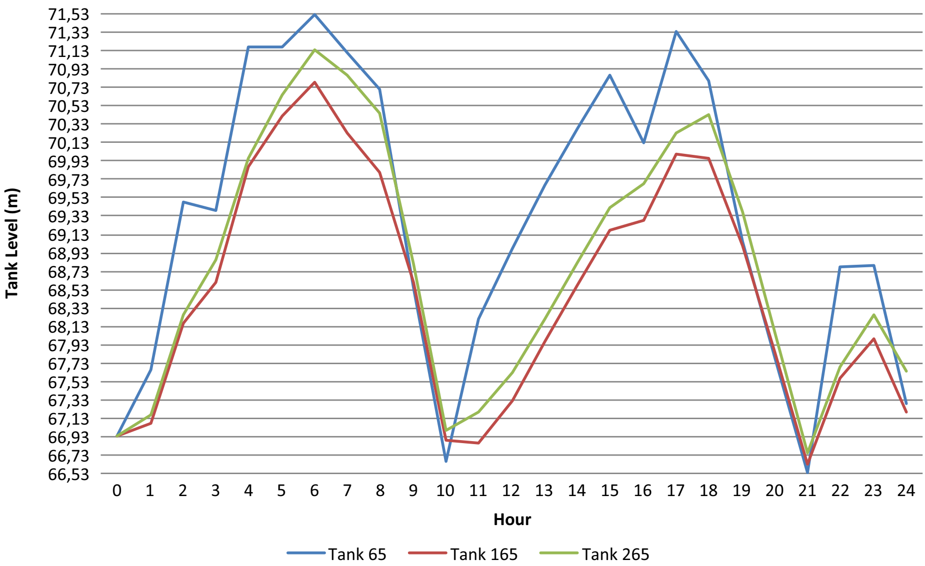


Fig. 8 Water-level variation for storage tanks

Table 2 Results of the alternative scenarios

Namax	Solution Time(sec) Cost ($)	pumps	hour																							
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
3	121211110022222121000210	1	■	■	■	■	■	■	■	■			■	■	■	■	■	■	■	■				■	■	
	292040	2		■		■							■	■	■	■	■									
	3578,67	3																	■					■		
2	122111111022212121000111	1	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■				■	■	■
	36914	2		■	■								■	■	■											
	3618,59	3															■		■							
1	122111111112221111100111	1	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■	■
	425	2		■	■																					
	3916,98	3												■	■	■										

energy costs. Results of these nature can aid the decision makers of WSN, leading to a better energy efficiency solution for each system.

It is possible to observe that the limitation of number of actions can lead to solutions with lower global costs, because the maintenance costs tend to decrease with a minor number of actions. In this sense, the proposed approach can propitiate fast and high quality results, with the flexibility of testing several operational scenarios.

In the B&B algorithm for integer programming, the more constrained is the problem better for the method because the amount of bounded solutions increases, reducing the number of evaluated solutions (Wolsey 1998). In relation to the pump scheduling, the constraint of maximal number of actions is the main reason for the efficiency of the proposed approach. The lower the number of actions, the lower the amount of feasible solutions in the search space, reducing substantially the computational costs. In pumping stations with a large amount of pumps, the method can be applied with a reduced amount of pump schedules.

6 Conclusions

In this research, an innovative Branch-and-Bound algorithm is presented in the search for the operational strategy of lower energy costs for WSN. The proposed approach was validated in the hypothetical network presented in specialized literature AT (M). To the best of the authors' knowledge, there is no other competitive exact method for the determination of optimal operational procedures in WSN in the available literature.

On this research, we were able to find the global optimum solution of the hypothetical network AT (M). In addition to the proof of global optimum, the proposed B&B algorithm found all the feasible solutions for the evaluated network. In practice, for different demand profiles, these results can be stored in a database and used by managers, improving the efficiency of real systems.

The developed algorithm enumerates all the feasible solutions for a given problem, excluding a large amount of infeasible solutions which not need to be simulated. The filtering procedure of the feasible solutions has demonstrated to be efficient because the pump scheduling is severely constrained. The restriction of the maximal number of pump actions takes the problem even more constrained, increasing substantially the amount of infeasible solutions which will be eliminated of the search process. For pumping stations with a small amount of pumps or for systems in which the managers aim to reduce the number of pump actions and consequently minimize the maintenance costs, the authors consider the proposed approach robust.

Although the proposed approach propitiates the guarantee of the optimality in the achieved solution, the computational times were longer than the times presented by other authors. Three main strategies could be used to mitigate this limitation: (i) the use of parallel computing (López-Ibáñez et al. 2008); (ii) the use of Artificial Neural Networks (ANN) (Rao and Alvarruiz 2007) to replace the hydraulic simulator EPANET; (iii) the use of harder constraints, aiming at the increase of the number of branches pruned in the search process; and (iv) network simplification by means of a skeletization strategy (Grayman and Rhee 2000).

As a natural development of this research work, the authors are currently working with the development of lower bounds for the water supply network problems, aiming at a performance improvement of the Branch-and-Bound algorithm, such as the evaluation of a Depth-First-Search strategy in the abovementioned problem.

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