On The Use of Higher Order Statistics for Blind Source Separation

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***Abstract*— The use of higher order statistics in blind source separation problem is analyzed in this work. Despite the well known fact that they are necessary to provide source separation in a general framework, their impact on the performance of adaptive solutions is a still open research field. In order to provide new elements for the comparison of using one or more higher order moments on adaptive solutions, two constrained algorithms are investigated. The multiuser kurtosis algorithm (MUK) and the multiuser constrained fitting probability density function algorithm (MU-CFPA) are used due to the desired characteristics of different higher order statistics involved in their design. Simulation results are carried out to basis our analysis.**

***Index Terms*— Blind source separation, higher order moments, pdf estimation, kurtosis maximization, constrained criteria.**

I. Introduction

Blind source separation (BSS) has been gained increasing attention in the signal processing community due to its wide applicability in many fields such as digital communications, biomedical engineering and financial data analysis among others [1, 2].

Since the milestone work by Hérault *et al* in 1985 [3] much effort has been done in order to construct suitable statistical criteria that reflect some known structural properties of the sources [4]. A common characteristic of all those criteria is the use of higher order statistics (HOS) since second order statistics (SOS) are not sufficient to solve the separation problem for general sources [2].

The information-theoretic approach has been introduced by Donoho in [5], who has treated the BSS problem by an entropy minimization view point. Other well known method to solve BSS problems is the use of contrast functions introduced by Comon [6], where a contrast function is a cumulant-based function of the separation filter outputs that is maximized if and only if separation is achieved [2, 4].

Those works have provided important results on the issue of necessary and sufficient conditions to provide perfect separation. Despite the development of techniques that rely directly on HOS cumulants, some single user techniques, such as constant modulus (CM) and Shalvi-Weinstein criteria, have been proposed to BSS in a single-stage and multistage context [2, 7, 8].

Papadias proposed in [4, 8] a source separation approach that is based on the Shalvi-Weinstein criterion. The proposal is called multiuser kurtosis (MUK) and consists on the kurtosis maximization, constrained to an orthogonal global response. It has been a great advance on the field of BSS because it has proved global convergence for an arbitrary number of users, what has not been done so far.

We have previously proposed a source separation criterion based on the estimation of the probability density function (pdf) of the ideally recovered signals [9]. The criterion also takes profit from the MUK approach by considering the constraint over the global response in order to provide correct source separation.

Our objective in this work is to evaluate the differences on adaptive solutions when the algorithm considers only one higher order moment, as in MUK case, or *all* higher order moments as our approach in [9]. This aspect has not been exploited in the literature and, as will shown in the sequel, can provide significant improvements on the performance of adaptive BSS algorithms.

The rest of the paper is organized is follows. Section II presents the mathematical formulation of the problem. In Section III, the necessary conditions to provide source separation are presented, as well as the two fundamental strategies to be analyzed in this paper. Simulation results that bases our analysis are presented in Section IV and, finally, in Section V our conclusions are stated.

II. Problem Formulation

We consider that K independent and identically distributed (i.i.d.) and also mutually independent zero mean discrete sequences $a_k(n)$, $k = 1, \ldots, K$, that share the same statistical properties, are transmitted over a MIMO linear memoryless channel that introduces interuser interference.

If we consider M sensors in the receiver we can represent the received signal at time instant n as

$$\mathbf{x}(n) = \mathbf{H}\mathbf{a}(n) + \mathbf{v}(n), \quad (1)$$

where $\mathbf{a}(n) = \begin{bmatrix} a_1(n) & \cdots & a_K(n) \end{bmatrix}^T$ is the vector of sources, $\mathbf{H}$ is the $M \times K$ channel matrix, $\mathbf{v}(n)$ is the $M \times 1$

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vector of additive gaussian noise and $\mathbf{x}(n)$ is the $M \times 1$ vector of received signals.

The received signals are then precessed by the MIMO equalizer given by the matrix $\mathbf{W}(n) = \left[\begin{array}{ccc} \mathbf{w}_1(n) & \cdots & \mathbf{w}_K(n) \end{array} \right]$, which produces a $K \times 1$ vector $\mathbf{y}(n)$ that consists of the estimative of the sources. The receiver output can be mathematically written as

$$\begin{aligned} \mathbf{y}(n) &= \mathbf{W}^H(n)\mathbf{x}(n) = \mathbf{W}^H(n)\mathbf{H}\mathbf{a}(n) + \mathbf{v}'(n) \\ &= \mathbf{G}(n)\mathbf{a}(n) + \mathbf{v}'(n), \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathbf{G} = \mathbf{W}^H(n)\mathbf{H} &= \left[\begin{array}{ccc} \mathbf{g}_1 & \cdots & \mathbf{g}_K \end{array} \right] \\ &= \left[\begin{array}{ccc} g_{11} & \cdots & g_{K1} \\ \vdots & \ddots & \vdots \\ g_{1K} & \cdots & g_{KK} \end{array} \right]_{K \times K} \end{aligned}$$

is the global response matrix and $\mathbf{v}'(n) = \mathbf{W}^H(n)\mathbf{v}(n)$ is the filtered noise at the receiver output. Figure 1 depicts the above-described system.

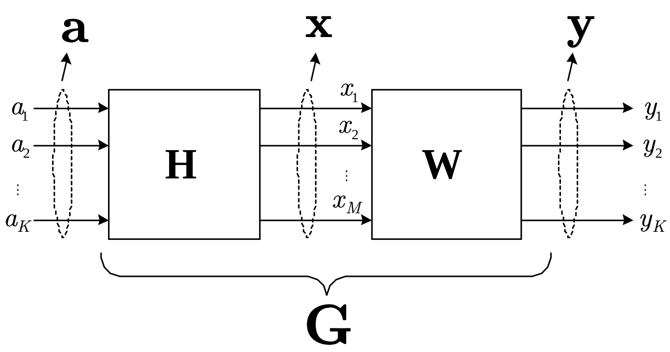


Fig. 1. General blind source separation scheme.

Ideally, the global response system should be in such a way that the K signals on the receiver output, $y_k(n)$, $k = 1, \ldots, K$, should match the source (transmitted) signals $a_q(n)$, $q = 1, \ldots, K$. However, by its blind characteristic, the problem has an indeterminacy w.r.t. to scaling and order of separated sources [2, 10]. This makes the solution become:

$$\mathbf{y}(n) = \mathbf{PDa}(n), \tag{3}$$

where $\mathbf{P}$ is a permutation matrix and $\mathbf{D}$ is a diagonal matrix.

We will focus our attention to the case of source separation in an arbitrary order, since order recovering can be provided by an appropriate post-processing.

To solve the overall problem a large variety of algorithms can be used. Just to cite a few [11-16] and for a very good (and huge) list on BSS solutions, see [2] and references therein.

Our interest concerns adaptive BSS techniques with no explicit statistics estimation neither information-theoretic measures. Due to its simplicity and on-line capability, we chose MUK to be compared with our previous proposal (MU-CFPA), in order to evaluate the influence of the number of higher order moments on the performance of adaptive solutions to BSS problems. Both criteria are discussed and analyzed in next section.

III. Some Strategies: MUK and MU-CFPA

The presented strategies are based on the well known Shalvi-Weinstein (SW) criterion [17] proposed to the single user (equalization) case. From the SW criterion we know that if the received power (after equalization) is assured to be equal to the transmitted one, it is sufficient to equalize one higher-order moment to achieve equalization, except by a phase rotation. Generalization of this theorem to the multiuser case is done by the insertion of the condition that the recovered sources must be different. Then, the following conditions must hold to assure source separation [4]:

- C1. $a_k(n)$ is i.i.d. and zero mean $(k = 1, \ldots, K)$;
- C2. $a_k(n)$ and $a_q(n)$ are statistically independent for $k \neq q$ and have the same pdf;
- C3. $|\kappa[y_k(n)]| = |\kappa_a| \quad (k = 1, \ldots, K)$;
- C4. $E\left\{|y_k(n)|^2\right\} = \sigma_a^2 \quad (k = 1, \ldots, K)$;
- C5. $E\left\{y_k(n)y_q^*(n)\right\} = 0, \quad k \neq q$.

where $a_k(n)$ is the transmitted sequence by the k-th source, $E\{\cdot\}$ stands for expectation, κ_a is the kurtosis, σ_a^2 is the variance of the transmitted sequence, and $\kappa[\cdot]$ is the kurtosis operator.

In order to prove the sufficiency of the conditions above, we may express the variance and kurtosis of each output as

$$E\left\{|y_k(n)|^2\right\} = \sigma_a^2 \sum_{k=1}^{K} |g_{kl}|^2 \tag{4}$$

and

$$\kappa[y_k(n)] = \kappa_a \sum_{k=1}^{K} |g_{kl}|^4 . \tag{5}$$

Then, from Equation (5) and Condition C3 we have

$$\sum_{k=1}^{K} |g_{kl}|^4 = 1, \tag{6}$$

and from Equation (4) and Condition C4 we obtain

$$\sum_{k=1}^{K} |g_{kl}|^2 = 1. \tag{7}$$

Therefore, based on the fact that

$$\sum |g_k|^4 \leq \left(\sum |g_k|^2\right)^2,$$

Equations (6) and (7) state that $\mathbf{g}_k$ must be in the form

$$\mathbf{g}_k = \left[\begin{array}{cccccc} 0 & \cdots & 0 & e^{j\phi_k} & 0 & \cdots & 0 \end{array} \right]^T, \tag{8}$$

where the single nonzero element can be at any position and ϕ_k is an arbitrary phase rotation. Then, by combining Condition C5 with the noiseless case of Equation (2) we obtain:

$$\mathbf{g}_k^H \mathbf{g}_q = 0, \qquad k \neq q. \tag{9}$$

Equations (8) and (9) dictate the important property that the nonzero position of the solution vectors $\mathbf{g}_k$ and $\mathbf{g}_q$ cannot be the same. Hence, the unique solution that satisfies the problem corresponds to the K solution vectors $\mathbf{g}_k$ be different "Dirac"-type vectors, as given in Equation (8).

In the sequel, we present two algorithms that perform BSS, in according to the conditions given above.

A. Multiuser Kurtosis Algorithm (MUK)

A multiuser algorithm based on the maximization of the kurtosis for blind signals recovering is proposed in [8, 4]. MUK is a constrained criterion that maximizes the kurtosis of the signals subject to the constraint of normalized global response, it means,

$$\begin{cases} \max_{\mathbf{G}} J_{\text{MUK}}(\mathbf{G}) = \sum_{j=1}^{K} \left|\kappa\left[y_k\right]\right| \\ \text{subject to: } \mathbf{G}^H\mathbf{G} = \mathbf{I} \end{cases}, \qquad (10)$$

where $\mathbf{I}$ is the identity matrix.

The criterion divides the separation task into two parts: the equalization step, that maximizes the kurtosis, and the separation one, that performs the decorrelation of the outputs. For each task, we denote the beamformers by $\mathbf{W}^e$ for equalization part, and $\mathbf{W}$ for the later one. The constraint step corresponds to a Gram-Schimdt orthogonalization of matrix $\mathbf{W}^e$ [4]. Therefore, those two parts can be, respectively, written in their adaptive versions by [4, 8]:

$$\mathbf{W}^e(n+1) = \mathbf{W}(n) + \mu\,\text{sign}\left(\kappa_a\right)\mathbf{x}^*(n)\mathcal{Y}(n), \qquad (11)$$

where $\mathcal{Y}(n) = \left[\ |y_1(n)|^2 y_1(n) \quad \cdots \quad |y_K(n)|^2 y_K(n)\ \right]$ and Equation (11) corresponds to the equalization step. For the orthogonalization one, we have, for the j-th user

$$\mathbf{w}_j(n+1) = \frac{\mathbf{w}_j^e(n+1) - \sum_{l=1}^{j-1}\left(\mathbf{w}_l^H(n+1)\mathbf{w}_j^e(n+1)\right)\mathbf{w}_l(n+1)}{\left\|\mathbf{w}_j^e(n+1) - \sum_{l=1}^{j-1}\left(\mathbf{w}_l^H(n+1)\mathbf{w}_j^e(n+1)\right)\mathbf{w}_l(n+1)\right\|}. \qquad (12)$$

The MUK algorithm is summarized in Table I [4].

TABLE I

MUK ALGORITHM.

1. Initialize $\mathbf{W}(0)$
2. for $n > 0$
3. Obtain $\mathbf{W}^e(n+1)$ from Equation (11)
4. Obtain $\mathbf{w}_1(n+1) = \frac{\mathbf{w}_1^e(n+1)}{\left\|\mathbf{w}_1^e(n+1)\right\|}$
5. for $j = 2 : K$
6. Compute $\mathbf{w}_j(n+1)$ from Equation (12)
7. Go to 5
8. Go to 2

This algorithm considers only the fourth order moment (unnormalized kurtosis) to provide source separation. Next section presents an algorithm that uses all higher order moments.

B. Multiuser Constrained Fitting Probability Algorithm (MU-CFPA)

The MU-CFPA [9] is a constrained version of the algorithm proposed for single-user equalization in [18] that has a multiuser version reported in [19]. The original one is based on the estimation of the pdf of an ideally equalized signal at each output, by means of a parametric model that fits the system order and pdf features.

Then, we can construct the criterion in order to minimize the "*distance*" between the desired pdf (ideally equalized one) and the parametric model. Thus, the well known *Kullback-Leibler divergence* (KLD) [20] is used to minimize the divergence between both functions, since both are positive definite functions. The criterion may be written, for the k-th user, as [21]

$$J_{\text{FP}}(\mathbf{w}_k) = D_{p_{Y,\text{ideal}}(y_k)\|\Phi(y,\sigma_r^2)}, \qquad (13)$$

where $D_{\circ\|\bullet}$ is the KLD between the pdfs, $p_{Y,\text{ideal}}$ is the pdf of the ideally equalized signal and $\Phi(y,\sigma_r^2)$ is the parametric model given by

$$\Phi(y_k) = \frac{1}{\sqrt{2\pi\sigma_r^2}}\sum_{i=1}^{S}\exp\left(-\frac{\left|\mathbf{w}_k^H(n)\mathbf{x}(n) - a_i\right|^2}{2\sigma_r^2}\right), \qquad (14)$$

where σ_r^2 is the variance of each Gaussian in the model, S is the number of symbols in the transmitted constellation and a_i is the i-th symbol from the alphabet of transmitted symbols. It worths to mention that minimize Equation (13) corresponds to maximize the log-likelihood function [20] and also to find the entropy of y if $\Phi\left(y,\sigma_r^2\right) = p_{Y,\text{ideal}}(y)$ [18, 19, 22].

In a previous work we have used the explicit decorrelation procedure proposed in [23] in order to provide decorrelation between the outputs. This results in the following multiuser criterion:

$$J_{\text{MU-FP}}\left(\mathbf{w}_k\right) = J_{\text{FP}}\left(\mathbf{w}_k\right) + \gamma\sum_{i=1}^{K}\sum_{\substack{j=1\\ j\neq i}}^{K}\left|r_{ij}\right|^2, \qquad (15)$$

where γ is a regularization parameter and r_{ij} is the correlation term between the i-th and j-th outputs. This criterion and its stochastic adaptive algorithm presents good results but suffers from the strong trade-off between the number of lost users and steady state error w.r.t. the parameter γ.

In [9] the explicit decorrelation procedure has been dropped and the orthogonalization procedure of the MUK is inserted in order to cope with the previous described problem. Then, the following constrained criterion has been derived:

$$\begin{cases} \min_{\mathbf{W}} J_{\text{FP}}(\mathbf{W}) = \sum_{k=1}^{K} D_{p_{Y,\text{ideal}}(y)\|\Phi(y_k,\sigma_r^2)} \\ \text{subject to: } \mathbf{G}^H\mathbf{G} = \mathbf{I} \end{cases}. \qquad (16)$$

and the adaptive version of the algorithm consists in replacing the step 3 in Table I by the following expression:

$$\mathbf{W}^e(n+1) = \mathbf{W}(n) - \mu\nabla J_{\text{FP}}\left(\mathbf{W}(n)\right), \qquad (17)$$

where $\nabla J_{\text{FP}}(\mathbf{W}(n))$ is given by

$$\nabla J_{\text{FP}}(\mathbf{W}(n)) = \frac{\sum_{i=1}^{S} \exp\left(-\frac{|\mathbf{y}(n)-\mathbf{a}_i|^2}{2\sigma_r^2}\right)(\mathbf{y}(n)-\mathbf{a}_i)}{\sigma_r^2 \cdot \sum_{i=1}^{S} \exp\left(-\frac{|\mathbf{y}(n)-\mathbf{a}_i|^2}{2\sigma_r^2}\right)} \mathbf{x}^* \quad (18)$$

where $\mathbf{a}_i$ is the $K \times 1$ vector with the a_i symbol from the transmitted alphabet in all positions.

The resulting algorithm is then called Multiuser Constrained FPA (MU-CFPA) and Table II summarizes the dynamic of the algorithm.

TABLE II

MU-CFPA.

1. Initialize $\mathbf{W}(0)$
2. for $n > 0$
3. Obtain $\mathbf{W}^e(n+1)$ from Equations (17) and (18)
4. Obtain $\mathbf{w}_1(n+1) = \frac{\mathbf{w}_1^e(n+1)}{\|\mathbf{w}_1^e(n+1)\|}$
5. for $j = 2 : K$
6. Compute $\mathbf{w}_j(n+1)$ from Equation (12)
7. Go to 5
8. Go to 2

The minimization of KLD in Equation (16) is achieved if and only if the two pdf are equal, which consists in matching all statistical moments of both pdfs. So, the MU-CFPA clearly respects the necessary conditions to provide source separation and kurtosis maximization is implicitly comprised in the equalization procedure.

Then, the following question arises: Are the additional higher order moments than kurtosis useful to achieve source separation?

In the sequence we try to answer this question by showing that those other higher order moments provide significant improvements in terms of adaptive performance.

IV. Simulation Results

We consider a simple case of a 2×2 unitary channel matrix [4]

$$\mathbf{H} = \begin{bmatrix} 0.701 + j0.172 & 0.629 + j0.286 \\ -0.274 - j0.634 & 0.159 + j0.704 \end{bmatrix}, \quad (19)$$

for the case of two independent QPSK inputs in a 30 dB signal-to-noise ratio (SNR) environment. The parameters of simulations were: $\mu_{\text{MUK}} = 2 \times 10^{-3}$, $\mu_{\text{MU-CFPA}} = 10^{-2}$, $\sigma_r^2 = 0.1$ and $\mathbf{W}(0) = \mathbf{W}^e(0) = \mathbf{I}$ for both algorithms.

In order to evaluate the performance of both algorithms we use the constant modulus error (CME) defined, for the k-th user, as follows:

$$\text{CME}_k(n) = \left(|y_k|^2 - R\right)^2. \quad (20)$$

The constant R is related to the power of the transmitted constellation. In our case we will assume a normalized power, it means, $R = 1$. It is important to mention that the step sizes were chosen as the highest ones that allow to reach the lowest CME for both algorithms.

Figure 2 shows the evolution of the mean CME (from both users) of the two algorithms. We can observe that the MU-CFPA outperforms the MUK in terms of convergence rate and reaches the same final steady state error (about -30 dB). The convergence of the MU-CFPA is reached about of 150 symbols and the MUK converges around the 1500 symbols. This is probably due to the consideration of all higher order moments that allows to increase the step size and also helps in the estimation of the sources by the filter.

After convergence the MU-CFPA reaches the following setting:

$$\mathbf{G} = \begin{bmatrix} -0.0015 - j0.0024 & -0.9990 + j0.0067 \\ 0.9992 - j0.0046 & -0.0017 + j0.0026 \end{bmatrix}, \quad (21)$$

that corresponds to the first source recovered on the second output and the second source on the first output. For the MUK we obtain:

$$\mathbf{G} = \begin{bmatrix} 0.0143 - j0.0087 & -0.9369 - j0.3463 \\ 0.3295 + j0.9431 & -0.0091 + j0.0139 \end{bmatrix}, \quad (22)$$

that corresponds to the same order of recovering that in the previous case. One should note that the residual interference, provided by the main diagonal terms in both cases, is higher for the MUK, when compared with the MU-CFPA case. This should imply that for a more complex case (more users, antenna array receiver, etc.) the MUK performance can be even more outperformed by the MU-CFPA due to the involved HOS.

Further, we can observe that a phase rotation is inserted in the MUK case. This is not observed in the MU-CFPA, due its characteristic of phase recovering [9, 18, 19]. Figure 3 illustrate the 10% last symbols constellations from both algorithm, so that the phase recovering capability of the MU-CFPA can be observed.

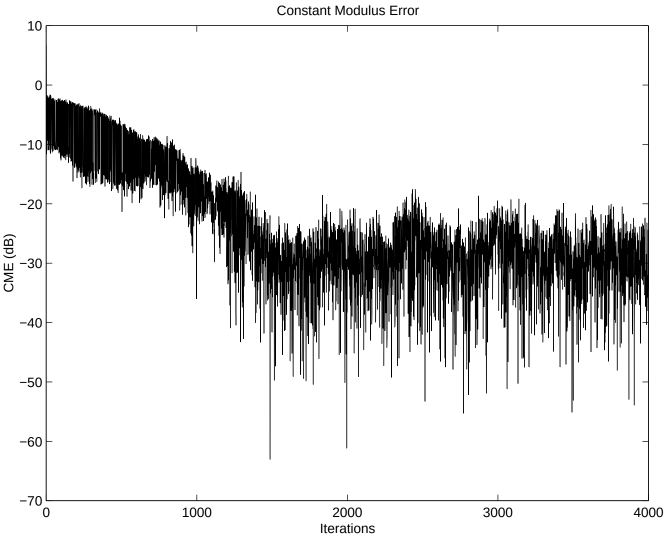
V. Conclusions

We have presented a brief analysis of the use of higher order moments in blind source separation.

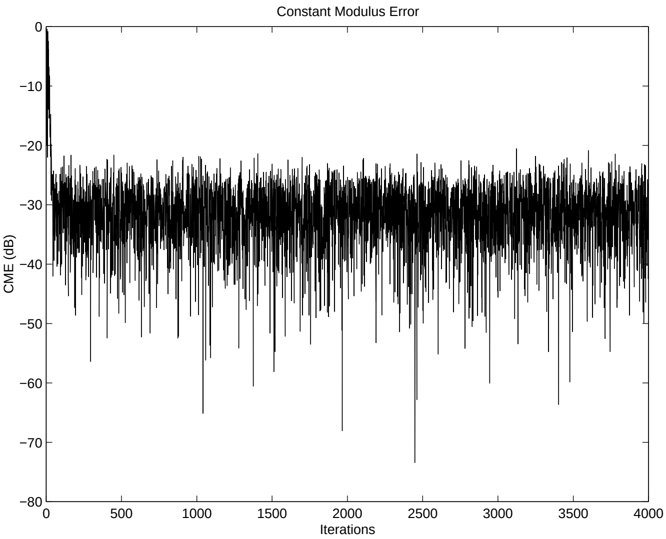
We have investigated the improvements on adaptive algorithms when they respect the sufficient and necessary conditions to perform blind source separation and use one or more higher order moments. For this sake, we have used two algorithms that are based on constraint filtering, namely the multiuser kurtosis algorithm (MUK) and the multiuser constrained fitting pdf algorithm (MU-CFPA).

The main feature we have observed is the improvement on the speed of convergence with a small increase in the computational complexity, in the particular case of the MU-CFPA.

A natural extension to this work is to derive mathematical analysis on the influence of the use of other higher order moments in adaptive BSS solutions.

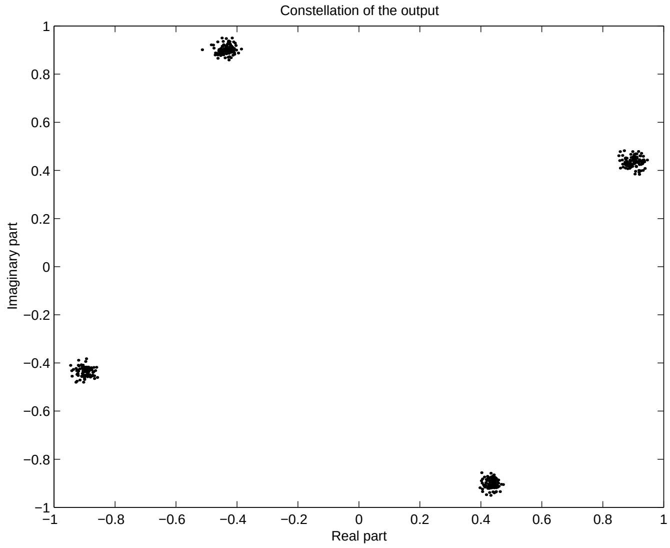


(a) MUK

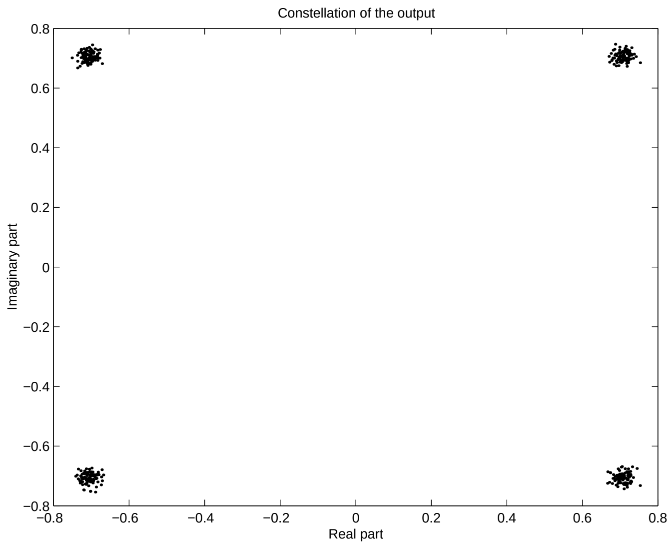


(b) MU-CFPA

Fig. 2. CME evolution for both algorithms.



(a) MUK



(b) MU-CFPA

Fig. 3. Constelation of the 10% last symbols of one output from both algorithms.

ACKNOWLEDGMENTS

The authors would like to thank Dr. C. B. Papadias by his help in better understanding the MUK algorithm.

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