

SPACE-TIME MULTIPLEXING CODES: A TENSOR MODELING APPROACH

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ABSTRACT

In this paper, we present new Space-Time Multiplexing Codes (STMC) for multiple-antenna transmissions, which rely on a three-dimensional tensor modeling of the transmitted/received signals. The proposed codes combine spatial multiplexing and space-time coding by spreading a linear combination of different sub-streams of data over the space and time dimensions. We show the STMC induces a tensor structure on the transmitted/received signal that can be modeled using a trilinear tensor decomposition. Tensor modeling is exploited at the receiver for a blind decoding of the transmitted sub-streams based on linear processing and without any ambiguity. The proposed approach also provides full diversity while benefiting from the maximum multiplexing gain offered by the multiple antennas. Simulation results show that the tensor-based STMC offer remarkable performance with good diversity-multiplexing trade-off.

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) antenna systems, which employ multiple antennas at both ends of the wireless link, promise increased spectral efficiency and performance gains compared to conventional systems that employ multiple antennas at the receiver only [1, 2]. Such gains come from the exploitation of the space dimension as a additional radio-resource in scattering-rich wireless environments.

In general, multiple transmit antennas are employed either to achieve high-data rates via spatial multiplexing or to improve link reliability through space-time coding. A popular spatial multiplexing approach is called V-BLAST (Vertical Bell Labs Layered Space-Time) [3]. It is based on transmitting several sub-streams of data on the individual antennas in a parallel way. These sub-streams are separated at the receiver using interference nulling and canceling techniques. On the other hand, Space-Time Codes (STC) rely on simultaneous coding across space and time to achieve diversity with a minimum bandwidth sacrifice [4, 5, 6]. Typical STC approaches are space-time block-codes [7, 5] based on orthogonal designs and space-time trellis-codes [4].

Spatial multiplexing and space-time coding are opposite approaches for exploiting the MIMO wireless channel, since the first one increases spectral efficiency sacrificing diversity while the latter improves link performance by limiting higher data-rates [8]. With the goal of achieving a good diversity-multiplexing trade-off, recent works have focused on the design and analysis of MIMO systems that combine the advantages of both spatial multiplexing and space-time coding [9, 10, 11, 12]. For example, in [9] the concept of Linear Dispersion Codes (LDC) is introduced

as a high-rate space-time block-coding concept that generalizes orthogonal designs to any number of transmit and receive antennas. They are based on spreading linear combinations of data sub-streams over space and time. LDC are designed to maximize the mutual information between the transmitted and received signals and do not guarantee the maximum transmit diversity. LDC also assume that the Channel State Information (CSI) is accurately estimated using training sequences. In [10], the co-called *Khatri-Rao* Space-Time Codes (KRSTC) are proposed for achieving variable rate-diversity trade-offs for any transmit-receive antenna configuration. KRSTC rely on the PARAFAC (Parallel Factors) tensor decomposition [13, 14] for modeling the received signal. By capitalizing on the PARAFAC structure for the received signal as well as on the model identifiability, blind decoding of the transmitted symbols is made possible with linear complexity. Despite of its diversity-rate flexibility, spreading of each sub-stream of data is done in the time dimension only, i.e. across consecutive symbol periods. There is no spreading in the space dimension, i.e., across the transmit antennas. On the other hand, LDC benefits from a full spreading of the data sub-streams over both the space and time dimensions.

We present a new STC concept for multiple-antenna transmissions that relies on a tensor modeling of the transmitted and received signals. It consists of spreading linear combinations of a certain number of data sub-streams over the space and time dimensions following the same principle of LDC. For this reason we prefer to use here the term Space-Time Multiplexing Codes (STMC) in place of Space-Time-Coding (STC). Tensor-based STMC can, in some sense, be interpreted as a sort of general LDC formulated using the tensor formalism. Contrarily to LDC, the tensor-based STMC are designed to obtain full diversity. From a tensor modeling point of view, our approach can also be interpreted as a sort of KRSTC performing full spreading over all transmit antennas. The proposed multiplexing structure is designed to cope with more multiplexed sub-streams than transmit antennas, providing full diversity while benefiting from the maximum multiplexing gain. The design of the tensorial multiplexing structure makes possible to control the diversity and coding gains in a simple way.

By modeling the STMC process using the tensor formalism, we also show that the received signal follows a third-order tensor model, which can be viewed as a generalization of the standard PARAFAC model. Thanks to the trilinear structure of the received signal, joint CSI recovery and blind decoding of the transmitted sub-streams are done in a relatively simple way using alternating least squares [14]. Simulation results show remarkable performances for the proposed STMC with good diversity-multiplexing trade-offs.

The paper is organized as follows. In Section 2, the general system model is introduced. In Section 3, the proposed STMC

approach is presented using the tensor formalism. A tensor model of STMC is also developed in this section. In Section 4, the design criterion and the choice of the multiplexing structure are shown. Section 5 presents a blind receiver for joint CSI and symbol recovery. Simulation results are presented in Section 6 and the paper is concluded in Section 7.

Notation: Some notations and properties used throughout the paper are now defined. $\mathbf{A}^T, \mathbf{A}^H$ and \mathbf{A}^\dagger stand for transpose, Hermitian transpose and pseudo-inverse of \mathbf{A} , respectively; The operator $\text{Diag}(\mathbf{a})$ forms a diagonal matrix from its vector argument; \otimes and \diamond denote the Kronecker product and the Khatri-Rao product, respectively:

$$\mathbf{A} \diamond \mathbf{B} = [\mathbf{a}_1 \otimes \mathbf{b}_1, \dots, \mathbf{a}_R \otimes \mathbf{b}_R],$$

where $\mathbf{A} = [\mathbf{a}_1 \dots \mathbf{a}_R] \in \mathbb{C}^{I \times R}$ and $\mathbf{B} = [\mathbf{b}_1 \dots \mathbf{b}_R] \in \mathbb{C}^{J \times R}$. We make use of the following property of the Khatri-Rao product:

$$\text{vec}(\mathbf{A}\mathbf{D}\mathbf{B}^T) = (\mathbf{B} \diamond \mathbf{A})\mathbf{d}(\mathbf{D}), \quad (1)$$

where \mathbf{D} is a diagonal matrix, $\text{vec}(\cdot)$ stacks the columns of its matrix argument in a vector, and $\mathbf{d}(\cdot)$ forms a column-vector from the diagonal of its matrix argument. The following property involving the Kronecker product of matrices is used:

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{A}\mathbf{C} \otimes \mathbf{B}\mathbf{D}. \quad (2)$$

2. SYSTEM MODEL

We adopt the same general system model described in [9, 10]. Consider a MIMO antenna system with M transmit antennas and K receive antennas. Assume that the wireless channel is frequency-flat and quasi-static during N time-slots, where each time-slot corresponds to P symbol periods, where P is the code length. The discrete-time baseband equivalent model for the received signal is given by:

$$\mathbf{X}_n = \sqrt{\frac{\rho}{M}} \mathbf{H} \mathbf{C}_n + \mathbf{V}_n, \quad n = 1, \dots, N, \quad (3)$$

where $\mathbf{X}_n \in \mathbb{C}^{K \times P}$ is received signal matrix during the n -th symbol period or *time-slot*, $\mathbf{H} \in \mathbb{C}^{K \times M}$ denotes the channel matrix, $\mathbf{C}_n \in \mathbb{C}^{M \times P}$ represents the n -th transmitted code matrix and $\mathbf{V}_n \in \mathbb{C}^{K \times P}$ is the additive noise matrix. The additive noise is assumed to be spatially and temporally white. The channel matrix is assumed to have i.i.d. entries following a zero-mean unit-variance complex-Gaussian distribution, so that $E[\text{Tr}(\mathbf{H}\mathbf{H}^H)] = MK$. We also have $E[\text{Tr}(\mathbf{C}_n \mathbf{C}_n^H)] = PM$, $n = 1, \dots, N$, where $\text{Tr}(\cdot)$ is the trace operator. $\sqrt{\rho/M}$ is a transmit power normalization factor ensuring that ρ is the signal-to-noise ratio at each receive antenna.

3. SPACE-TIME MULTIPLEXING CODES: TENSOR MODELING

In this work, Space-Time Multiplexing Codes (STMC) consists of i) spreading Q co-channel data sub-streams over the M transmit antennas and P symbol periods and ii) linearly combining the Q spread signals to transmit them through the wireless channel. The proposed approach interprets the STMC as a transformation involving two tensor spaces, the core of this transformation being represented by a *multiplexing tensor* $\mathcal{W} \in \mathbb{C}^{P \times M \times Q}$. As will be seen shortly, the idea is to obtain the desired balance

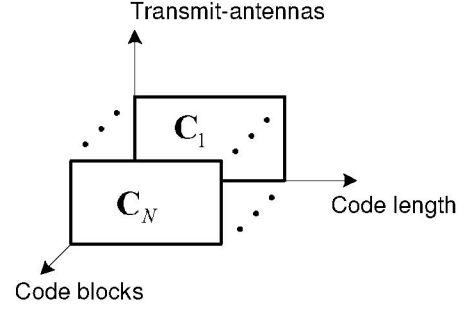


Fig. 1. Organization of the STMC signal in the 3D bbb bb axis.

between diversity and rate by varying the three dimensions of the multiplexing tensor. The approach proposed herein, suggests to interpret the transmitted code matrices $\mathbf{C}_1, \dots, \mathbf{C}_N$ as N slabs or *slices* of a transmitted *code tensor* $\mathcal{C} \in \mathbb{C}^{P \times M \times N}$. The code tensor has three dimensions: the first one equal to the number of transmit-antennas, the second one corresponds to the code-length while the third-one is equal to the number of code-blocks. Figure 1 illustrates this concept.

Let $\mathbf{S} = [\mathbf{s}_1 \dots \mathbf{s}_N] \in \mathbb{C}^{Q \times N}$, with $\mathbf{s}_n = [s_n^{(1)} \dots s_n^{(Q)}]^T \in \mathbb{C}^Q$, be a symbol matrix that concatenates Q data sub-streams that are simultaneously transmitted. The entries of \mathbf{S} are chosen from an arbitrary J -Phase Shift-Keying (PSK) or J -Quadrature Amplitude Modulation (QAM) constellation and satisfy the power constraint $E[\text{Tr}(\mathbf{S}^H \mathbf{S})] = NQ$. STMC are defined as a one-to-one mapping:

$$\mathcal{W} : \mathbf{S} \rightarrow \mathcal{C}.$$

Using multi-indexed tensor notation, this can be represented as:

$$c_{m,p,n} = \sum_{q=1}^Q w_{m,p,q} s_{q,n}, \quad (4)$$

where $c_{m,p,n} = [\mathcal{C}]_{m,p,n}$, $w_{m,p,q} = [\mathcal{W}]_{m,p,q}$ and $s_{q,n} = [\mathbf{S}]_{q,n}$ are typical elements of the code tensor, multiplexing tensor and symbol matrix respectively. The received signal can be modeled as:

$$\begin{aligned} x_{k,p,n} &= \sum_{m=1}^M h_{k,m} c_{m,p,n} + v_{k,p,n} \\ &= \sum_{m=1}^M h_{k,m} \sum_{q=1}^Q w_{m,p,q} s_{q,n} + v_{k,p,n}, \end{aligned} \quad (5)$$

where $x_{k,p,n} = [\mathcal{X}]_{k,p,n}$ is the received signal sample at the k -th receive antenna, and associated to the p -th time-slot of the n -th code block, and $h_{k,m} = [\mathbf{H}]_{k,m}$. The signal part of model (5) follows a tensor decomposition known in the tensor-related literature by the name of *Tucker2* [15]. Figure 2 illustrates the tensor decomposition of the received signal (in absence of noise), as a function of the channel matrix, symbol matrix and STMC tensor.

In order to write (5) in a more usual matrix form, let us define a set of Q spreading matrices $\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(Q)}$ as $w_{p,m}^{(q)} = w_{p,m,q}$. These matrices are the Q slices obtained by slicing the tensor \mathcal{W} shown in Fig. 2 along its third dimension and let $s_n^{(q)} = s_{n,q}$. Taking these definitions into account, (4) can be expressed in matrix form

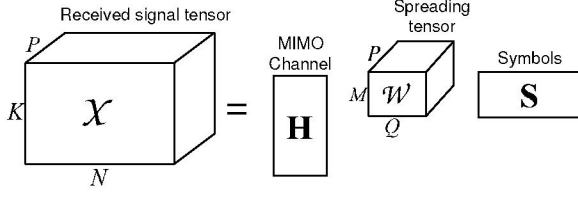


Fig. 2. Tensor decomposition of the received signal

as:

$$\mathbf{C}_n = \sum_{q=1}^Q s_n^{(q)} \mathbf{W}^{(q)} = (\mathbf{s}_n^T \otimes \mathbf{I}_M) \mathbf{W}^T, \quad (6)$$

where

$$\mathbf{W} = [\mathbf{W}^{(1)} \dots \mathbf{W}^{(Q)}] \in \mathbb{C}^{P \times M \times Q} \quad (7)$$

is the *overall multiplexing matrix*. Note that (6) looks like a generalized linear dispersion code, stated using the tensor formalism. Now, substituting (6) into (3), we get the following model for the received signal:

$$\mathbf{X}_n = \sqrt{\frac{\rho}{M}} \mathbf{H} (\mathbf{s}_n^T \otimes \mathbf{I}_M) \mathbf{W}^T + \mathbf{V}_n. \quad (8)$$

Note that the term $\mathbf{s}_n^T \otimes \mathbf{I}_M$ clearly indicates that every symbol is spread across the M transmit antennas. The Rate (R) and Multiplexing Gain (MG) are respectively equal to:

$$R = \left(\frac{Q}{P}\right) \log_2(J) \text{ bits/channel use}, \quad \text{MG} = \frac{Q}{M},$$

J being the modulation cardinality. Note that the rate is independent on the number of transmit antennas, but dependent on the number of multiplexed sub-streams instead.

3.1. Block-PARAFAC Model for Space-Time-Multiplexing

Model (8) can be interpreted as a special case of a Block-PARAFAC decomposition (this decomposition is detailed in [16, 17]). In this work, we adopt the Block-PARAFAC approach for formulating the STMC model in a more compact matrix notation. The obtained model will be effectively exploited at the receiver for the blind recovery of the CSI and the transmitted sub-streams. In order to arrive at the proposed model, we start from (8) and note that

$$\begin{aligned} \mathbf{H} (\mathbf{s}_n^T \otimes \mathbf{I}_M) &= [\mathbf{H} \mathbf{s}_n^{(1)} \mathbf{I}_M \dots \mathbf{H} \mathbf{s}_n^{(Q)} \mathbf{I}_M] \\ &= [\mathbf{H} \dots \mathbf{H}] \begin{bmatrix} \mathbf{s}_n^{(1)} \mathbf{I}_M & & 0 \\ & \ddots & \\ 0 & & \mathbf{s}_n^{(Q)} \mathbf{I}_M \end{bmatrix} \\ &= (\mathbf{1}_Q^T \otimes \mathbf{H}) \text{Diag}(\mathbf{s}_n \otimes \mathbf{1}_M), \end{aligned}$$

which allows us to rewrite (8) as:

$$\mathbf{X}_n = \sqrt{\frac{\rho}{M}} (\mathbf{1}_Q^T \otimes \mathbf{H}) \text{Diag}(\mathbf{s}_n \otimes \mathbf{1}_M) \mathbf{W}^T + \mathbf{V}_n \quad (9)$$

Using property (1) of the Khatri-Rao product, we can express (9) as:

$$\text{vec}(\mathbf{X}_n) = \sqrt{\frac{\rho}{M}} [\mathbf{W} \diamond (\mathbf{1}_Q^T \otimes \mathbf{H})] (\mathbf{s}_n \otimes \mathbf{1}_M) + \text{vec}(\mathbf{V}_n)$$

Now, collect N received code blocks in the following matrix:

$$\mathbf{X} = [\text{vec}(\mathbf{X}_1) \dots \text{vec}(\mathbf{X}_N)] \in \mathbb{C}^{KP \times N}. \quad (10)$$

It follows that:

$$\mathbf{X} = \sqrt{\frac{\rho}{M}} [\mathbf{W} \diamond (\mathbf{1}_Q^T \otimes \mathbf{H})] [(\mathbf{s}_1 \otimes \mathbf{1}_M) \dots (\mathbf{s}_N \otimes \mathbf{1}_M)] + \mathbf{V}.$$

Now, using property (2) of the Kronecker product, the term $\mathbf{1}_Q^T \otimes \mathbf{H}$ can be rewritten as:

$$\begin{aligned} \mathbf{1}_Q^T \otimes \mathbf{H} &= \mathbf{1} \mathbf{1}_Q^T \otimes \mathbf{H} \mathbf{I}_M = (\mathbf{1} \otimes \mathbf{H}) (\mathbf{1}_Q^T \otimes \mathbf{I}_M) \\ &= \mathbf{H} \underbrace{(\mathbf{1}_Q^T \otimes \mathbf{I}_M)}_{\Psi} = \mathbf{H} \Psi. \end{aligned}$$

Similarly, the term $[(\mathbf{s}_1 \otimes \mathbf{1}_M) \dots (\mathbf{s}_N \otimes \mathbf{1}_M)]$ can be simplified as:

$$\begin{aligned} [(\mathbf{s}_1 \otimes \mathbf{1}_M) \dots (\mathbf{s}_N \otimes \mathbf{1}_M)] &= [\mathbf{s}_1 \dots \mathbf{s}_N] \otimes \mathbf{1}_M \\ &= \mathbf{S} \otimes \mathbf{1}_M = (\mathbf{I}_Q \mathbf{S} \otimes \mathbf{1}_M) = (\mathbf{I}_Q \otimes \mathbf{1}_M) (\mathbf{S} \otimes \mathbf{1}) \\ &= \underbrace{(\mathbf{I}_Q \otimes \mathbf{1}_M)}_{\Phi} \mathbf{S} = \Phi \mathbf{S}, \end{aligned}$$

where we have defined $\Psi = \mathbf{1}_Q^T \otimes \mathbf{I}_M$ and $\Phi = \mathbf{I}_Q \otimes \mathbf{1}_M$. We finally get a compact expression for the received signal:

$$\mathbf{X} = \sqrt{\frac{\rho}{M}} (\mathbf{W} \diamond \mathbf{H} \Psi) \Phi \mathbf{S} + \mathbf{V}. \quad (11)$$

4. DESIGN CRITERION AND MULTIPLEXING STRUCTURE

Our analysis is based on model (8). We assume that the receiver has perfect knowledge of both the CSI and the overall multiplexing matrix \mathbf{W} . Recall that the channel matrix is assumed to have independent entries. The signal multiplexing structure is chosen to maximize the transmit diversity (instead of the mutual information as in [9]). In the following, we briefly discuss the design criterion and the choice of the multiplexing structure.

4.1. Design Criterion

Suppose that \mathbf{C}_n and $\tilde{\mathbf{C}}_n$ are two different STMC matrices. The pairwise error probability between \mathbf{C}_n and $\tilde{\mathbf{C}}_n$ related to the two symbol vectors \mathbf{s}_n and $\tilde{\mathbf{s}}_n$ can be upper bounded by [4]:

$$P(\mathbf{C}_n \rightarrow \tilde{\mathbf{C}}_n) \leq \binom{2r-1}{r} \left(\prod_{i=1}^r \lambda_i \right)^{-1} \left(\frac{\rho}{M} \right)^{-r},$$

where r is the rank of $(\mathbf{C}_n - \tilde{\mathbf{C}}_n)(\mathbf{C}_n - \tilde{\mathbf{C}}_n)^H$, and $\lambda_1, \lambda_2, \dots, \lambda_r$ are its nonzero eigenvalues. According to the *rank criterion* of space-time code design, the maximum achievable diversity gain, manifested at high SNR, is at most Kr . This is achieved when $\mathbf{C}_n - \tilde{\mathbf{C}}_n$ is full rank. Let us call $\mathbf{E}_n = \mathbf{C}_n - \tilde{\mathbf{C}}_n$, and recall (6), which gives the following expression for \mathbf{E}_n

$$\mathbf{E}_n = [(\mathbf{s}_n - \tilde{\mathbf{s}}_n)^T \otimes \mathbf{I}_M] \mathbf{W}^T.$$

$\forall \mathbf{s}_n \neq \tilde{\mathbf{s}}_n, \text{rank}((\mathbf{s}_n - \tilde{\mathbf{s}}_n)^T \otimes \mathbf{I}_M) = \min(M, MQ) = M$. Provided that \mathbf{W} is full rank, the maximum diversity gain of STMC are $Kr \leq K \min(P, M)$. Note that when $Q > 1$, not only full diversity KM is achieved but the system supports more sub-streams than transmit antennas, i.e., it has $\text{MG} > 1$.

4.2. Choice of the Multiplexing Structure

The choice of $\mathbf{W} = [\mathbf{W}^{(1)} \dots \mathbf{W}^{(Q)}]$ is crucial for achieving the maximum diversity gain, since it should have full rank in order to satisfy the above design criterion. We adopt the same approach of [10] and choose \mathbf{W} to be a $P \times MQ$ Vandermonde matrix defined as:

$$[\mathbf{W}]_{p,m'} = e^{j2\pi(m'-1/MQ)(p-1)},$$

where $m' = (q-1)M + m$, $m = 1, \dots, M$, $q = 1, \dots, Q$. \mathbf{W} satisfies the following condition:

$$\mathbf{W}\mathbf{W}^H = M\mathbf{I} \rightarrow \text{Tr}(\mathbf{W}\mathbf{W}^H) = \frac{PM}{Q}.$$

This means that \mathbf{W} is a scaled semi-unitary matrix that is full rank and satisfies transmit power constraint. Using the tensor formalism, the entries of the overall multiplexing matrix are related to those of the multiplexing tensor by:

$$[\mathbf{W}]_{p,(q-1)M+m} = [\mathcal{W}]_{p,m,q}.$$

As pointed out in [10], the particular Vandermonde structure of \mathbf{W} simplifies the diversity-rate control. In the case of our STMC approach, it is possible to control three spreading-multiplexing parameters which are the number of transmit antennas, code block length, and the number of spatially-multiplexed sub-streams by varying the number of columns/rows of \mathbf{W} , or similarly, by varying the three dimensions of the multiplexing tensor \mathcal{W} . This particular multiplexing structure can provide full diversity while benefiting at the same time from the maximum multiplexing gain offered by the multiple antennas.

5. TENSOR-BASED BLIND RECEIVER

We present a blind receiver that makes use of the Block-PARAFAC model for the received signal for a joint channel/symbol estimation. The receiver is based on the Alternating Least Squares (ALS) algorithm and follows the same principles of that of [10], except that in our case, it fits a different tensor model.

In general for the trilinear case, ALS consists in computing the decomposition by estimating in an alternating way the three matrices that define the tensor model. At each iteration, one matrix is updated using least squares regression, while the two other matrices defining the model are fixed to their values obtained in previous estimation steps. In our context, \mathbf{W} is known and the receiver processing consists of alternated estimations of \mathbf{H} and \mathbf{S} . The estimation criterion can be formulated as follows:

$$J(\mathbf{H}, \mathbf{S}) = \|\mathbf{X} - (\mathbf{W} \diamond \mathbf{H}\Psi)\Phi\mathbf{S}\|_F^2,$$

where we have dropped the term $\sqrt{\frac{P}{M}}$ to simplify notation, and $\|\cdot\|_F$ denotes the Frobenius norm. Recall (10) and (11), and note that \mathbf{X} contains the full tensor $\mathcal{X} \in \mathbb{C}^{M \times K \times P}$ organized in matrix form, which is a collection of N received code blocks. Due to the symmetry of the trilinear model, we can express the same information of the received signal tensor in two other matrix forms: $\mathbf{X}' = [\text{vec}(\mathbf{X}'_1) \dots \text{vec}(\mathbf{X}'_K)] \in \mathbb{C}^{PN \times K}$ and $\mathbf{X}'' = [\text{vec}(\mathbf{X}''_1) \dots \text{vec}(\mathbf{X}''_P)] \in \mathbb{C}^{NK \times P}$. Consequently, three equivalent matrix expressions for the received signal are possible, and they are:

$$\begin{aligned} \mathbf{X} &= (\mathbf{W} \diamond \overline{\mathbf{H}})\overline{\mathbf{S}} + \mathbf{V} \\ \mathbf{X}' &= (\overline{\mathbf{S}}^T \diamond \mathbf{W})\overline{\mathbf{H}}^T + \mathbf{V}' \\ \mathbf{X}'' &= (\overline{\mathbf{H}} \diamond \overline{\mathbf{S}}^T)\mathbf{W}^T + \mathbf{V}'' \end{aligned}$$

where $\overline{\mathbf{H}} = \mathbf{H}\Psi$ and $\overline{\mathbf{S}} = \Phi\mathbf{S}$. Ψ and Φ are known constraint matrices of the Block-PARAFAC model (they only depend on the multiplexing parameters M and Q) and this is exploited during the estimation process.

At the beginning of the algorithm, an initial estimate for $\widehat{\mathbf{H}}_{i=0}$ is obtained from a singular value decomposition of \mathbf{X}' . At the i -th iteration, the update equations for $\widehat{\mathbf{S}}_i$ and $\widehat{\mathbf{H}}_i$ are given by:

$$\begin{aligned} \widehat{\mathbf{S}}_i &= [(\mathbf{W} \diamond \widehat{\mathbf{H}}_{i-1}\Psi)\Phi]^\dagger \mathbf{X}, \\ \widehat{\mathbf{H}}_i^T &= [(\widehat{\mathbf{S}}_i^T \Phi^T \diamond \mathbf{W})\Psi^T]^\dagger \mathbf{X}' \end{aligned}$$

At the end of the i -th iteration, an overall error measurement between the estimated model and the received signal tensor can be obtained from the following equation:

$$e_i = \left\| \mathbf{X} - (\mathbf{W} \diamond \widehat{\mathbf{H}}_i\Psi)\Phi\widehat{\mathbf{S}}_i \right\|_F,$$

We declare that the algorithm has converged at the i -th iteration when $|e_i - e_{i-1}| \leq 10^{-6}$. The knowledge of \mathbf{W} , which is fixed during the whole estimation process, helps us to achieve the global minimum faster compared to the case where it is not known. The estimation of both \mathbf{H} and \mathbf{S} are affected by a scaling factor. Following [10], we assume that the first symbol of each data sub-stream is a known ID symbol in order to eliminate the scaling factor that is inherent to the blind estimation process.

6. SIMULATION RESULTS

The performance of the proposed tensor-based STMC are now evaluated. We want to illustrate that the proposed approach achieves full diversity and also takes advantage of the multiplexing gain offered by the multiple-antennas. The results are shown in terms of Bit-Error-Rate (BER) versus Signal-to-Noise Ratio (SNR) per receive antenna. BER values are calculated from 5000 independent random realizations of the channel matrix \mathbf{H} . In all results, $N = 50$ transmitted code blocks/sub-stream are used at the receiver for blind detection. The BER shown in the figures is the average BER over the Q sub-streams.

6.1. BER versus multiplexing gain

We evaluate the BER of the proposed STMC as a function of the multiplexing gain, which is given by the ratio Q/M . The SNR is fixed to 20dB. $M=2$ and $P = 4$ are also fixed while Q is varied, which results in $\text{MG} = 3; 5/2; 2$ and $3/2$. We consider 4PSK and 8PSK with $K = 2$ and 4. It can be noted from Fig. (3) that the higher the multiplexing gain, the more limited is the BER performance, as expected. Note that the proposed approach copes with a $\text{MG} > 1$, at a relatively low BER. For example, at target BER values between 10^{-3} and 10^{-2} , $Q = 4$ sub-streams can be detected using only $M = 2$ transmit antennas, i.e. $\text{MG} = 2$ and $R = 2$. This represents a good trade-off between multiplexing gain and rate.

6.2. Comparison with KRSTC [10]

The BER performance of the proposed STMC is compared with that of KRSTC [10]. This comparison is interesting since both systems employ PARAFAC-based blind detection, although they are different tensor-based space-time coding approaches. [10] suggests the use of both linear constellation precoding and sphere decoding in KRSTC. We do not consider this here, in order to have an adequate

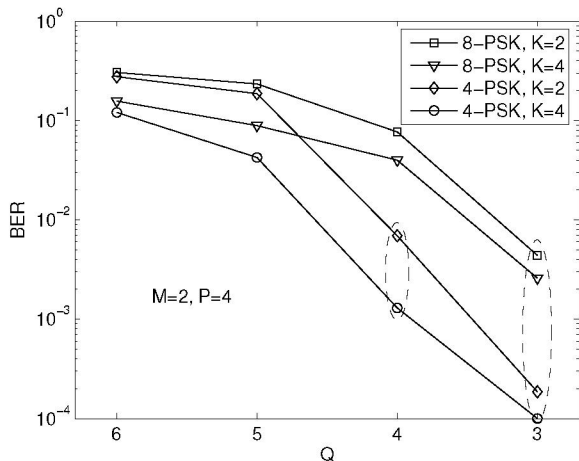


Fig. 3. Performance as a function of the multiplexing gain.

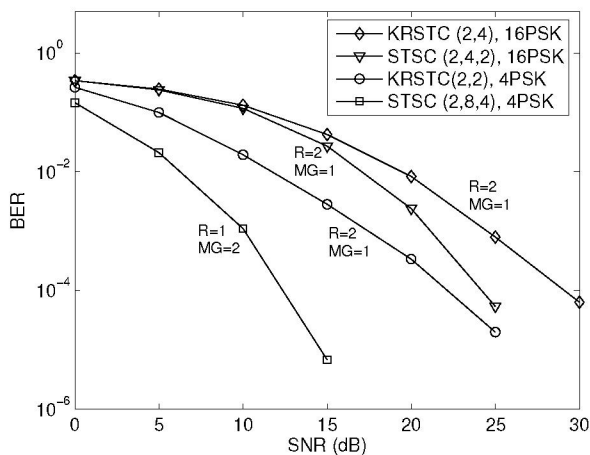


Fig. 4. Comparison between STMC and KRSTC.

comparison with the proposed approach under the same conditions. Two cases are studied and we set $K = 2$ for both approaches. First we compare KRSTC with $(M, P) = (2, 4)$ against STMC with $(M, P, Q) = (2, 4, 2)$. In this case, 16PSK is used for both, so that $R = 2$. Figure 4 shows that STMC outperforms KRSTC, due to its higher diversity gain. In a second case, we compare $(2, 2)$ -KRSTC with $(2, 8, 4)$ -STMC. It can be seen that STMC is considerably better than KRSTC, but this is achieved with a rate reduction by a factor of two for STMC. However, note that the MG of STMC is twice that of KRSTC. Such performance gains confirm that the full spreading of sub-streams across the transmit antennas in STMC provides higher diversity and multiplexing gains.

7. CONCLUSIONS

We have proposed a new approach to combined space-time coding and spatial multiplexing in multiple-antenna transmissions based on tensor modeling. Tensor-based Space-Time Multiplexing Codes (STMC) consist of spreading linear combinations of a certain number of data sub-streams multiplexed in space and time. They

are designed with a flexible multiplexing structure that provides full diversity while benefiting from the maximum multiplexing gain offered by the multiple antennas. We have shown that the received signal follows a third-order (Block-PARAFAC) tensor model. At the transmitter, tensor-based STMC allow a variable and controllable diversity/coding gain (by varying the dimensions of the tensorial multiplexing structure) and are designed to cope with more multiplexed sub-streams than transmit antennas. From a receiver processing perspective, tensor modeling is exploited for a joint CSI recovery and blind detection of the transmitted sub-streams. Simulation results have shown that the proposed STMC approach offers remarkable bit-error-rate performances with a good diversity-multiplexing trade-off.

8. REFERENCES

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