Predicting the Shear Strength of Unfilled Rock Joints with the First-Order Takagi-Sugeno Fuzzy Approach

Y.M.P. Matos, S.A. Dantas Neto, G.A. Barreto

Abstract. As a result of a number of studies, some analytical models have been developed to predict the shear behavior of unfilled rock joints, but they all present a purely deterministic nature because their input variables are defined without considering the uncertainties inherent in the formation processes of the rock masses and related discontinuities. This work aims to present a model for predict the shear strength of unfilled rock joints by incorporating uncertainties in the variables that govern its shear behavior with a First-Order Takagi-Sugeno fuzzy controller. The model is developed based on the results of 44 direct shear tests carried out on different types of joints. The model input variables are the normal boundary stiffness and initial normal stress acting on the joint, its roughness (expressed by the JRC value), the uniaxial compressive strength, the basic friction angle of the intact rock and the shear displacement imposed to the joint. The results show that the predicted shear strength of unfilled rock joints obtained by the fuzzy model fits satisfactorily the experimental data and allows the shear behavior of the discontinuities to be defined. A practical application of the model in a stability analysis of a rock mass is also presented.

Keywords: Fuzzy, shear strength, Takagi-Sugeno, unfilled rock joints.

1. Introduction

One of the main difficulties with analyzing and designing geotechnical structures in rock is predicting the behavior of the rock masses correctly because it depends on the shear strength of the existing discontinuities. The shear behavior of unfilled discontinuities depends on their boundary conditions, *i.e.*, constant normal loading (CNL) or constant normal stiffness (CNS) conditions, their roughness, and on the properties of the intact rock (Patton, 1966; Barton, 1973; Benmokrane & Ballivy, 1989; Skinas *et al.*, 1990; Papaliangas *et al.*, 1993; Indraratna *et al.*, 1998, 1999, 2005, 2008, 2010a, 2010b, 2015; Indraratna & Haque, 2000, among others).

Several analytical models have been used to predict the shear strength of unfilled discontinuities (Patton, 1966; Barton, 1973; Barton & Choubey, 1977; among others). However, these models can only predict the peak shear strength of discontinuities that has been developed from shear tests conducted under CNL conditions, which many times do not represent the behavior of the discontinuity due the confinement imposed by the surrounding rock mass leading it to a CNS condition. Barton & Bandis (1990) presented the JRC-JCS method which allows the definition of the complete shear stress-displacement behavior of unfilled rock joints by considering the concept of the mobilized JRC (roughness), providing a more realistic prediction for the nonlinear shear behavior of rock joints. Barton (2013, 2016) and Prassetyo *et al.* (2017) warn for the need to consider the nonlinearity for the shear behavior of rock joints. According to these authors, the dilation which occurs during the shearing process leads to a degradation of the joint asperities represented by the variation of JRC mobilized resulting in a nonlinearity in the shear behavior of the unfilled rock discontinuities.

Results of a number of direct shear strength tests indicate that normal boundary stiffness affects the shear behavior of unfilled rock joints as it increases their shear strength and reduces dilation in the shearing process (Skinas *et al.*, 1990; Papaliangas *et al.*, 1993; Indraratna *et al.*, 1998, 1999, 2005, 2008, 2010a, 2010b, 2015). Indraratna & Haque (2000) presented an analytical model where the shear strength of unfilled rock joints is estimated as a function of the boundary conditions (CNL or CNS) of the discontinuity; it is expressed by the initial normal stress and normal boundary stiffness of the joint such as its roughness which is expressed by the asperity inclination angle and the basic friction angle. The model of Indraratna & Haque (2000) is one of the most advanced models used to predict the shear strength of unfilled rock joints because unlike some traditional models, the shear stress and shear displacement in CNL and CNS conditions can be predicted. However, this model is somewhat laborious to use because the variation of rock joint dilation with shear displacement must be

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known, and since they are obtained through large-scale direct shear, they are not always available under the same boundary conditions acting on the discontinuity.

Dantas Neto *et al.* (2017) proposed a model to predict the shear behavior of unfilled rock joints developed using artificial neural networks. Since the model proposed by Indraratna & Haque (2000), this neural model enables the shear behavior of discontinuities to be completely defined without the need for any special laboratory test. The results obtained using this model fit the experimental data of a wide variety of rock types better than the model by Indraratna & Haque (2000).

Despite these mentioned models being able to predict the shear behavior of unfilled rock joints quite well, they still do not consider any existing uncertainties in the input parameters along a certain discontinuity because there is no consideration on how the rock mass and discontinuities were formed. In this scenario of uncertainties, the Fuzzy Sets Theory (Zadeh, 1965) is a useful tool to model complex real systems with input parameters involving uncertainty, such as those observed in geotechnical works designed and built in rock masses.

The use of Fuzzy Sets Theory in a logical context to solve practical problems is known as Fuzzy Logic; Fuzzy Logic enables phenomena to be modelled by mathematical equations and also allows heuristics to be adopted to explain real problems. The heuristic method determines the solution of a given problem according to previous specialist experience or frequently used inference rules. These rules can be applied by expert systems that according to Grima (2000), aim to provide solutions for complex engineering problems without resorting to mathematical models. These expert systems are known as fuzzy controllers that use past experiences, and theoretical knowledge of the investigated phenomenon to determine the fuzzy inference rules which will provide solutions to the problem.

Several studies related to the application of fuzzy controllers in Rock Mechanics have been developed, such as Grima & Babuska (1999), Gokceoglu (2002), Kayabasi *et al.* (2003), Nefeslioglu *et al.* (2003), Sonmez *et al.* (2003), Gokceoglu & Zorlu (2004), Sonmez *et al.* (2004), Daftaribesheli *et al.* (2011), Monjezi & Rezaei (2011), Akgun *et al.* (2012), Asadi (2016), and Sari (2016). However, since none of them can study the behavior of unfilled rock discontinuities during shearing, they provided the motivation for developing this present work.

This paper will therefore present the results of predicting the shear strength in unfilled rock joints as a function of the main variables that influence this phenomenon such as normal boundary stiffness, the initial normal stress acting on the discontinuity, joint roughness represented by the joint roughness coefficient (JRC), the intact rock properties such as the compressive strength and basic friction angle, as well as the shear displacement imposed onto the discontinuity. Thus, the results of 44 direct shear tests from different joints and boundary conditions were used. This model was developed using a First-Order Takagi-Sugeno fuzzy controller. The results from predicting the shear strength of unfilled rock joints by the actual fuzzy model fit the experimental results used in the model development quite well, while also considering how the model responded to variability or uncertainty of the input variable of the studied phenomenon. A practical application of the model in a slope stability analysis of a rock mass is also presented.

2. Literature Review

2.1 Fuzzy logic

The Fuzzy Sets Theory conceived by Zadeh (1965) is a more general case of the classical Theory of Sets since it allows to consider the vague aspect of information, while admitting that a certain variable can assume a set of possible values rather than a single and unique one. Fuzzy Logic therefore uses Fuzzy Sets Theory in a logical context to solve practical problems.

Unlike classical (bivalent) logic, with Fuzzy Logic the existing sets do not have precise boundaries so the degree of membership (μ) of an element measures the possibility that that element belongs to a given set (see Fig. 1), *i.e.*, this degree of membership of a variable can vary between zero and one, depending on how much that one belongs to the analyzed set data. That is a fundamental difference between fuzzy and crisp sets, once in crisp sets the values of some element are unique and they do not consider the uncertainties possibly involved on that variable definition.

Fuzzy Logic is very useful when the number of data available is not enough to characterize the uncertainty involved in the studied phenomenon using the Theory of Probability. By making an analogy of it, Ganoulis (1994) states that fuzzy numbers are equivalent to random variables and that membership functions correspond to proba-

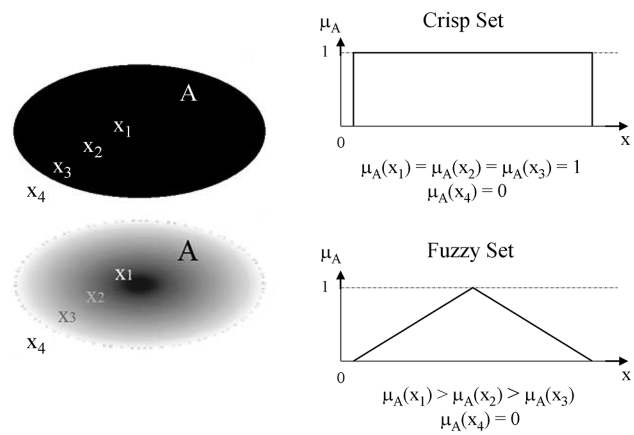

Figure 1 - Difference between classic and fuzzy logic (Jalalifar *et al.*, 2011).

bility density functions. However, the basic arithmetical rules of fuzzy sets are quite different from the Theory of Probability.

The membership function establishes the relationship between the values of a variable and their respective degrees of membership with regard to a given set, and since there are several types of membership functions, the most common are triangular, trapezoidal, Gaussian, and sigmoidal. The definition of membership functions of any variable is based on the knowledge of a specialist or on the analysis of a known series of observed values of the regarded variable. The delimitation of these functions is fundamental to the use of fuzzy controllers.

2.2 Fuzzy controller

A fuzzy controller is a system that contains a set of "IF ... THEN" inference rules that define the controlling actions based on different ranges of values that the governing variables of the problem can assume. Systems constructed in this way are even more interesting when the response of the existing mathematical model is subject to their input variables uncertainties.

Unlike conventional controllers where control is described analytically through a deterministic mathematical model, fuzzy controllers use logical rules to control a process where the modelled phenomenon can involve the human experience and intuition. These systems use fuzzy sets to describe the input and output variables, so instead of an exact value for the variables, possible sets of values could be adopted. It is important to mention that the fuzzy controllers allow to express the human experience and intuition, and therefore the uncertainty of a certain value, by considering the fuzzy set as a linguistic variable to which values as "low", "high", "very high" can be assigned.

Figure 2 presents a fuzzy controller which relates the uniaxial compressive strength of intact rock (σ_c) and the JRC with the shear strength (τ_h) of an unfilled rock joint. This example illustrates that the shear strength is not defined by unique values for the uniaxial compressive strength and JRC values but considering the uncertainties expressed by the range of values of each fuzzy linguistic membership function. The Boolean operator is called the antecedent part of the inference rule and its function is to combine the influence of the input variables on the fuzzy output, which is the consequent one.

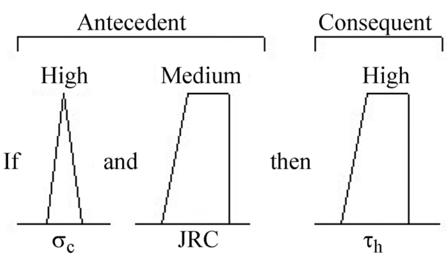

Figure 2 - Example of an inference rule using fuzzy numbers.

Simões & Shaw (2007) state that the basic structure of a fuzzy controller represents the transformation that occurs from the real domain to the fuzzy domain, known as the fuzzification step, where a set of fuzzy inference rules is used for decision-making that will provide the fuzzy outputs. At the end of the process, these outputs, which are currently fuzzy numbers, must be transformed into real numbers by a defuzzification process.

2.3 Takagi-Sugeno fuzzy model

Different fuzzy controllers may differ with regard to how the operators use them in their implementation and how they represent the fuzzy outputs of each specific rule. One of the most common types of fuzzy controllers is the interpolation model presented by Takagi & Sugeno (1983); it is known as the Takagi-Sugeno controller. The Takagi-Sugeno controller establishes that only the antecedent of the rules (premise part) is formed by fuzzy variables, and the output of each rule (consequent part) is defined as a function of these input variables. The operation of this controller is illustrated in Fig. 3.

The first step taken by a Takagi-Sugeno controller is the fuzzification process in which the membership functions for each input variable (x and y) are established, and the i rules of inference are defined based on the judgment of specialists. In the activation of each R_i rule of inference, a Boolean operator AND or OR is defined to establish how the input variables x and y are combined to define the response z of the model. When a connector AND is used, at each R_i rule the multiplication of degrees of membership of the input variables (μ_{xi} and μ_{yi}) is performed and a weight W_i is then obtained. Otherwise, when a connector "OR" is used, the highest value of the degree of membership of the input variables is adopted. Analyzing the rule R_1 presented in Fig. 3 and adopting real values for the two inputs x and y, it was observed that x belongs to the fuzzy set A_1 with degree of membership μ_{x1}, and y belongs to the fuzzy set B_1 with degrees of membership μ_{y1}. Therefore, using the connector AND to combine the variables x and y, the weight W_1 can be determined by multiplying μ_{x1} and μ_{y1}.

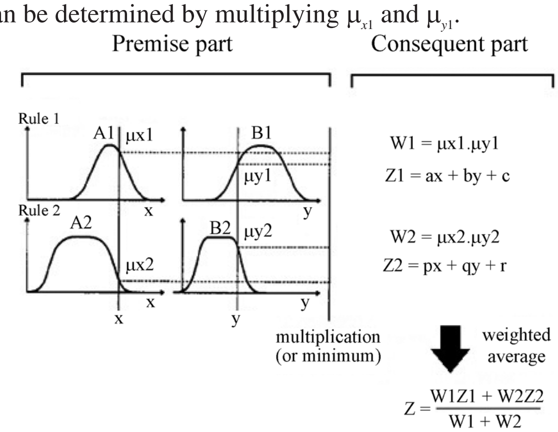

Figure 3 - Functioning of a Takagi-Sugeno fuzzy controller (Jang, 1993).

The implication step consists of defining a linear function that relates the consequents of rules z_i to the input variables x and y. This definition does not force the use of a specific implication function and may even be a constant value, but linear equations are normally adopted, as shown in Fig. 3, to present the functions for z_1 e z_2. These linear equations are obtained by interpolating inside the dataset representing the experience on the modelled phenomenon the output variable as a function of the input variables in order to obtain the parameters a, b, c, p, q and r presented in Fig. 3.

Finally, the output z is the weighted average of the consequent of each rule, evaluated by the respective membership values that result from processing the antecedent of the rule (W_1 and W_2).

According to MathWorks (2006), Takagi-Sugeno controllers are computationally efficient and better suited for mathematically analyzing phenomena because adjustments to customize the membership functions and implication functions can be used to improve the fuzzy system.

Regarding the use of Takagi-Sugeno fuzzy controllers in Rock Mechanics, Grima & Babuska (1999) developed a fuzzy system to predict the uniaxial compressive strength of rock samples. The authors found that the Takagi-Sugeno fuzzy model could potentially model complex, non-linear and multivariable geological engineering systems. Grima & Babuska (1999) highlight the importance of intelligent computational systems that can be applied to Rock Mechanics because vague and imprecise information can be used about the materials and data whose physical meaning is not obvious.

3. Fuzzy Model Development

The proposed fuzzy model uses logical implications to describe the relationships between control variables and the physical phenomenon analyzed, *i.e.*, the shear strength in discontinuities of rock masses. This model was built based on a dataset of 44 direct shear tests presented by Benmokrane & Ballivy (1989), Skinas *et al.* (1990), Papaliangas *et al.* (1993), Indraratna & Haque (2000), and Indraratna *et al.* (2010a), performed in different types of discontinuities (saw-tooth, tension-model, field-model and field-natural) and distinct boundary conditions.

The model was developed using 673 examples as the dataset, while considering as input variables the main factors governing the shear behavior of unfilled rock joints: the normal boundary stiffness (k_n), the initial normal stress (σ_{n0}) acting on the discontinuity, the JRC, the uniaxial compressive strength of the intact rock (σ_c), the basic friction angle (ϕ_b), and the shear displacement (δ_h) having as its response the shear strength of the discontinuity (τ_h).

The model was implemented using MATLAB and consists of a Takagi-Sugeno fuzzy controller (Takagi & Sugeno, 1983), where the linear (first-order) equations of the input variables are implied, and the shear strength is the weighted average of the consequent of each rule that varies according to a combination of values assumed by the inputs as previously explained.

To develop this model, the membership function of each input variable had to be defined, *i.e.*, the type of function and its parameters. From the types of functions available, the authors used trapezoidal functions at the edges of the intervals of each variable and triangular functions to fill in the remaining values not comprised by the trapezoidal functions.

The parameters of the membership functions were defined by considering some values provided in the literature (when available), the results of direct shear tests, and the judgment of specialists. The membership functions of JRC, σ_c and ϕ_b were defined by considering the suggestions made by Barton & Choubey (1977), Bieniawski (1984) and Barton (1973), respectively. Due to the lack of data in literature regarding other variables, the parameters of the membership functions of k_n, σ_{n0} and δ_h are based on the results of direct shear tests only, and on the previous experience of specialists. The membership functions for each input variable presented in Figs. 4 to 9 cover the entire range of variables in the available dataset.

After defining all the membership functions for each variable, were also defined 57 fuzzy inference rules by analyzing how the input variables affected the shear strength values available in the experimental dataset used to develop the fuzzy model presented in this paper. An example of one of these rules is: if k_n is VERY HIGH and σ_{n0} is MEDIUM

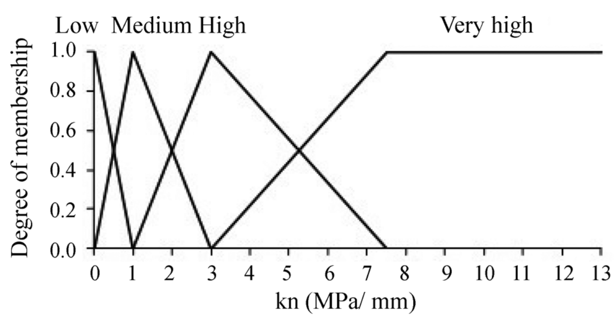

Figure 4 - Membership functions for the normal boundary stiffness (k_n) variable.

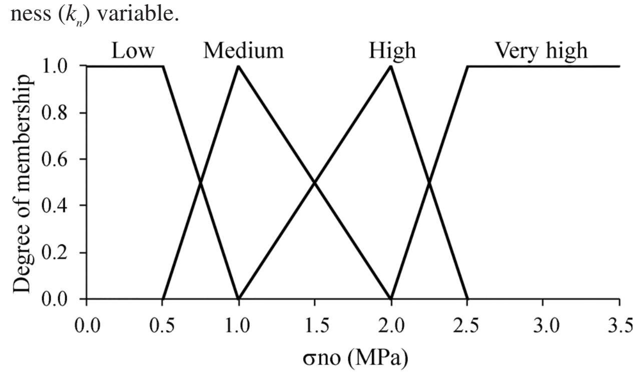

Figure 5 - Membership functions for the initial normal stress (σ_{n0}) variable.

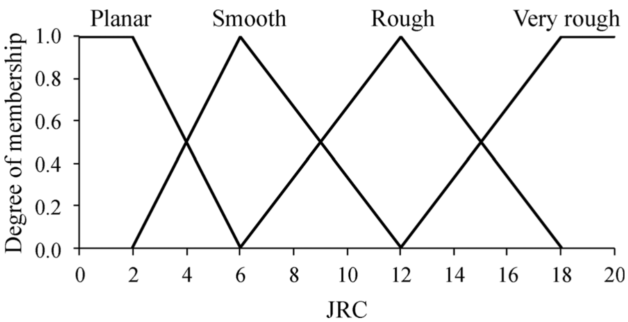

Figure 6 - Membership functions for the JRC variable.

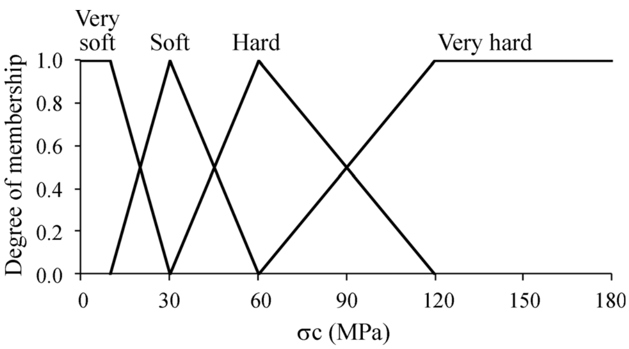

Figure 7 - Membership functions for the uniaxial compressive strength (σ_c) variable.

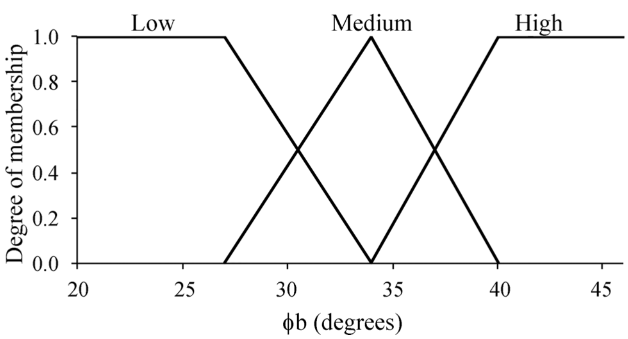

Figure 8 - Membership functions for the basic friction angle (ϕ_b) variable.

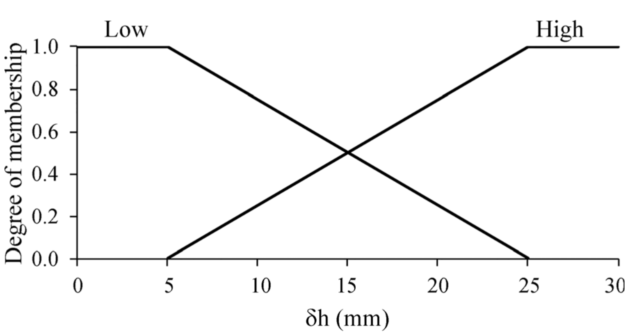

Figure 9 - Membership functions for the horizontal displacement (δ_h) variable.

and JRC is VERY ROUGH and σ_c is HARD and ϕ_b is MEDIUM and δ_h is LOW then τ_h is HIGH.

The coefficients of the implication functions were obtained by multiple linear regressions of the results of direct shear tests. The prediction of the shear strength of an unfilled rock joint by using the Takagi-Sugeno controller is a result of the defuzzification procedure of a membership function obtained by combining all the established inference rules.

4. Results and Discussion

Figures 10 to 12 present comparisons between the experimental data and values predicted by the Takagi-Sugeno model to evaluate whether the model can represent the influence of the governing parameters on the shear behavior of an unfilled rock joint with values for the uniaxial compressive strength and basic friction angle of 12 MPa and 37.5°, respectively.

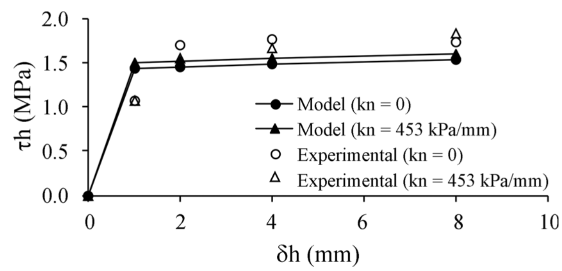

Figure 10 - Influence of normal boundary stiffness on the shear strength of unfilled rock joints.

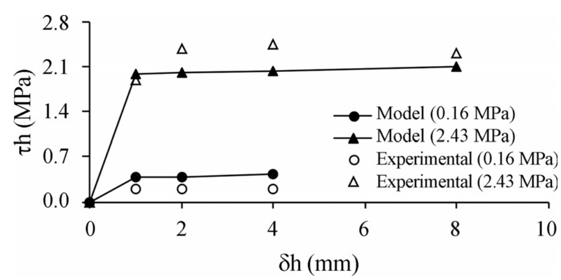

Figure 11 - Influence of the initial normal stress on the shear strength of unfilled rock joints.

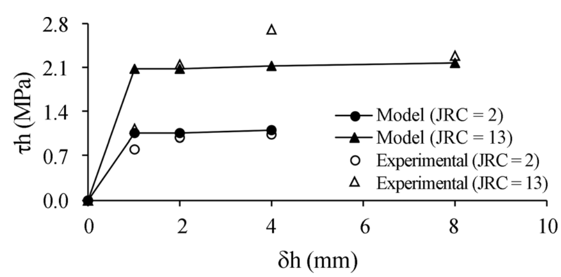

Figure 12 - Influence of the JRC values on the shear strength of unfilled rock joints.

The results in Figs. 10 to 12 show that the First-Order Takagi-Sugeno fuzzy controller fits the experimental data very well. Moreover, the model also represents the influence of the input variable on the shear behavior of the unfilled rock joints considered, as shown by an increase in the shear strength as the normal boundary stiffness, the roughness of the joint, and the initial normal stress also increased.

Figure 13 shows the correlation between experimental and predicted values of τ_h obtained for the fuzzy model. The fuzzy model has a high value of 0.85 for the coefficient of determination, which means it is a useful tool for predicting the shear behavior of unfilled rock joints and present as an advantage in relation to the existing models the fact of considering the uncertainties of their input variables.

5. Practical Application of the Fuzzy Model in a Rock Slope Stability Analysis

The initial application of the fuzzy model was made by assuming the general configuration of a rock slope subjected to a surcharge *F*, with height *H*, inclination α_s and whose potential slip surface is defined by an unfilled discontinuity with angle α_j, as shown in Fig. 14. The presence of the force *T* applied by the bolts defines the constant normal stiffness condition for the discontinuity.

The weight of the rock wedge (*W*) delimited by the rock discontinuity considering its unit weight (γ) can be determined according to Eq. 1.

$$W = 0.5\gamma H^2(\cot\alpha_j - \cot\alpha_s) \quad (1)$$

The normal stress (σ_n) acting on the discontinuity can be determined as a function of the increase in the normal force (*N*) which acts on the discontinuity, due to the CNS

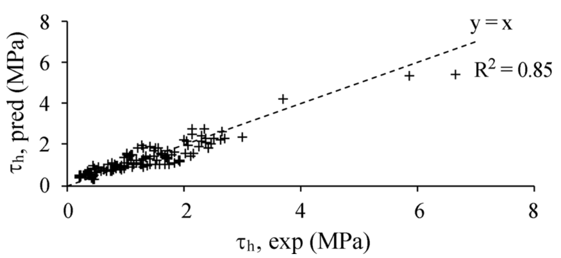

Figure 13 - Comparison between the experimental data with the shear strength predicted by the First-Order Takagi-Sugeno fuzzy model.

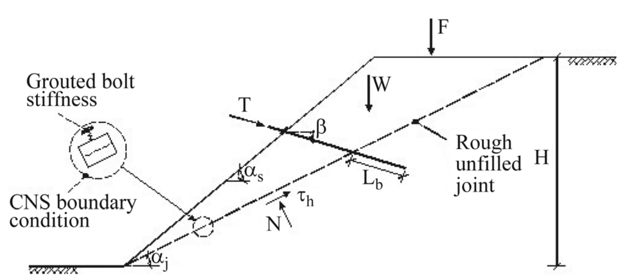

Figure 14 - Stability analysis of rock slope (Indraratna *et al.*, 2010a).

boundary condition imposed by the bolts. This increase in the normal force depends on the number of bolts inserted in the slope (*n*), and their horizontal spacing (s_h) and inclination (β), as well as the initial normal force (N_0) acting on the discontinuity.

$$\sigma_n = \frac{N\sin\alpha_j}{H} \quad (2)$$

$$N = N_0 + \frac{n}{s_h}T\sin(\alpha_j + \beta) \quad (3)$$

If there are no bolts, the normal force is constant and calculated according to Eq. 4.

$$N_0 = (W + F)\cos\alpha_j \quad (4)$$

The value of *T* can be calculated by using Eq. 5, which considers the characteristics of the bolts and the discontinuity dilation (δ_v), and whose measurement is obtained by laboratory tests or by using the Dantas Neto *et al.* (2017) neural model.

$$T = \frac{E_b A_b}{L_b}\frac{\delta_v}{\sin(\alpha_j + \beta)} \quad (5)$$

where E_b is the modulus of elasticity of the bolts; A_b is the cross-sectional area of the bolts; and L_b is the length of the ground anchored section of the bolts.

The normal boundary stiffness acting on the discontinuity can be defined by the elastic properties of the bolts and the geometry of the discontinuity (Eq. 6).

$$k_n = \frac{nE_b A_b \sin\alpha_j}{HL_b s_h \sin(\alpha_j + \beta)} \quad (6)$$

Finally, the factor of safety (FS) is obtained by the relation between the resisting forces acting on the wedge and the forces that cause its failure.

$$FS = \frac{\tau_h\left(\frac{H}{\sin\alpha_j}\right) + \left(\frac{n}{s_h}\right)T\cos(\alpha_j + \beta)}{(W + F)\sin\alpha_j} \quad (7)$$

The shear strength (τ_h) can be determined by laboratory tests or estimated by any available calculation methodology. In this paper, the analytical model of Indraratna & Haque (2000) and the neural model proposed by Dantas Neto *et al.* (2017) are used to predict the shear behavior of the unfilled rock joint in the rock slope stability analysis presented. A comparison of the results obtained by applying the First-Order Takagi-Sugeno fuzzy model is also presented.

Based on results of CNL and CNS direct shear tests, Indraratna & Haque (2000) proposed that the shear strength of an unfilled rock joint, presented in Eq. 8, can be defined as a function of the characteristics of the discontinuity, the normal boundary stiffness, the initial normal stress acting on the joint, and the shear displacement.

$$\tau_h = \left(\sigma_{n0} + \frac{k_n \delta_v(\delta_h)}{A_j}\right)\left(\frac{\tan(\varphi_b) + \tan(i_0)}{1 - \tan(\varphi_b)\tan(i_h)}\right) \quad (8)$$

where A_j is the surface area of the discontinuity; $\delta_v(\delta_h)$ is the dilation during shearing; ϕ_b is the basic friction angle; i_0 is the initial asperity angle of the discontinuity; and i_h is the dilation angle at the horizontal displacement δ_h.

To use the analytical model proposed by Indraratna & Haque (2000), the dilation during shearing must be measured in large-scale direct shear tests. Once their values are known, the variation of dilation with the shear displacement to be inserted in Eq. 8 can be represented using a Fourier series, as presented in Eq. 9.

$$\delta_v(\delta_h) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n \delta_h}{T_F}\right) + b_n \sin\left(\frac{2\pi n \delta_h}{T_F}\right)\right] \quad (9)$$

where a_0, a_n e b_n are the coefficients of the Fourier series; n is the number of harmonics; and T_F is the period of the Fourier series.

The terms a_0, a_n, b_n e T_F are determined by interpolating the dilation *vs.* shear displacement curve, as obtained by direct shear tests.

Indraratna *et al.* (2005, 2010a, 2010b) and Oliveira & Indraratna (2010) have shown that the model proposed by Indraratna & Haque (2000) can predict the shear behavior of unfilled rock discontinuities, but they also highlight the difficulties involved in obtaining its parameters because the results of laboratory tests are required and may not be easily available. Note also that the experimental data can only represent the field behavior if the boundary conditions imposed in laboratory tests are the same as those observed in the field, a fact that is not always possible, due to the limitations of the test equipment and the sampling process (Dantas Neto *et al.*, 2017).

In this practical application, the parameters representing the rock mass are: $H = 30.5$ m, $\alpha_s = 80°$, and $\alpha_j = 50°$, $\gamma = 27.5$ kN/m^3 and $F = 25{,}000$ kN. The bolts are 63.5 mm in diameter by $L_b = 1.0$ m long, are inclined at $\beta = 15°$ to the horizontal. The horizontal spacing of $s_h = 1.4$ m is assumed. Assuming $E_b = 200$ GPa and $n = 30$ bolts leads the discontinuity to an initial normal stress and boundary normal stiffness of 540 kPa and 380 kPa/mm, respectively.

The Indraratna & Haque (2000) model is used by applying the results of a direct shear test in a saw-tooth unfilled rock joint with $\sigma_c = 12$ MPa, $\phi_b = 37.5°$ and JRC = 12 conducted under $k_n = 453$ kPa/mm and $\sigma_{n0} = 0.56$ MPa to obtain the coefficients of the Fourier series presented in Table 1. A saw-tooth unfilled rock joint was adopted to facilitate the calculations for applying the analytical model of Indraratna & Haque (2000). Note that the results of the direct shear test were obtained under boundary conditions

Table 1 - Fourier coefficients used in the stability analysis.

	Fourier coefficients							
σ_{n0} (MPa)	T_F	a_0	a_1	a_2	a_3	b_1	b_2	b_3
0.56	35.67	2.16	-1.14	0.04	0.00	-0.08	0.15	0.00

that differed from those imposed on the unfilled rock joint considered in the rock slope stability analysis. However, the fuzzy model proposed in this paper allows the shear strength of rock joints for the actual conditions of the rock slope to be evaluated, *i.e.*, normal boundary stiffness of 380 kPa/mm, and initial normal stress of 540 kPa.

This is one of the main advantages of this fuzzy model because it can predict the shear strength of unfilled discontinuities when carrying out laboratory tests to reproduce field boundary conditions that become difficult or unfeasible. Likewise, the neuronal model of Dantas Neto *et al.* (2017) also allows for a direct application, and it does not require laboratory tests.

Figure 15 shows the variation of the factor of safety with the shear displacement of the unfilled rock joint obtained by applying the First-Order Takagi-Sugeno fuzzy model and the results of shear strength obtained with models by Indraratna & Haque (2000) and Dantas Neto *et al.* (2017). The use of shear stresses provided by laboratory tests under boundary conditions, other than those imposed onto the analyzed rock slope, may have overestimated the factor of safety in most of the tangential displacements considered.

Other than what has been portrayed in the models proposed by Indraratna & Haque (2000) and Dantas Neto *et al.* (2017), the displacements could not initiate the degradation of the joint asperities, a phenomenon that leads to a loss of shear strength during shearing. This is possibly due to the previously established fact that the model provides predictions close to the residual strength of the joints.

Furthermore, to apply the fuzzy model to practical problems of rock slopes under CNS conditions, the dilation of the discontinuity must be determined in order to estimate the force applied by the bolts T, which defines the normal boundary stiffness of the discontinuity. In their analysis the authors used the dilations obtained by the model of Indraratna & Haque (2000).

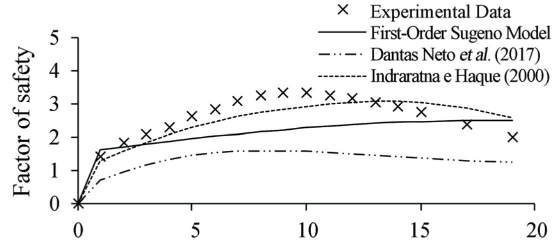

Figure 15 - Factors of safety *vs.* shear displacement for the analyzed rock slope.

6. Conclusions

The proposed fuzzy model is a Takagi-Sugeno controller with linear (first-order) implication functions used in the prediction of the shear strength of unfilled discontinuities; it was developed using a robust data set with 673 examples and was defined based on previous studies that identified the main factors that govern the shear behavior of unfilled joints. The proposed fuzzy model fits the experimental data very well, presenting a coefficient of correlation of 0.85. It presents as advantage in relation to the existing models the fact of considering the uncertainties of their input variables in its response, *i.e.*, in the shear strength of unfilled rock discontinuities, leading to more rational and safer analyses and design or structures in rock masses.

By analyzing the errors, the proposed Takagi-Sugeno model can explain the shear behavior of unfilled rock joints because it only needs some information about the characteristics of the discontinuities, the intact rock, and the boundary conditions imposed onto them.

In the rock slope stability problem presented, this limitation was confirmed, and the fuzzy model did not portray the degradation of joint asperities that can occur during the wedge movement which reduces the factor of safety. However, the model was very useful for analyzing the rock slope stability and for predicting the residual strength of unfilled discontinuities, especially where laboratory tests would be difficult or unfeasible, and the joint is subject to Constant Normal Loading (CNL) conditions.

Finally, it is important to mention that the main limitations of this fuzzy model are the domains of its input variables, which are defined during its construction, *i.e.*, they do not allow the insertion of values that are outside their pre-defined range of occurrence as input data. In the present work, the Takagi-Sugeno controller was conditioned to the domain of the measurements of direct shear tests for most of its parameters, but they can be adjusted as new data sets become available.

A suggestion for future studies would be to develop a Takagi-Sugeno fuzzy controller to predict the dilation of unfilled discontinuities of rock masses in order to apply the proposed model to practical problems of rock slopes under CNS conditions. Another interesting alternative would be to use neuro-fuzzy techniques to fully predict the shear behavior of unfilled rock joints.

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