MULTIPATH PARAMETER ESTIMATION OF TIME-VARYING SPACE-TIME COMMUNICATION CHANNELS USING PARALLEL FACTOR ANALYSIS

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ABSTRACT

In this work we propose a new method for estimation of multipath parameters in the context of mobile communications. The proposed method is an effective way to exploit the fact that the paths amplitudes are fast-varying while angles and delays are slowly-varying over multiple time-slots. By relying on such a multislot invariance of angles and delays and by periodically extending a training sequence over multiple time-slots, we show that the received signal can be modeled as a third-order (3D) tensor, which follows a Parallel Factor (PARAFAC) model. An accelerated alternating least squares (ALS) algorithm is used for a joint estimation of the angles of arrival, delays and fading amplitudes of the multipaths. Numerical results from computer simulations show that the proposed PARAFAC-based estimator is capable of estimating the multipath channel parameters with good accuracy using short training sequences and with fewer receiver antennas than multipaths.

1. INTRODUCTION

The issue of parametric multipath channel estimation has been exploited in several works [1, 2, 3]. Most of approaches are based on subspace methods, which exploit shift-invariance properties and/or the knowledge of the pulse shape function. Simultaneous estimation of angles of arrival and delays benefits from the fact that paths amplitudes are fast-varying while angles and delays are slowly-varying over multiple transmission blocks or time-slots. In [1, 2, 3], the angles and delays are blindly-estimated using a collection of previous estimates of the space-time channel impulse response. As in [2, 3], the linear-phase variation property of the frequency domain transformed version of the known pulse shape function is exploited. Training-sequence-based space-time channel estimation methods exploiting the multislot invariance of angles and delays have been proposed recently in [4].

In this paper we develop a new approach to multipath parameter estimation of time-varying space-time channels using Parallel Factor (PARAFAC) analysis [5, 6]. PARAFAC was introduced in the context of wireless communications by N. D. Sidiropoulos as a modeling toll for several signal processing problems (see [7] and associated references therein). In this work, we use the fact that the variation of multipath amplitudes over multiple time-slots is faster than that of angles and delays for showing that the received signal can be modeled as a third-order (3D) tensor. The proposed PARAFAC model arises thanks to the use of a training sequence which is periodically extended over multiple time-slots, which are jointly processed at the receiver. By tapping on the powerful identifiability properties of the PARAFAC decomposition, the proposed method performs joint estimation of the angles of arrival, the time-delays and the fading amplitudes of the multipaths without any ambiguity. An accelerated alternating least squares (ALS) algorithm is used for this purpose. Numerical results from computer simulations show that the PARAFAC-based estimator is capable of estimating the triplet angle-delay-amplitude for each multipath with good accuracy even for short training sequences, provided that the number of time-slots processed is enough. The proposed estimator also performs well with fewer receiver antennas than multipaths.

This paper is organized as follows. Section 2 introduces the signal and multipath channel models. Section 3 presents the PARAFAC modeling approach to the problem of multipath parameter estimation. Identifiability issues of the proposed model are discussed in Section 4. In Section 5, the PARAFAC estimator is proposed. Numerical results are evaluated in Section 6 while Section 7 contains the conclusions.

2. SIGNAL AND CHANNEL MODELS

Let us consider a wireless communication system in which a digital signal is transmitted in a specular multipath environment. The receiver is equipped with an array of M antennas spaced half wavelength or closer. We focus on the case of a single-user transmission. The transmitted information symbols are organized into I blocks or time-slots. Assume that the time-slots are sufficiently short so that the channel fading can be regarded as stationary over a time-interval necessary for the transmission of a whole time-slot and it varies independently from slot to slot. This is typically the case of Time Division Multiple Access (TDMA)-based systems [4]. We assume that the considered system is training-sequence-based, with the particular characteristic that consists in reusing the training sequence: A known training sequence of N symbols is periodically extended over multiple time-slots that are jointly processed at the receiver. The idea of processing multiple time-slots, based on training sequence reuse is also known as *multislot processing*.

Let $\{s(n)\}_{n=1}^{N}$ be the known training sequence. During the training period, the received baseband discrete-time signal impinging the antenna array at the n-th symbol period for the i-th slot, $\mathbf{x}_i(n) = [x_{1,i}(n), \ldots, x_{M,i}(n)]^T \in \mathbb{C}^M$ can be written as the convolution of the training sequence and the i-th channel response:

$$\mathbf{x}_i(n) = \sum_{k=0}^{K-1} \mathbf{h}_i(kT)s(n-k) + \mathbf{v}_i(n), \tag{1}$$

where T is the symbol period and $\mathbf{v}_i(n)$ is the additive noise, which is assumed to be Gaussian with variance σ_v^2, irrespective of the slot. The temporal support of the channel impulse response is $(0, KT]$.

A matrix model for the received signal can be obtained from (1):

$$\mathbf{X}_i = \mathbf{H}_i \mathbf{S}^T + \mathbf{V}_i, \tag{2}$$

where the received samples are collected into the matrix $\mathbf{X}_i = [\mathbf{x}_i(0), \ldots, \mathbf{x}_i(N-1)] \in \mathbb{C}^{M\times N}$, the space-time channel impulse response matrix is $\mathbf{H}_i = [\mathbf{h}_i(0), \ldots, \mathbf{h}_i(K-1)T] \in \mathbb{C}^{M\times K}$ and $\mathbf{S}$ is a Toeplitz matrix, $[\mathbf{S}]_{n,k} = s(n-k)$. Finally, $\mathbf{V}_i = [\mathbf{v}_i(0), \ldots, \mathbf{v}_i(N-1)] \in \mathbb{C}^{M\times N}$ is the discrete-time noise, assumed to be a Gaussian, temporally and spatially uncorrelated sequence.

The multipath channel within the I slots can be modeled as a combination of L paths, each one of them being characterized by an angle of arrival θ_l, a relative propagation delay τ_l and a complex valued amplitude (fading coefficient) $\beta_l(i)$ that accounts for the channel variations over the I slots:

$$\mathbf{h}_i(kT) = \sum_{l=1}^{L} \beta_l(i) \mathbf{a}(\theta_l) g(kT - \tau_l). \tag{3}$$

The known waveform or pulse shape function is the convolution of the transmitter and receiver matched filters and $\mathbf{a}(\theta_l) \in \mathbb{C}^M$ is the array response to a narrowband signal impinging the array from an angle of arrival θ_l. For a uniform linear array of half-wavelength-spaced omnidirectional antennas, we have $\mathbf{a}(\theta_l) = [1, e^{-j\pi \sin(\theta_l)}, \ldots, e^{-j\pi(M-1)\sin(\theta_l)}]^T$. The variations of angles and delays of the paths over the I slots can be considered as negligible so that the set of parameters $\{\theta_l, \tau_l\}_{l=1}^L$ can be considered constant, i.e., slot-independent. In mobile communication systems, this assumption is reasonable if the number of slots I is chosen according to the mobile speed and multipath geometry. Furthermore, the path amplitudes $\{\beta_l(i)\}_{l=1}^L$ are assumed to be uncorrelated from slot to slot, although this is not a necessary assumption in the context of this work. The space-time channel matrix (3) can be factored as

$$\mathbf{H}_i = \mathbf{A}(\boldsymbol{\theta})\, Diag(\boldsymbol{\beta}(i))\, \mathbf{G}^T(\boldsymbol{\tau}) \tag{4}$$

where $\mathbf{g}(\tau_l) = [g(-\tau_l), g(T-\tau_l), \ldots, g((K-1)T - \tau_l)]^T \in \mathbb{R}^K$ is the sampled delayed pulse shape function and $\mathbf{G}(\boldsymbol{\tau}) \in \mathbb{R}^{K\times L}$ collects L vectors for the set of delays $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^T$. Similarly, the matrix $\mathbf{A}(\boldsymbol{\theta}) \in \mathbb{C}^{M\times L}$ collects L array responses for the set of angles $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_L]^T$. The operator $Diag(\boldsymbol{\beta}(i))$ forms a diagonal matrix that holds the vector of fading amplitudes $\boldsymbol{\beta}(i) = [\beta_1(i), \ldots, \beta_L(i)]^T \in \mathbb{R}^L$ for the i-th slot on its diagonal.

3. PARAFAC MODEL

The multipath channel model (4) can be alternatively interpreted using the PARAFAC formalism. Let us write the slot-dependent space-time channel response in a scalar form as follows

$$h_{m,i,k} = \sum_{l=1}^{L} a_{m,l} \beta_{i,l} g_{k,l}. \tag{5}$$

$h_{m,i,k}$ is interpreted here as the (m,i,k)-th element of a three-way array or third-order *tensor* $\mathcal{H} \in \mathbb{C}^{M\times I\times K}$. Note that $a_{m,l} = [\mathbf{A}(\boldsymbol{\theta})]_{m,l}$, $g_{k,l} = [\mathbf{G}(\boldsymbol{\tau})]_{k,l}$ and $\beta_{i,l} = [\mathbf{B}]_{i,l}$, where $\mathbf{B} \in \mathbb{C}^{I\times L}$ collects the fading amplitudes for all slots. The model (5) is recognized as an L-factor PARAFAC decomposition [5], which decomposes each element $h_{m,i,k}$ as a sum of L rank-1 triple products. The three dimensions or *modes* of the tensor $\mathcal{H}$ are *space*, *slot* and *time*. It is also possible to represent (5) using matrix notation, as a function of matrices $\mathbf{A} = \mathbf{A}(\boldsymbol{\theta})$, $\mathbf{B}$ and $\mathbf{G} = \mathbf{G}(\boldsymbol{\tau})$. The space-time channel $\mathbf{H}_i$ of (4) can be regarded as the i-th matrix slice of the tensor $\mathcal{H}$, which is obtained by slicing the tensor along the *slot* dimension:

$$\mathbf{H}_{\cdot\, i\, \cdot} = \mathbf{A}\, D_i(\mathbf{B})\, \mathbf{G}^T, \ i = 1, \ldots, I. \tag{6}$$

where operator $D_i(\mathbf{B}) = \mathrm{Diag}(\boldsymbol{\beta}(i))$ forms a diagonal matrix from the i-th row of $\mathbf{B}$ and $\mathbf{H}_{\cdot\, i\, \cdot}$ is the slice notation for the $\mathbf{H}_i$ matrix. By defining $\overline{\mathbf{H}} = [\mathbf{H}_{\cdot\,1\,\cdot}^T, \ldots, \mathbf{H}_{\cdot\,I\,\cdot}^T]^T \in \mathbb{C}^{MI\times K}$ as a matrix collecting the I slices of the space-time channel, we get:

$$\overline{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{\cdot\,1\,\cdot} \\ \vdots \\ \mathbf{H}_{\cdot\,I\,\cdot} \end{bmatrix} = \begin{bmatrix} \mathbf{A}D_1[\mathbf{B}] \\ \vdots \\ \mathbf{A}D_I[\mathbf{B}] \end{bmatrix} \mathbf{G}^T = (\mathbf{B} \diamond \mathbf{A})\mathbf{G}^T \tag{7}$$

where $\diamond$ denotes the Khatri-Rao (column-wise Kronecker) matrix product, i.e., $\mathbf{B} \diamond \mathbf{A} = [\mathbf{b}_1 \otimes \mathbf{a}_1, \ldots, \mathbf{b}_L \otimes \mathbf{a}_L] \in \mathbb{C}^{MI\times L}$.

Now, let us go back to model (2), which expresses the received signal matrix for the i-th slot. We concatenate the received signal of the I slots by stacking column-wise the matrices $\mathbf{X}_1, \ldots, \mathbf{X}_I$. Using the slice notation in (7) with $\mathbf{X}_{\cdot\,i\,\cdot} = \mathbf{X}_i$, we have the following model

$$\overline{\mathbf{X}}_1 = \begin{bmatrix} \mathbf{X}_{\cdot\,1\,\cdot} \\ \vdots \\ \mathbf{X}_{\cdot\,I\,\cdot} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{\cdot\,1\,\cdot} \\ \vdots \\ \mathbf{H}_{\cdot\,I\,\cdot} \end{bmatrix} \mathbf{S}^T = (\mathbf{B} \diamond \mathbf{A})\mathbf{G}^T\mathbf{S}^T + \overline{\mathbf{V}}_1 \tag{8}$$

where the noise matrix $\overline{\mathbf{V}}_1 \in \mathbb{C}^{MI\times N}$ is defined in the same way as $\overline{\mathbf{X}}_1$. In this work we make use of the fact that a Fourier transform maps a delay to a certain phase shift. This fact will be exploited for an unambiguous multipath parameter estimation as will be shown latter. If the pulse shape function is bandlimited and sampled at or above the Nyquist rate, the Discrete Fourier Transform (DFT) of $\mathbf{g}(\tau)$ can be expressed as $\mathrm{DFT}(\mathbf{g}(\tau)) = \mathbf{f}(\phi)$ [2], where $\mathbf{f}(\phi) = [1, \phi, \ldots, \phi^{K-1}]^T \in \mathbb{C}^K$ and $\phi = e^{-j(2\pi/K)\tau}$. By taking the Discrete Fourier Transform (DFT) at each receiver antenna, the model (8) turns to the following one:

$$\check{\mathbf{X}}_1 = \begin{bmatrix} \check{\mathbf{X}}_{\cdot\,1\,\cdot} \\ \vdots \\ \check{\mathbf{X}}_{\cdot\,I\,\cdot} \end{bmatrix} = (\mathbf{B} \diamond \mathbf{A})\mathbf{F}^T\mathbf{S}^T + \check{\mathbf{V}}_1 \tag{9}$$

where $\mathbf{F}$ is a Vandermonde matrix

$$\begin{aligned} \mathbf{F} &= \begin{bmatrix} 1 & \cdots & 1 \\ \phi_1 & \cdots & \phi_L \\ \vdots & & \vdots \\ \phi_1^{(K-1)} & \cdots & \phi_L^{(K-1)} \end{bmatrix} \\ \phi_l &= e^{-j(2\pi/K)\tau_l}, \ \ l = 1, \ldots, L. \end{aligned} \tag{10}$$

From (9) and defining $\mathbf{C} = \mathbf{S}\mathbf{F} \in \mathbb{C}^{N\times L}$, the DFT of the received signal over multiple time-slots can be interpreted as a PARAFAC tensor where one of its factor matrices is itself the result of the product of two other matrices. The DFT-transformed received signal can thus be expressed as

$$\check{\mathbf{X}}_1 = \begin{bmatrix} \check{\mathbf{X}}_{\cdot\,1\,\cdot} \\ \vdots \\ \check{\mathbf{X}}_{\cdot\,I\,\cdot} \end{bmatrix} = (\mathbf{B} \diamond \mathbf{A})\mathbf{C}^T + \check{\mathbf{V}}_1 \tag{11}$$

The same information contained in (11) can also be rearranged as $\check{\mathbf{X}}_2 = (\mathbf{A} \diamond \mathbf{C})\mathbf{B}^T + \check{\mathbf{V}}_2 \in \mathbb{C}^{NM\times I}$ or as $\check{\mathbf{X}}_3 = (\mathbf{C} \diamond \mathbf{B})\mathbf{A}^T + \check{\mathbf{V}}_3 \in \mathbb{C}^{IN\times M}$.

4. IDENTIFIABILITY

At this point, we focus on the identifiability conditions of model (11) and discuss its link to the problem of multipath parameter estimation.

Definition 1 (Kruskal rank): *Given* $\mathbf{A} \in \mathbb{R}^{I\times R}$*,* $r_{\mathbf{A}} = rank(\mathbf{A}) = r$ *iff it contains* at least *a collection of* r *linearly independent columns but no collection of* $r+1$ *linearly independent columns. The* Kruskal-*rank (or* k*-rank) of* $\mathbf{A}$*, denoted by* $k_{\mathbf{A}}$ *is equal to* r *if* every *set of* r *columns of* $\mathbf{A}$ *is linearly independent. As a consequence,* $k_{\mathbf{A}} \leq r_{\mathbf{A}} \leq min(I, R)$*, i.e., the k-rank is always less than or equal to the conventional matrix rank. If* $\mathbf{A}$ *is full-rank it is also full k-rank.*

For the set of I matrices $\mathbf{X}_{\cdot\,i\,\cdot} = \mathbf{A}D_i(\mathbf{B})\mathbf{C}^T, \quad i = 1, \cdots, I$, obtained from the slice representation of model (11), if

$$k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2(L+1), \tag{12}$$

the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are unique up to permutation and scaling of columns. This means that any matrices $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$ and $\overline{\mathbf{C}}$ satisfying the model $\mathbf{X}_{\cdot\,i\,\cdot}$, $i = 1, \cdots, I$, are linked to $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ by

$$\overline{\mathbf{A}} = \mathbf{A}\mathbf{\Pi}\mathbf{\Delta}_1, \quad \overline{\mathbf{B}} = \mathbf{B}\mathbf{\Pi}\mathbf{\Delta}_2, \quad \overline{\mathbf{C}} = \mathbf{C}\mathbf{\Pi}\mathbf{\Delta}_3, \tag{13}$$

where $\mathbf{\Pi}$ is a permutation matrix and $\mathbf{\Delta}_1$, $\mathbf{\Delta}_2$ and $\mathbf{\Delta}_3$ are diagonal (scaling) matrices satisfying the condition

$$\mathbf{\Delta}_1\mathbf{\Delta}_2\mathbf{\Delta}_3 = \mathbf{I}. \tag{14}$$

It is worth mentioning that the permutation ambiguity does not need to be solved in the context of the multipath parameter estimation problem, since the ordering of multipath spatial and temporal responses is not important. Concerning the scaling ambiguity, it can be eliminated from our model thanks to the Vandermonde structure of $\mathbf{A}$ and $\mathbf{F}$. In other words, by using the fact that all the entries in the first row of these matrices are equal to one, the scaling matrices $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_3$ are known, allowing the determination of $\mathbf{\Delta}_1$ from (14).

In the present context, we make the following assumptions concerning the multipath channel structure. 1) The array manifold is known and the multipath signals arrive at the array at distinct angles; 2) The multipaths undergo independent fading and vary independently from slot to slot and 3) The multipaths have distinct propagation delays to the receiver. Under these assumptions, the identifiability condition (12) can be further simplified if some additional structure of the model is taken into account. Let us first state the following Lemmas:

Lemma 1 (Vandermonde k-rank Lemma [8]): *A Vandermonde matrix* $\mathbf{V} \in \mathbb{C}^{m\times n}$ *with distinct nonzero generators* $\phi_1, \phi_2, \ldots, \phi_n \in \mathbb{C}$ *is not only full rank but also full k-rank, i.e.,* $k_{\mathbf{V}} = r_{\mathbf{V}} = min(m, n)$. ■

Lemma 2: *Let* $\mathbf{A} \in \mathbb{C}^{p\times m}$ *and* $\mathbf{B} \in \mathbb{C}^{m\times n}$ *be two matrices. If* $\mathbf{A}$ *is full column rank, then* $r_{\mathbf{AB}} = r_{\mathbf{B}}$*. If* $\mathbf{B}$ *is a Vandermonde matrix with distinct nonzero generators* $\phi_1, \phi_2, \ldots, \phi_n \in \mathbb{C}$*, then the full column rank condition of* $\mathbf{A}$ *implies that* $r_{\mathbf{AB}} = min(m, n)$. ■

Note that the matrix of spatial array responses $\mathbf{A}$ is Vandermonde, for which Lemma 1 applies, i.e., $k_{\mathbf{V}} = \min(M, L)$. Let us study the structure of matrix $\mathbf{C}$ in (11). This matrix is factored as the product of a Toeplitz matrix $\mathbf{S}^T$ and a Vandermonde matrix $\mathbf{F}$. Without loss of generality, let us assume that $K \geq L$. Under the condition of "persistence of excitation" of the training symbols, matrix $\mathbf{S}$ is full column rank (also full k-rank). Thus, Lemma 2 can be directly applied to $\mathbf{C} = \mathbf{SF}$, which means that $k_{\mathbf{C}} = \min(K, L) = L$. Finally, the matrix of fading amplitudes $\mathbf{B}$ is also full k-rank with probability one, under the condition of independent multipath fading variation [9]. Thus, the identifiability condition (12) can be equivalently stated as

$$\min(M, L) + \min(I, L) \geq L + 2. \tag{15}$$

By studying the condition (15), we can distinguish two cases:

1. If $I \geq L$ then $M \geq 2$ receiver antennas are sufficient for estimating angle, delay and amplitudes of the L multipaths.
2. If $M \geq L$ then $I \geq 2$ slots are sufficient for estimating the set of multipath parameters.

5. PARAFAC-BASED ESTIMATOR

The receiver algorithm for the joint estimation of angles, delays and amplitudes of the multipaths fully exploits the trilinear structure of the multipath channel model. It is based on the alternating least squares (ALS) principle coupled with an efficient method for accelerating the convergence of the PARAFAC estimator.

Recall from Section 3 that the received multi-slot signal admits three matrix representations $\breve{\mathbf{X}}_1 \in \mathbb{C}^{MI\times N}$, $\breve{\mathbf{X}}_2 \in \mathbb{C}^{NM\times I}$ and $\breve{\mathbf{X}}_3, \in \mathbb{C}^{IN\times M}$. The ALS algorithm consists in estimating in an alternating way the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ from matrices $\breve{\mathbf{X}}_{i=1,2,3}$. In presence of additive Gaussian noise, the estimated matrices respectively optimize three independent least squares criteria:

$$J_{ALS}(\widehat{\mathbf{C}}) = \|\breve{\mathbf{X}}_1 - (\mathbf{B}\diamond\mathbf{A})\mathbf{C}^T\|_F^2, \tag{16}$$

$$J_{ALS}(\widehat{\mathbf{B}}) = \|\breve{\mathbf{X}}_2 - (\mathbf{A}\diamond\mathbf{C})\mathbf{B}^T\|_F^2, \tag{17}$$

$$J_{ALS}(\widehat{\mathbf{A}}) = \|\breve{\mathbf{X}}_3 - (\mathbf{C}\diamond\mathbf{B})\mathbf{A}^T\|_F^2, \tag{18}$$

where $\|\cdot\|_F$ denotes the Frobenius norm of its matrix argument. One complete iteration of the ALS has three updating steps. The basic idea is to update one factor matrix using the least squares algorithm, conditioned on previously obtained estimates for the remaining factor matrices that define the decomposition. This process is repeated until convergence in the least squares fit. The ALS algorithm is monotonically convergent but sometimes it requires a large number of iterations to converge [10]. In this work we make use of the recently developed Enhanced Line Search (ELS) method [11] to speed up the convergence of the ALS estimator. The ELS principle consists in predicting in an optimal way the value of the factor matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ a certain number of iterations ahead. Due to lack of space, see [11] for further algorithmic details.

At the end of the j-th iteration, an overall error measurement between the estimated model and the received signal tensor can be obtained, for example, from the following equation:

$$e^{(i)} = \|\breve{\mathbf{X}}_1 - (\widehat{\mathbf{B}}^{(i)}\diamond\widehat{\mathbf{A}}^{(i)})(\widehat{\mathbf{C}}^{(i)})^T\|_F^2. \tag{19}$$

We declare that the algorithm has converged at the i-th iteration when $|e^{(i)} - e^{(i-1)}| \leq 10^{-6}$. At this point, we show how the scaling ambiguity is eliminated. At the end of the ALS-based estimation stage, an estimate of the DFT-transformed pulse shape response is obtained as $\widehat{\mathbf{F}} = \left(\mathbf{S}^T\right)^{\dagger}\widehat{\mathbf{C}}$. The final (unambiguous) estimate of $\mathbf{A}$ (array responses), $\mathbf{B}$ (fading amplitudes) and $\mathbf{F}$ (delay responses) are linked to the ALS-estimated matrices as:

$$\widehat{\mathbf{A}} = \widetilde{\mathbf{A}}\mathbf{\Delta}_1, \quad \widehat{\mathbf{B}} = \widetilde{\mathbf{B}}\mathbf{\Delta}_2, \quad \widehat{\mathbf{F}} = \widetilde{\mathbf{F}}\mathbf{\Delta}_3. \tag{20}$$

The *a priori* knowledge of the Vandermonde structure of $\mathbf{A}$ and $\mathbf{F}$ means that the first row of both matrices have unity entries, i.e., $\mathbf{A}(1,:) = [1, \ldots, 1]$ and $\mathbf{F}(1,:) = [1, \ldots, 1]$. This allows a unique determination of the scaling matrices $\mathbf{\Delta}_{i=1,2,3}$ satisfying (14) as $\mathbf{\Delta}_1 = D_1(\widehat{\mathbf{A}})$, $\mathbf{\Delta}_3 = D_1(\widehat{\mathbf{F}})$ and $\mathbf{\Delta}_2 = (\mathbf{\Delta}_1\mathbf{\Delta}_3)^{-1}$, from which the final estimates $\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{B}}$ and $\widetilde{\mathbf{F}}$ are obtained.

6. NUMERICAL RESULTS

In this section, the performance of the PARAFAC-based multipath parameter estimator is evaluated through computer simulations. The training sequence to be used over the I slots is randomly generated at each run, following a normal distribution with unity variance. The pulse shape function is a raised cosine with roll-off factor 0.35. The temporal support of the channel is $K = 5$. A multipath scenario with $L = 3$ paths is assumed. The angles of arrival and time delays are $\{\theta_1, \theta_2, \theta_3\} = \{-10°, 0, 20°\}$ and $\{\tau_1, \tau_2, \tau_3\} = \{0, 1.1T, 2T\}$. The paths are assumed to have the same average power. The results are averaged over 100 Monte Carlo runs. For each run, multipath fading amplitudes for the I time-slots are redrawn from an i.i.d. Rayleigh generator. For the combined ALS-ELS algorithm, random initialization is used. If convergence is not achieved within 100 iterations, we re-start the algorithm from a different initialization point. It has been observed however, that convergence is achieved within 20-30 iterations in most of the runs. The Root Mean Square Error (RMSE) between the estimated and true matrices is used here as the evaluation metric for the estimator performance.

Figure 1 depicts the RMSE versus SNR for the estimation of the array (angle) and pulse shape (delay) responses, considering $M = 2$ antennas and $N = 8$ training samples. The results are shown for $I = 5$ and $I = 10$ time-slots. It is seen that the proposed estimator exhibits a linear decrease in its RMSE as SNR increases. This is valid for both angle and delay RMSE. The performance gap between angle and delay estimation is due to the fact that the raised cosine pulse shape function is not bandlimited, which leads to some delay estimation bias. As expected, the estimator performance improves as the number of time-slots processed increases. Although not displayed in the figure, the RMSE results for the fading amplitudes are very close to those for the delay responses. Note that these performance results are achieved with fewer antennas than multipaths and with a very short training sequence, which is interesting characteristic of the proposed PARAFAC-based estimator.

7. CONCLUSIONS

This paper has developed a tensor modeling approach to the problem of multipath parameter estimation of time-varying wireless channels, which is based on PARAFAC analysis. The proposed model relies on the fact that the fading amplitudes of multipaths are fast-varying while its angles and delays can be considered as stationary across multiple time-slots. Under this assumption, we have shown that a third-order PARAFAC model for the received signal arises thanks to a periodically extension of a training sequence over multiple slots.

Based on the PARAFAC modeling, a multipath parameter estimation algorithm has been presented. The proposed estimator is based on multislot processing, fully exploiting the time-varying structure of the multipath channel for joint estimation of angles of arrival, delays and fading amplitudes of multipaths without any ambiguity. The identifiability conditions for the proposed tensor model have been discussed. An accelerated alternating least squares algorithm was considered for parameter estimation. The performance of the proposed estimator has been evaluated from computer simulations. According to the results, the PARAFAC-based estimator performs joint estimation of the triplet angle-delay-amplitude with good accuracy, using short training sequences and with fewer receiver antennas than multipaths.

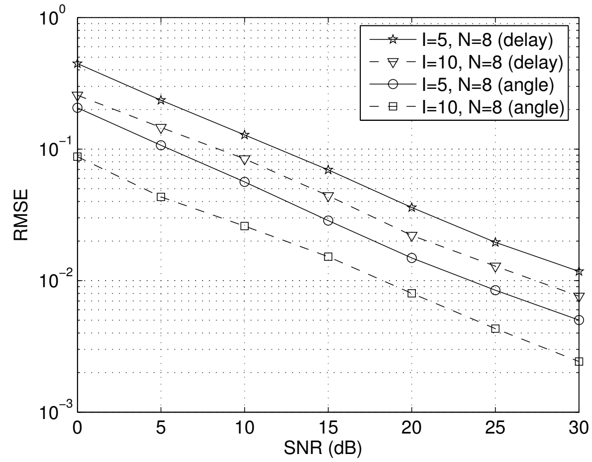

Fig. 1. RMSE versus SNR results.

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