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A probabilistic CNOT gate for coherent state qubits

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ABSTRACT

We propose a scheme for implementing a probabilistic controlled-NOT (CNOT) gate for coherent state qubits using only linear optics and a particular four-mode state. The proposed optical setup works, as a CNOT gate, near-faithful when $|\alpha|^2 \geqslant 25$ and independent of the input state. The key element for realizing the proposed CNOT scheme is the entangled four-mode state.

1. Introduction

After Knill et al. [1] showed that linear optics alone would suffice to implement efficient quantum computing, quantum optics, that had proved to be a fertile field for experimental tests of quantum science, brought a great perspective to quantum information processing (QIP).

In [1] efficient quantum computation is achieved using single photon sources and single photon detectors, but the alternate idea of encoding quantum information on continuous variables [2] has lead to a number of proposals for realizing multi-photon [3–7] and hybrid (coherent states and single photon) [8] quantum computations. The hybrid scheme proposed in [8] is, actually, more efficient than pure linear optical and pure coherent state quantum computers.

The main drawback of proposals [3–5] is that “hard”, non-linear interactions are required in-line of the computation, and these would be difficult to implement in practice.

The elegant scheme proposed in [6] requires only relatively simple linear optical networks and photon counting, but, unfortunately, the amplitude of the required superpositions of coherent states is prohibitively large. On the other hand, the scheme proposed in [7], that was built on the idea found in [6], requires only “easy”, linear in-line interactions, since all the hard interactions are only required for off-line production of resource states, and is based on much smaller superposition states.

The universal set of gates presented in [7] is composed by a phase rotation gate, a superposition gate (that implements a rotation of $\pi/2$ about X) and a two-qubit controlled phase gate. If a CNOT gate using coherent states is proposed, the universal set of gates for [7] can be simplified, since any quantum circuit can be built using single qubit gates and CNOTs. Our goal here is to propose a scheme for implementing probabilistically a CNOT gate for coherent state encoded qubits using an entangled four-mode state, beam splitters and photon number counters.

Several proposals and experimental implementations of a CNOT gate for single photon qubits have been done in the last years [9,10]. Pittman et al. describe in [9] a quantum parity check and a quantum encoder and show how they may be combined to implement a CNOT gate using polarizing beam splitters and polarization single photon qubits. The experimental demonstration of this gate can be found in [11]. It is described in [12] the operation and tolerances of a nondeterministic, coincidence basis, quantum CNOT gate for photonic qubits. The gate is constructed using linear optical elements and requires only a two-photon source for its demonstration. Its success probability is 1/9.

An unambiguous experimental demonstration and comprehensive characterization of quantum CNOT operation in an optical system using four entangled Bell states as a function of only the input qubits' logical values, for a single operating condition of the gate, is found in [13]. The gate is probabilistic, but with the addition of linear optical quantum non-demolition measurements, it is equivalent to the CNOT gate required for scalable all-optical quantum computation.

In [14] it is reported an experimental demonstration of teleportation of a CNOT gate assisted with linear optical manipulations, photon entanglement produced from parametric down-conversion, and postselection from the coincidence measurements. The average fidelity for the teleported gate is 0.84. Zhao et al. detail in [10] a proof-of-principle experimental demonstration of a nondestructive

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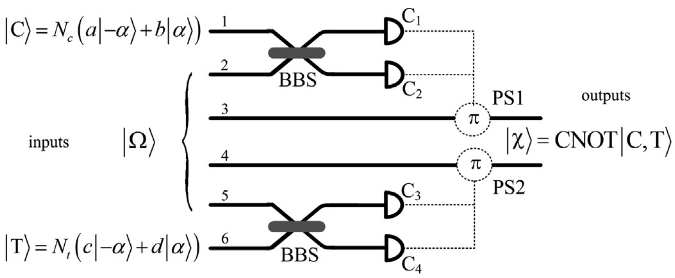

Fig. 1. Optical setup for performing the CNOT gate for coherent state qubits probabilistically.

CNOT gate for two independent photons using only linear optical elements in conjunction with single-photon sources and conditional dynamics.

All the examples given above are probabilistic gates. A deterministic CNOT is still not available due to the need of non-linear operation [15,16]. Here we present a proposal for implementing probabilistically a CNOT gate inspired by the scheme presented in [9].

This Letter is outlined as follows: in Section 2 we present the optical setup for a probabilistic CNOT gate for coherent state encoded qubits; in Section 3 brings the analysis of success and fidelity of the proposed CNOT gate; and, at last, we make our concluding remarks in Section 4.

2. Optical setup for a probabilistic CNOT gate

We intend to perform a CNOT gate between the qubits $|C\rangle = a|0\rangle + b|1\rangle$ and $|T\rangle = c|0\rangle + d|1\rangle$, where $|C\rangle$ and $|T\rangle$ are the control and the target qubits, respectively. In a coherent state quantum computer (CSQC), the qubit is encoded as $|0\rangle_L = |-\alpha\rangle$ and $|1\rangle_L = |\alpha\rangle$ where α is assumed to be real. In this case, we have $|\langle 0|1\rangle|^2 = |\langle -\alpha|\alpha\rangle|^2 = \exp(-4|\alpha|^2)$, which ensures the orthogonality if $\alpha \geqslant 2$ [3–6]. Thus, the states $|C\rangle$ and $|T\rangle$ for coherent state qubits are: $|C\rangle = N_c(a|-\alpha\rangle + b|\alpha\rangle)$ and $|T\rangle = N_t(c|-\alpha\rangle + d|\alpha\rangle)$, where $N_c = [1 + 2\cdot\Re(a^*b)\exp(-2|\alpha|^2)]^{-1/2}$ and $N_t = [1 + 2\cdot\Re(c^*d)\exp(-2|\alpha|^2)]^{-1/2}$ are normalization constants.

Schematic of the optical setup for our proposed CNOT gate is showed in Fig. 1. The state $|\Omega\rangle$ in Fig. 1 is an four-mode state given by $|\Omega\rangle = N_\Omega(|-\alpha,-\alpha,-\alpha,-\alpha\rangle + |-\alpha,-\alpha,\alpha,\alpha\rangle + |\alpha,\alpha,\alpha,-\alpha\rangle + |\alpha,\alpha,-\alpha,\alpha\rangle)$, where the normalization constant is $N_\Omega = 4[1+\exp(-4|\alpha|^2) + 2\cdot\exp(-6|\alpha|^2)]^{-1/2}$. This state can be generated by the quantum circuit shown in Fig. 2 and can be implemented nondeterministically from the optical scheme proposed in [26]. The success probability of this scheme is 1/4.

In Fig. 1, BS, PS and C are beam splitters, phase shifters and photon counters, respectively. The set of beam splitters and photon counters are used to perform Bell-state measurements [17,18]. The unitary operator of a lossless balanced beam splitter is $\hat{B} = \exp[\pi(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2)/4]$. If we send two coherent states $|\alpha\rangle_1$ and $|\beta\rangle_2$ through the BS, the total state at the output is given by:

$$\hat{B}|\alpha,\beta\rangle_{1,2} = \big|(\alpha-\beta)/\sqrt{2}, (\alpha+\beta)/\sqrt{2}\big\rangle_{1,2}. \tag{1}$$

The PS, by its turn, adds a phase θ to the signal passing through it. Its unitary operator is $\hat{U}(\theta) = \exp(j\theta\hat{a}^\dagger\hat{a})$, such that:

$$\hat{U}|\alpha\rangle = \big|e^{j\theta}\alpha\big\rangle. \tag{2}$$

If $\theta = \pi$, the PS is a NOT or X gate for a CSQC because if the light entering the PS is a coherent state $|\alpha\rangle(|-\alpha\rangle)$, the output state will be $|-\alpha\rangle(|\alpha\rangle)$.

Still referring to Fig. 1, mode 1 is the control qubit $|C\rangle$, mode 6 is the target qubit $|T\rangle$ and modes 2 to 5 correspond to the auxiliary resource state $|\Omega\rangle$. Before the photon counters, the state $|\psi\rangle$, resulting from the evolution of the input state $|C\rangle_1 \otimes |\Omega\rangle_{2-5} \otimes |T\rangle_6$ through the optical setup, is given by:

$$\begin{aligned}
|\psi\rangle = N\big[&ac\big(|0,-\sqrt{2}\alpha,-\alpha,-\alpha,0,-\sqrt{2}\alpha\rangle \\
&+ |0,-\sqrt{2}\alpha-\alpha,\alpha,\sqrt{2}\alpha,0\rangle \\
&+ |-\sqrt{2}\alpha,0,\alpha,\alpha,0,-\sqrt{2}\alpha\rangle \\
&+ |-\sqrt{2}\alpha,0,\alpha,-\alpha,\sqrt{2}\alpha,0\rangle\big) \\
&+ ad\big(|0,-\sqrt{2}\alpha,-\alpha,-\alpha,-\sqrt{2}\alpha,0\rangle \\
&+ |0,-\sqrt{2}\alpha-\alpha,\alpha,0,\sqrt{2}\alpha\rangle \\
&+ |-\sqrt{2}\alpha,0,\alpha,\alpha,-\sqrt{2}\alpha,0\rangle \\
&+ |-\sqrt{2}\alpha,0,\alpha,-\alpha,0,\sqrt{2}\alpha\rangle\big) \\
&+ bc\big(|\sqrt{2}\alpha,0,-\alpha,-\alpha,0,-\sqrt{2}\alpha\rangle \\
&+ |\sqrt{2}\alpha,0,-\alpha,\alpha,\sqrt{2}\alpha,0\rangle \\
&+ |0,\sqrt{2}\alpha,\alpha,\alpha,0,-\sqrt{2}\alpha\rangle + |0,\sqrt{2}\alpha,\alpha,-\alpha,\sqrt{2}\alpha,0\rangle\big) \\
&+ bd\big(|\sqrt{2}\alpha,0,-\alpha,-\alpha,-\sqrt{2}\alpha,0\rangle \\
&+ |\sqrt{2}\alpha,0,-\alpha,\alpha,0,\sqrt{2}\alpha\rangle \\
&+ |0,\sqrt{2}\alpha,\alpha,\alpha,-\sqrt{2}\alpha,0\rangle \\
&+ |0,\sqrt{2}\alpha,\alpha,-\alpha,0,\sqrt{2}\alpha\rangle\big)\big],
\end{aligned} \tag{3}$$

where $N = N_c \cdot N_\Omega \cdot N_t$. When the photon counter C_x registers n_x photons, we obtain one of the following states on modes 3 and 4:

$$\begin{aligned}
|\chi\rangle_{3,4} &= {}_{1,2,5,6}\langle 0,n_2,0,n_4|\psi\rangle_{1-6} \\
&\simeq ac(-1)^{n_2+n_4}|-\alpha,-\alpha\rangle + ad(-1)^{n_2}|-\alpha,\alpha\rangle \\
&\quad + bc(-1)^{n_4}|\alpha,\alpha\rangle + bd|\alpha,-\alpha\rangle,
\end{aligned} \tag{4}$$

$$\begin{aligned}
|\chi\rangle &= \langle n_1,0,n_3,0|\psi\rangle \\
&\simeq ac(-1)^{n_1}|\alpha,-\alpha\rangle + ad(-1)^{n_1+n_3}|\alpha,\alpha\rangle \\
&\quad + bc|-\alpha,\alpha\rangle + bd(-1)^{n_3}|-\alpha,-\alpha\rangle,
\end{aligned} \tag{5}$$

$$\begin{aligned}
|\chi\rangle &= \langle 0,n_2,n_3,0|\psi\rangle \\
&\simeq ac(-1)^{n_2}|-\alpha,\alpha\rangle + ad(-1)^{n_2+n_3}|-\alpha,-\alpha\rangle \\
&\quad + bc|\alpha,-\alpha\rangle + bd(-1)^{n_3}|\alpha,\alpha\rangle,
\end{aligned} \tag{6}$$

$$\begin{aligned}
|\chi\rangle &= \langle n_1,0,0,n_4|\psi\rangle \\
&\simeq ac(-1)^{n_1+n_4}|\alpha,\alpha\rangle + ad(-1)^{n_1}|\alpha,-\alpha\rangle \\
&\quad + bc(-1)^{n_4}|-\alpha,-\alpha\rangle + bd|-\alpha,\alpha\rangle.
\end{aligned} \tag{7}$$

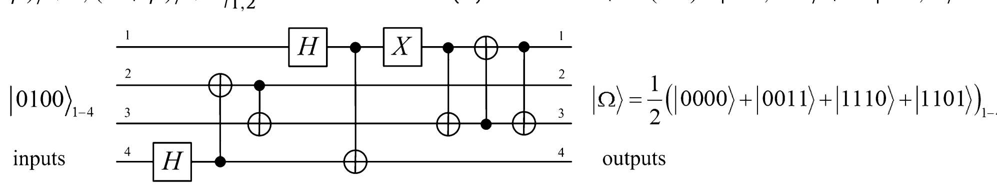

Fig. 2. Circuit to generate a four-partite entangled state $|\Omega\rangle$ for single photon qubits.

From (4), we note that photon counters C_1 and C_3 registered zero photons whereas photon counters C_2 and C_4 registered a non-zero number of photons n_2 and n_4, respectively. A similar analysis can be done in (5)–(7). The schematic optical setup here presented will work correctly if the output state is equal to

$$|\lambda\rangle = \mathrm{CNOT}|\mathrm{C},\mathrm{T}\rangle$$
$$= N_\lambda\big(ac|-\alpha,-\alpha\rangle + ad|-\alpha,\alpha\rangle + bc|\alpha,\alpha\rangle + bd|\alpha,-\alpha\rangle\big)$$

or a state that can be converted to $|\lambda\rangle$ by a unitary operator, with

$$N_\lambda = \big\{1 + 2\cdot\big[\Re\{c^*d\}\cdot\big(1 + 2\cdot\Re\{a^*b\}\big) + \Re\{a^*b\}\cdot\exp\big(-2|\alpha|^2\big)\big]\cdot\exp\big(-2|\alpha|^2\big)\big\}^{-1/2}.$$

Therefore, the optical system is successful when measurements in the photon counters correspond to one of the following mutually exclusive situations:

(i) $n_1 = n_3 = 0$, both n_2 and n_4 are even and both PS's are disabled;
(ii) $n_2 = n_4 = 0$, both n_1 and n_3 are even and only PS1 is activated;
(iii) $n_1 = n_4 = 0$, both n_2 and n_3 are even and only PS2 is activated;
(iv) $n_2 = n_3 = 0$, both n_1 and n_4 are even and both PS's are activated.

3. Analysis of probability of success and fidelity of the optical setup

In this section, we analyze the probability of success of the proposed CNOT gate, considering each of the four situations listed. For simplicity, we assume a, b, c, d and α real.

The probability of success for situation (i), $p_i = |_{1,2,5,6}\langle 0, n_2, 0, n_4 \mid \psi\rangle_{1-6}|^2$, is given by

$$p_i = \frac{|N|^2}{4|N_\lambda|^2}\big(1 - e^{-2\alpha^2}\big)^2. \tag{8}$$

It is easy to verify using Eq. (8) that the probability of one successful event is 1/16. The same result is obtained for other situations ($p = p_i = p_{ii} = p_{iii} = p_{iv}$). Therefore, the total probability of success is 1/4.

We can use an appropriate displacement operator in the cases where the CNOT gate fails, achieving the so-called near-faithful operation, i.e., the fidelity of the collapsed state can be almost 1 for a large enough $|\alpha|^2$, as expected. Suppose that in (4) n_2 and n_4 are odd, resulting in a state

$$|\phi_1\rangle = N_1\big(ac|-\alpha,-\alpha\rangle - ad|-\alpha,\alpha\rangle - bc|\alpha,\alpha\rangle + bd|\alpha,-\alpha\rangle\big),$$
$$N_1 = \big\{1 - 2\big[cd(1-2ab) + abe^{-2|\alpha|^2}\big]e^{-2|\alpha|^2}\big\}^{-1/2}. \tag{9}$$

The state given by Eq. (9) is neither $|\lambda\rangle$ nor it be can converted in $|\lambda\rangle$ by a unitary operator and its fidelity should be less than 1. Therefore, let us apply the displacement operator $\hat{D}_2(\beta) = \exp(\beta\hat{a}_2^\dagger - \beta^*\hat{a}_2)$ on mode 2 of the state in (9) to increase the fidelity of the collapsed state. If $\beta = -j\pi/(4\alpha)$, the resulting state is

$$|\phi_1'\rangle = \hat{D}_2\left(-\frac{j\pi}{4\alpha}\right)|\phi_1\rangle$$
$$= N_1 e^{j\pi/4}\left(ac\left|-\alpha, -\frac{j\pi}{4\alpha} - \alpha\right\rangle + jad\left|-\alpha, -\frac{j\pi}{4\alpha} + \alpha\right\rangle + jbc\left|\alpha, -\frac{j\pi}{4\alpha} + \alpha\right\rangle + bd\left|\alpha, -\frac{j\pi}{4\alpha} - \alpha\right\rangle\right). \tag{10}$$

Such that the fidelity of the state given by Eq. (10) is

$$F_1' = \big|\langle\phi'_1 \mid \lambda\rangle\big|$$
$$= |N_\lambda|\cdot|N_1|\cdot e^{-\pi^2/(32|\alpha|^2)}\big(1 + 4\cdot abcd\cdot e^{-2|\alpha|^2}\big). \tag{11}$$

Analyzing (11), we notice that the fidelity is almost 1 for a large enough $|\alpha|^2$ and the probability of success in this case is

$$p_1' = \frac{|N|^2}{4|N_1|^2}\big(1 - e^{-2\alpha^2}\big)^2. \tag{12}$$

Now suppose that in Eq. (4) n_2 and n_4 are even and odd, respectively, resulting in

$$|\phi_2\rangle = N_2\big(-ac|-\alpha,-\alpha\rangle + ad|-\alpha,\alpha\rangle - bc|\alpha,\alpha\rangle + bd|\alpha,-\alpha\rangle\big),$$
$$N_2 = \big\{1 - 2\big[cd(1+2ab) - abe^{-2|\alpha|^2}\big]e^{-2|\alpha|^2}\big\}^{-1/2}. \tag{13}$$

If we apply the displacement operator $\hat{D}(\beta)$ on both modes of (13), the following state is obtained:

$$|\phi_2'\rangle = \hat{D}_1\left(-\frac{j\pi}{4\alpha}\right)\otimes\hat{D}_2\left(-\frac{j\pi}{4\alpha}\right)|\phi_2\rangle$$
$$= N_2\left(-jac\left|-\frac{j\pi}{4\alpha} - \alpha, -\frac{j\pi}{4\alpha} - \alpha\right\rangle + ad\left|-\frac{j\pi}{4\alpha} - \alpha, -\frac{j\pi}{4\alpha} + \alpha\right\rangle + jbc\left|-\frac{j\pi}{4\alpha} + \alpha, -\frac{j\pi}{4\alpha} + \alpha\right\rangle + bd\left|-\frac{j\pi}{4\alpha} + \alpha, -\frac{j\pi}{4\alpha} - \alpha\right\rangle\right). \tag{14}$$

The fidelity and probability of success in this case are, respectively,

$$F_2' = \big|\langle\phi'_2 \mid \lambda\rangle\big|$$
$$= |N_\lambda|\cdot|N_2|e^{-\pi^2/(16|\alpha|^2)}\big(1 + 2\big(d^2 - c^2\big)ab\cdot e^{-4|\alpha|^2}\big), \tag{15}$$

$$p_2' = \frac{|N|^2}{4|N_2|^2}\big(1 - e^{-2\alpha^2}\big)^2. \tag{16}$$

For the case that n_2 and n_4 are odd and even, respectively, the projected state with the displacement operator applied on mode 1 of (4) is

$$|\phi_3'\rangle = -N_3\left(ac\left|-\frac{j\pi}{4\alpha} - \alpha, -\alpha\right\rangle + ad\left|-\frac{j\pi}{4\alpha} - \alpha, \alpha\right\rangle + jbc\left|-\frac{j\pi}{4\alpha} + \alpha, +\alpha\right\rangle + jbd\left|-\frac{j\pi}{4\alpha} + \alpha, -\alpha\right\rangle\right),$$
$$N_3 = \big\{1 + 2\big[cd(1-2ab) - abe^{-2|\alpha|^2}\big]e^{-2|\alpha|^2}\big\}^{-1/2}. \tag{17}$$

The fidelity and probability of success are then:

$$F_3' = \big|\langle\phi'_3 \mid \lambda\rangle\big|$$
$$= |N_\lambda|\cdot|N_3|e^{-\pi^2/(32|\alpha|^2)}\cdot\big(1 + 2cd\cdot e^{-2|\alpha|^2}\big), \tag{18}$$

$$p_3' = \frac{|N|^2}{4|N_3|^2}\big(1 - e^{-2\alpha^2}\big)^2. \tag{19}$$

Table 1 shows all sixteen possible situations where the proposed CNOT is successful and the operations that we must perform depending on the number of registered photons. Therefore, the

Table 1
All sixteen possible situations (distinguished by recorded photon numbers n_x and turning on-off of PS's) and the corresponding recovery operator necessary for the successful performance of the CNOT.

Possible situations				Collapsed state	Phase shifters		Recovery operator	Fidelity	Probability of success
n_1	n_2	n_3	n_4		PS_1	PS_2			
0	even	0	even	Eq. (4)	off	off	$I \otimes I$	1	Eq. (8)
0	even	0	odd	Eq. (4)	off	off	$\hat{D}(\beta) \otimes \hat{D}(\beta)$	Eq. (15)	Eq. (16)
0	odd	0	even	Eq. (4)	off	off	$\hat{D}(\beta) \otimes I$	Eq. (18)	Eq. (19)
0	odd	0	odd	Eq. (4)	off	off	$I \otimes \hat{D}(\beta)$	Eq. (11)	Eq. (12)
even	0	even	0	Eq. (5)	on	off	$I \otimes I$	1	Eq. (8)
even	0	odd	0	Eq. (5)	on	off	$\hat{D}(\beta) \otimes \hat{D}(\beta)$	Eq. (15)	Eq. (16)
odd	0	even	0	Eq. (5)	on	off	$\hat{D}(\beta) \otimes I$	Eq. (18)	Eq. (19)
odd	0	odd	0	Eq. (5)	on	off	$I \otimes \hat{D}(\beta)$	Eq. (11)	Eq. (12)
0	even	even	0	Eq. (6)	off	on	$I \otimes I$	1	Eq. (8)
0	even	odd	0	Eq. (6)	off	on	$\hat{D}(\beta) \otimes \hat{D}(\beta)$	Eq. (15)	Eq. (16)
0	odd	even	0	Eq. (6)	off	on	$\hat{D}(\beta) \otimes I$	Eq. (18)	Eq. (19)
0	odd	odd	0	Eq. (6)	off	on	$I \otimes \hat{D}(\beta)$	Eq. (11)	Eq. (12)
even	0	0	even	Eq. (7)	on	on	$I \otimes I$	1	Eq. (8)
even	0	0	odd	Eq. (7)	on	on	$\hat{D}(\beta) \otimes \hat{D}(\beta)$	Eq. (15)	Eq. (16)
odd	0	0	even	Eq. (7)	on	on	$\hat{D}(\beta) \otimes I$	Eq. (18)	Eq. (19)
odd	0	0	odd	Eq. (7)	on	on	$I \otimes \hat{D}(\beta)$	Eq. (11)	Eq. (12)

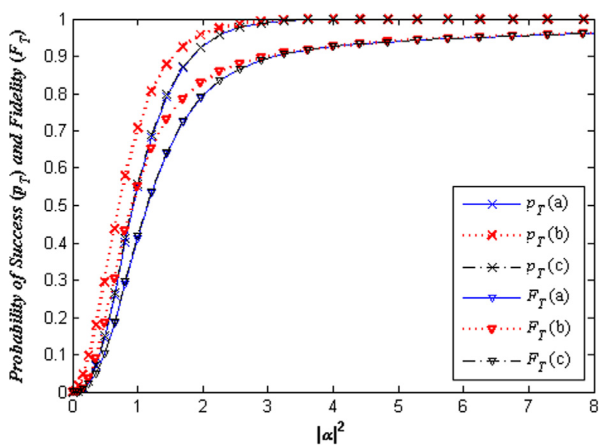

Fig. 3. Total probability of success and fidelity versus $|\alpha|^2$ for a lossless optical setup and ideal photon number counters. (a) $\theta = \pi/4$ and $\phi = \pi/4$; (b) $\theta = \pi/4$ and $\phi = 2\pi/3$; (c) $\theta = \pi/3$ and $\phi = 2\pi/3$.

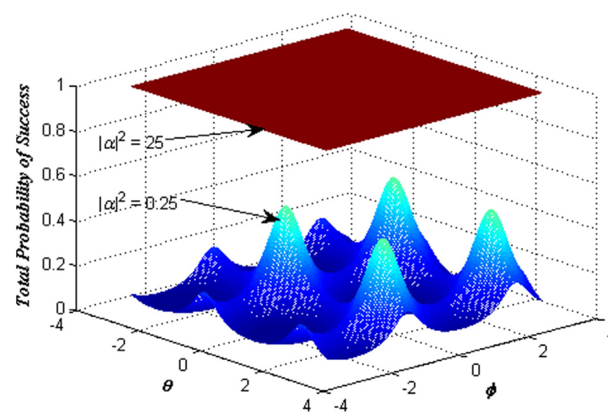

Fig. 4. Total probability of success as a function of θ and ϕ for $|\alpha|^2 = 0.25$ and $|\alpha|^2 = 25$.

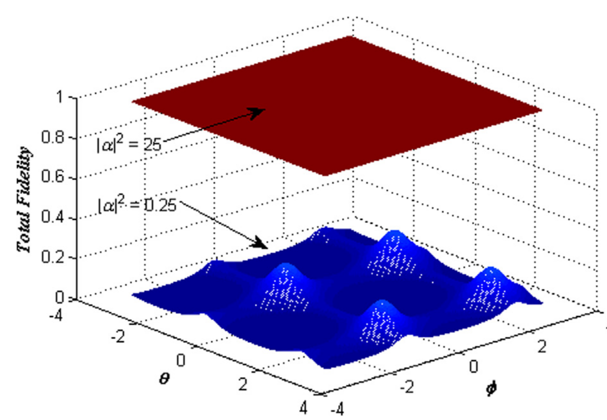

Fig. 5. Total fidelity as a function of θ and ϕ for $|\alpha|^2 = 0.25$ and $|\alpha|^2 = 25$.

total probability of success and fidelity of the optical system are, respectively,

$$p_T = 4 \cdot (p + p_1{}' + p_2{}' + p_3{}'), \tag{20}$$

$$F_T = 4 \cdot (p \cdot 1 + p_1{}' \cdot F_1{}' + p_2{}' \cdot F_2{}' + p_3{}' \cdot F_3{}'). \tag{21}$$

Figs. 3, 4 and 5 show plots of the total probability of success and fidelity as a function of $|\alpha|^2$, θ and ϕ, where $a = \sin(\theta)$, $b = \cos(\theta)$, $c = \sin(\phi)$, $d = \cos(\phi)$ and α is assumed to be real.

As we can see in Fig. 3, if we consider a lossless optical setup and ideal photon number counters, there is a monotonic relation between the total probability of success and fidelity, given by Eqs. (20) and (21), and the average number of photons $|\alpha|^2$, for several values of θ and ϕ. Both p_T and F_T asymptotically approach 1 in the limit of $|\alpha|^2 \to \infty$.

In Figs. 4 and 5, we can see that the proposed CNOT gate is near-faithful when $|\alpha|^2 \geqslant 25$ and independent of θ and ϕ, i.e., independent of the input states, $|C\rangle$ and $|T\rangle$.

As mentioned before, the CNOT gate here proposed may be used to simplify the universal set of gates for a coherent state based quantum computer such as described in [7]. Alternatively, a CNOT gate could be implemented by using a Controlled-Z gate and two Hadamard gates as proposed in [7] and [5], respectively. The main drawbacks to this alternative is the fact that we would need non-linearity (for the Hadamard gates) and small values of α, demanding quantum Zeno effect or multiple use of teleportations, increasing the number of Bell-state measurements, beam splitters and resource states.

In general, the proposed scheme has the following advantages compared to gates proposed in [5,7] for coherent state qubits: it does not require (1) Bell-state measurements with arbitrarily high precision (that needs three beam splitters and four detectors) [5], (2) Hadamard gates [5], and (3) beam splitters with reflectivity dependent on the average number of photons [7]; and it uses only two NOT gates instead of three as used in [5]. Furthermore, the implementation of this CNOT gate becomes viable with the present development of silicon photonics technology and multi-pixel counters (MPPC) that are able to distinguish between $1, 2, \ldots, 10$ photons [http://www.hamamatsu.com].

In a more recent work, Lund et al. [19] propose a set of gates for coherent state qubits and study fault tolerance under the effects of small amplitudes and loss. Their chosen universal set of quantum gates is composed by a X gate, an arbitrary Z rotation, a Hadamard gate, and a controlled-Z gate. They show that using error correction only small amplitudes are required for fault-tolerant quantum computing. As in the previous work by Ralph et al. [7], in [19] a CNOT gate is not proposed.

The CNOT gate here presented may be used with one of the two schemes [7,19] as an alternative to the universal set of quantum gates. As a future work, one may study if in this case, using error correction, small amplitudes could be enough for fault-tolerant computation.

4. Conclusion

We presented a proposal for implementing a probabilistic CNOT gate for coherent state qubits. The proposed optical setup uses only linear optical devices, photon number counters and a special entangled four-mode state as an auxiliary resource. An appropriate displacement operator can be used when the CNOT gate fails, such that it can work near-faithful, when $|\alpha|^2 \geqslant 25$, independent of the input states. The total efficiency of the optical setup is $1/4$, considering that the entangled four-mode state is supplied.

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