Trilinear Space-Time-Frequency Codes for Broadband MIMO-OFDM Systems

André L. F. de Almeida, Gérard Favier, João C. M. Mota

Abstract **A new multi-antenna coding framework is proposed for Space-Time-Frequency (STF) transmissions over broadband Multiple Input Multiple Output (MIMO) systems based on Orthogonal Frequency Division Multiplexing (OFDM). A tensor decomposition known as PARAFAC is used as the core of a multi-stream space-time-frequency coder that jointly multiplex and spreads several input streams over space (transmit antennas), time (symbol periods) and frequency (subcarriers). We coin the term *trilinear* STF codes since each input symbol is coded over a space-time-frequency grid by a triple product code: each trilinearly coded symbol is interpreted as an element of a third-order tensor, which can be decomposed using PAFAFAC analysis. Trilinear STF codes are designed for an arbitrary number of transmit and receive antennas. They afford a variable degree of multiplexing-spreading over each one of the three signal dimensions while providing full diversity gain for each multiplexed stream. At the receiver, a direct blind decoding based on a relatively simple linear processing is made possible thanks to the PARAFAC modeling of the received signal. Computer simulation results are provided for performance assessment of the proposed codes in a variety of con gurations.**

Index Terms **Blind detection, MIMO, multiplexing, OFDM, PARAFAC, space-time-frequency coding, spreading, tensor modeling, trilinear coding, wireless communications.**

I. Introduction

Multiple Input Multiple Output (MIMO) antenna systems employing multiple antennas at both the transmitter and receiver can provide an increased spectral efficiency compared to systems that employ multiple antennas at the receiver only [1], [2]. Such gains come from the exploitation of the space dimension as an additional radio-resource in scattering-rich wireless environments. On the other hand, due to the increasing demand for high data rate and reliable transmissions over broadband wireless links, orthogonal frequency division multiplexing (OFDM) is been considered as a primary candidate for next generation broadband wireless systems.

The combination of MIMO and OFDM has been focus of a large number of recent works [3]. In MIMO-OFDM, the transmit antennas can be employed to achieve high data rates via spatial multiplexing as well as to improve link reliability through space-time/space-frequency or Space-Time-Frequency (STF) coding [4], [5]. In [4], space-time codes were proposed for frequency-selective channels. Design criteria for full-diversity space-frequency codes were derived in [5]. A few space-frequency designs were proposed recently in [6]–[8] for MIMO-OFDM, which can guarantee full-diversity, full-rate and good coding gain. The transmission framework of these designs maximizes the performance of a single data stream, and suffers from low multiplexing gain in the sense that no more than one signal/user is transmitted at the same space-time-frequency slot. Moreover, performance is always evaluated assuming perfect channel knowledge at the receiver, which is an optimistic assumption in practice. The decoding complexity of these codes is considerably high and prohibitive in some cases.

Recently, several STF coding transceivers were proposed relying on a combination of direct- sequence spread spectrum and multicarrier modulations, to enable orthogonal multiple-access in multiuser multi-antenna systems. [9] proposed space-frequency spreading codes for the downlink of a multiuser MIMO-OFDM system. The transmission is designed to support more multiplexed signals than transmit antennas and to provide full-diversity for each multiplexed signal. Another spread spectrum-based STF transmission framework is proposed in [10] for Multicarrier Spread Spectrum Multiple Access (MC SSMA). With the idea of fully spreading each user symbol over space, time and frequency, space-time-frequency diagonal spreading sequences are used as the STF coding structure. Despite the spectral efficiency gains achieved, the design of [10] was restricted to the case where the number of transmit and receive antennas is equal to the spreading gain. In [11], a STF transmit diversity strategy was proposed for Multicarrier Direct Sequence Code Division Multiple Access (MC DS-CDMA), which is based on the concatenation of a space-time spreading code with a frequency-domain spreading code. A common characteristic in all these works, is that a perfect channel knowledge is always assumed at the receiver.

In this work, we present a new STF coding framework for broadband MIMO-OFDM systems. A tensor decomposition known as PARAFAC [12], [13] is used as the core of a multi-stream space-time-frequency coder that jointly multiplex and spreads several input streams over space (transmit antennas), time (symbol periods) and frequency (sub-carriers). In the proposed framework, the total number of subcarriers is divided into parallel STF coding groups of F subcarriers each, so that each STF codeword spans F subcarriers, M transmit antennas and P OFDM symbols. Within each subcarrier group, R input symbols are jointly coded over a space-time-frequency grid by a triple product coder. The STF coder follows a trilinear PARAFAC model, and for this reason, we coin the term *trilinear* STF codes. Each STF-coded symbol is interpreted as an element of a third-order tensor, which can be decomposed using PAFAFAC analysis.

The trilinear STF coder structure is designed to afford a variable degree of multiplexing-spreading over each one of the three signal dimensions and to provide full diversity gain in

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frequency-selective MIMO channels. Also, they are valid for an arbitrary number of transmit and receive antennas. A key feature of trilinear STF coding is its inherent blind decodability. This is possible since the received signal is also interpreted as a third-order tensor following a trilinear PARAFAC model. Thanks to uniqueness properties of PARAFAC models, direct blind decoding is made possible with linear complexity, by means of an alternating least squares algorithm.

Tensor modeling approaches for Space-Time (ST) and/or STF coding have recently been proposed in the literature relying on PARAFAC analysis and its generalizations. [14] presents blindly-decodable space-time codes based on the Khatri-Rao product. By formulating the received signal as a PARAFAC model, blind channel estimation and decoding are achieved. A generalization of [14] was recently proposed in [15] for flat-fading MIMO channels, and in [16] for frequency-selective MIMO-OFDM channels, with the idea of performing full space spreading of each symbol using all the available transmit antennas. The distinguish feature of the trilinear STF codes when compared to the previous tensor-based codes is its higher flexibility for controlling the spreading-multiplexing pattern over space, time and frequency dimensions, thanks to the use of a PARAFAC structure for the STF coder itself.

This paper is organized as follows. Section II presents the channel and system models. The trilinear STF coding concept is presented in Section III. In Section IV, the diversity gain of the proposed STF codes is analyzed, and the choice of the code structure is discussed. Section V studies the blind decoding of the trilinear codes based on PARAFAC analysis. Uniqueness conditions for blind decodability are investigated and a simple blind receiver algorithm is presented in this section. Section VI presents some simulation results for performance evaluation. The paper is concluded in Section VII.

Notation: Some notations used throughout the paper are now defined. $\mathbf{A}$, $\mathbf{A}^T$, $\mathbf{A}^H$ and $\mathbf{A}^\dagger$ stand for conjugate, transpose, transconjugate, and pseudo-inverse of $\mathbf{A}$, respectively; $\circ$ represents the outer product; $\otimes$ and denote the Knonecker product and the Khatri-Rao product, respectively: $\mathbf{A}\ \ \mathbf{B} = [\mathbf{a}_1 \otimes \mathbf{b}_1, \ldots, \mathbf{a}_R \otimes \mathbf{b}_R]$, where $\mathbf{A} = [\mathbf{a}_1 \ldots \mathbf{a}_R] \in \mathbb{C}^{I\ \ R}$ and $\mathbf{B} = [\mathbf{b}_1 \ldots \mathbf{b}_R] \in \mathbb{C}^{J\ \ R}$. We use the following property of the Kronecker product:

$$vec(\mathbf{A}\mathbf{C}\mathbf{B}^T) = (\mathbf{B} \otimes \mathbf{A})vec(\mathbf{C}), \qquad (1)$$

where $vec(\ )$ stacks the columns of its matrix argument in a vector. The following properties are also used:

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}, \qquad (2)$$

$$(\mathbf{A}\ \ \mathbf{B})^T(\mathbf{A}\ \ \mathbf{B}) = (\mathbf{A}^T\mathbf{A}) \odot (\mathbf{B}^T\mathbf{B}), \qquad (3)$$

where $\odot$ is the Schur-Hadamard (element-wise) product.

II. Channel and System Models

Consider a MIMO-OFDM system of M transmit and K receive antennas, and using N_c subcarriers. We suppose that the frequency-selective fading channels between each pair of transmit and receive antennas have L independent delay paths and the same power-delay profile. The channel impulse

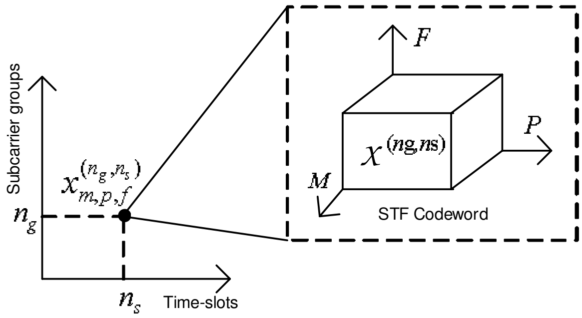

Fig. 1. Illustration of the STF transmission model.

response from the m-th transmit antenna to the k-th receive antenna is denoted by:

$$\hbar_{k,m}(\ \ ) = \sum_{l=1}^{L} \alpha_{k,m,l}\ \ (\ \ -\ \ _l), \qquad (4)$$

where $_l$ is the delay of the l-th path, and $\alpha_{k,m,l}$ denotes the complex amplitude of the l-th path between the m-th transmit antenna and the k-th receive antenna. The $\alpha_{k,m,l}$'s are modeled as zero-mean complex Gaussian random variables, and independent for any (k, m, l), $l = 1, \ldots, L$, $k = 1, \ldots, K$, $m = 1, \ldots, M$. This implies that path gains are spatially uncorrelated. The average power of each path is denoted by:

$$\ _l^2 = E[|\alpha_{k,m,l}|^2] > 0, \forall(k, m, l), \qquad (5)$$

where $E[\ ]$ stands for the mathematical expectation. They are normalized such that $\ _1^2 + \ldots + \ _L^2 = 1$. The frequency response of the channel (4) can be expressed as:

$$h_{k,m,n_c} = \sum_{l=1}^{L} \alpha_{k,m,l} e^{\ \ j2\ \ \left(\frac{n_c\ \ 1}{N_c}\right)\ \ _l}, \qquad (6)$$

where N_c represents the length of the discrete Fourier transform.

The transmission is interpreted as a concatenation of *tensor codewords* over a 3-D space-time-frequency grid (see Fig. 1). Each codeword is represented by a $M \times P \times F$ tensor spanning P OFDM symbols, F subcarriers and M transmit antennas. In Fig. 1, each point $x^{(n_g,n_s)}_{m,p,f}$ in the n_g versus n_s cordinate system on the left, is an element of a 3-D tensor codeword $\mathcal{X}^{(n_g,n_s)}$ of dimensions $M \times P \times F$, shown on the right. The construction of the 3-D codeword is explained in the next section. A total of $N_g = N_c/F$ subcarrier groups and N_s time-slots are available for transmission. It is assumed that the channel is constant over a time necessary for the transmission of the N_s time-slots. At the transmitter, the input data stream is parsed into R parallel data substreams, each one composed of N_gN_s information symbols partitioned into N_g subcarrier groups and N_s time-slots. The r-th data substream occupying the n_g-th subcarrier group and n_s-th time-slot is defined as $s_r^{(n_g,n_s)} = s\left(r + (n_g + n_s - 2)R\right)$, $r = 1, \ldots, R$, $n_g = 1, \ldots, N_g$, $n_s = 1, \ldots, N_s$. At the output of the STF coding block, Inverse Fast Fourier Transform (IFFT) is applied to the resulting signal from the N_g subcarrier groups followed by the insertion of a Cyclic Prefix (CP) before transmission, at each transmit antenna. At the receiver, perfect timing and synchronization is assumed. After baseband conversion, the CP is removed and Fast Fourier Transform (FFT) is applied at each receive antenna. The channel state information is *unknown* at both transmitter and receiver. STF decoding of the R transmitted data substreams is done without resorting to training sequences, i.e. in a blind way.

III. TRILINEAR STF CODES

Let us call $\mathcal{X} \in \mathbb{C}^{M\ \ P\ \ F}$ the tensor codeword. STF coding is defined as the following one-to-one mapping:

$$\{\mathcal{T}_1, \ldots, \mathcal{T}_R\} : \{s_1^{(n_g,n_s)}, \ldots, s_R^{(n_g,n_s)}\} \rightarrow \mathcal{X}^{(n_g,n_s)},$$

where $\mathcal{T}_r \in \mathbb{C}^{M\ \ P\ \ F}$ is the r-th constituent *coding tensor*. The STF codeword transmitted at the n_g-th subcarrier group and n_s-th time-slot is a linear combination over the R constituent terms:

$$\mathcal{X}^{(n_g,n_s)} = \sum_{r=1}^{R} s_r^{(n_g,n_s)} \mathcal{T}_r.$$

The coding tensor $\mathcal{T}_r$ is modeled as the outer product of three coding vectors, in the following manner:

$$\mathcal{T}_r = \mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{w}_r, \quad r = 1, \ldots, R,$$

where $\{\mathbf{u}_r\} \in \mathbb{C}^M$, $\{\mathbf{v}_r\} \in \mathbb{C}^P$ and $\{\mathbf{w}_r\} \in \mathbb{C}^F$ are three sets of R coding vectors, which spread the R information symbols over the space, time and frequency dimensions. This allows us to express the (n_g, n_s)-th tensor codeword as the following decomposition,

$$\mathcal{X}^{(n_g,n_s)} = \sum_{r=1}^{R} s_r^{(n_g,n_s)} (\mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{w}_r), \tag{7}$$

which is recognized as a Parallel Factor (PARAFAC) decomposition [12], [13], where each outer product contribution is scaled by the corresponding information symbol. The PARAFAC decomposition expresses the third-order tensor $\mathcal{X}^{(n_g,n_s)}$ as a scaled sum of R trilinear terms (rank-1 tensors), each one of them being given by the outer product of three vectors. In (7), the r-th term contributing to the generation of the resulting tensor codeword can be interpreted as a *modulated* version of the space-, time- and frequency-domain coding vectors, the modulating factor being the r-th information symbol $s_r^{(n_g,n_s)}$. Hence, trilinear STF coding can be seen as a trilinear space-time-frequency modulation. Figure 2 illustrates the PARAFAC decomposition of the (n_g, n_s)-th tensor codeword.

Let us define three *coding matrices* $\mathbf{U} = [\mathbf{u}_1\mathbf{u}_2 \quad \mathbf{u}_R] \in \mathbb{C}^{M\ \ R}$, $\mathbf{V} = [\mathbf{v}_1\mathbf{v}_2 \quad \mathbf{v}_R] \in \mathbb{C}^{P\ \ R}$ and $\mathbf{W} = [\mathbf{w}_1\mathbf{w}_2 \quad \mathbf{w}_R] \in \mathbb{C}^{F\ \ R}$ with typical elements $[\mathbf{U}]_{m,r} = u_{m,r}$, $[\mathbf{V}]_{p,r} = v_{p,r}$ and $[\mathbf{W}]_{f,r} = w_{f,r}$. A vector containing the symbols of the R substreams transmitted on the n_g-th subcarrier group and n_s-th time-slot, is defined as:

$$\mathbf{s}^{(n_g,n_s)} = [s_1^{(n_g,n_s)} s_2^{(n_g,n_s)} \quad s_R^{(n_g,n_s)}]^T \in \mathbb{C}^R.$$

In order to satisfy the transmit power constraint, the input symbols are normalized so that $E[\|\mathbf{s}^{(n_g,n_s)}\|^2] = 1$. Based on these definitions, the overall STF coding is given by the following one-to-one trilinear mapping:

$$f(\mathbf{U}, \mathbf{V}, \mathbf{W}) : \mathbf{s}^{(n_g,n_s)} \rightarrow \mathcal{X}^{(n_g,n_s)}.$$

Defining $x_{m,p,f}^{(n_g,n_s)}$ as the (m, p, f)-th entry of $\mathcal{X}^{(n_g,n_s)}$, we can express (7) in scalar form:

$$x_{m,p,f}^{(n_g,n_s)} = \sum_{r=1}^{R} s_r^{(n_g,n_s)} u_{m,r} v_{p,r} w_{f,r}. \tag{8}$$

Now, let us "unfold" the tensor codeword into the matrix $\mathbf{X}^{(n_g,n_s)} \in \mathbb{C}^{FM\ \ P}$, in the following manner:

$$[\mathbf{X}^{(n_g,n_s)}]_{(f\ \ 1)M+m,p} = x_{m,p,f}^{(n_g,n_s)}. \tag{9}$$

Note that each column of $\mathbf{X}^{(n_g,n_s)}$ is a stacking of the space and frequency dimensions of the tensor codeword for a fixed OFDM symbol. Defining

$$\mathbf{S}^{(n_g)} = [\mathbf{s}^{(n_g,1)}\mathbf{s}^{(n_g,2)} \quad \mathbf{s}^{(n_g,N_s)}]^T \in \mathbb{C}^{N_s\ \ R}, \tag{10}$$

as a matrix that concatenates the input symbols of the n_g-th subcarrier group during N_s time-slots, it can be shown that $\mathbf{X}^{(n_g,n_s)}$ can be factored as:

$$\mathbf{X}^{(n_g,n_s)} = (\mathbf{W} \quad \mathbf{U}) D_{n_s}(\mathbf{S}^{(n_g)}) \mathbf{V}^T. \tag{11}$$

The data-rate of the trilinear STF code (without the cyclic prefix) is given by:

$$Rate = \quad \frac{R}{FP} \quad \log_2(\quad) \text{bits/channel use}, \tag{12}$$

where is the cardinality of either a -Phase Shift Keying (PSK) or a -Quadrature Amplitude Modulation (QAM).

Remark 1: *Note that (11) can be viewed as a trilinear PARAFAC model, decomposed as a function of the set* $\{(\mathbf{W} \quad \mathbf{U}), \mathbf{V}, \mathbf{S}\}$, *i.e., where one of the three component matrices has a Khatri-Rao structure. In this work, we adopt this interpretation.*

In order to arrive at the received signal model, let us partition the frequency-domain MIMO channel into N_g group channel blocks of dimension F, in the following manner:

$$h_{k,m,f}^{(n_g)} = h_{k,m,(n_g\ \ 1)F+f},$$

where $h_{k,m,(n_g\ \ 1)F+f}$ is defined in (6), with $n_c = (n_g - 1)F + f$. This definition allows us to write the n_g-th group MIMO channel at the f-th subcarrier as:

$$\mathbf{H}_{\cdot\cdot f}^{(n_g)} = \begin{bmatrix} h_{1,1,f}^{(n_g)} & h_{1,2,f}^{(n_g)} & & h_{1,M,f}^{(n_g)} \\ h_{2,1,f}^{(n_g)} & h_{2,2,f}^{(n_g)} & & h_{2,M,f}^{(n_g)} \\ \vdots & \vdots & \vdots & \vdots \\ h_{K,1,f}^{(n_g)} & h_{K,2,f}^{(n_g)} & & h_{K,M,f}^{(n_g)} \end{bmatrix} \in \mathbb{C}^{K\ \ M}. \tag{13}$$

The n_g-th group space-frequency MIMO channel can be written as the following block-diagonal matrix:

$$\mathbf{H}^{(n_g)} = \begin{bmatrix} \mathbf{H}_{\cdot\cdot 1}^{(n_g)} & & \\ & \ddots & \\ & & \mathbf{H}_{\cdot\cdot F}^{(n_g)} \end{bmatrix} \in \mathbb{C}^{FK\ \ FM}. \tag{14}$$

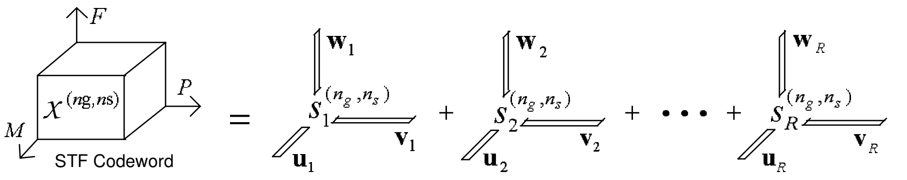

Fig. 2. PARAFAC decomposition of the STF codeword for the n_g-th subcarrier group and n_s-th time-slot.

Taking these definitions into account, the received signal $\mathbf{Y}^{(n_g,n_s)} \in \mathbb{C}^{FK\ \ P}$ can be expressed as:

$$\begin{aligned} \mathbf{Y}^{(n_g,n_s)} &= \mathbf{H}^{(n_g)}\mathbf{X}^{(n_g,n_s)} + \mathbf{N}^{(n_g,n_s)} \\ &= \mathbf{G}^{(n_g)} D_{n_s}(\mathbf{S}^{(n_g)})\mathbf{V}^T + \mathbf{N}^{(n_g,n_s)}, \quad (15) \end{aligned}$$

where

$$\mathbf{G}^{(n_g)} = \mathbf{H}^{(n_g)}(\mathbf{W}\ \ \ \mathbf{U}) \in \mathbb{C}^{FK\ \ R}, \quad (16)$$

is a combined channel-code matrix, which is a transformed version of the original space-frequency MIMO channel matrix, and $\mathbf{N}^{(n_g,n_s)}$ is the additive noise matrix, with entries modeled by i.i.d zero-mean complex Gaussian random variables.

IV. Diversity Gain and Code Structure

The diversity gain of the trilinear STF code is now evaluated, and the choice of the structure of the coding matrices is presented. Conditions for achieving the maximum diversity gain are now given. For reasons of space, we have suppressed the proof. It will be presented in more complete version of this work. According the diversity analysis, full diversity MKL can be achieved if the following condition is satisfied:

$$min \quad \lfloor \frac{FP}{M} \rfloor, F \qquad L. \quad (17)$$

As consequence of the above condition is that for $F \quad L$ subcarriers, coding over $P \quad M$ OFDM symbols is necessary to achieve the maximum diversity gain. The diversity gain can be evaluated for the pair of codewords $\mathbf{X}^{(n_g,n_s)}$ and $\mathbf{X}'^{(n_g,n_s)}$. Note that

$$\begin{aligned} &\mathbf{X}^{(n_g,n_s)} - \mathbf{X}'^{(n_g,n_s)} = \\ &(\mathbf{W}\ \ \ \mathbf{U}) \left[D_{n_s}(\mathbf{S}^{(n_g)}) - D_{n_s}(\mathbf{S}'^{(n_g)}) \right] \mathbf{V}^T. \quad (18) \end{aligned}$$

It is assumed that $D_{n_s}(\mathbf{S}^{(n_g)}) - D_{n_s}(\mathbf{S}'^{(n_g)})$ is full rank for every distinct $\mathbf{S}^{(n_g)}$ and $\mathbf{S}'^{(n_g)}$. This assumption holds if a constellation rotation method is applied by pre-multiplying $\mathbf{S}^{(n_g)}$ by a rotation matrix $\mathbf{\Theta} \in \mathbb{C}^{R\ \ R}$ (see [17], [18] for further details). The main goal of constellation rotation is to guarantee that the elements on the diagonal of $D_{n_s}(\mathbf{\Theta}\mathbf{S}^{(n_g)})$ and $D_{n_s}(\mathbf{\Theta}\mathbf{S}'^{(n_g)})$ are all different, thus avoiding the rank deficiency of (18). In the present analysis, we consider that an appropriate constellation rotation has been used, so that the full rank condition for $D_{n_s}(\mathbf{S}^{(n_g)}) - D_{n_s}(\mathbf{S}'^{(n_g)})$ holds, $\forall(n_g, n_s)$. Under this assumption, maximum diversity thus depends on the structure of the three coding matrices $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{W}$ characterizing the trilinear STF code. In this work, we restrict ourselves to the design of these coding matrices, and bypass the constellation rotation design.

We are interested in designing full-rank matrices $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$ that do not influence the maximum diversity gain MKL. Let us define an unfolded matrix representation $\mathbf{T} \in \mathbb{C}^{FPM\ \ R}$ of the STF coding tensor as:

$$\mathbf{T} = \mathbf{W}\ \ \ \mathbf{V}\ \ \ \mathbf{U}. \quad (19)$$

We propose to choose semi-unitary matrices $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$ such that $\mathbf{T}$ satisfies the following condition:

$$\mathbf{T}^H\mathbf{T} = FPM\mathbf{I}_R. \quad (20)$$

Substituting (19) into (20), and using property (3), we get:

$$\begin{aligned} \mathbf{T}^H\mathbf{T} &= (\mathbf{W}\ \ \ \mathbf{V}\ \ \ \mathbf{U})^H(\mathbf{W}\ \ \ \mathbf{V}\ \ \ \mathbf{U}) \\ &= \ \ (\mathbf{W}\ \ \ \mathbf{V})^H(\mathbf{W}\ \ \ \mathbf{V})\ \ (\mathbf{U}^H\mathbf{U}) \\ &= (\mathbf{W}^H\mathbf{W}) \odot (\mathbf{V}^H\mathbf{V}) \odot (\mathbf{U}^H\mathbf{U}) \quad (21) \end{aligned}$$

The design condition (20) is satisfied if *at least* one of the three coding matrices is square or tall. The other two matrices can be "fat" matrices, provided that

$$\max(M, P, F) \quad\ R. \quad (22)$$

It is worth mentioning that (17) is a necessary condition while (22) gives a sufficient condition (but not necessary) for achieving maximum diversity gain. In fact, one could choose P, M and F to be greater than R without influencing the maximum diversity gain, provided that this choice satisfies(17). However, we should keep in mind that increasing P and/or F much beyond R reduces the overall data-rate that is proportional to R/FP.

Several approaches exist for designing unitary $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{W}$ so that the design criterion (20) is satisfied. In this work, the three coding matrices are chosen as Vandermonde matrices with complex roots $e^{j2\ \ (\frac{r\ \ 1}{R})}$, $r = 1, \ldots, R$, defined as:

$$\begin{aligned} [\mathbf{U}]_{m,r} &= e^{j2\ \ (\frac{r\ \ 1}{R})(m\ \ 1)}, \\ [\mathbf{V}]_{p,r} &= e^{j2\ \ (\frac{r\ \ 1}{R})(p\ \ 1)}, \\ [\mathbf{W}]_{f,r} &= e^{j2\ \ (\frac{r\ \ 1}{R})(f\ \ 1)}. \quad (23) \end{aligned}$$

The Vandermonde design is flexible in the sense that the three signal spreading dimensions, i.e., transmit antennas, OFDM symbols and subcarriers/group, can be controlled by simple truncation of $\mathbf{U}$, $\mathbf{V}$ and/or $\mathbf{W}$, respectively. The Vandermonde structure was considered for the design of Khatri-Rao Space-Time (KRST) codes in [14]. Another possible design approach is to choose a Hadamard structure for the coding matrices. Hadamard matrices are real orthogonal matrices with entries belonging to $\{+1, -1\}$, existing for one, two, and all dimensions multiples of 4 [19]. The Hadamard design is useful for reducing the peak-to-mean envelope power ratio, which results from the joint spreading-multiplexing of the R symbols over the different transmit antennas. The Hadamard structure was addressed in the design of Diagonal Algebraic Space-Time (DAST) codes in [19], [20]. We have experimentally verified that Vandermonde and Hadamard give essentially the same diversity performance.

Remark 2: *The trilinear STF coding framework can be applied to Multi-Carrier Direct Sequence Code Division Multiple Access (MC-DS-CDMA) systems. In this case, the coding matrix* $\mathbf{V}$ *is designed as a Hadamard matrix, the columns of which containing* R time-domain spreading *sequences of length* P *and duration* PT_c*, where* T_c *is the chip period. This spreading is applied at each subcarrier. The coding matrix* $\mathbf{W}$ *would be a Hadamard matrix concatenating* R frequency-domain spreading *sequences of length* F*, where each chip is associated to a different spreading subcarrier. In this context, each frequency-domain spreading sequence is of duration* $(F/N_g)\ T$*, where* T *is the OFDM symbol duration. In this context, trilinear STF coding can be considered as an competing approach to those proposed in [11], [18] and [19].*

V. Blind Decoding using PARAFAC and ALS

In the previous section, the diversity analysis of the trilinear STF codes was carried out by assuming perfect channel knowledge at the receiver. Here, we present a direct blind receiver algorithm for decoding the information symbols. The receiver decodes the transmitted symbols by fitting a PARAFAC model to the received signal given in (15). This is possible since the received signal can be seen as a set of third-order tensors $\mathcal{Y}^{(n_g)} \in \mathbb{C}^{FK\ \ P\ \ N_s}$, $n_g = 1, \ldots, N_g$, each one of them following a trilinear PARAFAC model characterized by the component matrices $\mathbf{G}^{(n_g)}$, $\mathbf{V}$ and $\mathbf{S}^{(n_g)}$. Otherwise stated, the first dimension of the received signal tensor $\mathcal{Y}^{(n_g)}$ is a combined *space-frequency* dimension given by the product between the number of subcarriers per group and the number of receive antennas. The second one corresponds to the number of OFDM symbols per time-slot, and the third one is equal to the number of time-slots.

In order to formulate the received signal as a PARAFAC model, let us define $\mathbf{Y}^{(n_g)}_{\cdot\cdot n_s} = \mathbf{Y}^{(n_g, n_s)} \in \mathbb{C}^{FK\ \ P}$, which can be viewed as the n_s-th matrix-slice of the received signal tensor $\mathcal{Y}^{(n_g)} \in \mathbb{C}^{FK\ \ P\ \ N_s}$. Due to the symmetry of the trilinear model, we can also define two other matrix-slices $\mathbf{Y}^{(n_g)}_{k'\cdot\cdot} \in \mathbb{C}^{P\ \ N_s}$ where $k' = (f-1)K + k$, and $\mathbf{Y}^{(n_g)}_{\cdot p\cdot} \in \mathbb{C}^{N_s\ \ FK}$, obtained by slicing $\mathcal{Y}^{(n_g)}$ along its first and second dimensions, respectively. The set of matrix-slices $\mathbf{Y}^{(n_g)}_{k'\cdot\cdot}$, $k' = 1, \ldots, FK$, $\mathbf{Y}^{(n_g)}_{\cdot p\cdot}$, $p = 1, \ldots, P$, and $\mathbf{Y}^{(n_g)}_{\cdot\cdot n_s}$, $n_s = 1, \ldots, N_s$, can be factored as:

$$
\begin{aligned}
\mathbf{Y}^{(n_g)}_{k'\cdot\cdot} &= \mathbf{V} D_{k'}(\mathbf{G}^{(n_g)})\mathbf{S}^{(n_g)T} + \mathbf{N}^{(n_g)}_{k'\cdot\cdot}, \\
\mathbf{Y}^{(n_g)}_{\cdot p\cdot} &= \mathbf{S}^{(n_g)} D_p(\mathbf{V})\mathbf{G}^{(n_g)T} + \mathbf{N}^{(n_g)}_{\cdot p\cdot}, \\
\mathbf{Y}^{(n_g)}_{\cdot\cdot n_s} &= \mathbf{G}^{(n_g)} D_{n_s}(\mathbf{S}^{(n_g)})\mathbf{V}^T + \mathbf{N}^{(n_g)}_{\cdot\cdot n_s},
\end{aligned}
\tag{24}
$$

where $\mathbf{N}^{(n_g)}_{k'\cdot\cdot}$, $\mathbf{N}^{(n_g)}_{\cdot p\cdot}$ and $\mathbf{N}^{(n_g)}_{\cdot\cdot n_s}$ are matrix-slices of the noise tensor. Stacking columnwise $\mathbf{Y}^{(n_g)}_{\cdot\cdot n_s}$, $n_s = 1, \ldots, N_s$ into an $N_sFK \times P$ matrix $\mathbf{Y}^{(n_g)}_1$, $\mathbf{Y}^{(n_g)}_{\cdot p\cdot}$, $p = 1, \ldots, P$, into an $PN_s \times FK$ matrix $\mathbf{Y}^{(n_g)}_2$, and $\mathbf{Y}^{(n_g)}_{k'\cdot\cdot}$, $k' = 1, \ldots, FK$, into an $FKP \times N_s$ matrix $\mathbf{Y}^{(n_g)}_3$, we get:

$$
\begin{aligned}
\mathbf{Y}^{(n_g)}_1 &= (\mathbf{S}^{(n_g)}\quad \mathbf{G}^{(n_g)})\mathbf{V}^T + \mathbf{N}^{(n_g)}_1, \\
\mathbf{Y}^{(n_g)}_2 &= (\mathbf{V}\quad \mathbf{S}^{(n_g)})\mathbf{G}^{(n_g)T} + \mathbf{N}^{(n_g)}_2, \\
\mathbf{Y}^{(n_g)}_3 &= (\mathbf{G}^{(n_g)}\quad \mathbf{V})\mathbf{S}^{(n_g)T} + \mathbf{N}^{(n_g)}_3.
\end{aligned}
\tag{25}
$$

$\mathbf{Y}^{(n_g)}_{i=1,2,3}$ are the unfolded matrices of $\mathcal{Y}^{(n_g)}$, which are different rearrangements of the full information contained in the received signal tensor. Figure 3 provides an illustration of the PARAFAC decomposition of the received signal tensor in absence of noise, where $\mathbf{g}^{(n_g)}_r$ and $\mathbf{s}^{(n_g)}_r$ are the r-th column of $\mathbf{G}^{(n_g)}$ and $\mathbf{S}^{(n_g)}$, respectively. This figure shows that the received signal is a sum of R signal components, each one of them being a channel-transformed contribution of a given coded multiplexed data stream.

One of the key features of the PARAFAC model is its uniqueness [12], [13]. When fitting a PARAFAC model from noisy data, it is necessary to investigate under which conditions the model is identifiable. In the present context, when ensuring the identifiability of the model, it is possible to blindly decode the transmitted symbols. Assuming that the component matrices $\mathbf{G}^{(n_g)}, \mathbf{S}^{(n_g)}$ and $\mathbf{V}$, $n_g = 1, \ldots, N_g$, are full rank, a sufficient condition for their identifiability, up to permutation and scaling of their columns, is given by [13]:

$$
\min(FK, R) + \min(N_s, R) + \min(P, R) \quad 2(R+1). \tag{26}
$$

This condition links the received signal dimensions (F, K, N_s, P) to the number R of multiplexed data streams. We assume that the number of used time-slots is such that $N_s \quad R$, which simplifies (26) to:

$$
\min(FK, R) + \min(P, R) \quad R + 2. \tag{27}
$$

By studying together (27), (17) and (22), we can distinguish the following particular cases:

1) When $M = K = 1$ (single-transmit/receive antenna case), blind decoding is achieved with $F \quad L$ subcarriers per group, provided that $P \quad R$; a diversity gain of L is obtained.
2) When $F = L = 1$ (single-carrier flat-fading case), blind decoding is achieved with $K = 2$ receiver antennas regardless of the number of used transmit antennas, provided that $P \quad R$. In this case, the maximum diversity gain MK can be obtained when $P \quad M$.

Due to the presence of the product FK in (27), the system can support an arbitrary number of multiplexed data streams and/or transmit antennas with only $K = 1$ receive antenna, and a diversity gain ML is obtained, provided that F and P satisfy (27), (17) and (22). In this case, blind decoding can be carried out from noisy observations of the received signal tensor, by using the Alternating Least Squares (ALS) algorithm [12], [13]. In its general form, the ALS algorithm consists in computing a third-order PARAFAC model by alternating among the least square estimations of the three component matrices that define the model. The idea is to update one component matrix at each iteration, using the least

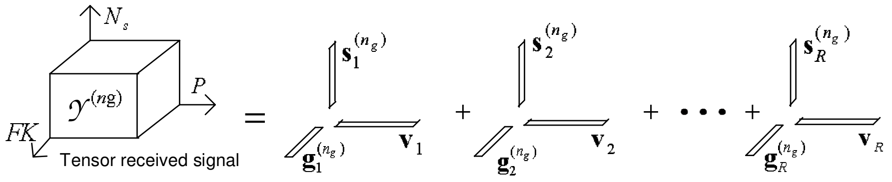

Fig. 3. PARAFAC decomposition of the received signal tensor for the n_g-th subcarrier group.

squares algorithm, while the two other ones are fixed to their values obtained at the previous estimation steps.

In the present context, the knowledge of the coding matrices $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$ at the transmitter/receiver can be exploited for improving the receiver performance (although this is not necessary for the estimation of the transmitted symbols). In this case, each iteration of the ALS algorithm has only two estimation steps instead of three, since $\mathbf{V}$ is known. Hence, at the i-th iteration, the two update equations for $\mathbf{G}^{(n_g)}$ and $\mathbf{S}^{(n_g)}$, $n_g = 1, \ldots, N_g$, are given by:

$$\begin{aligned} \widehat{\mathbf{G}}^{(n_g)T}(i) &= [\mathbf{V} \quad \mathbf{S}^{(n_g)}(i-1)]^{\dagger}\mathbf{Y}_2^{(n_g)}, \\ \mathbf{S}^{(n_g)T}(i) &= [\mathbf{G}^{(n_g)}(i) \quad \mathbf{V}]^{\dagger}\mathbf{Y}_3^{(n_g)}. \end{aligned} \tag{28}$$

At the end of the i-th iteration, an overall error measurement between the estimated model and the received signal can be calculated using the following equation:

$$e_i^{(n_g)} = \quad \mathbf{Y}_1^{(n_g)} - (\mathbf{S}^{(n_g)}(i) \quad \mathbf{G}^{(n_g)}(i))\mathbf{V}^T \quad _F \, .$$

We declare that the ALS algorithm has converged at the i-th iteration when $|e_i - e_{i \quad 1}| \quad 10^{\quad 6}$. The estimation of each column of $\widehat{\mathbf{G}}^{(n_g)}$ and $\mathbf{S}^{(n_g)}$ is affected by a scaling factor ambiguity. In order to eliminate this ambiguity, we assume that the first transmitted symbol vector at all subcarrier groups are equal to one: $\mathbf{s}^{(n_g,1)} = [1\ 1 \quad 1]^T$, $n_g = 1, \ldots, N_g$. Exploiting this knowledge, a final unambiguous estimate of $\mathbf{S}^{(n_g)}$ is obtained by post-multiplying it by a diagonal matrix formed from its first row [14].

VI. Simulation Results

The performance of the proposed trilinear STF codes is evaluated in this section. In all results, the Bit-Error-Rate (BER) is shown as a function of the Signal-to-Noise Ratio (SNR) per bit. Each plotted BER curve is an average over the over 1000 independent Monte Carlo runs. We consider $N_c = 128$ sub-carriers over a total bandwidth of 1MHz, which means that the OFDM symbol duration is $T = 128 \quad s$ without the cyclic prefix. We recall that the ALS algorithm described in the previous section (c.f. (28)) is applied for each set of N_c/F orthogonal sub-carriers in order to estimate the overall transmitted streams.

Figure 4 shows the BER versus SNR performance of trilinear STF codes for different transmit/receive configurations. We consider three modulation schemes, which are 4-QAM, 16-QAM and 64-QAM, leading to three spectral efficiency levels, with data-rates 1, 2 or 3 bits/channel use, respectively. At the receiver, $K = 1$ or 2 receive antennas are considered. In this figure, we have assumed a two-ray equal-power profile ($L = 2$) with a delay of $1 \quad s$ between the two rays. The trilinear STF code is characterized by $Q = 2$ and $(M, P, F) = (2, 2, 2)$, and $N = 20$ time-slots are used at the receiver for the blind decoding of the transmitted symbols. It can be seen that the trilinear STF codes with ALS-based blind decoding provide good results in all considered configurations. Note that the combination of the proposed STF codes with different modulation schemes offer the flexibility for trading off performance and rate, while achieving the maximum diversity gain.

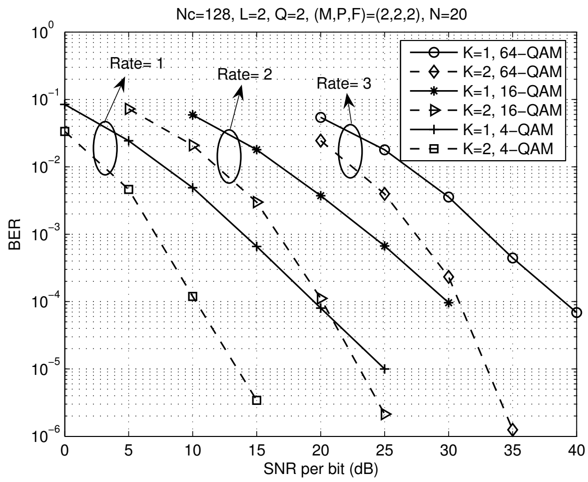

Fig. 4. BER versus SNR for $K = 1, 2$ and 4-QAM, 16-QAM and 64-QAM.

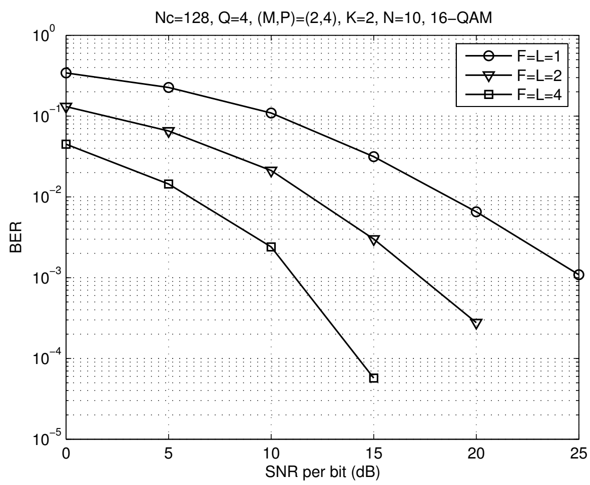

Fig. 5. BER versus SNR for $F = L = 1, 2$ and 4.

In Fig. 5, we evaluate the performance of a fixed configuration by varying the number L of resolvable multipaths, as well

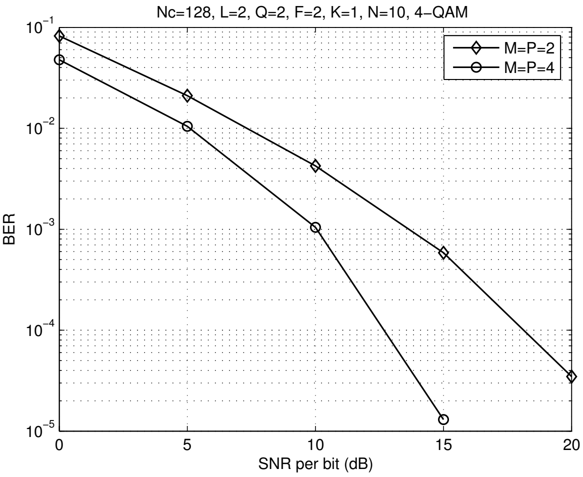

Fig. 6. BER versus SNR for $M = 2$ and 4.

as the number F of coded subcarriers per group. The delay of each multipath is assumed to be equal to $\ _l = (l-1)T/N_c$, $l = 1, \ldots L$. For example, for $L = 2$ we have $\ _1 = 0$ and $\ _2 = T/N_c$. As expected, as L increases the BER performance is improved. This means that the trilinear STF codes efficiently exploit multipath diversity.

Now, we evaluate the influence of the number of used transmit antennas on the performance. We consider $M = 2$ or 4. The other parameters of the trilinear STF code are $Q = 2$ and (F, P)=(2,2), and $N = 10$, $K = 1$ and $L = 2$. According to Fig. 6, a higher diversity gain is achieved as more transmit antennas are used for coding. Of course, such a diversity gain comes at the expense of a sacrifice in data-rate.

VII. Conclusions

We have presented a new multiantenna coding framework for broadband MIMO-OFDM systems, which we have called *trilinear space-time-frequency codes*. The idea consists in using a trilinear PARAFAC structure for the STF coder, in order to jointly multiplex and spread several input streams over space (transmit antennas), time (symbol periods) and frequency (sub-carriers). Due to the PARAFAC structure of the coder, each codeword is as an element of a third-order tensor, which can also be decomposed using PAFAFAC analysis. The trilinear codes afford a variable degree of multiplexing-spreading over each one of the three signal dimensions, and are valid for an arbitrary number of transmit/receive antennas. Trilinear STF codes achieve full diversity MKL in a frequency-selective MIMO channel. At the receiver, blind decoding of the trilinear STF codes is possible with linear complexity, due to uniqueness properties of the resulting PARAFAC model.

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