

# Construction and PID Control for Stability of an Unmanned Aerial Vehicle of the Type Quadrotor

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**Abstract**— This paper presents the development of an unmanned aerial vehicle of type quadrotor, its dynamic model, besides simulations and tests of a PID controller for the projected structure embedded stabilization (vertical direction motion). This vehicle is characterized by having four motors, which are responsible for generating the platform movement. This work objective is to show a structure, type quadrotor, development (design and construction), presenting a mathematical model of quadrotor based on the Newton-Euler formalism and a microcontroller system project used for the PID controller, used for stabilization and driving the motors, as well as the classical PID controller simulation in Simulink®/Matlab environment and the tests in the developed structure.

**Keywords**- Quadrotor; Unmanned Aerial Vehicle; PID Controller; Dynamic Model.

## I. INTRODUCTION

In the last years, there is a growing interest by researchers and companies in autonomous platforms that can replace man in certain activities that may be considered dangerous for him. The use of Unmanned Aerial Vehicles (UAV) has become an alternative for many of these activities. Among UAV rotorcraft structures types, the ones with four helices are gaining prominence. Numerous studies have been performed on the dynamic modeling and control methods development for automatic flight stabilization and regulation.

A quadrotor is a helicopter type which is held by four propulsion engines. As the four propellers are positioned horizontally, adjacent propellers rotate in the opposite direction, not requiring a motor spray to compensate the thrusters angular momentum. Being a dynamic system, when its motors speed changes, position will also change. Such vehicles are inherently unstable and underactuated systems [1].

Among all quadrotor applications, some are the use for security and surveillance, military, filming and photography. There is no particular application for the structure to be built. However, it is intend to use it for educational purposes and as a platform for testing control algorithms.

This article's objectives are listed below: to present a quadrotor design to meet the size and weight requirements, be able to take off and take extra payloads, Fig.1, developing a

simulator capable of supporting several different control algorithms to be tested in platform, the dynamic model showing the constructed platform using Simulink®/Matlab, developing a controller capable of stabilizing the quadrotor in a given reference position. The task is to make the quadrotor stay at a certain height, defined by three angular positions from the proportional, integrative and derivative control (PID).



Figure 1. A picture of the developed UAV-type quadrotor.

The paper remainder is organized as follows. Section 2 describes the structures. The dynamic quadrotor model is described in Section 3. Section 4 shows some developed platform details. The control used for stabilization is presented in section 5. Results and conclusions are presented in sections 6 and 7 respectively.

## II. QUADROTORS

In recent years, due to advancement in technology on miniaturization of sensors and processors, many small size UAVs have been developed, both for commercial purposes and for research. Several research laboratories and universities began quadrotors projects, but autonomous flight complete development in all environments still is a challenge [2]. It is

common practice that each research group develops its own device.

The AR.Drone [3] and the Draganflyer RC Toys [4] are some of the most famous commercially sold. The X4-Flyer [5], STARMARC [6], OS4 [1] are some of the most popular used for research around the world, and several types of controls tested on platforms.

In Brazil, there are several research groups conducting research on quadrotors. The Federal University of Espírito Santo dissertation is limited to the design, hardware and firmware construction that gives grant to implement some control technique [7], another quadrotor was developed by a partnership between the Federal University of Rio Grande do Norte and the State University of Feira de Santana, the platform will be used in monitoring oil facilities, distribution systems and electric power transmission [8], among other platforms.

### III. QUADROTOR DYNAMIC MODEL

A quadrotor operation is possible thanks to four rotors. These rotors are comprised of four motors and two propellers pairs, one pair is used to turn in one direction and other one is used in reverse direction. Thus, two rotors spin clockwise and the other two in a counterclockwise direction. The motors correct arrangement eliminates the torque effect generated by them. Then, adjacent helices should rotate in opposite directions, as used in 1907 by brothers Breguet [9]. Thus, an engine torque eliminates the other one effect causing the platform to stay in balance at a given position when the motors are at the same angular velocity.

In Fig. 2 the possible movements are illustrated, its arrow width is proportional to the rotors speed. Motors 1 and 3 rotate counterclockwise and motors 2 and 4 turn clockwise. Setting motors 2 and 4 speed and by increasing motor 3 speed relative to motor 1 speed, moves quadrotor freight, as shown in Fig. 2 (a). The opposite occurs when motor 3 speed is greater than engine 1 speed, Fig. 2 (b).

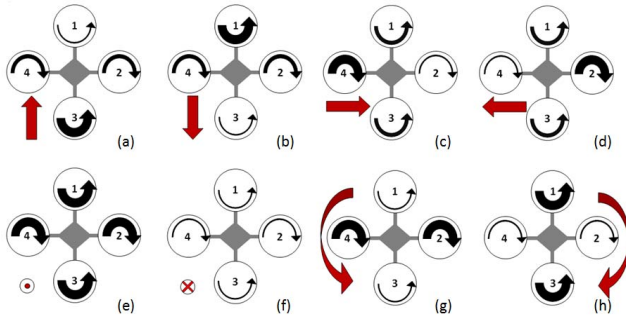


Figure 2. Illustrative propellers rotation (black arrows) and quadrotor resulting movement (red arrows) sketches.

Left and right movements occur when motors 1 and 3 speed remain the same and motor 2 varies in relation to motor 4 speed, Fig. 2 (c) and (d).

Setting two motors speed on the same axis and increasing or decreasing other motors speed makes the quadrotor move forward or backward Fig 2 (a) and (b). The left and right movements occurs when motors 1 and 3 speed remain the same and motor 2 varies in relation to motor 4 speed, Fig 2 (c) and (d).

#### A. Newton-Euler model

The model used to represent the quadrotor behavior in this work consists in the rigid bodies dynamic model using forces and torques [5] [10].

Fig. 3 has a coordinate systems illustration used to describe motion equations. Considering coordinate E system fixed to the ground and coordinate B system fixed on quadrotor center.

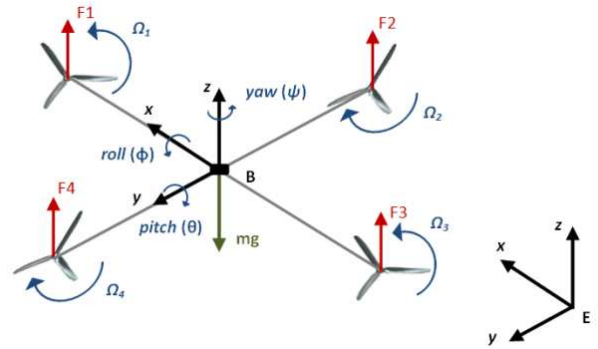


Figure 3. Coordinates system.

The equations developed assume that the quadrotor structure and the propellers are rigid, the gravity center is at the structure center, the structure is symmetrical, the motors are identical and aerodynamic lift and drag are proportional to the square of rotational speed of the motors.

The Euler angular orientation provides forces and moments for height control and position system [11]. The Euler-Newton equation for rigid bodies describes rotation and translation combination given by [12]:

$$\begin{bmatrix} mI_{3 \times 3} & 0 \\ 0 & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} \dot{v}_{3 \times 3} \\ \dot{w}_{3 \times 3} \end{bmatrix} + \begin{bmatrix} w_{3 \times 3} \times m v_{3 \times 3} \\ w_{3 \times 3} \times I_{3 \times 3} w_{3 \times 3} \end{bmatrix} = \begin{bmatrix} F_{3 \times 3} \\ \tau_{3 \times 3} \end{bmatrix} \quad (1)$$

where  $m$  is the system total mass,  $I \in \mathbb{R}^{3 \times 3}$  is the quadrotor inertia matrix,  $v = [v_x \ v_y \ v_z]^T$  and  $w = [\omega_x \ \omega_y \ \omega_z]^T$  are respectively linear and angular rigid body velocity and  $x$  the cross product. The force vector produced by the propellers is given by  $F = [f_x \ f_y \ f_z]^T$  and the torques vector  $\tau = [\tau_x \ \tau_y \ \tau_z]^T$ . The torques  $\tau_x$ ,  $\tau_y$  and  $\tau_z$  which are applied along the axes may be described as the difference between the generated torque of each motor for each axis.

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} lb(\Omega_4^2 - \Omega_2^2) \\ lb(\Omega_3^2 - \Omega_1^2) \\ d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{pmatrix} \quad (2)$$

where  $l$  is the distance between the quadrotor center and the propeller center,  $d$  is the drag coefficient,  $b$  is the thrust coefficient and  $\Omega_i$ ,  $i=1, \dots, 4$ , represents the  $i$ th propeller angular speed.

The matrix  $R$  projects the B coordinate system vectors in the E coordinate system for inclination angles. This matrix is defined as:

$$R = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi + s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - s_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \quad (3)$$

where  $C$  and  $S$  stand for the cosine and sine functions, respectively.

Thus, the quadrotor dynamic model as a result of pitch or roll rotations using the matrix  $R$  together with Newton's second law:

$$\ddot{x} = (\cos \phi \cdot \text{sen } \theta \cdot \cos \psi + \text{sen } \phi \cdot \text{sen } \psi) \frac{1}{m} U_1 \quad (4)$$

$$\ddot{y} = (\cos \phi \cdot \text{sen } \theta \cdot \text{sen } \psi - \text{sen } \phi \cdot \cos \psi) \frac{1}{m} U_1 \quad (5)$$

$$\ddot{z} = -g + (\cos \phi \cdot \cos \theta) \frac{1}{m} U_1 \quad (6)$$

$$\ddot{\phi} = \dot{\theta} \dot{\psi} \left( \frac{I_y - I_z}{I_x} \right) - \frac{J_r}{I_x} \dot{\theta} \Omega + \frac{1}{I_x} U_2 \quad (7)$$

$$\ddot{\theta} = \dot{\phi} \dot{\psi} \left( \frac{I_z - I_x}{I_y} \right) + \frac{J_r}{I_y} \dot{\phi} \Omega + \frac{1}{I_y} U_3 \quad (8)$$

$$\ddot{\psi} = \dot{\phi} \dot{\theta} \left( \frac{I_x - I_y}{I_z} \right) + \frac{J_r}{I_z} U_4 \quad (9)$$

where  $J_r$  is the rotor inertia,  $\phi$ ,  $\theta$  and  $\psi$  are angles in roll, pitch and yaw, respectively, and  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  the axes  $x$ ,  $y$  and  $z$  inertia moments. System inputs are presented in  $U_1$ ,  $U_2$ ,  $U_3$  and  $U_4$ , representing entries control for altitude, roll, pitch and yaw, respectively, shown in the following equations:

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (10)$$

$$U_2 = b(\Omega_4^2 - \Omega_2^2) \quad (11)$$

$$U_3 = b(\Omega_3^2 - \Omega_1^2) \quad (12)$$

$$U_4 = b(\Omega_2^2 + \Omega_4^2 - \Omega_3^2 - \Omega_1^2) \quad (13)$$

#### IV. PLATFORM DEVELOPED

The used structure design and components were determined according to other platforms successes, such as those described in [7];[13];[14];[15];[16];[17], besides the platforms already presented in the second section. The design and structure components developed in this article are described below.

We opted for an aluminum profile in "U" form as the main structure base, commonly used as door frame and some glasses corner. The aluminum profile was arranged in a cross and the edge were connected by steel cords to prevent problems, contributing to the increased rigidity, thus avoiding using more aluminum profiles, which eventually would make the structure heavier [18].

The propulsion system was made by four engines *Brushless Direct Current (BLDC)* from EMAX manufacturer, model CF2822, and propellers with three blades from GWS manufacturer, model HD-9050x3, for two normal clockwise direction and two reverse (with inverted attack angle) for counterclockwise direction. With electronic speed control (ESCs) circuit for each motor from Hobby King, of the 30A.

The circuit responsible for passing commands to the quadrotor motors was developed using the dsPIC30F4011 microcontroller, Microchip family.

The commands are sent by computer via radio frequency to another radio that is connected to a microcontroller in the structure. This handles incoming data and sends commands to motors ESCs. Commands sent are altitude and flight direction. The radio is Easy-Radio ER400TRS. The system operating diagram is shown in Fig. 4.

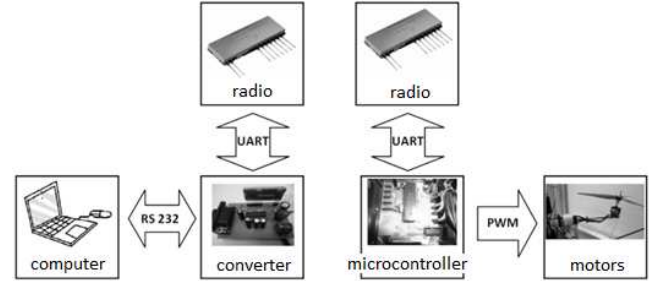


Figure 4. A basic diagram of the main parts comprising the developed quadrotor-type UAV.

#### V. STABILIZATION CONTROL

The simulator starts at initial conditions sub-block, the block is used to control desired positions, and subsequently propellers speeds are estimated in the dynamic engine block. Estimated rotor speed feeds the block aerodynamic dynamic system that generates forces and moments of each propeller. Together with dynamic system block measured state checks the new state. Propellers speed, accelerations and structure graphics can be viewed in the respective green blocks as seen in Fig. 5.

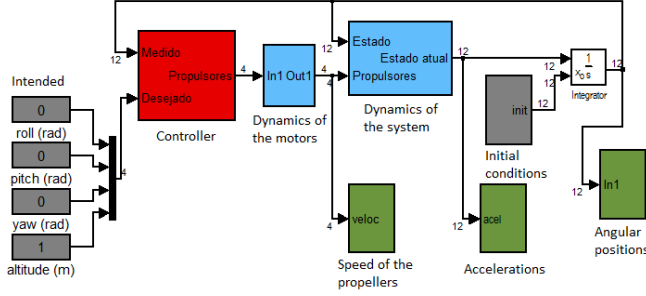


Figure 5. Simulator block diagram developed in the Matlab Simulink.

Initially, in order to stabilize the quadrotor angles we chose to use a separate controller PID for each angle. The error is the difference between the desired angle and the estimated angle. The scheme representing the closed loop control of each angle analyzing the error signal is shown in Fig. 6.

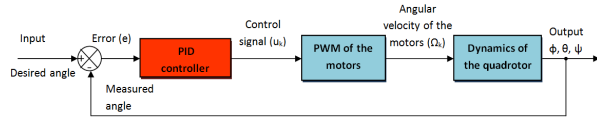


Figure 6. Angle control scheme.

The continuous error signal at controller input is collected and converted into digital signals, while the controller output is converted to an analog signal feeding the continuous process. The controller inputs are positions sent by the computer and the values measured by the sensors. The controller output contains the angles adjusted values and the altitude value that are converted to Pulse Width Modulation PWM signals of each motor. Quadrotor position is estimated using the sensor inertia moment readings fixed in the structure.

A controller with proportional, integral and derivative (PID) action is a controller that covers the three control modes. In continuous time domain, the input-output PID controller relationship to the  $k$ -th angle,  $k = 1, 2$  and  $3$ , is given by:

$$u_k(t) = u_{prop}(t) + u_{int}(t) + u_{der} = K_p^{(k)} e_k(t) + K_i^{(k)} \int_0^t e_k(t) dt + K_d^{(k)} \frac{de_k(t)}{dt} \quad (14)$$

Three controllers parameters at each position (nine in all) were selected manually by closed loop system using poles transfer function analysis.

In (10) to (13) it can be written as a linear system as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -bl & 0 & bl \\ -bl & 0 & bl & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \quad (15)$$

With outputs linearization, depending on motors angular speeds is possible to apply the classical control on structures of this type. Researchers like [19], [20] and [21] used this control type strategy adjusting the parameters according to their structure built. Although each research group had build their own structure, the PID control to stabilize showed good results both in simulation and in tests checked in real structures.

## VI. RESULTS

The tests were performed indoors and started from the construction beginning. The quadrotor as a whole is the result of design methodology and meets size and weight requirements.

Controller efficiency was simulated for various error values, altering the structure behavior in various ways, in order to also check the time that the controller takes to stabilize the vehicle. Its know that the difference between the measured and the desired angle would be of 0 radians Fig. 7.

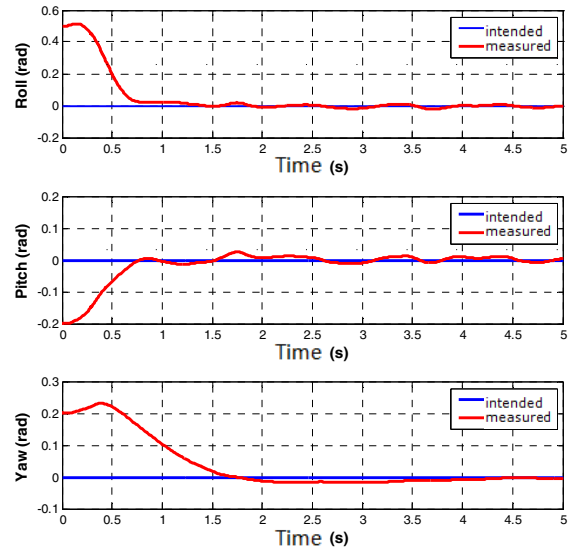


Figure 7. System response using classical PID controller to stabilize, with an error of 0.5 rad in the initial roll angle, pitch angle -0.2 and 0.2 in the yaw angle, with added noise at the system output, using a filter lowpass. Coordinates System.

The stabilization controller was implemented using the discrete PID control equation. To minimize sensors noise we used a moving average filter, checking the platform angular position using accelerometers and gyroscopes (MMA7361L and LPR510AL respectively). Fig. 8 shows a few results presented on the platform embedded controller.



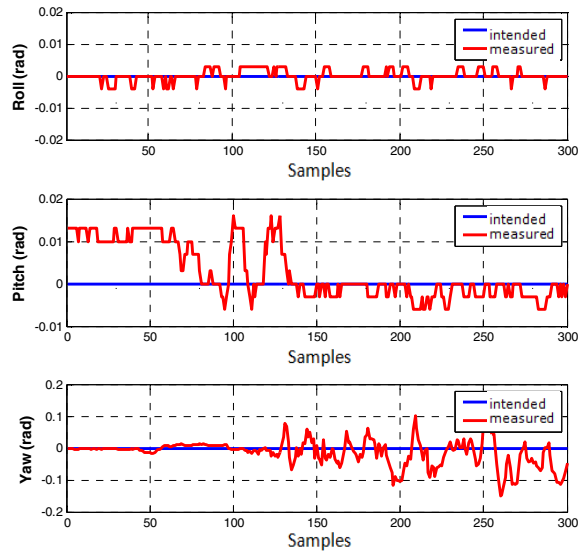


Figure 8. Angles of the platform stabilized by PID control embedded in the microcontroller.

## VII. CONCLUSIONS AND FUTURE WORK

It was possible to verify the proposed controller efficiency to stabilize the platform developed. However, it stills need improvement in sensor readings, because the real platform sensors chosen and how it was done, the reading became rather noisy.

As future work is proposed improve the sensors and verify variations of the PID controller and test other controllers. So, the quadrotor developed can be used to implemented and test various control algorithms.

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