BLIND MIMO SYSTEM IDENTIFICATION USING CONSTRAINED FACTOR DECOMPOSITION OF OUTPUT GENERATING FUNCTION DERIVATIVES

André L. F. de Almeida[(1)*], *Xavier Luciani*[(2)], *Alwin Stegeman*[(3)], *Pierre Comon*[(2)]

[(1)]Department of Teleinformatics Engineering, Federal University of Ceará,Brazil.
[(2)]I3S Laboratory, University of Nice-Sophia Antipolis (UNS), CNRS, France.
[(3)]Heijmans Institute for Psychological Research, University of Groningen, The Netherlands.
E-mails: andre@gtel.ufc.br, {luciani,comon}@i3s.unice.fr, a.w.stegeman@rug.nl

ABSTRACT

This work addresses the blind identification of complex MIMO systems driven by complex input signals using a new tensor decomposition approach. We show that a collection of successive second-order derivatives of the second generating function of the system outputs can be stored in a higher-order tensor following a constrained factor (CONFAC) decomposition. The proposed decomposition captures the repeated linear combinations involving real and imaginary components of the MIMO system matrix arising from the successive differentiation of output's generating function derivatives. By exploiting different derivative forms computed at multiple points of the observation space, an "extended" CONFAC decomposition enjoying essential uniqueness is obtained. Thanks to this uniqueness property, a blind estimation of the MIMO system response matrix is possible.

Index Terms— Blind identification, MIMO systems, generating function, tensor decomposition.

1. INTRODUCTION

The connection between tensor decompositions and signal processing has led to several solutions to the blind MIMO identification problem. When the diversity of the observations is not sufficient, one can resort to a second class of tensor-based methods that rely on the multilinearity properties of high-order statistics (HOS) [1–5]. A large majority of these methods solves the blind MIMO identification problem by means of the PARAFAC decomposition of a tensor storing the cumulants of the system outputs (see e.g. [1, 6, 7]). This is the case, for instance, of FOOBI/FOOBI2 [4, 5], and 6-BIOME [3] algorithms, which use 4th and 6th order output cumulants, respectively, by capitalizing on the parallel factor (PARAFAC) decomposition [8, 9]. A particular class of blind identification methods exploiting the second characteristic function of the system outputs has been proposed in a few works [10, 11]. For instance, in [11], the authors show that partial derivatives of the second characteristic function can be stored in a symmetric tensor, the PARAFAC decomposition of which provides a direct estimation of the mixing matrix up to trivial indeterminacies. In a recent work [12], we have considered a more general scenario where the inputs are assumed to be complex.

In this work, we show that the blind MIMO system identification problem can be addressed by means of a constrained factor (CONFAC) decomposition [13]. Under the assumption of complex system matrix and complex input signals, we show that a collection of successive second-order derivatives of the second generating function of the system outputs can be stored in a higher-order tensor following a CONFAC decomposition, which arise by combining differentiation w.r.t. real and imaginary components of the second generating function of the system outputs. As we will show, the profile of 1's and 0's of the CONFAC constraint matrices captures the linear combination patterns involving real and imaginary components of the successive characteristic function derivatives. By exploiting different derivative forms, we can obtain an "extended" CONFAC tensor decomposition which is shown to be essentially unique under some conditions. Thanks to this uniqueness property, a blind estimation of the MIMO system response matrix is possible.

Notations: In the following, vectors, matrices and tensors are denoted by lower case boldface ($\mathbf{a}$), upper case boldface ($\mathbf{A}$) and upper case calligraphic ($\mathcal{A}$) letters respectively. a_i is the i-th coordinate of vector $\mathbf{a}$ and $\mathbf{a}_i$ is the i-th column of matrix $\mathbf{A}$. The (i, j) entry of matrix $\mathbf{A}$ is denoted A_{ij} and the (i, j, k) entry of the third order tensor $\mathcal{A}$ is denoted $\mathcal{A}_{ijk}$. Complex objects are underlined, their real and imaginary parts are denoted $\Re\{\cdot\}$ and $\Im\{\cdot\}$ respectively. E[.] denotes the expected value of a random variable.

2. PROBLEM FORMULATION

We consider a linear MIMO system with K inputs and N outputs. The system matrix is defined by $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{R}^{N\times K}$. Define $\mathbf{z}(m) = [z_1(m), \ldots, z_N(m)]^T \in \mathbb{R}^N$, $\mathbf{s}(m) = [s_1(m), \ldots, s_K(m)]^T \in \mathbb{R}^K$ and $\mathbf{n}(m) \in \mathbb{R}^N$ as the m^{th} discrete-time realizations of the output, input and noise vectors, respectively, $m = 1, \ldots, M$. According to this model we have:

$$\mathbf{z}(m) = \mathbf{H}\mathbf{s}(m) + \mathbf{n}(m). \quad (1)$$

The input signals can be real or complex. Our goal is to estimate $\mathbf{H}$ from the only knowledge of the system output. The approach presented here resorts to partial derivatives of the second generating function of the output. Specifically, the problem consists in finding $\widehat{\mathbf{H}}$ such that $\widehat{\mathbf{H}} = \mathbf{H}\mathbf{\Lambda}\mathbf{\Pi}$, where $\mathbf{\Lambda}$ is a diagonal matrix and $\mathbf{\Pi}$ is a permutation matrix. The identification of $\mathbf{H}$ relies on the following assumptions:

H1. $\mathbf{H}$ does not contain pairwise collinear columns;

H2. The inputs $s_1, \ldots, s_K$ are non-Gaussian and mutually statistically independent;

H3. The number of inputs K is known.

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The second generating function of the system output, Φ_z, can be decomposed into a sum of marginal second generating functions of the inputs, φ_k, $k = 1 \cdots K$. We start by defining Φ_z and φ_k in the complex field. Since generating functions of a complex variable are defined by assimilating $\mathbb{C}$ to $\mathbb{R}^2$, the second generating function of the k-th input φ_k taken at a point x of $\mathbb{C}$ can be compactly written as

$$\varphi_k(\Re\{x\}, \Im\{x\}) = \log \mathrm{E}[\exp(\Re\{x^* s_k\})]. \tag{2}$$

Likewise, Φ_z taken at the point $\mathbf{w}$ of $\mathbb{C}^N$ is defined in $\mathbb{R}^{2N}$. Denoting $\mathbf{x} = \Re\{\mathbf{z}\}$ and $\mathbf{y} = \Im\{\mathbf{z}\}$, it can be shown that

$$\Phi_z(\mathbf{w}) \quad = \quad \log \mathrm{E}[\prod_k \exp(\Re\{\mathbf{w}^{\mathrm{H}}\mathbf{h}_k s_k\})], \tag{3}$$

where $\mathbf{h}_k$ is the k-th column of $\mathbf{H}$. Using the hypothesis of mutual statistical independence of the inputs, (2) yields

$$\Phi_z(\Re\{\mathbf{w}\}, \Im\{\mathbf{w}\}) = \sum_k \varphi_k\left(\Re\{\mathbf{w}^{\mathrm{T}}\mathbf{h}_k^*\}, \Im\{\mathbf{w}^{\mathrm{T}}\mathbf{h}_k^*\}\right). \tag{4}$$

Finally, by defining $\mathbf{A}$ and $\bar{\mathbf{A}}$ so that $\mathbf{H} = \mathbf{A} + \jmath\bar{\mathbf{A}}$, we arrive at

$$\Phi_z(\mathbf{w}) = \sum_k \varphi_k\left(g_1(\mathbf{w})\,,\, g_2(\mathbf{w})\right), \tag{5}$$

where $\mathbf{w} = (\mathbf{u}, \mathbf{v}) \in \mathbb{R}^{2N}$, $\mathbf{u} = \Re\{\mathbf{w}\}$, $\mathbf{v} = \Im\{\mathbf{w}\}$ and

$$\begin{aligned} g_1(\mathbf{w}) \quad &= \quad \sum_n A_{nk}u_n + \bar{A}_{nk}v_n \\ g_2(\mathbf{w}) \quad &= \quad \sum_n A_{nk}v_n - \bar{A}_{nk}u_n \end{aligned}$$

Let us define

$$\begin{aligned} g:\quad \mathbb{R}^{2N} &\longrightarrow \mathbb{R}^2 \\ \mathbf{w} &\longmapsto g(\mathbf{w}) = (g_1(\mathbf{w}), g_2(\mathbf{w})). \end{aligned}$$

and a mapping φ_k from $\mathbb{R}^2$ to $\mathbb{R}$, as

$$\begin{aligned} \varphi_k:\quad \mathbb{R}^2 &\longrightarrow \mathbb{R} \\ g &\longmapsto \varphi_k(g). \end{aligned}$$

A more compact representation of (5) can thus be obtained:

$$\Phi_z(\mathbf{w}) = \sum_k \varphi_k\left(g(\mathbf{w})\right).$$

Note that defining φ_k and Φ_z in $\mathbb{R}^{2N}$ and $\mathbb{R}^2$, respectively, instead of $\mathbb{C}^N$ and $\mathbb{C}^2$, allows their differentiation.

3. THE CONFAC DECOMPOSITION

For a third-order tensor $\mathcal{X} \in \mathbb{C}^{P\times Q\times R}$, the constrained factor (CONFAC) decomposition of $\mathcal{X}$ with F factor combinations is defined in scalar form as:

$$X_{pqr} = \sum_{f=1}^{F}\sum_{f_1=1}^{F_1}\sum_{f_2=1}^{F_2}\sum_{f_3=1}^{F_3} A_{pf_1}B_{qf_2}C_{rf_3}\Theta_{f_1f}\Psi_{f_2f}\Omega_{f_3f}, \tag{6}$$

$$\text{with } F \geq \max\left(F_1, F_2, F_3\right),$$

where $A_{pf_1} \doteq [\mathbf{A}]_{p,f_1}$, $B_{qf_2} \doteq [\mathbf{B}]_{q,f_2}$, $C_{rf_3} \doteq [\mathbf{C}]_{r,f_3}$ are entries of three *factor matrices* $\mathbf{A} \in \mathbb{C}^{P\times F_1}$, $\mathbf{B} \in \mathbb{C}^{Q\times F_2}$, $\mathbf{C} \in \mathbb{C}^{R\times F_3}$, respectively, and $\Theta_{f_1f} \doteq [\boldsymbol{\Theta}]_{f_1f}$, $\Psi_{f_2f} \doteq [\boldsymbol{\Psi}]_{f_2f}$, $\Omega_{f_3f} \doteq [\boldsymbol{\Omega}]_{f_3,f}$ are entries of first-, second- and third-mode constraint matrices $\boldsymbol{\Theta}, \boldsymbol{\Psi}$ and $\boldsymbol{\Omega}$, respectively. These constraint matrices are full row-rank matrices. In this work, we assume that the entries of these matrices belong to the set $\{-1, 0, 1\}$. Note that the CONFAC decomposition with $F_i = F$, $i = 1, 2, 3$, and $\boldsymbol{\Theta} = \boldsymbol{\Psi} = \boldsymbol{\Omega} = \mathbf{I}_F$ coincides with the F-factor PARAFAC decomposition [8, 9]. Uniqueness results have been reported in a recent contribution [14].

The CONFAC decomposition can be represented in an equivalent matrix form by unfolding the information contained in the tensor $\mathcal{X} \in \mathbb{C}^{P\times Q\times R}$. For instance, it can be shown that the matrix unfolding $\mathbf{X}_1 \in \mathbb{C}^{PQ\times R}$ admits the following factorization [13]:

$$\mathbf{X}_1 \quad = \quad \big((\mathbf{A}\boldsymbol{\Theta}) \odot (\mathbf{B}\boldsymbol{\Psi})\big)\big(\mathbf{C}\boldsymbol{\Omega}\big)^T, \tag{7}$$

where $\odot$ is the Khatri-Rao (column-wise Kronecker) product.

4. EXPANDING SECOND-ORDER DERIVATIVES USING THE CONFAC DECOMPOSITION

For a fixed differentiation order, the number of derivative equations can be increased by computing partial derivatives of Φ_z in R different points of $\mathbb{R}^{2N}$, denoted here as $\mathbf{w}^{(r)} = (u^{(r)}, v^{(r)})$, $r = 1 \cdots R$. In this work, we limit ourselves to the second-order case for simplicity, being understood that equations associated with higher differentiation orders can be similarly obtained.

By successively differentiating (6) twice with respect to $u_p^{(r)}$ and $u_q^{(r)}$, $p = 1, \ldots, N$, $q = 1, \ldots, N$, $r = 1, \ldots, R$, we get:

$$\begin{aligned} \frac{\partial^2\Phi_z(\mathbf{w}^{(r)})}{\partial u_p^{(r)}\partial u_q^{(r)}} \quad = \quad \sum_{k=1}^{K}\Big(&A_{pk}A_{qk}G_{rk}^{11} - A_{pk}\bar{A}_{qk}G_{rk}^{12} \\ &-\bar{A}_{pk}A_{qk}G_{rk}^{12} + \bar{A}_{pk}\bar{A}_{qk}G_{rk}^{22}\Big), \end{aligned} \tag{8}$$

where

$$G_{r,k}^{ij} = \frac{\partial^2\psi_k(g(\mathbf{w}^{(r)}))}{\partial g_i(\mathbf{w}^{(r)})\partial g_j(\mathbf{w}^{(r)})} \quad i = 1, 2 \quad j = 1, 2,$$

and we have used the fact that $G_{rk}^{12} = G_{rk}^{21}$. Similarly, by differentiating (6) twice with respect to $v_p^{(r)}$ and $v_q^{(r)}$, yields:

$$\begin{aligned} \frac{\partial^2\Phi_z(\mathbf{w}^{(r)})}{\partial v_p^{(r)}\partial v_q^{(r)}} \quad = \quad \sum_{k=1}^{K}\Big(&\bar{A}_{pk}\bar{A}_{qk}G_{rk}^{11} + \bar{A}_{pk}A_{qk}G_{rk}^{12} \\ &+A_{pk}\bar{A}_{qk}G_{rk}^{12} + A_{pk}A_{qk}G_{rk}^{22}\Big). \end{aligned} \tag{9}$$

Finally, the differentiation of (6) twice with respect to $u_p^{(r)}$ and $v_q^{(r)}$, yields:

$$\begin{aligned} \frac{\partial^2\Phi_z(\mathbf{w}^{(r)})}{\partial u_p^{(r)}\partial v_q^{(r)}} \quad = \quad \sum_{k=1}^{K}\Big(&A_{pk}\bar{A}_{qk}G_{rk}^{11} + A_{pk}A_{qk}G_{rk}^{12} \\ &-\bar{A}_{pk}\bar{A}_{qk}G_{rk}^{12} - \bar{A}_{pk}A_{qk}G_{rk}^{22}\Big). \end{aligned} \tag{10}$$

Let $\{\Phi_z(\mathbf{w}^{(1)}), \Phi_z(\mathbf{w}^{(2)}), \ldots, \Phi_z(\mathbf{w}^{(R)})\}$ be the set containing the second generating function evaluated at R different points of the observation space, with $\mathbf{w}^{(r)} = (\mathbf{u}^{(r)}, \mathbf{v}^{(r)})$. Define the three third-order tensors $\mathcal{X}^{\Phi_1} \in \mathbb{C}^{N\times N\times R}$, $\mathcal{X}^{\Phi_2} \in \mathbb{C}^{N\times N\times R}$

and $\mathcal{X}^{\Phi_3} \in \mathbb{C}^{N\times N\times R}$ storing the second-order derivatives of $\Phi_z(\mathbf{w}^{(r)})$ w.r.t. (u_p, u_q), (v_p, v_q) and (u_p, v_q), respectively, as:

$$\mathcal{X}^{\Phi_1}_{pqr} \overset{\text{def}}{=} \frac{\partial^2 \Phi_z(\mathbf{w}^{(r)})}{\partial u_p^{(r)} \partial u_q^{(r)}}, \quad \mathcal{X}^{\Phi_2}_{pqr} \overset{\text{def}}{=} \frac{\partial^2 \Phi_z(\mathbf{w}^{(r)})}{\partial v_p^{(r)} \partial v_q^{(r)}},$$
$$\mathcal{X}^{\Phi_3}_{pqr} \overset{\text{def}}{=} \frac{\partial^2 \Phi_z(\mathbf{w}^{(r)})}{\partial u_p^{(r)} \partial v_q^{(r)}} \tag{11}$$

We call $\mathcal{X}^{\Phi_1}$, $\mathcal{X}^{\Phi_2}$ and $\mathcal{X}^{\Phi_3}$ simply as "derivative tensors" that result by successively differentiating the second generating function of the outputs in three different manners. Let $\mathbf{A}^{(k)} \in \mathbb{R}^{N\times 2}$ and $\mathbf{G}^{(k)} \in \mathbb{R}^{R\times 3}$, $k = 1, \dots, K$, be defined as:

$$\mathbf{A}^{(k)} \overset{\text{def}}{=} \begin{bmatrix} A_{1k} & \bar{A}_{1k} \\ \vdots & \vdots \\ A_{Nk} & \bar{A}_{Nk} \end{bmatrix} = [\mathbf{a}_k,\, \bar{\mathbf{a}}_k]$$

$$\mathbf{G}^{(k)} \overset{\text{def}}{=} \begin{bmatrix} G^{11}_{1k} & G^{12}_{1k} & G^{22}_{1k} \\ \vdots & \vdots & \vdots \\ G^{11}_{Rk} & G^{12}_{Rk} & G^{22}_{Rk} \end{bmatrix} = [\mathbf{g}_{1,k},\, \mathbf{g}_{2,k},\, \mathbf{g}_{3,k}]$$

4.1. CONFAC formulation

Using these definitions, we can show that (8)-(10) can be written as a CONFAC decomposition of the s-th derivative tensor $\mathcal{X}^{\Phi_s} \in \mathbb{C}^{N\times N\times R}$, $s = 1, 2, 3$, which is given by a sum of K derivative tensor "blocks", i.e. $\mathcal{X}^{\Phi_s}_{pqr} = \sum_{k=1}^{K} \mathcal{X}^{\Phi_s(k)}_{pqr}$, The CONFAC decomposition of the k-th derivative tensor is given by:

$$\mathcal{X}^{\Phi_s(k)}_{pqr} = \sum_{f=1}^{4}\sum_{f_1=1}^{2}\sum_{f_2=1}^{2}\sum_{f_3=1}^{3} A^{(k)}_{pf_1} A^{(k)}_{qf_2} G^{(k)}_{rf_3} \Theta^{(s)}_{f_1 f} \Psi^{(s)}_{f_2 f} \Omega^{(s)}_{f_3 f} \tag{12}$$

where the constraint matrices have the following structure:

$$\mathbf{\Theta}^{(s)} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \; \mathbf{\Psi}^{(s)} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \tag{13}$$

$$\mathbf{\Omega}^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; \mathbf{\Omega}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{\Omega}^{(3)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{14}$$

Note that the k-th tensor block in (12) is given by a sum of F outer products involving repeated linear combinations of the columns of real and imaginary parts $\mathbf{A}^{(k)}$ and $\mathbf{G}^{(k)}$ of the system matrix. The joint structure of $\mathbf{\Theta}^{(s)}$, $\mathbf{\Psi}^{(s)}$ and $\mathbf{\Omega}^{(s)}$ determine such a linear combination pattern and depends on the pair of differentiation variables with respect to which the second generating function $\Phi_z(\mathbf{w}^{(r)})$ is successively derived. The pairs of differentiation variables are $(u_p^{(r)}, u_q^{(r)})$, $(v_p^{(r)}, v_q^{(r)})$ and $(u_p^{(r)}, v_q^{(r)})$, for $s = 1, 2$ and 3, respectively. Note also that the first- and second-mode constraint matrices do not depend on the differentiation variables, the dependence being confined in the third-mode constraint matrix.

Let us define the block matrices

$$\mathbf{A} = [\mathbf{A}^{(1)} \,|\, \dots \,|\, \mathbf{A}^{(K)}] \in \mathbb{R}^{N\times 2K}, \tag{15}$$
$$\mathbf{G} = [\mathbf{G}^{(1)} \,|\, \dots \,|\, \mathbf{G}^{(K)}] \in \mathbb{R}^{R\times 3K}, \tag{16}$$

which concatenate the contributions of the K system inputs. Define also block-diagonal constraint matrices

$$\bar{\mathbf{\Theta}} = \mathbf{I}_K \otimes \mathbf{\Theta} \in \mathbb{R}^{2K\times 4K}, \tag{17}$$
$$\bar{\mathbf{\Psi}} = \mathbf{I}_K \otimes \mathbf{\Psi} \in \mathbb{R}^{2K\times 4K}, \tag{18}$$
$$\bar{\mathbf{\Omega}}^{(s)} = \mathbf{I}_K \otimes \mathbf{\Omega}^{(s)} \in \mathbb{R}^{3K\times 4K}. \tag{19}$$

With these definitions, we can treat the derivative tensors $\mathcal{X}^{\Phi_s} \in \mathbb{C}^{N\times N\times R}$, $s = 1, 2, 3$, as a CONFAC decomposition with K blocks. From (7), we can deduce the following correspondences:

$$(\mathbf{A}, \mathbf{B}, \mathbf{C}) \leftrightarrow (\mathbf{A}, \mathbf{A}, \mathbf{G}), \quad (\mathbf{\Theta}, \mathbf{\Psi}, \mathbf{\Omega}) \leftrightarrow (\bar{\mathbf{\Theta}}, \bar{\mathbf{\Psi}}, \bar{\mathbf{\Omega}}^{(s)}),$$
$$(F_1, F_2, F_3, F) \leftrightarrow (2K, 2K, 3K, 4K),$$
$$(P, Q, R) \leftrightarrow (N, N, R),$$

4.2. Combining all derivative tensors

Note that the CONFAC tensors $\mathcal{X}^{\Phi_1} \in \mathbb{C}^{N\times N\times R}$, $\mathcal{X}^{\Phi_2} \in \mathbb{C}^{N\times N\times R}$ and $\mathcal{X}^{\Phi_3} \in \mathbb{C}^{N\times N\times R}$ differ on the structure of the third-mode constraint matrix $\bar{\mathbf{\Omega}}_s$, $s = 1, 2, 3$. Here, we take all the three types of second-order derivatives into account by means of the following bigger CONFAC model

$$\bar{\mathbf{X}}_1 = \begin{bmatrix} \big((\mathbf{A}\bar{\mathbf{\Theta}}) \odot (\mathbf{A}\bar{\mathbf{\Psi}})\big)\big(\mathbf{G}\bar{\mathbf{\Omega}}^{(1)}\big)^T \\ \big((\mathbf{A}\bar{\mathbf{\Theta}}) \odot (\mathbf{A}\bar{\mathbf{\Psi}})\big)\big(\mathbf{G}\bar{\mathbf{\Omega}}^{(2)}\big)^T \\ \big((\mathbf{A}\bar{\mathbf{\Theta}}) \odot (\mathbf{A}\bar{\mathbf{\Psi}})\big)\big(\mathbf{G}\bar{\mathbf{\Omega}}^{(3)}\big)^T \end{bmatrix} \tag{20}$$

which can be rewritten compactly in the standard form (7), as

$$\bar{\mathbf{X}}_1 = \big((\tilde{\mathbf{A}}\tilde{\mathbf{\Theta}}) \odot (\mathbf{A}\tilde{\mathbf{\Psi}})\big)(\mathbf{G}\tilde{\mathbf{\Omega}})^T, \tag{21}$$

where $\tilde{\mathbf{A}} = \mathbf{I}_3 \otimes \mathbf{A} \in \mathbb{C}^{3N\times 6K}$, $\tilde{\mathbf{\Theta}} = \mathbf{I}_3 \otimes \bar{\mathbf{\Theta}} \in \mathbb{C}^{6K\times 12K}$, $\tilde{\mathbf{\Psi}} = \mathbf{1}_3^T \otimes \bar{\mathbf{\Psi}} \in \mathbb{C}^{2K\times 12K}$, $\tilde{\mathbf{\Omega}} = \left[\bar{\mathbf{\Omega}}^{(1)}, \bar{\mathbf{\Omega}}^{(2)}, \bar{\mathbf{\Omega}}^{(3)}\right] \in \mathbb{R}^{3K\times 12K}$.

5. UNIQUENESS ISSUES

Note that (21) can be rewritten as:

$$\bar{\mathbf{X}}_1 = (\mathbf{I}_3 \otimes \mathbf{A} \otimes \mathbf{A})\; \mathbf{T}\; \mathbf{G}^T, \tag{22}$$

with $\mathbf{T} = [\mathbf{T}^{(1)T}, \mathbf{T}^{(2)T}, \mathbf{T}^{(3)T}]^T$ and

$$\mathbf{T}^{(s)} = [(\mathbf{I}_K \otimes \mathbf{\Theta}) \odot (\mathbf{I}_K \otimes \mathbf{\Psi})]\; (\mathbf{I}_K \otimes \mathbf{\Omega}^{(s)})^T \tag{23}$$

$s = 1, 2, 3$. It can be shown that $\mathbf{T}^{(s)}$ has full column rank by definition. We assume that $N^2 \geq K$ and that $\mathbf{G}$ have full column rank (which implies and $R \geq 3K$). The latter restriction is not severe since R is the number of included points of the observation space. Denote alternative component matrices to $\mathbf{A}$ and $\mathbf{G}$, respectively, as $\mathbf{F}$ and $\mathbf{L}$, with $\mathbf{F} = [\mathbf{F}_1 \,|\, \dots \,|\, \mathbf{F}_K]$ and $\mathbf{F}_k = [\mathbf{f}_k \,|\, \bar{\mathbf{f}}_k]$. Recall that $\mathbf{A}_k$ contains the real part $\mathbf{a}_k$ and imaginary part $\bar{\mathbf{a}}_k$ of the k-th column of the complex-valued mixing matrix $\mathbf{H}$, i.e. $\mathbf{H} = [\mathbf{a}_1 \dots \mathbf{a}_K] + \jmath\,[\bar{\mathbf{a}}_1 \dots \bar{\mathbf{a}}_K]$. $\mathbf{H}$ is called *essentially unique* if for any alternative $\hat{\mathbf{H}}$ the relation $\hat{\mathbf{H}} = \mathbf{H}\,\mathbf{\Pi}\,\mathbf{\Lambda}$ holds, with $\mathbf{\Pi}$ a permutation matrix and $\mathbf{\Lambda}$ a complex-valued nonsingular diagonal matrix. Multiplying the k-th column of $\mathbf{H}$ by $\alpha_k + \jmath\,\beta_k$ yields $(\alpha_k\, \mathbf{a}_k - \beta_k\, \bar{\mathbf{a}}_k) + \jmath\,(\beta_k\, \mathbf{a}_k + \alpha_k\, \bar{\mathbf{a}}_k)$. Thus, $\mathbf{H}$ is essentially unique if for any alternative $\mathbf{F} = [\mathbf{F}_1 \,|\, \dots \,|\, \mathbf{F}_K]$, there holds

$$\mathbf{F}_k = \mathbf{A}_{\pi(k)} \begin{bmatrix} \alpha_k & \beta_k \\ -\beta_k & \alpha_k \end{bmatrix}, \qquad k = 1, \dots, K, \tag{24}$$

with $\pi(\cdot)$ a permutation of $\{1,\ldots,K\}$, and α_k and β_k not both zero, $k=1,\ldots,K$. Under the assumptions above, we have proven that condition (24) is satisfied for system configurations $(N,K)=(3,2)$, $(N,K)=(3,3)$, $(N,K)=(4,3)$ and $(N,K)=(4,4)$. The procedure adopted to prove uniqueness is long and the details will be provided in an extended version of this paper. We still do not have the proof for the underdetermined case $N<K$ (i.e. more inputs than outputs), although our numerical experiments have demonstrated uniqueness for $N=3$ and $K=4$.

6. NUMERICAL RESULTS

The blind estimation of the MIMO system matrix consists in fitting the CONFAC model (21) to the derivative tensor. In this work, we use the alternating least squares (ALS) algorithm [9], which alternates between estimates of $\mathbf{A}$ and $\mathbf{G}$ exploiting the unfolded matrix representations of the proposed CONFAC model. The details of the ALS algorithm have been omitted due to lack of space. After convergence, by properly combining pairs of columns of the final estimate of the real-valued $\mathbf{A}$, an estimate of the complex-valued MIMO system matrix $\mathbf{H}$ can be obtained.

In Figure 1, we compare the proposed method, named "CONFAC-ALS-2", with the LEMACAFC-2 method [12] and the 6-BIOME method [3]. Note that both CONFAC-ALS-2 and LEMACAFC-2 rely on second-order derivatives of the system outputs. The 6-BIOME exploits the hexacovariance of the system outputs. The normalized mean square error (NMSE)

$$f_H(\mathbf{H},\hat{\mathbf{H}})=\frac{\text{vec}(\mathbf{H}-\hat{\mathbf{H}})^T\text{vec}(\mathbf{H}-\hat{\mathbf{H}})}{\text{vec}(\mathbf{H})^T\text{vec}(\mathbf{H})} \tag{25}$$

is plotted as a function of the number of observations for $N=3$, $K=4$, $R=10$ and SNR=80dB. Each of the NMSE curves represents an average of 50 independent Monte Carlo runs. It can be noted that the proposed CONFAC-ALS-2 method offers improved performance over the LEMACAFC-2 one using the same data tensor. The 6-BIOME method offers the best performance. This comes from the additional diversity provided by the use of sixth-order statistics.

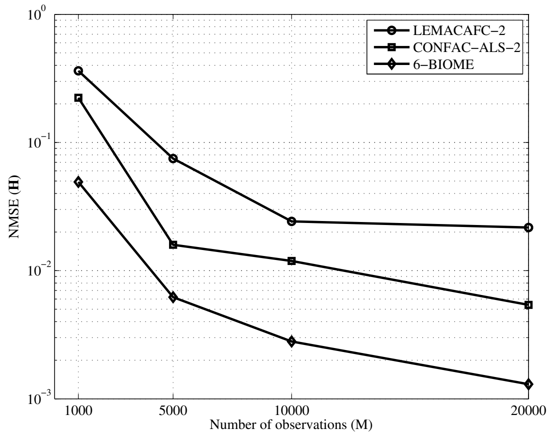

Fig. 1. RMSE vs. M

7. CONCLUSION

The problem of blind identification of complex MIMO systems driven by complex input signals can be addressed by resorting to the CONFAC decomposition. By combining three types of output second-order generating function derivatives taken at different points of the observation space, a CONFAC decomposition is proposed. Thanks to its essential uniqueness, a blind estimate of the system matrix can be obtained using the ALS algorithm. An extended version of this paper should consider the use of third-order derivatives as a means to improve performance of the proposed blind estimation method. The discussion of the uniqueness properties of the underlying CONFAC model and more efficient estimation algorithms will also be addressed in a future contribution.

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