Generalized PARAFAC Model for Multidimensional Wireless Communications with Application to Blind Multiuser Equalization

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***Abstract*— In this work we develop a new tensor modeling approach for multiuser wireless communication systems where the received signal has a multidimensional nature. The proposed tensor model follows from a third-order (3D) Block-Parallel Factor (Block-PARAFAC) decomposition with factor interactions, which can be viewed as a more general model than the standard model [1], [2]. The proposed tensor decomposition aims at unifying the received signal modeling for i) Temporally-Oversampled, ii) Direct-Sequence Code Division Multiple Access (DS-CDMA) and iii) Orthogonal Frequency Division Multiplexing (OFDM) systems. This modeling approach assumes a receiver antenna array, specular multipath propagation and frequency-selectivity. We show that the model for each of the considered systems can be derived from the Block-PARAFAC model by making appropriate choices in its dimensions and/or structure. As an application of the proposed tensor model to blind multiuser separation/equalization, a new receiver algorithm is derived.**

I. INTRODUCTION

In several wireless communication systems, the received signal is *multidimensional* in nature and can be interpreted as a tensor, although it is not always treated as such. In a seminal paper [3], N. D. Sidiropoulos et al. showed that the received signal in a Direct-Sequence Code Division Multiple Access (DS-CDMA) system exhibits a trilinear structure and can be modeled as a third-order (3D) tensor, following a Parallel Factor (PARAFAC) model [1], [2]. They first derived a link between PARAFAC and the problem of blind multiuser separation, from which the benefits of using a tensor approach in place of a matrix one were made clear. Still in the context of DS-CDMA systems, some other works have addressed the problem of multiuser separation/equalization using PARAFAC modeling approaches [4], [5], [6].

In this work, we develop a new tensor model for the received signal in multiuser wireless communication systems, which is a generalization of PARAFAC. Contrarily to previously developed PARAFAC models, it is assumed here that the frequency-selective wireless channel is characterized by a sum of a small number of multipaths, which is also known in the literature as the specular multipath assumption. Interestingly, we show that the same model is valid for i) Temporally-Oversampled, ii) DS-CDMA and iii) Orthogonal Frequency Division Multiplexing (OFDM) systems, where the model for each particular system can be derived from the same tensor model by making appropriate choices in its dimensions and/or structure. The proposed 3D tensor model is called here "Block-PARAFAC", since it models the received signal as a sum of PARAFAC tensor-blocks, the number of blocks being equal to the number of interfering users in the system. In each PARAFAC block, interactions among factors of different dimensions or *modes* are modeled with the aid of *interaction matrices*, the structure/dimension of which depends on the number of specular multipaths and on the temporal support of the channel impulse response.

We also present an application of the Block-PARAFAC model to blind multiuser separation/equalization, where a new receiver algorithm is derived. Simulation results are shown to illustrate the Bit-Error-Rate (BER) performance of the proposed blind receiver.

The rest of this paper is summarized as follows. In Section 2, the wireless channel model is described and the received signal for each considered system (Temporally-Oversampled, DS-CDMA and OFDM) is formulated using tensor notation. In Section 3, a Block-PARAFAC decomposition is introduced. Section 4 links the Block-PARAFAC decomposition to the tensor modeling of the received signal for the considered systems. In Section 5, we derive a blind multiuser separation/equalization receiver as an application of the developed model. Some simulation results are presented in this section for performance evaluation. Finally, Section 6 concludes this paper.

II. TENSOR SIGNAL MODELING

Let us consider a linear uniform spaced array of M antennas receiving signals from Q co-channel users. Assume that the signal transmitted by each interfering user is subject to multipath propagation and that the received signal is a superposition of a finite number L of multipaths (see Fig. 1). The propagation channel is assumed to be time-dispersive.

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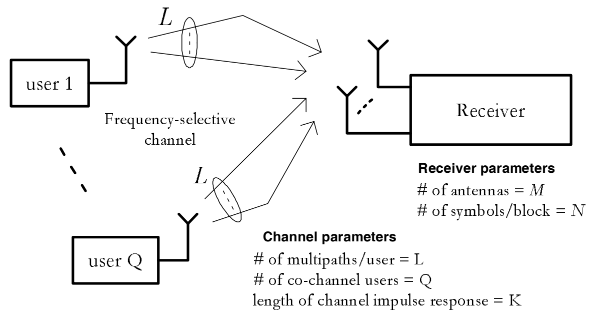

Fig. 1. Multiuser model with specular multipath propagation.

It is considered that multipath delay spread exceeds the inverse of the coherence bandwidth of the system, meaning that multipath fading is frequency-selective. The temporal support of the channel impulse response is finite and equal to K symbol periods. Assuming also that the transmitted signals are narrowband with respect to the array aperture, the receiver antennas are spaced half-wavelength apart and that multipath is directional (which is generally the case when multipath reflectors are local to the receiver), the discrete-time baseband representation of the signal received at the m-th antenna at the n-th symbol interval is given by:

$$x_m(n) = \sum_{q=1}^{Q}\sum_{l=1}^{L} b_{lq} a_m(\theta_{lq}) \sum_{k=0}^{K-1} g(k-\tau_{lq}) s_q(n-k) + v_m(n),$$

where b_{lq} is the complex fading gain of the l-th path of the q-th user. The term $a_m(\theta_{lq})$ is the response of the m-th antenna to the l-th path of the q-th user, θ_{lq} being the angle of incidence or Direction Of Arrival (DOA). Similarly, the term τ_{lq} denotes the propagation delay (normalized by the symbol period T) and the term $g(k-\tau_{lq})$ represents the k-th component of the pulse-shape function. It is assumed that $K \geq max(\tau_{lq})$, in such a way that most of the multipath energy is captured in our frequency-selective channel impulse response model. Finally, $s_q(n)$ is the symbol transmitted by the q-th user at the n-th time instant.

For the considered wireless communication systems, the received signal is now formulated in scalar form using tensor notation. Each scalar component of the received signal is interpreted as a *three-way array* or 3D tensor, characterized by three indices. For all cases, the first two indices are for the *space* and *time* dimensions while the third one depends on the considered system and can be an *oversampling*, a *spreading* or a *frequency* dimension. The use of the tensor formalism for the received signal models will make the introduction of the proposed Block-PARAFAC model more natural in the sequel.

A. Temporally-Oversampled system

At the output of each receiver antenna, the signal is sampled at a rate that is P times the symbol rate. Due to temporal oversampling, the resolution of the pulse-shape function is increased by a factor P, which also increases the temporal resolution of the received signal by the same factor. Here, oversampling is interpreted as a third dimension for the received signal. We define $a_{m,l}^{(q)} = a_m(\theta_{lq})$, $b_l^{(q)} = b_{lq}$ and $s_{n,k}^{(q)} = s_q(n-k)$. The p-th *oversample* associated with the k-th component of the pulse-shape function is defined as the scalar $g_{p,lk}^{(q)} = g(k-1+(p-1)/P - \tau_{l,q})$. Taken these definitions into account, the received signal can thus be interpreted as a 3D tensor $\mathcal{X} \in \mathbb{C}^{M\times N\times P}$, of which the (m,n,p)-th scalar component can be written as:

$$x_{m,n,p} = \sum_{q=1}^{Q}\sum_{l=1}^{L} a_{m,l}^{(q)}\, b_l^{(q)} \sum_{k=1}^{K} s_{n,k}^{(q)}\, g_{p,lk}^{(q)} + v_{m,n,p}, \quad (1)$$

where $v_{m,n,p}$ is a scalar component of the additive noise, which is assumed to be a zero-mean complex Gaussian random variable. The four scalar quantities in (1) are defined as elements of associated *factor matrices*, in which the proposed tensor model will be decomposed, i.e., $a_{m,l}^{(q)} = [\mathbf{A}^{(q)}]_{m,l}$, $b_l^{(q)} = [\mathbf{B}^{(q)}]_{l,l}$, $s_{n,k}^{(q)} = [\mathbf{S}^{(q)}]_{n,k}$ and $g_{p,lk}^{(q)} = [\mathbf{G}^{(q)}]_{p,(l-1)K+k}$.

B. DS-CDMA system

At the transmitter, each symbol is spread by a signature (spreading code) sequence of length J with period $T_c = T/J$, with T denoting the symbol period. The spreading sequence associated with the q-th user is denoted here by $\mathbf{c}^{(q)} = [c_q(1)c_q(2)\cdots c_q(J)]^T \in \mathbb{C}^J$. As a result of spreading operation, each symbol to be transmitted is converted into J *chips*. Considering the l-th path of the q-th user, we define $c_j^{(q)} = c_q(j)$ and the response of the (chip-sampled) pulse-shape function in scalar form as $g_{j,lk}^{(q)} = g(j-1+(k-1)J-\tau_{lq})$. The received signal can also be interpreted as a 3D tensor $\mathcal{X} \in \mathbb{C}^{M\times N\times J}$, with associated (m,n,j)-th scalar component given as:

$$x_{m,n,j} = \sum_{q=1}^{Q}\sum_{l=1}^{L} a_{m,l}^{(q)}\, b_l^{(q)} \sum_{k=1}^{K}\sum_{j'=1}^{J} s_{n,k}^{(q)}\, g_{j-j',lk}^{(q)}\, c_{j'}^{(q)} + v_{m,n,j}. \quad (2)$$

Defining: $u_{j,lk}^{(q)} = g_{j,lk}^{(q)} * c_j^{(q)}$ as the convolution between the pulse-shape function and the spreading code, we can rewrite (2) as:

$$x_{m,n,j} = \sum_{q=1}^{Q}\sum_{l=1}^{L} a_{m,l}^{(q)}\, b_l^{(q)} \sum_{k=1}^{K} s_{n,k}^{(q)}\, u_{j,lk}^{(q)} + v_{m,n,j}. \quad (3)$$

The scalar quantities in (3) are equally defined as those of (1) (as well as the associated matrices), except for $u_{j,lk}^{(q)} = [\mathbf{U}^{(q)}]_{j,(l-1)K+k}$, which is itself a result of a convolution.

C. OFDM system

In an OFDM system, the symbol sequence to be transmitted is organized into blocks of F symbols (serial-to-parallel conversion), i.e., $\mathbf{s}_q(n) = [s_q(nF-F+1), \ldots, s_q(nF)]^T \in \mathbb{C}^F$, $n = 1, \ldots, N$. The so-called *multicarrier modulation* consists in linearly combining each block of F symbols using an Inverse Fast Fourier Transform (IFFT) matrix $\mathbf{\Gamma} \in \mathbb{C}^{F\times F}$, $[\mathbf{\Gamma}]_{i,j} = (1/\sqrt{F})\, e^{j\frac{2\pi}{F}(i-1)(j-1)}$,

$i, j = 1, \ldots, F$. After the IFFT stage, a cyclic prefix (CP) of minimum length K is inserted at the beginning of each block of F symbols , before transmission. At the receiver, inverse processing is done. The CP is removed and each received OFDM block is linearly combined using an FFT matrix $\mathbf{\Gamma}^H$. Thanks to the use of IFFT/FFT together with insertion/removal of the CP, it can be shown that the wireless channel can be represented as a set of F scalar (non-convolutive) channels [7], i.e., the wireless channel for each subcarrier is frequency-flat. For the l-th path of the q-th user, the frequency-response of the channel for the f-th subcarrier is denoted by $w_{f,l}^{(q)}$. The received signal can be interpreted as a 3D tensor $\mathcal{X} \in \mathbb{C}^{M \times N \times F}$, the (m, n, f)-th scalar component of which is written as follows:

$$x_{m,n,f} = \sum_{q=1}^{Q} \sum_{l=1}^{L} a_{m,l}^{(q)} b_l^{(q)} s_{n,f}^{(q)} w_{f,l}^{(q)} + v_{m,n,f}, \qquad (4)$$

where $s_{n,f}^{(q)} = s_q(nF - F + f) = [\mathbf{S}^{(q)}]_{n,f}$ and $w_{f,l}^{(q)} = [\mathbf{W}^{(q)}]_{f,l}$ are respectively the scalar components of the symbol matrix and the frequency-domain pulse-shape response matrix.

Comparing (4) with (1) and (3), it can be noted that the inner summation over K, representing the convolution between $s_{n,k}^{(q)}$ and $g_{p,lk}^{(q)}$ (or between $s_{n,k}^{(q)}$ and $u_{j,lk}^{(q)}$ for the DS-CDMA case) gives place to an element-by-element multiplication involving $s_{n,f}^{(q)}$ and $w_{f,l}^{(q)}$, for each frequency.

III. A Block-PARAFAC decomposition

We propose a Block-PARAFAC decomposition, where the tensor is decomposed as a sum of PARAFAC blocks. Matrices defining each block are allowed to have different number of factors (columns), contrarily to the conventional PARAFAC model, in which all matrices have the same number of factors. Within each PARAFAC block, there are interactions between factors of the different matrices that define the model.

Let $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ be a third-order tensor and define three sets of matrices $\{\mathbf{A}^{(1)}, \ldots, \mathbf{A}^{(Q)}\} \in \mathbb{C}^{I_1 \times R_1}$, $\{\mathbf{B}^{(1)}, \ldots, \mathbf{B}^{(Q)}\} \in \mathbb{C}^{I_2 \times R_2}$ and $\{\mathbf{C}^{(1)}, \ldots, \mathbf{C}^{(Q)}\} \in \mathbb{C}^{I_3 \times R_1 R_2}$ with typical elements $\alpha_{i_1,r_1}^{(q)} = [\mathbf{A}^{(q)}]_{i_1,r_1}$, $\beta_{i_2,r_2}^{(q)} = [\mathbf{B}^{(q)}]_{i_2,r_2}$ and $\xi_{i_3,r_1 r_2}^{(q)} = [\mathbf{C}^{(q)}]_{i_3,(r_1-1)R_2+r_2}$. The Block-PARAFAC decomposition of $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ with interactions between factors of the *first mode* (columns of $\mathbf{A}^{(q)}$) and factors of the *second mode* (columns of $\mathbf{B}^{(q)}$), $q = 1, \ldots, Q$, is given in scalar form as:

$$x_{i_1,i_2,i_3} = \sum_{q=1}^{Q} \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} \alpha_{i_1,r_1}^{(q)} \beta_{i_2,r_2}^{(q)} \xi_{i_3,r_1 r_2}^{(q)}. \qquad (5)$$

The i_3-th matrix slice $\mathbf{X}_{\cdot\cdot i_3} \in \mathbb{C}^{I_1 \times I_2}$ of $\mathcal{X}$, i.e. $[\mathbf{X}_{\cdot\cdot i_3}]_{i_1,i_2} = x_{i_1,i_2,i_3}$, can be written as:

$$\mathbf{X}_{\cdot\cdot i_3} = \sum_{q=1}^{Q} \mathbf{A}^{(q)} \mathbf{\Psi} D_{i_3}(\mathbf{C}^{(q)}) (\mathbf{B}^{(q)} \mathbf{\Phi})^T, \qquad (6)$$

where $\mathbf{\Psi} \in \mathbb{C}^{R_1 \times R_1 R_2}$ and $\mathbf{\Phi} \in \mathbb{C}^{R_2 \times R_1 R_2}$ are defined here as *constraint matrices* that have the following structure:

$$\mathbf{\Psi} = \mathbf{I}_{R_1} \otimes \mathbf{1}_{R_2}^T, \quad \mathbf{\Phi} = \mathbf{1}_{R_1}^T \otimes \mathbf{I}_{R_2}, \qquad (7)$$

where $\otimes$ represents the Kronecker product. Note that the post-multiplication of $\mathbf{A}^{(q)}$ by $\mathbf{\Psi}$ generates a matrix where each column of $\mathbf{A}^{(q)}$ is repeated R_2 times, while the post-multiplication of $\mathbf{B}^{(q)}$ by $\mathbf{\Phi}$ results in matrix that has $\mathbf{B}^{(q)}$ repeated R_1 times. Analyzing (6), it is seen that every column of $\mathbf{A}^{(q)}$ *interacts* (i.e., forms the outer product) with everyone of the columns of $\mathbf{B}^{(q)}$ and *vice-versa.* There is a total of Q blocks of $R_1 R_2$ outer-product terms. The third-mode matrix $\mathbf{C}^{(q)}$ can thus be interpreted as a matrix, the elements of which represent the magnitude (or importance) of the interactions within the q-th block, while $\mathbf{\Psi}$ and $\mathbf{\Phi}$ contains the *pattern* of these interactions, which is the same for all the blocks. Figure 2 illustrates the decomposition of the received signal tensor in a Block-PARAFAC form. It is relatively straightforward to generalize (5) to cases where the within-block interaction pattern as well as the number of interactions differ from block to block[1]. Defining $\mathbf{X}_1 \in \mathbb{C}^{I_3 I_1 \times I_2}$, $[\mathbf{X}_1]_{(i_3-1)I_1+i_1,i_2} = x_{i_1,i_2,i_3}$, as a matrix that stacks the I_3 slices $\mathbf{X}_{\cdot\cdot 1}, \ldots, \mathbf{X}_{\cdot\cdot I_3}$, $\mathbf{X}_2 \in \mathbb{C}^{I_1 I_2 \times I_3}$, $[\mathbf{X}_2]_{(i_1-1)I_2+i_2,i_3} = x_{i_1,i_2,i_3}$, as a matrix that stacks the I_1 slices $\mathbf{X}_{1\cdot\cdot}, \ldots, \mathbf{X}_{I_1\cdot\cdot}$, and $\mathbf{X}_3 \in \mathbb{C}^{I_2 I_3 \times I_1}$, $[\mathbf{X}_3]_{(i_2-1)I_3+i_3,i_3} = x_{i_1,i_2,i_3}$, as a matrix that stacks the I_2 slices $\mathbf{X}_{\cdot 1\cdot}, \ldots, \mathbf{X}_{\cdot I_2\cdot}$, we get the following matrix representations for the tensor $\mathcal{X}$: i) $\mathbf{X}_1 = (\mathbf{C} \diamond \mathbf{A}\mathbf{\Psi})(\mathbf{B}\mathbf{\Phi})^T$, ii) $\mathbf{X}_2 = (\mathbf{A}\mathbf{\Psi} \diamond \mathbf{B}\mathbf{\Phi})\mathbf{C}^T$, iii) $\mathbf{X}_3 = (\mathbf{B}\mathbf{\Phi} \diamond \mathbf{C})(\mathbf{A}\mathbf{\Psi})^T$, where $\diamond$ is the Khatri-Rao (column-wise Kronecker) product.

The computation of the Block-PARAFAC decomposition allows the resolution (or separation) of the Q blocks. Although between-block resolution can be achieved, partial uniqueness within each block may exist in some cases. Such within-block partial uniqueness means that some matrix factors can be completely determined (up to permutation and scaling) while the determination of the other ones is also affected by a nonsingular matrix multiplication, although the subspaces spanned by these factors are uniquely determined. This leaves rotational indeterminacy. However, uniqueness of the factors involved in the rotational indeterminacy can be restored in some cases by exploiting prior knowledge about the structure of the factor matrices defining the model.

In general, identification of the model (5) is affected by the following ambiguities:

$$\mathbf{A}' = \mathbf{A}\mathbf{\Pi}_a \mathbf{T}_a, \quad \mathbf{B}' = \mathbf{B}\mathbf{\Pi}_b \mathbf{T}_b, \quad \mathbf{C}' = \mathbf{C}\mathbf{\Pi}_c \mathbf{T}_c, \qquad (8)$$

where $\mathbf{\Pi}_a = BlockDiag(\mathbf{\Pi}_{R_1}^{(1)}, \ldots, \mathbf{\Pi}_{R_1}^{(Q)})(\mathbf{\Pi}_Q \otimes \mathbf{I}_{R_1})$, $\mathbf{\Pi}_b = \mathbf{\Pi}_Q \otimes \mathbf{I}_{R_2}$ and $\mathbf{\Pi}_c = BlockDiag(\mathbf{\Pi}_{R_1}^{(1)} \otimes \mathbf{I}_{R_2}, \ldots, \mathbf{\Pi}_{R_1}^{(Q)} \otimes \mathbf{I}_{R_2})(\mathbf{\Pi}_Q \otimes \mathbf{I}_{R_1 R_2})$ are block-permutation

[1]Indeed, allowing PARAFAC blocks with different interaction structure is useful when the number/structure of multipaths differ from user to user.

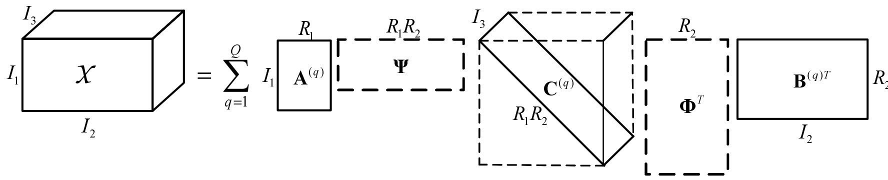

Fig. 2. Block-PARAFAC decomposition.

matrices and

$$\begin{aligned}\mathbf{T}_a &= BlockDiag(\mathbf{T}_a^{(1)}, \dots, \mathbf{T}_a^{(Q)}),\\ \mathbf{T}_b &= BlockDiag(\mathbf{T}_b^{(1)}, \dots, \mathbf{T}_b^{(Q)}), \qquad (9)\\ \mathbf{T}_c &= BlockDiag((\mathbf{T}_a^{(1)}\mathbf{T}_b^{(1)})^{-1}, \dots, (\mathbf{T}_a^{(Q)}\mathbf{T}_b^{(Q)})^{-1}),\end{aligned}$$

are nonsingular block-diagonal matrices. The block-diagonal structure of $\mathbf{T}_a$, $\mathbf{T}_b$ and $\mathbf{T}_c$ means that the rotational indeterminacy independently affect Q sets of R_1 columns in $\mathbf{A}$, Q sets of R_2 columns in $\mathbf{B}$ and Q sets of R_1R_2 columns in $\mathbf{C}$. This is due to the fact that, in our model all the Q sets of factors that are collinear in one mode are linearly independent in the other two modes and thus define distinct subspaces in the solution. This is the reason why the computation of the Block-PARAFAC decomposition separates the Q blocks. Further details on the uniqueness conditions of the Block-PARAFAC model will be given in a more complete version of this paper.

IV. Unified tensor modeling

Let us rewrite the scalar representations for the received signal given by (1), (3) and (4) for the Temporally-Oversampled, DS-CDMA and OFDM systems respectively:

$$\begin{aligned}x_{m,n,p} &= \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{k=1}^{K} a_{m,l}^{(q)}\, s_{n,k}^{(q)}\, \overline{g}_{p,lk}^{(q)} + v_{m,n,p},\\ x_{m,n,j} &= \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{k=1}^{K} a_{m,l}^{(q)}\, s_{n,k}^{(q)}\, \overline{u}_{j,lk}^{(q)} + v_{m,n,j}, \qquad (10)\\ x_{m,n,f} &= \sum_{q=1}^{Q}\sum_{l=1}^{L}\sum_{f'=1}^{F} a_{m,l}^{(q)}\, s_{n,f'}^{(q)} \overline{w}_{f,lf'}^{(q)} + v_{m,n,f},\end{aligned}$$

where $\overline{g}_{p,lk}^{(q)} = b_l^{(q)} g_{p,lk}^{(q)}$, $\overline{u}_{j,lk}^{(q)} = b_l^{(q)} u_{j,lk}^{(q)}$ and $\overline{w}_{f,lf'}^{(q)} = b_l^{(q)} w_{f,l}^{(q)} \delta_{ff'}$. In the third line of (10), we have introduced an artificial summation over the frequency domain without modifying the meaning of the model, since $\overline{w}_{f,lf'}^{(q)}$ is equal to zero for $f \neq f'$ (see Section II-C).

The received signal models in (10) for the three systems are quite similar. Note that the basic difference is on the meaning of the third dimension of the received signal tensor, which corresponds to the oversampling (spreading) factor for the Temporally-Oversampled (DS-CDMA) system or to the number of subcarriers for the OFDM system. By comparing (10) with (5), we notice that the received signal in each system follows a third-order Block-PARAFAC model. In the following a brief summary is provided, showing how the physical channel and system parameters are linked to the general decomposition (5):

- I_1: number of antennas (M)
- I_2: number of symbols per block (N)
- I_3: $\begin{cases}\text{oversampling factor } (P)\\ \text{spreading factor } (J)\\ \text{number of subcarriers } (F)\end{cases}$
- R_1 : number of multipaths per user (L)
- R_2 : $\begin{cases}\text{length of the channel impulse response } (K)\\ \text{number of subcarriers } (F)\end{cases}$
- Q: number of active users in the system
- First-mode matrices $\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(Q)}$:

$$[\mathbf{A}^{(q)}]_{i_1,r_1} = a_m(\theta_{lq}) \qquad (11)$$

- Second-mode matrices $\mathbf{B}^{(1)}, \dots, \mathbf{B}^{(Q)}$:

$$[\mathbf{B}^{(q)}]_{i_2,r_2} = \begin{cases} s_q(n-k);\\ s_q(nF-F+f);\end{cases} \qquad (12)$$

- Third-mode matrices $\mathbf{C}^{(1)}, \dots, \mathbf{C}^{(Q)}$:

$$[\mathbf{C}^{(q)}]_{i_3,r_1r_2} = \begin{cases} b_l^{(q)} g(k-1+(p-1)/P-\tau_{lq});\\ b_l^{(q)} u(j-1+(k-1)J-\tau_{lq});\\ b_l^{(q)} w_{f,l}^{(q)} \delta_{ff'}\end{cases} \qquad (13)$$

A. Uniqueness

The interpretation of (8) from a signal processing point of view is that between-block uniqueness/resolution of the Block-PARAFAC model allows the blind separation of the Q users. After the separation of the Q blocks, within-block uniqueness of the second-mode factor matrix (i.e. the symbol matrix) will guarantee the symbol recovering (equalization) of each user sequence. From (8) and (12) we have:

$$\mathbf{S}^{(q)'} = \mathbf{S}^{(q)}\mathbf{T}_b^{(q)}, q = 1, \dots, Q. \qquad (14)$$

For the Temporally-Oversampled/DS-CDMA system, the matrix factorization (14) is unique up to a scaling factor due to the Toeplitz structure of the symbol matrix $\mathbf{S}^{(q)}$ [8], i.e. $\mathbf{T}_b^{(q)} = \delta^{(q)}\mathbf{I}_K$. For the OFDM system the above factorization is also unique. From (8) and (13), we have:

$$\mathbf{W}^{(q)'} = \mathbf{W}^{(q)}\mathbf{T}_c^{(q)}, q = 1, \dots, Q. \qquad (15)$$

Due to the diagonal structure of the $\mathbf{W}^{(q)}$'s, ambiguity is reduced to $\mathbf{T}_c^{(q)} = \delta^{(q)}\mathbf{I}_F$, allowing the determination of the frequency-domain channel responses.

V. Application to Blind Multiuser Equalization

As an application of the proposed tensor model to the problem of blind multiuser equalization, a receiver algorithm combining the Block-PARAFAC modeling and a subspace method is now presented. We consider the Temporally-Oversampled system for this application. Multiuser signal separation is done in the 3D tensor space, exploiting *space*, *time* and *oversampling* dimensions of the received signal tensor. An alternating least squares (ALS) algorithm [1] is used for this purpose. After the ALS stage, user-by-user equalization is done in the 2D matrix space via a subspace method [8]. At the i-th iteration, the ALS algorithm consists in estimating three factor matrices $\widehat{\mathbf{Z}}_1(i), \widehat{\mathbf{Z}}_2(i)$ and $\widehat{\mathbf{Z}}_3(i)$ in the following manner:

1) $\widehat{\mathbf{Z}}_1^T(i) = \left[\left(\widehat{\mathbf{Z}}_2(i-1) \diamond \widehat{\mathbf{Z}}_3(i-1)\mathbf{\Psi}\right)\mathbf{\Phi}^T\right]^{\dagger}\mathbf{X}_1,$
2) $\widehat{\mathbf{Z}}_2^T(i) = \left[\left(\widehat{\mathbf{Z}}_3(i-1)\mathbf{\Psi} \diamond \widehat{\mathbf{Z}}_1(i)\mathbf{\Phi}\right)\right]^{\dagger}\mathbf{X}_2,$
3) $\widehat{\mathbf{Z}}_3^T(i) = \left[\left(\widehat{\mathbf{Z}}_1(i)\mathbf{\Phi} \diamond \widehat{\mathbf{Z}}_2(i)\right)\mathbf{\Psi}^T\right]^{\dagger}\mathbf{X}_3.$

After convergence, equalization of each user symbol sequence is performed by solving Q independent sets of Toeplitz matrix factorization problems as follows:

$$\widehat{\mathbf{Z}}_1^{(1)} = \widehat{\mathbf{S}}^{(1)}\mathbf{T}_s^{(1)} \quad ,\ldots, \quad \widehat{\mathbf{Z}}_1^{(Q)} = \widehat{\mathbf{S}}^{(Q)}\mathbf{T}_s^{(Q)}. \qquad (16)$$

A final estimate of the Q symbol sequences is obtained by properly averaging over the rows of the respective Toeplitz symbol matrices. The inherent scaling ambiguity on each estimated symbol sequence is eliminated by considering differential detection as suggested in [3]. For purposes of performance evaluation, we ignore permutation ambiguity.

Simulation Results

The performance of the proposed blind PARAFAC-based receiver is evaluated through computer simulations. Results are shown in terms of bit-error-rate (BER) versus signal-to-noise ratio (SNR) curves, which are obtained from an average over 100 Monte Carlo runs. The number of users is $Q = 2$, the number of multipaths/user is $L = 2$ and the temporal support of the channel $K = 2$. For each run, multipath fading gains are redrawn from an i.i.d. Rayleigh generator. The multipath angles are $(\theta_{11}, \theta_{21})$=$(0, 30^o)$ and $(\theta_{12}, \theta_{22})$=$(-10^o, 15^o)$ and the delays are (τ_{11}, τ_{21})=(τ_{12}, τ_{22})=$(0, T)$. The pulse-shape function is a raised-cosine filter with a roll-off factor of 0.35. Symbol sequences are redrawn from an i.i.d. distribution and follow a binary-phase shift keying (BPSK) modulation. The number of users is $Q = 2$. For each run, a block of $N = 50$ received samples is processed at the receiver and the BER is averaged over the two users. The ALS algorithm is randomly initialized at each run.

Figure 3 shows the results of the proposed blind PARAFAC-based receiver, compared to that of the Minimum Mean Square Error (MMSE) receiver with perfect channel knowledge. We consider $M = 4$ antennas and an oversampling factor of $P = 4$. Note that the PARAFAC-based receiver performs close to the MMSE one, with a performance gap of 3 dB approximately.

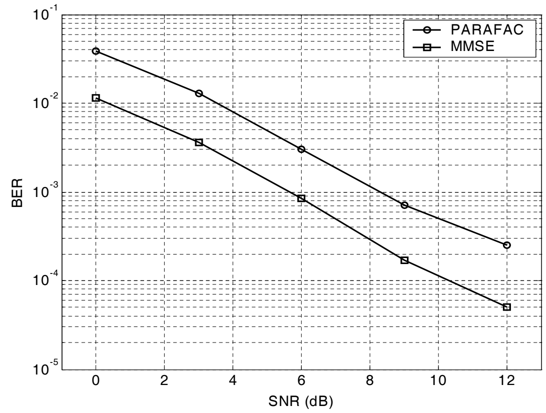

Fig. 3. BER versus SNR results. $M = P = 4$.

VI. Conclusions and further work

We have presented a new tensor modeling approach for Temporally-Oversampled/DS-CDMA and OFDM wireless communication systems, from a unified perspective. The proposed model assumes frequency-selective channel with specular multipath propagation, and is based on a Block-PARAFAC decomposition with factor interactions. We have shown that the Block-PARAFAC model can be applied to the problem of blind multiuser separation and equalization, where the between-block resolution of the Block-PARAFAC model enables the separation of users' transmissions while user-by-user equalization is made possible from the within-block uniqueness property of one of the factor matrices of the model due to its Toeplitz structure. Simulation results have illustrated the performance of the proposed blind receiver. Following this work, we will address more general propagation scenarios, where users have different multipath structure.

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