A Lower Bound on the Ergodic Capacity of Jointly Correlated Rician Fading Channels

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***Abstract*—In this paper, we investigate the ergodic capacity of multiple-input multiple-output (MIMO) Rician fading channels in the presence of spatial correlation at both the transmitter and the receiver with perfect channel state information (CSI) at the receiver. Based on majorization theory and the distribution of quadratic forms in normal random variables, we propose a tight lower bound on the ergodic capacity in terms of Meijer G-function. Our analytical results are validated through extensive Monte Carlo simulations using the exponential correlation model.**

I. Introduction

In wireless communication systems, the transmitted signals are attenuated by various phenomena including *shadowing* due to large objects in the signal path, such as buildings and hills, and *fading* due to multi-path propagation, both yielding a great challenge for reliable communication. Recently, the use of multiple-input multiple-output (MIMO) systems has gained considerable attention in the combat against such phenomena of the wireless medium. Among the benefits of multi-antenna communication, we stand out the multiplexing gain, where multiple streams can be sent simultaneously as a means to improve the capacity of the communication system, and the diversity in order to enhance the link reliability.

The multiplexing gain calculation is related to the *ergodic capacity*, which is a fundamental metric of performance that determines an information-theoretical bound on the achievable average rate for reliable communication. Since the pioneer work of [1], [2], which shows that in independent and identically distributed (i.i.d.) Rayleigh fading MIMO channels, the ergodic capacity increases linearly with the number of receive and transmit antennas, various papers have investigated the ergodic capacity of MIMO systems under different settings to understand the fundamental restrictions of multi-antenna communication.

In this work, we take into consideration the line-of-sight (LoS) component of the channel between the transmitter and the receiver and investigate the ergodic capacity under Rician fading. Modeling the channel fading statistics according to a Rice distribution encompasses the Rayleigh distribution. Specifically, we study the ergodic capacity on jointly correlated Rician fading MIMO channels and we derive a closed-form expression for a lower bound on the ergodic capacity, assuming uniform power allocation across transmit antennas and perfect channel state information (CSI) at the receiver.

There are several papers in the literature focusing on exact expressions, closed-form bounds and approximations on the ergodic capacity. In general, such results are derived from the complex non-central Wishart distribution which has a complex mathematical treatment. For example, for uncorrelated Rician MIMO channels, a converging series expression was achieved to represent the ergodic mutual information in [3]. In turn, [4] obtained a lower bound for i.i.d. uncorrelated Rician fading MIMO channels using the Bartlett decomposition. In [5], [6], the impact of spatial fading correlation was investigated and tight upper and lower bounds were provided. In turn, an analytical upper bound of dual MIMO systems was proposed in [7] together with an explicit asymptotic expression. Most recently, following as well an asymptotic approach, [8] presented a good approximation for the ergodic capacity of separately correlated Rician fading MIMO channels with co-channel interfering multiple-antenna systems.

In this paper, we take on the approach in [9] for Nakagami-m fading MIMO channels and obtain tight bounds on the ergodic capacity based on *majorization theory* [10], avoiding thus direct use of the involved non-central Wishart distribution. Such results are derived from infinite series of Meijer G-functions. It is important to mention that the Meijer G-function has been explored in Rician literature. For instance, [11] has been presented some exact results on the capacity of MIMO Rician channels from the moment generating function (MGF) of the mutual information.

In turn, the analysis developed here is also different from previous work on Rician fading MIMO channels due the application of majorization theory and the consideration of Weichselberger channel model [12], [13].

The rest of this paper is organized as follows. In Section II, a brief overview of the distribution of complex non-central quadratic forms and basic results from majorization theory are presented, which are required in the remainder of the paper. In Section III, we introduce the system model, the Rician fading MIMO channel as well as the Weichselberger correlation channel model. In Section IV, we derive a closed-form expression for a lower bound as well as tight approximations on the ergodic capacity for spatially correlated Rician MIMO

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channels. Numerical results are provided in Section V, while the paper is concluded in Section VI.

We shall use the following notations throughout the paper. Vectors and matrices will be indicated by bold lowercase and uppercase, respectively. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the transpose conjugated. We use $\mathbf{I}_n$ to represent the $n \times n$ identity matrix, $\det(\cdot)$ stands for the determinant of a square matrix, and $\mathrm{diag}(a_1, a_2, \cdots, a_n)$ is a diagonal matrix with diagonal entries $a_1, a_2, \cdots, a_n$. In turn, $\mathrm{vec}(\mathbf{A})$ denotes a vector obtained by staking the columns of $\mathbf{A}$. The symbol $\otimes$ denotes the Kronecker product and $\odot$ is the element-wise product of two matrices. The expectation operator is represented by $\mathbb{E}\{\cdot\}$. Finally, $x \sim \mathcal{N}(\mu, \sigma^2)$ implies that the random variable x follows a complex normal distribution with mean μ variance σ^2, while $\mathbf{x} \sim \mathcal{CN}_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ indicates an $n \times 1$ vector with a multivariate complex normal distribution with mean vector $\boldsymbol{\mu}$ $(n \times 1)$ and covariance matrix $\boldsymbol{\Sigma}$ $(n \times n)$.

II. Mathematical Preliminaries

In this section, we briefly present the distribution of quadratic forms in normal random variables and some results from majorization theory. Majorization theory is a very useful tool for deriving inequalities and it have been recently received significant attention for analyzing the performance of wireless communication systems among other applications [14]–[16].

A. Distribution of Quadratic Forms

We present the probability density function (PDF) of linear combinations of non-central chi-squared random variables or, equivalently, the PDF of quadratic forms in normal variables. Let $\mathbf{x}$ be the complex p-dimensional normal random vector with mean $\boldsymbol{\mu} \in \mathbb{C}^{p\times 1}$ and covariance matrix $\boldsymbol{\Sigma} \in \mathbb{C}^{p\times p}$, or $\mathbf{x} \sim \mathcal{CN}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. The random variable $Y = \mathbf{x}^H\mathbf{x}$ has a non-central chi-squared distribution with p degrees of freedom and non-centrality parameter $s^2 = \boldsymbol{\mu}^H\boldsymbol{\mu}$. Furthermore, the PDF of Y is given by [17]

$$p_Y(y) = \sum_{l=0}^{\infty} c_l \frac{y^{\frac{p}{2}+l-1} \exp\left(-\frac{p}{2}y\right)}{(2\beta)^{\frac{p}{2}+l}\,\Gamma\left(\frac{p}{2}+l\right)}, \tag{1}$$

where β is an arbitrary positive constant, and $\Gamma(\cdot)$ is the Euler gamma function [18]. The coefficients c_l are obtained recursively by

$$c_0 = \exp\left(-\frac{1}{2}\sum_{i=1}^{p} |b_i|^2\right) \prod_{i=1}^{p} \left(\frac{\beta}{\widetilde{\lambda}_i}\right)^{1/2}, \tag{2a}$$

$$c_l = \frac{1}{2l}\sum_{r=0}^{l-1} d_{l-r} c_r, \quad l \geq 1 \tag{2b}$$

with

$$d_l = \sum_{i=1}^{p}\left(1-\frac{\beta}{\widetilde{\lambda}_i}\right)^{l} + l\beta\sum_{i=1}^{p}\frac{|b_i|^2}{\widetilde{\lambda}_i}\left(1-\frac{\beta}{\widetilde{\lambda}_i}\right)^{l-1}. \tag{2c}$$

Here, $\widetilde{\lambda}_1, \widetilde{\lambda}_2, \cdots, \widetilde{\lambda}_p$ are the eigenvalues of the covariance matrix $\boldsymbol{\Sigma}$ and b_i is the i-th element of the complex vector $\mathbf{b} = \boldsymbol{\Sigma}^{-\frac{1}{2}}\boldsymbol{\mu}$.

B. Majorization theory

For any vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^{n\times 1}$, $\mathbf{x}$ is said to be majorized by $\mathbf{y}$, denoted by $\mathbf{x} \prec \mathbf{y}$, if

$$\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]}, \quad 1 \leq k \leq n-1 \tag{3a}$$

$$\sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]} \tag{3b}$$

where $x_{[i]}$ and $y_{[i]}$ denote the i-th largest components of $\mathbf{x}$ and $\mathbf{y}$, respectively. For example, if $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{s} = (s_1, 0, \ldots, 0)$ are vectors in $\mathbb{R}^{n\times 1}$, with $s_1 = \sum_{i=1}^{n} x_i$, then the vector $\mathbf{x}$ is majorized by $\mathbf{s}$, i.e.

$$\mathbf{x} \prec \mathbf{s}. \tag{4}$$

Here, it is worthwhile to mention that the above result will be instrumental in this work.

We present now a class of real-valued functions which changes the partial order relation among the entries of the vectors previously majorized. In other words, such functions transform a majorization relationship into a numerical inequality. Specifically, a real-valued function $\mathcal{S}(\cdot)$ on $\mathbb{R}^{n\times 1}$ is said to be Schur-concave if

$$\mathcal{S}(\mathbf{x}) \geq \mathcal{S}(\mathbf{y}) \text{ for all } \mathbf{x} \prec \mathbf{y}. \tag{5}$$

An important case of Schur-concave functions is given by

$$\mathcal{S}(\mathbf{x}) \;=\; \sum_{i=1}^{n} g(x_i), \tag{6}$$

since, $g : \mathbb{R} \to \mathbb{R}$ is a concave function. For more details about majorization theory, the interested reader is referred to [10].

III. System Model

We consider a single-user flat-fading MIMO system with n_R receive antennas and n_T transmit antennas with perfect channel knowledge at the receiver but not at the transmitter. For convenience, we define $r = \min\{n_R, n_T\}$ and $t = \max\{n_R, n_T\}$, and we assume that the number of receive antennas does not exceed the number of transmit antennas. The input-output relation of the system is given by

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{y} \in \mathbb{C}^{r\times 1}$ and $\mathbf{x} \in \mathbb{C}^{t\times 1}$ are the received and transmitted signal vectors, respectively, while $\mathbf{n} \in \mathbb{C}^{r\times 1}$ is the complex additive white Gaussian noise (AWGN) vector with $\mathbf{n} \sim \mathcal{CN}_r(\mathbf{0}, N_0\mathbf{I}_r)$. Here, we assume that the transmitted signal vector satisfies the power constraint $\mathbb{E}\left\{\mathbf{x}^H\mathbf{x}\right\} \leq P_T$. In turn, $\mathbf{G} \in \mathbb{C}^{r\times t}$ is the random channel matrix whose elements g_{ij} represent the complex fading parameter between the j-th transmit and i-th receive antenna. The channel gain undergoes Rician fading with spatial correlation occurring at both ends of the MIMO link. As such, the channel matrix $\mathbf{G}$ is modeled according to the Kronecker model

$$\mathbf{G} = \mathbf{R}_{R_x}^{1/2}\mathbf{H}(\mathbf{R}_{T_x}^{1/2})^H. \tag{8}$$

We consider the entries of the matrix $\mathbf{H} = [h_{kl}]$ to be i.i.d. and expressed in terms of the in-phase and quadrature components as $h_{kl} = h_{kl}^I + jh_{kl}^Q$, where the in-phase component satisfies the condition $h_{kl}^I \sim \mathcal{N}(s, \sigma^2)$, while the quadrature component is also a Gaussian random variable with zero mean and variance σ^2, i.e. $h_{kl}^Q \sim \mathcal{N}(0, \sigma^2)$. Thus, the envelope

$$|h_{kl}| = \sqrt{\left(h_{kl}^I\right)^2 + \left(h_{kl}^Q\right)^2} \tag{9}$$

is Rician distributed.

In order to characterize a measure of the line-of-sight (LoS) component of the environment, the Rician distribution is commonly described in terms of a fading parameter $\mathcal{K}$, defined as

$$\mathcal{K} = \frac{s^2}{2\sigma^2}, \tag{10}$$

where s is referred to as the non-centrality parameter of the envelope. Specially, when $\mathcal{K} = 0$ the Rician faded envelope reduces to the Rayleigh fading channel, while $\mathcal{K} \to \infty$ corresponds to free space propagation.

A. Weichselberger MIMO Channel Model

The well-known separable transmit and receive correlation model described in eq. (8) is a particular case of the Weichselberger model, which is able to model the correlation properties at the transmitter and receiver jointly. The equivalent representation of the Kronecker model is written as follows [12]

$$\mathbf{G} = \mathbf{U}_{R_x}(\mathbf{\Omega} \odot \mathbf{H})\mathbf{U}_{T_x}^H, \tag{11}$$

where $\mathbf{U}_{R_x}$ and $\mathbf{U}_{T_x}$ are deterministic unitary matrices obtained from the eigenvalue decomposition of the transmit and receive correlation matrices $\mathbf{R}_{T_x} = \mathbf{U}_{T_x}\mathbf{\Lambda}_{T_x}\mathbf{U}_{T_x}^H$ and $\mathbf{R}_{R_x} = \mathbf{U}_{R_x}\mathbf{\Lambda}_{R_x}\mathbf{U}_{R_x}^H$, respectively, with

$$\mathbf{\Lambda}_{T_x} = \text{diag}\left(\lambda_1^{T_x}, \lambda_2^{T_x}, \ldots, \lambda_t^{T_x}\right) \tag{12a}$$

$$\mathbf{\Lambda}_{R_x} = \text{diag}\left(\lambda_1^{R_x}, \lambda_2^{R_x}, \ldots, \lambda_r^{R_x}\right). \tag{12b}$$

In turn, the matrix $\mathbf{\Omega}$ is given by

$$\mathbf{\Omega} = \boldsymbol{\lambda}_{R_x}^{1/2}\left(\boldsymbol{\lambda}_{T_x}^{1/2}\right)^T. \tag{13}$$

Here the vectors $\boldsymbol{\lambda}_{T_x}^{1/2}$ and $\boldsymbol{\lambda}_{R_x}^{1/2}$ are defined as containing the square root of the eigenvalues of $\mathbf{R}_{T_x}$ and $\mathbf{R}_{R_x}$, respectively.

B. MIMO Channel Capacity

In the sequel, we assume that channel state information is available only at the receiver side. Thus, the power along transmit antennas is equally allocated. Based on eq. (11), the ergodic capacity of the Weichselberger MIMO model previously described is given by

$$\overline{C} = \mathbb{E}\left\{\log_2\left[\det\left(\mathbf{I}_r + \frac{\rho}{t}(\mathbf{\Omega} \odot \mathbf{H})(\mathbf{\Omega} \odot \mathbf{H})^H\right)\right]\right\}, \tag{14}$$

where $\rho \triangleq \frac{P_T}{N_0}$ is the received signal-to-noise ratio (SNR). Equivalently, the ergodic capacity can be written in terms of the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_r$ of $\mathbf{\Delta} \triangleq \mathbf{\Omega} \odot \mathbf{H}$ as

$$\overline{C} = \mathbb{E}\left\{\sum_{i=1}^{r}\log_2\left(1 + \frac{\rho}{t}\lambda_i\right)\right\}, \tag{15}$$

where r is equal to the rank of $\mathbf{\Delta}$. Without loss of generality, we assume that the matrix $\mathbf{\Delta}$ has full rank and the eigenvalues λ_i are in decreasing order, i.e. $\lambda_i \geq \lambda_{i+1}$.

IV. Lower-Bound on Ergodic Capacity

In this section, we shall derive a new closed-form expression for a lower bound on the ergodic capacity of jointly correlated Rician MIMO channels using results from the distribution of quadratic forms and majorization theory. Moreover, from the chi-squared series expansion derived in our analytical lower bound, we also propose approximations to the ergodic capacity with a truncated version of these series.

A. Analytical Lower-Bound

In order to obtain the analytical expression, we establish the following intermediate derivations. First, we define in $\mathbb{R}^{r\times 1}$ the vectors: $\mathbf{d}\left(\mathbf{\Delta}\mathbf{\Delta}^H\right) = (d_1, d_2, \ldots, d_r)$, $\boldsymbol{\lambda}\left(\mathbf{\Delta}\mathbf{\Delta}^H\right) = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ and $\mathbf{\Lambda} = (\sum_{i=1}^r \lambda_i, 0, \ldots, 0)$. Here, d_i corresponds to the i-th diagonal element of $\mathbf{\Delta}\mathbf{\Delta}^H$ while λ_i represents the i-th eigenvalue.

Next we define the following real-valued $\mathcal{S}(\cdot)$ on $\mathbb{R}^{r\times 1}$

$$\mathcal{S}(\mathbf{x}) = \sum_{i=1}^{r}\log_2\left(1 + \frac{\rho}{t}x_i\right). \tag{16}$$

Since $g(x) = \log_2\left(1 + \frac{\rho}{t}x\right)$ is a concave function, then $\mathcal{S}(\cdot)$ is a Schur concave function as seen in eq. (6). Thus, we can apply this result on the majorization relationship $\boldsymbol{\lambda}\left(\mathbf{\Delta}\mathbf{\Delta}^H\right) \prec \mathbf{\Lambda}$ (see eq.(4)) to obtain the following fundamental numerical inequality for our purposes: $\mathcal{S}(\mathbf{\Lambda}) \leq \mathcal{S}\left(\boldsymbol{\lambda}\left(\mathbf{\Delta}\mathbf{\Delta}^H\right)\right)$. Consequently, the ergodic capacity presented in eq. (15) can be given by $\overline{\mathcal{C}} = \mathbb{E}\left\{\mathcal{S}\left(\boldsymbol{\lambda}\left(\mathbf{\Delta}\mathbf{\Delta}^H\right)\right)\right\}$ and lower bounded by $\mathcal{C}^{(\text{lo})} \triangleq \mathbb{E}\{\mathcal{S}(\mathbf{\Lambda})\}$, i.e. $\mathcal{C}^{(\text{lo})} = \mathbb{E}\{\mathcal{S}(\mathbf{\Lambda})\} \leq \overline{\mathcal{C}}$.

Expressing now the last operator expectation $\mathbb{E}\{\cdot\}$ in integral form we have

$$\begin{aligned}\mathcal{C}^{(\text{lo})} &= \mathbb{E}\left\{\log_2\left(1 + \frac{\rho}{t}\sum_{i=1}^{r}\lambda_i\right)\right\} \\ &= \mathbb{E}\left\{\log_2\left(1 + \frac{\rho}{t}\sum_{i=1}^{r}d_i\right)\right\} \\ &= \frac{1}{\ln 2}\int_0^\infty \ln\left(1 + \frac{\rho}{t}\varepsilon\right)p_S(\varepsilon)d\varepsilon,\end{aligned} \tag{17}$$

where $p_S(\cdot)$ is the PDF of the random variable

$$S = \sum_{i=1}^{r}\sum_{j=1}^{t}\lambda_i^{R_x}\lambda_j^{T_x}|h_{ij}|^2. \tag{18}$$

The random variable S can be conveniently represented as following the distribution of quadratic form in normal variables. Indeed, $S = \mathrm{vec}\left(\boldsymbol{\Delta}\right)^H \mathrm{vec}\left(\boldsymbol{\Delta}\right)$, where $\mathrm{vec}\left(\boldsymbol{\Delta}\right)$ is distributed as a $(t \cdot r)$-variate nonsingular normal random vector with

$$\mathrm{vec}\left(\boldsymbol{\Delta}\right) \sim \mathcal{CN}_{t \cdot r}\left(s\,\mathrm{vec}\left(\mathbf{G}\right), 2\sigma^2 \boldsymbol{\Lambda}_{T_x} \otimes \boldsymbol{\Lambda}_{R_x}\right). \tag{19}$$

Thus, based on eq. (1), the PDF of $p_S(\cdot)$ is given by

$$p_S(\varepsilon) = \sum_{l=0}^{\infty} c_l \frac{\varepsilon^{\frac{rt}{2}+l-1} \exp\left(-\frac{1}{2\beta}\varepsilon\right)}{(2\beta)^{\frac{rt}{2}+l}\,\Gamma\left(\frac{rt}{2}+l\right)}. \tag{20}$$

With the objective of accelerating the convergence of such series, we choose $\beta = 2\sigma^2\beta_*$, with $\beta_* = \min\left\{\lambda_i^{R_x}\lambda_j^{T_x}\right\}$ [19]. Additionally, the coefficients c_l can be obtained recursively by

$$c_0 = \exp\left(-\frac{rt\mathcal{K}}{2}\right) \prod_{i=1}^{r}\prod_{j=1}^{t}\left(\frac{\beta_*}{\lambda_i^{R_x}\lambda_j^{T_x}}\right)^{1/2}, \tag{21a}$$

$$c_l = \frac{1}{2l}\sum_{r=0}^{l-1} d_{k-r} c_r, \quad l \geq 1 \tag{21b}$$

with

$$\begin{aligned} d_l = &\sum_{i=1}^{r}\sum_{j=1}^{t}\left(1 - \frac{\beta_*}{\lambda_i^{R_x}\lambda_j^{T_x}}\right)^{l} + \\ &l\mathcal{K}\sum_{i=1}^{r}\sum_{j=1}^{t}\frac{\beta_*}{\lambda_i^{R_x}\lambda_j^{T_x}} \times \left(1 - \frac{\beta_*}{\lambda_i^{R_x}\lambda_j^{T_x}}\right)^{l-1}. \end{aligned} \tag{21c}$$

Here we use the Rician $\mathcal{K}$-factor as presented in eq. (10).

Now, substituting eq. (20) into eq. (17), we can rewrite the ergodic capacity lower bound $\mathcal{C}^{(\mathrm{lo})}$ as

$$\begin{aligned} \mathcal{C}^{(\mathrm{lo})} = &\frac{1}{\ln 2}\sum_{l=0}^{\infty}\frac{c_l}{(2\beta)^{\frac{rt}{2}+l}\,\Gamma\left(\frac{rt}{2}+l\right)} \times \\ &\int_0^{\infty} \ln\left(1 + \frac{\rho}{t}\varepsilon\right)\varepsilon^{\frac{rt}{2}+l-1}\exp\left(-\frac{1}{2\beta}\varepsilon\right) d\varepsilon. \end{aligned} \tag{22}$$

Finally, based on [9, Eq. 42], together with basic properties of the Meijer G-function $G_{p,q}^{m,n}\left[\cdot\,|\,\cdot\right]$ [18], the last integral can be easily calculated after some algebraic manipulations. Hence, the ergodic capacity of spatially correlated Rician MIMO channels is lower bounded by

$$\mathcal{C}^{(\mathrm{lo})} = \frac{1}{\ln 2}\sum_{l=0}^{\infty}\frac{c_l}{\Gamma\left(\frac{rt}{2}+l\right)} G_{3,2}^{1,3}\left[\frac{2\beta\rho}{t}\Big|_{1,0}^{1-\frac{rt}{2}-l,1,1}\right]. \tag{23}$$

B. Truncation Error

In practice, we consider a truncated version of the infinite series in eq. (20) as

$$p_S(\varepsilon|L) = \sum_{l=0}^{L} c_l \frac{\varepsilon^{\frac{rt}{2}+l-1}\exp\left(-\frac{1}{2\beta}\varepsilon\right)}{(2\beta)^{\frac{rt}{2}+l}\,\Gamma\left(\frac{rt}{2}+l\right)}. \tag{24}$$

Therefore, we obtain the following approximation to the lower bound on the ergodic capacity in terms of the truncation factor L

$$\mathcal{C}^{(\mathrm{lo})}(L) = \frac{1}{\ln 2}\sum_{l=0}^{L}\frac{c_l}{\Gamma\left(\frac{rt}{2}+l\right)} G_{3,2}^{1,3}\left[\frac{2\beta\rho}{t}\Big|_{1,0}^{1-\frac{rt}{2}-l,1,1}\right]. \tag{25}$$

In order to specify an accurate truncation factor, we define the following truncation error

$$e(L) = \int_0^{\infty} p_S(\varepsilon) d\varepsilon - \int_0^{\infty} p_S(\varepsilon|L) d\varepsilon, \tag{26}$$

which represents an approximation error of the area under the PDF $p_S(\cdot)$. Note that the last expression can be rewritten as

$$\begin{aligned} e(L) &= 1 - \sum_{l=0}^{L}\frac{c_l}{(2\beta)^{\frac{rt}{2}+l}\,\Gamma\left(\frac{rt}{2}+l\right)} \\ &\quad\times \int_0^{\infty}\varepsilon^{\frac{rt}{2}+l-1}\exp\left(-\frac{1}{2\beta}\varepsilon\right) d\varepsilon. \end{aligned} \tag{27}$$

Now, introducing the integration result [18, Eq. (8.312-2)], the approximation error $e(\cdot)$ is given by $e(L) = 1 - \sum_{l=0}^{L} c_l$. The numerical details in the truncation factor L and the respective approximation error $e(L)$ are described in the next section. Another truncation version of the infinite series (20) can be found in [20].

V. Numerical Results

In this section, we validate the analytical results presented in the previous section via Monte Carlo simulations. Specifically, we evaluate the lower bound on the ergodic capacity in different system configurations. We adopt in all systems the same Rician $\mathcal{K}$-factor, with $\mathcal{K} = 1$, and we assume the exponential correlation model [6] with transmit and receive correlation coefficients equal to $\delta_T = 0.3$ and $\delta_R = 0.1$, respectively.

In Fig. 1, the approximation error from the area under the chi-squared series PDF in eq. (20) versus the truncation factor is depicted. We observe that that with small number of terms in each series, we obtain tight approximations to the lower bound on the ergodic capacity. Specifically, for a single-input single-output (SISO) system, the number of series terms required is $L = 3$, $L = 10$ for 1×4 and 2×2 channels, while in 3×3 MIMO systems, the truncation factor required is $L = 32$.

In Fig. 2, the empirical ergodic capacity obtained through Monte Carlo simulations is compared with the analytical lower bound approximation in the low SNR regime. From the truncation factors investigated in the last paragraph, we obtain an accurate closed-form approximation to the ergodic capacity, mainly for SISO and MISO (multiple-input single-output) systems, where the respective curves overlapping.

Finally, for a fixed SNR value, Fig. 3 shows the behavior of the ergodic capacity as the Rician $\mathcal{K}$-factor grows large. Here, we set $\rho = -5$ dB. For all values of $\mathcal{K}$, it can be verified that the gap between the theoretical and the simulation results is small, justifying the validity of the derived closed-form expressions as well as the truncation factor method.

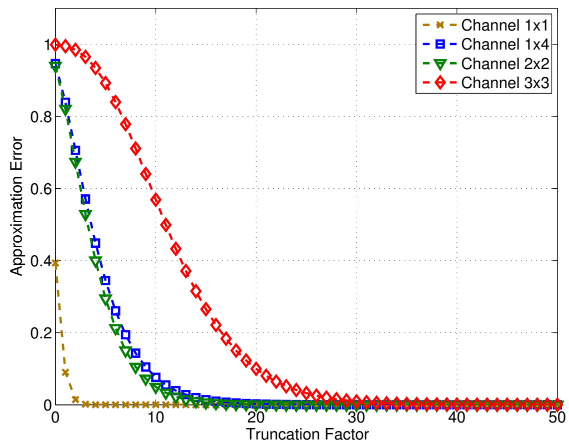

Fig. 1. Approximation error of the area under the PDF $p_S(\cdot)$ vs. truncation factor.

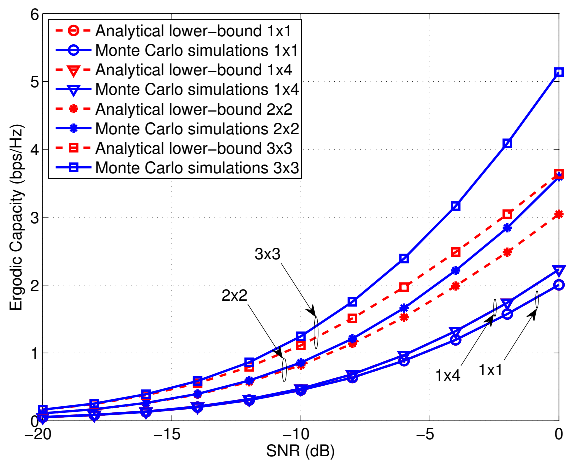

Fig. 2. Comparison of the empirical ergodic capacity and analytical lower bounds for correlated Rician fading channels.

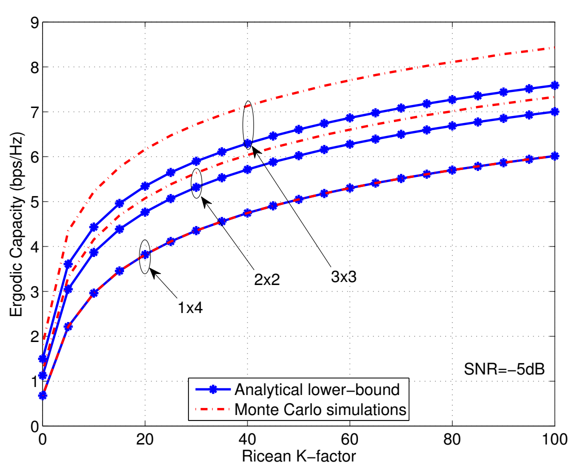

Fig. 3. Ergodic capacity for different Rician $\mathcal{K}$-factors and $\rho = -5$ dB.

VI. Conclusions

In this paper, we derived an analytical lower bound expression on the ergodic capacity of MIMO system under spatially correlated Rician fading channels and perfect CSI only at the receiver side. Using majorization theory, our analytical result can be expressed in a quadratic form in normal random variables. Furthermore, based on the analytical lower bound expression, we proposed tight approximations to the lower bound on the ergodic capacity as a function of the truncation factor of the chi-squared series PDF, associating an effective approximation error for such series. Finally, our analytical results have been verified through Monte Carlo simulations. Future work will focus on the investigation of the ergodic capacity for Nakagami-m fading channels under the Weichselberger correlation model.

References

[1] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Europ. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov. 1999.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, pp. 311–335, 1998.

[3] A. Lozano, A. M. Tulino, and S. Verdu, "Mutual Information and Eigenvalue Distribution of MIMO Ricean Channels," in *Proc. Int. Symp. Information Theory and Its Applications (ISITA'04)*, 2004.

[4] S. Jin and X. Gao, "Tight Upper Bound on the Ergodic Capacity of the Rician Fading MIMO Channels," in *IEEE Wireless Communications and Networking Conference (WCNC)*, Mar. 2005, vol. 1, pp. 402 – 407.

[5] M. R. McKay and I. B. Collings, "General Capacity Bounds for Spatially Correlated Rician MIMO Channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3121–3145, Sept. 2005.

[6] S. Jin, X. Gao, and X. You, "On the Ergodic Capacity of Rank-1 Ricean-Fading MIMO Channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 2, pp. 502–517, Feb. 2007.

[7] M. Matthaiou, Y. Kopsinis, D.I. Laurenson, and A.M. Sayeed, "Ergodic Capacity Upper Bound for Dual MIMO Ricean Systems: Simplified Derivation and Asymptotic Tightness," *IEEE Trans. Commun.*, vol. 57, no. 12, pp. 3589 – 3596, Dec. 2009.

[8] G. Taricco and E. Riegler, "On the Ergodic Capacity of Correlated Rician Fading MIMO Channels With Interference," *IEEE Trans. Inform. Theory*, vol. 57, no. 7, pp. 4123 – 4137, July 2011.

[9] C. Zhong, K-K. Wong, and S. Jin, "Capacity Bounds for MIMO Nakagami-m Fading Channels," *IEEE Trans. Signal Processing*, vol. 57, no. 9, pp. 3613–3623, Sept. 2009.

[10] A. W. Marshall, I. Olkin, and B. C. Arnold, *Inequalities: Theory of Majorization and Its Applications*, Springer, 2011.

[11] M. Kang and M.-S. Alouini, "Capacity of MIMO Rician Channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 1, pp. 112–122, Jan 2006.

[12] W. Weichselberger, M. Herdin, H. Ozcelik, and E. Bonek, "A Stochastic MIMO Channel Model With Joint Correlation of Both Link Ends," *IEEE Trans. Wireless Commun.*, vol. 5, no. 1, pp. 90 – 100, Jan. 2006.

[13] N. Costa and S. Haykin, *Multiple-Input Multiple-Output Channel Models: Theory and Practice*, Wiley, 2010.

[14] D. P. Palomar and Y. Jiang, "MIMO Transceiver Design via Majorization Theory," *Found. Trends Commun. Inf. Theory*, vol. 3, no. 4-5, pp. 331–551, 2006.

[15] E. Jorswieck and H. Boche, "Majorization and Matrix-Monotone Functions in Wireless Communications," *Found. Trends Commun. Inf. Theory*, vol. 3, no. 6, pp. 553–701, 2007.

[16] J. Wang and D. P. Palomar, *Majorization Theory and Applications*, chapter 16, CRC Press, Nov. 2011.

[17] A. M. Mathai and S. B. Provost, *Quadratic Forms in Random Variables: Theory and Applications*, M. Dekker, 1992.

[18] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, 7 edition, 2007.

[19] S. B. Provost and E. M. Rudiuk, "The Exact Distribution of Indefinite Quadratic Forms in Noncentral Normal Vectors," *Annals of the Institute of Statistical Mathematics*, vol. 48, pp. 381–394, 1996.

[20] G. Ropokis, A. Rontogiannis, and P. Mathiopoulos, "Quadratic Forms in Normal RVs: Theory and Applications to OSTBC over Hoyt Fading Channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5009 –5019, Dec. 2008.