Approaching Single-Photon Pulses with Weak Coherent States and Nonlinear Phase Modulation

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***Abstract*— Single-photons sources are crucial devices in the development of practical quantum communication and quantum computation. However, such sources are still a technological challenge. One option is to use weak coherent states to mimic single-photon pulses or to produce a pair of photons with parametric down conversion and to detect one of them to herald the presence of the other. In this direction, the present work shows how to produce better approximations of single-photon pulses by using a nonlinear phase modulator in a Mach-Zehnder interferometer. The proposed scheme is explained and applications are discussed.**

Keywords*— *Single-photon, photon counting, nonlinear phase modulation, quantum communication.

I. Introduction

Single-photon sources are crucial devices in the development of quantum communication and quantum computation. For example, the security of quantum key distribution (QKD) relies on the assumption that the important information to form the key is carried by single-photon pulses. On the other hand, multiphoton states like cluster states are important resources for quantum computation and they can be built by processing single-photons. The simplest example is the two-photon Bell state that can be probabilistically produced by using two single-photons, two polarization rotators and a polarizing beam splitter. Two two-photon states, by its turn can be transformed in a four-photon entangled state, and so on. However, to construct a true single-photon source is not a trivial task. There are several approaches using quantum dots, atomic ensembles, parametric down conversion and four-wave mixing in optical fibers, for example [1-6]. The last two are in fact photon pair sources, where one of them is detected to herald the presence of the other. By far, the easiest and cheapest way to mimic a single-photon source is to use weak coherent states (with mean photon number around 0.1). Such approach has been used in quantum key distribution setups (QKD). In this case, the multiphoton pulses that are eventually produced opens a loophole in the security. In order to avoid this problem, the QKD with decoy states is used, what increases the complexities of the optical hardware and classical data processing [7-12]. In this direction, the present work proposes a new device, a Mach-Zehnder interferometer employing a nonlinear phase modulator (NL_MZI) that improves the pseudo-single-photon source based on weak coherent states.

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This work is outlined as follows: in Section II the Mach-Zehnder interferometer using nonlinear phase modulator is described; In Section III some applications are discussed and the conclusions are drawn in Section IV.

II. Mach-Zehnder Interferometer with Nonlinear Modulation

In quantum optics, the phase modulation is described by the unitary operator $exp\left(i\theta\hat{N}\right)$, where $\hat{N}$ is the number operator. When the number state $|n\rangle$ is phase-modulated, the output state is

$$e^{i\theta\hat{N}}\left|n\right\rangle = e^{in\theta}\left|n\right\rangle. \quad (1)$$

Therefore, the phase modulation of the coherent state $|\alpha\rangle$ produces the following state

$$e^{i\theta\hat{N}}\left|\alpha\right\rangle = \sum_n e^{-\frac{|\alpha|^2}{2}}\frac{\left(\alpha e^{i\theta}\right)^n}{\sqrt{n!}}\left|n\right\rangle = \left|\alpha e^{i\theta}\right\rangle. \quad (2)$$

As one can note in eq. (2), the phase of the output state does not depend on the number of photons of the pulse. Hence, if the pulse sent by Alice is a multiphoton pulse ($n > 1$), all photons will have the same phase. The nonlinear phase modulation, by its turn, is an operation that introduces a phase in the number state that depends on the photon number. The nonlinear phase modulation is described by the unitary operator $U_x = \left(I - ix\hat{N}\right)/\left(I + ix\hat{N}\right)$. When the number state $|n\rangle$ is phase-modulated by U_x the output state is

$$U_x\left|n\right\rangle = \frac{I - ix\hat{N}}{I + ix\hat{N}}\left|n\right\rangle = \frac{1 - ixn}{1 + ixn}\left|n\right\rangle = e^{-i2\tan^{-1}(nx)}\left|n\right\rangle. \quad (3)$$

Therefore, the nonlinear phase modulation of the coherent state $|\alpha\rangle$ produces the following state

$$U_x\left|\alpha\right\rangle = \frac{I - ix\hat{N}}{I + ix\hat{N}}\left|\alpha\right\rangle = \sum_n e^{-\frac{|\alpha|}{2}}\frac{\alpha^n}{\sqrt{n!}}e^{-i2\tan^{-1}(nx)}\left|n\right\rangle. \quad (4)$$

As one can note in eq. (4), differently of what is described by eq. (2), the phase depends nonlinearly on the photon number n. Hence, the phase of the optical pulse will be different if the pulse has one or two photons, for example.

The device responsible for improving the pseudo-single-photon source is the Mach-Zehnder interferometer with nonlinear phase modulator shown in Fig. 1.

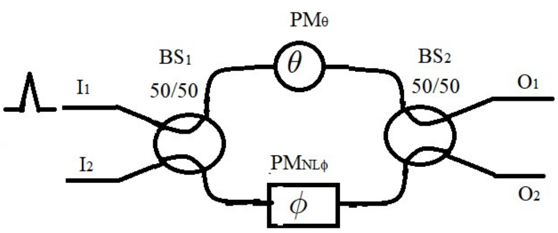

Fig. 1 - Mach-Zehnder interferometer with nonlinear phase modulator.

The optical pulse in the weak coherent state ($|\alpha|^2 \approx 0.1$) is divided by the balanced beam splitter BS_1. The pulse that takes the upper arm is phase modulated by PM_θ with $\theta = \pi/2$. The second pulse that takes the lower arm is nonlinearly phase-modulated by $PM_{NL\phi}$ with $\phi = -2\tan^{-1}[n\tan(-\pi/4)]$. Both pulses will arrive at BS_2 at the same time, one with phase θ and the other with phase ϕ, and they will suffer interference. The quantum state just before BS_2 is

$$\begin{aligned} |\psi\rangle &= U_{BS} e^{i\theta\hat{N}} \left|\alpha/\sqrt{2}\right\rangle U_x \left|i\alpha/\sqrt{2}\right\rangle \\ &= \sum_n \sum_m e^{-\frac{|\alpha|^2}{2}} \frac{i^m \left(\alpha/\sqrt{2}\right)^{n+m} e^{in\theta} e^{-i2\tan^{-1}(mx)}}{\sqrt{n!m!}} U_{BS} |n\rangle |m\rangle, \end{aligned} \tag{5}$$

where U_{BS} is the unitary operator that models the beam splitter. When the pulse at the input I_1 has only one photon one has $n + m = 1$, hence the state in (5) is reduced to

$$\begin{aligned} |\psi\rangle &= e^{-\frac{|\alpha|^2}{2}} \frac{\alpha}{\sqrt{2}} \left[e^{i\frac{\pi}{2}} U_{BS} |1\rangle|0\rangle + ie^{-i2\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)} U_{BS} |0\rangle|1\rangle \right] \\ &= e^{-\frac{|\alpha|^2}{2}} \alpha |1\rangle|0\rangle. \end{aligned} \tag{6}$$

Therefore, when the input state has only one photon it will always emerge at output O_1. When the pulse at the input I_1 has only two photons one has $n + m = 2$ and the state in (5) is reduced to

$$\begin{aligned} |\psi\rangle &= -e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^2}{2} \left[\frac{1}{\sqrt{2}} U_{BS} |2\rangle|0\rangle + iU_{BS}|1\rangle|1\rangle + (i)^2 e^{i0.7048\pi} U_{BS} |0\rangle|2\rangle \right] = \\ &= -e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^2}{2} \left[\left(\frac{1+e^{i0.7048\pi}+2i}{2\sqrt{2}} \right) |2\rangle|0\rangle + \left(\frac{1-e^{i0.7048\pi}}{2} \right) |1\rangle|1\rangle + \right. \\ &\qquad \left. + \left(\frac{1+e^{i0.7048\pi}-2i}{2\sqrt{2}} \right) |0\rangle|2\rangle \right]. \end{aligned} \tag{7}$$

The probabilities of the possible outcomes are: $|20\rangle \rightarrow \exp(-|\alpha|^2)|\alpha|^4/4$, $|11\rangle \rightarrow \exp(-|\alpha|^2)|\alpha|^4/5$, and $|02\rangle \rightarrow \exp(-|\alpha|^2)|\alpha|^4/20$. In other words, when the input pulse has two photons, with probability 0.5 both of them will emerge from the Mach-Zehnder at output O_1, with probability 0.4 one of them emerge from the Mach-Zehnder at output O_1 and the other at output O_2. At last, both photons will emerge from output O_2 with probability 0.1. The great advantage of the Mach-Zehnder with nonlinear modulation is the fact that it reduces the probability of two photons at the output without decreasing the single-photon probability at the output, what would occur, for example, if only a beam splitter is used.

III. Applications of the Mach-Zehnder Interferometer with Nonlinear Modulation

As explained since the beginning of this work, the first application of the NL_MZI is to improve the performance of a pseudo-single-photon source based on weak coherent states. For simplification, only the situations with 1 and 2 photons will be considered, since the input coherent state considered has mean photon number $|\alpha|^2 = 0.1$. Without using the NL_MZI, the probabilities of producing 1 (p_1) and 2 (p_2) photons (given that the pulse has photons) are, respectively, p_1 = 0.9508 and p_2 = 0.047. On the other hand, when the NL_MZI is used one gets the following probabilities: p_1 = 0.9696 and p_2 = 0.0235. Thus, the ration p_1/p_2 is twice better when the NL_MZI is used. This can be improved if NL_MZIs are used in cascade but this issue will not be discussed in this work

A second application of the NL_MZI is in photon counter devices [13-18]. The basic scheme is shown in Fig. 2, where a fiber ring resonator is used to implement a time multiplexing scheme.

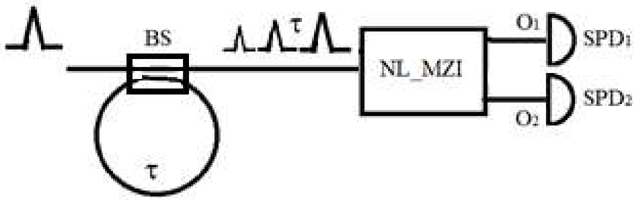

Fig. 2 – Photon counter using NL_MZI.

In the traditional photon counter with time (or spatial) multiplexing, the input multiphoton pulse is split in several small optical pulses having $|\alpha|^2 << 1$. These pulses are sent directly to single-photon detectors (SPD) that can detect one photon but cannot distinguish single-photons from multiphoton pulses. Therefore, multiphoton photon pulses are counted as they were single-photon pulses, causing an error in the photon counting. On the other hand, if one uses the NL_MZI as shown in Fig. 2, each two-photon pulse will be counted as a single-photon pulse with probability equal to 0.5 (instead of 1).

IV. Conclusions

Weak coherent states have been used to mimic single-photon pulses since the early stages of QKD development. However, the multiphoton pulses (mainly two-photon pulses that are the most frequently produced multiphoton pulse) decrease the security of QKD setups forcing the implementation of extra steps in the QKD protocol. On the other hand, the nonlinear phase modulator here described in a Mach-Zehnder interferometer transforms the input coherent state in a different quantum light state, eq. (5), that has a lower probability to have more than one photon. Hence, the usage of NL_MZI improves the security of the QKD setup.

To find out the physical implementation of the operator U_x given in eq. (3) and to know how much loss it will introduce are still open problems.

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