Study of constitutive models of DIANA FEA for long-term analysis of pre-stressed beams with bonded tendons

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Abstract. The work presented here was motivated by the existence of several constitutive models capable of numerically representing the time effect on prestressed beams. The main goal of this study is to analyze different models, comparing their influence on the long-term behavior in prestressed concrete beams. The study compares the behavior of prefabricated concrete beams with bonded post-tensioning using different constitutive models for concrete and steel tendons. The simulation was carried out using DIANA FEA. Some of the models adopted are from design codes, and others from models and theories available in literature. The study proposes a parametric study among the different models as well as the effect of the input parameters on the overall behavior of the structure. The nonlinear behavior of cracked concrete and the steel yield behavior in addition to the creep behavior of the concrete were considered. The results were compared with experimental results of prestressed beams tested for creep analysis for a long period of time. The results pointed out to the importance of choosing the appropriate constitutive model in simulations of prestressed elements.

Keywords: Constitutive models, Prestressing, DIANA.

1 Introduction

Modeling the material adopting adequate constitutive relationships is an essential step to obtain satisfactory analyzes results using the Finite Element Method (FEM) [1]. In Ultimate Limit State analyzes (ULS), for example, the choice of an adequate constitutive model will allow the correct study of concrete behavior in regions close to the material's failure and stress-strain relationship.

Many studies have been carried out in order to discuss how to model the long-term behavior of prestressed beams, considering that the creep of concrete and steel relaxation considerably affect the displacements of these beams over time. In these works, the constitutive models are always an important issue. A formulation for nonlinear analysis via FEM of beams with unbonded prestressing [2] was proposed, bringing in its implementation more than two models to represent concrete and steel, emphasizing each model advantages and disadvantages.

Another numerical model with several constitutive relationships to analyze the bending of structural elements with bonded and unbonded tendons was implemented [3]. The author adopted a low relaxation prestressing steel curve [4]. Creep and shrinkage of concrete, and relaxation of prestressed steel were represented by Maxwell Chain model [5]. Model validation was performed by comparing numerical and experimental results.

Considering the existence of different constitutive models, parametric studies to analyze the influence of some parameters in the analysis are important [3, 6]. An analysis of the beam ductility with unbonded prestressing was proposed [6], evaluating the influence of type of load, span/depth ratio, passive and active reinforcement ratios, compressive strength and passive reinforcement ratio. The study focused on the parameters that could influence the stress increase in the prestressing tendons.

A study on bonded prestressed concrete structures focusing on long-term behavior was presented [7]), keeping material properties fixed in the modeling, evaluating the influence of the element type to represent the long-term behavior of the material in Service Limit State (SLS). In this study, the concrete was modeled as a linear viscoelastic material ([8]), in which the creep and shrinkage effects are independent of each other. Shrinkage and creep are divided into two parts: the basic part and the curing period part. It was considered a bilinear curve for passive and active reinforcement.

A model to predict the behavior of partially prestressed beams over time [9], considering creep, shrinkage, cracking, relaxation and temperature variations, with a main focus on creep, was proposed. The model proposes 3 different numerical methods for considering creep, and all of them present good results when compared with experimental data [10]. In this study, the FIB model [8] was used to consider concrete creep and shrinkage.

Having presented the existence of several constitutive models to numerically represent prestressed beams, this work motivation is to analyze different models and compare their influence for creep behavior. To carry out this study, the *software* DIANA [11], which is a FEM program, was used. In this *software*, there are different constitutive models implemented. It also offers several models for concrete cracking. Two types of models are available: Fixed Distributed Crack Model (FDCM) and Rotational Distributed Crack Model (RDCM) , which differ in the post-cracking considerations.

Some researches have already been carried out similar studies using DIANA. A comparison of the constitutive models [12] available at *software* DIANA was performed using the FDCM to represent the effect of cracking in the material. A non-linear numerical analysis of prestressed concrete beams behavior was carried out applying a [13] softening curve to reproduce the tensile behavior of concrete. As for the behavior of compressed concrete, a stress-strain curve [14] was used for the DIANA software. In the steel modeling, it was assumed the linear-elastic behavior. Unlike the previous work, the RDCM was applied to represent the effect of material cracking [15]. Both used the 1993 Vecchio and Collins model to reduce the strength [16] to represent concrete under compression. Finally, for the prestressing tendons, it was used the elastic-plastic model combined with the Von-Mises yield criterion. Vecchio and Collins' 1993 model [16] presents an update in relation to another model presented by the same authors in 1986 [17]. What changes from one to another is the formula for considering strength reduction due to lateral cracks. Both are implemented in DIANA.

In view of the review herein presented, this work proposes to carry out a study of models for the analysis of beams with bonded prestressing. The beam simulations will be performed in DIANA software. To validate it, a comparison with experimental results will be performed. Experimental data for creep was found [18], [10], [19], among them, the one that presented more data to enable simulation in DIANA was Espion's 1993, which presents experimental results, material properties and explicit loading and prestressing methods, allowing it to be herein used for validation.

2 Constitutive models

Material properties were the focus of this study. For steel, only two models were adopted, the elastic and model of Menegotto and Pinto [20]. For concrete, different models were evaluated to study their influence on the beam behavior over time.

2.1 Menegotto and Pinto

This plasticity model is widely known, its main advantage is its simplicity combined with its efficiency. It is used in many works [2], [6], [21] for representing well the dynamic steel behavior in steel structures and in reinforcement [22]. This curve depends on dimensionless coefficients (ε^* and σ^*) that better represent experimental tests [11]. They are expressed in the strain-stress coordinates of the last reversal point ($\varepsilon_{\mathrm{r}}^n$,$\sigma_{\mathrm{r}}^n$) and in the updated yield point ($\varepsilon_{\mathrm{y}}^{n+1}$,$\sigma_{\mathrm{y}}^{n+1}$):

$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r^n}{\varepsilon_y^{n+1} - \varepsilon_r^n} \tag{1}$$

$$\sigma^* = \frac{\sigma - \sigma_r^n}{\sigma_y^{n+1} - \sigma_r^n} \tag{2}$$

The basic expression of the Menegotto-Pinto model is:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{\frac{1}{R}}} \tag{3}$$

whebe b is the hardening ratio and R is a curvature parameter that depends on material parameters the initial curvature R_0 and on the maximum plastic excursion developed during on half-cycle $\xi_{\mathrm{p}}^{\max}$:

$$R = R^0 - \frac{a_1 \xi_p^{max}}{a_2 + \xi_p^{max}} \tag{4}$$

To account for isotropic hardening A stress shift σ_{sh} in the linear yield asymptote was proposed to account for isotropic hardening, which depends on material properties (presented on Table 1 and on the maximum plastic strain ε^t_{max}:

$$\frac{\sigma_{sh}}{\sigma_{y0}} = a_3\,(\frac{\varepsilon^t_{max}}{\varepsilon_{y0}} - a_4) \tag{5}$$

Table 1. Coefficients of the Menegotto and Pinto curve [23]

R_0	20.00
a_1	18.50
a_2	0.15
a_3	0.01
a_4	7.00

2.2 FIB Model

This model is from a standard [8] and it is considered to be the simplest in terms of *software* implementation. The simplicity is due to the model being implemented in DIANA already, and few parameters need to be changed. When considering this model, the user has the option of considering or not some concrete behaviors, such as shrinkage, creep, Total Deformation Model, concrete hardening when it is still young, among others. In this first model, only creep and shrinkage were considered.

With basic data, such as f_{ck}, information about the start time of loading, curing period, temperature and humidity, this model makes considerations of the FIB code to calculate others parameters referring to the constitutive model, which are necessary to perform the analysis. Its curve is shown in Fig. 1. In this model, the average compression stress (f_{cm}) is given by:

$$f_{cm} = f_{ck} + \Delta f \tag{6}$$

where Δf = 8 MPa. The initial tangent modulus of elasticity (E_{ci}) is given by:

$$E_{ci} = E_{co}\alpha_E \left(\frac{f_{cm}}{f_{cmo}}\right)^{1/3} \tag{7}$$

where E_{co} = 2.5 x 10^4 MPa, f_{cmo} = 10 MPa and α_E depends on the aggregate type and it is equal 1 for quartz aggregates (which was considered). For materials with non-linear behavior, the initial plastic deformations must be taken into account, the modulus of elasticity must be reduced by a reducing factor α_i:

$$\alpha_i = 0.8 + 0.2\frac{f_{cm}}{88} \leq 1.0 \tag{8}$$

In this model, the coefficient of *Poisson* (ν) is considered fixed and equal to 0.2. The average tensile stress for concrete with strength less than 50 MPa is calculated by:

$$f_{ctm} = 0.3 = fck^{2/3} \tag{9}$$

FIB model- crack. Adding the cracking to the FIB model, DIANA itself makes the Total Deformations considerations automatically. In this case, there are no options for Fixed or Rotational Distributed Cracks because the program uses cracking equations from the FIB code itself for cracking.

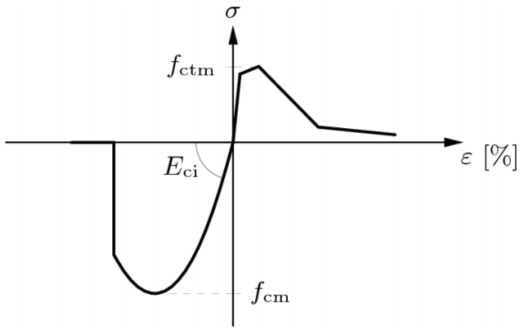

Figure 1. Concrete curve CEB-FIB Model 2010

2.3 Concrete DIANA

To implement this constitutive model, the user has to choose the "concrete and masonry" option in Diana, which does not use norm equations and data like the previous model. The properties adopted in this model were based on those of a study on prestressed beams [12]. Creep, shrinkage, cracking, reduced tensile and compression strength were considered. In cracking, it is possible to consider the inclination of the cracks to be fixed or rotational. The *software* manual [11] presents the complete formulation of the two models.

Total strain model – fixed crack. In the Total Deformations model, it was adopted that the cracks orientations were fixed. In addition, it is necessary to enter other material properties in the program. In this model, the orthotropic main axes remain fixed in the post-cracking phase, and parameters for shear retention are needed. The compression model was adopted as a parabolic curve, which needs the compression fracture energy as an input, which can be calculated according to guidelines[24].

As this is a resistance reduction model, it is necessary to choose according to which theory the DIANA will carry out this reduction. In the reference on which this model was based, the theory of Vecchio and Collins from 1993 [16] was used. For simplification, the 1986 model [17] was adopted, as it requires less input data, while the 1993 one requires as input the lower limit of the reduction curve, which is not found in the articles used as reference for this work. For the shear behavior, a constant reduction function equal to 0.2 was considered, this value is recommended for analyzes in more than one direction. For shrinkage, it was considered that it was a structure with time dependent behavior and for fluency the Kelvin chain model was considered.

Total strain model – rotating crack. In RDCM, the axes rotate on the same axis as the main deformations. In addition, the rotational model allows less stress locking and it handles better with localized cracks [1]. This model was implemented with exactly the same considerations as the previous one, however, in the crack orientation option, the rotational model was adopted.

3 ACI model

This model implementation [25] is similar to that of the FIB model, requiring little input from users. Its curve is shown in Fig. 2, having an important difference from the other curve [8], which are its straight sections. One of the differences is that the FIB model allows the user to implement creep behavior, choosing Kelvin or Maxwell chains, while in the ACI model the program automatically defines which theory to use and calculates important long-term data following its own code specifications. The compressive strength over time is given by:

$$f_c(t) = \frac{t}{\alpha + \beta y} f_{28} \tag{10}$$

where α and β are constants that depend on the cure and the type of cement. Tensile strength over time is given by:

$$f_t(t) = 0.0069\sqrt{w\, f_c(t)} \tag{11}$$

where w is the concrete density. And the basic relationship for considering fluency is given by:

$$J(t,t_0) = \frac{1}{E_c(t_0)}(1+\phi(t,t_0)) \tag{12}$$

where t_0 is the loading age and $\phi(t,t_0)$ is the creep coefficient.

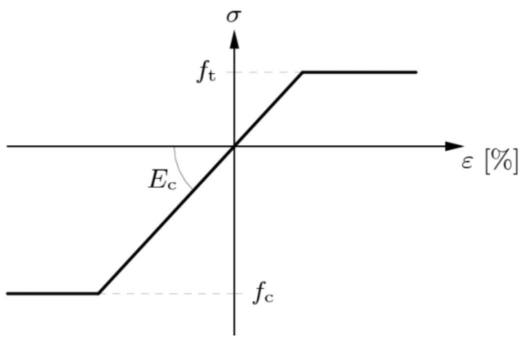

Figure 2. Concrete curve ACI209R-92

4 Results and discussion

The *benchmark* chosen was the same used to validate a numerical model that evaluates the behavior over time of reinforced and prestressed concrete beams [9, 10]. It is a simply supported beam (8m span), partially prestressed, with 50% of the loads balanced by the prestressing. Such beam has already been used to validate several programs and implementations, as it has creep results that are not easily obtained, considering that it is a time-dependent behavior.

4.1 Geometry and properties

Fig. 3 displays the geometry and characteristics of the cross section. The passive reinforcement is composed of 8 bars of 8 mm diameter and 5 of 18 mm in the lower part of the section. In total there are 5 prestressing tendons (PT), arranged as shown in Fig. 3. Table 2 presents the coordinates of the points for the prestressing tendons.

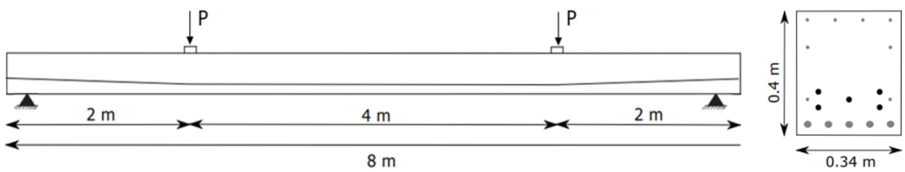

Figure 3. Geometry and cross-section characteristics

Table 2. Tendons coordinates

Tendons	Coordinate 1	Coordinate 2	Coordinate 3
PT1	[0,0.07, 0.2518]	[2,0.07, 0.140]	[4,0.07, 0.140]
PT2	[0,0.27, 0.2518]	[2,0.27, 0.140]	[4,0.27, 0.140]
PT3	[0,0.17, 0.2268]	[2,0.17, 0.115]	[4,0.17, 0.115]
PT4	[0,0.07, 0.2018]	[2,0.07, 0.090]	[4,0.07, 0.090]
PT5	[0,0.27, 0.2018]	[2,0.27, 0.090]	[4,0.27, 0.090]

The structure was tested at controlled temperature and humidity of 20ºC and 60%, respectively. To carry out the simulations, all the data mentioned above were used, as well as those present 4, it presents the concrete properties applied. Steel Young's Modulus (N/m^2) and mass density (Kg/m^3) are from 200E9 and 8000, respectively. Such properties were taken from the article that details the experimental testing of the beams [10], with the exception of the tensile fracture energy (G_f), which was calculated according to the Atena manual [26].

Table 3. Concrete Properties

Young's modulus (N/m^2)	30.75E9
Tensile strength (N/m^2)	3.00E6
Compressive strength (N/m^2)	33.9E6
Bulk density (Kg/m^3)	2350
Tensile fracture energy (N/m)	136.98
Compressive fracture energy (N/m)	13698

4.2 Loading and simulation

The loading, both prestressing and transverse loading, are not applied on the first day, for constructive reasons and to respect the curing period (1 day). The loading stage is presented in Table 4. To simulate the models in DIANA *software*, symmetry was used modeling only half of the beam and adding the symmetry condition in the center. To simulate the support and application of point load, two blocks were created, one simulating a support, and the other simulating a plate for applying the point load. In both, steel properties are used in order to represent the support stiffness. The beam was discretized into 480 CHX60 elements (Brick-20 solid quadratic elements - 20 nodes).

Table 4. Loading

Days	Prestress (kN/cable)	Load (kN)
$0\leq t<14$	0	0
$14\leq t<28$	122.8	0
$28\leq t<84$	122.8	16.5
$84\leq t<1642$	122.8	63.75

It was used the following models on the simulations:

1. FIB – Adopting its standard recommendations [8], considering creep and shrinkage.
2. FIB - *crack* – Adding cracking considerations to the previous model.
3. FF – Concrete model chosen by the user, adopting Fixed Distributed Crack Model.
4. FR – similar to the previous one, but adopting Rotating Distributed Crack Model.
5. ACI – Adopting ACI recommendations [25].

4.3 Results

As previously mentioned, this work focused on the long-term behavior. Thus, the displacement x time graph with all the results using the 5 different constitutive models for concrete is presented in Fig. 4. The differences in relation to the last displacement over time are presented in Table 5. Overall, most of the models represented the immediate behavior of the beam well, but they were not so good with respect to displacement over time, all of them showing results against safety.

The model that presented the best result regarding the last displacement was the FIB-*crack*. However, even this model having better represented this value over time, it did not represent well the immediate displacements, considering that it presented larger displacements than the experimental ones in the second stage of load application. However, the differences at the time of load application were in favor of safety.

The FIB and ACI models represented very well the immediate behavior in the moments of load application, not being so good for long duration, especially the ACI, which presented an error of 26.5% compared to the experimental one. As the only additional consideration of the FIB - *crack* in relation to the FIB was the consideration of the non-linearity of the concrete, it can be concluded that assuming the cracked properties of the section in the FIB model allowed better results over time. In the ACI model, the user has little option to enter data, so it can be considered that the model does not represent well the beam.

The worst results, both immediate and long-term, were presented by the FF and FR models, both presenting errors above 30%. One hypothesis to explain this behavior is that many parameters are needed in these models. Some were not found in the article that detailed the experiment, making it necessary to use formulas and suggestions from different authors, thus losing the analysis quality.

The considerations made here emphasize the importance of having a diversity of constitutive models in formulations [2, 3, 15]. Such models greatly influence the behavior of structures, and depending on each case and the known properties of the material, a certain model may be more suitable or not.

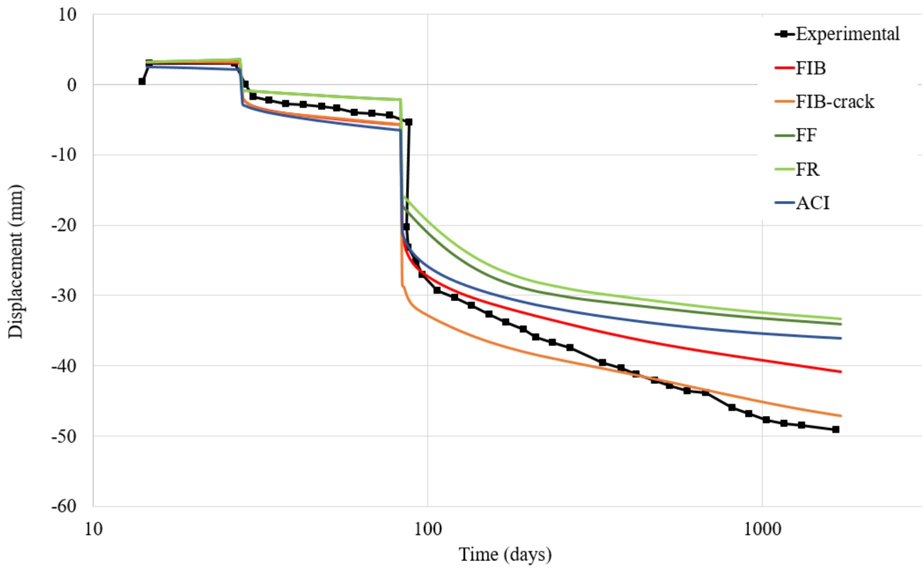

Figure 4. Displacement x Time with different models

Table 5. Error(%) in relation to the last experimental displacement

Constitutive models	Displacement	Error(%)
FIB	-40.90	16.8
FIB-crack	-47.18	4.0
FF	-34.15	30.5
FR	-33.32	32.2
ACI	-36.11	26.5

For steel, the only constitutive models of steel adopted were the linear elastic and the one by Menegotto and Pinto [20]. No difference in long-term behavior was observed. This is consistent, considering that with the application of prestressing, smaller displacements occur in the beam, preventing plastic deformations of the reinforcement. In this sense, without plastic deformations, the consideration of plasticity models really makes no difference. Therefore, to analyze the effect of the constitutive models for steel, it would be ideal to use partially prestressed beams with a lower percentage of balanced loads.

4.4 Conclusion

A beam with bonded prestressing was analyzed with different constitutive models implemented in DIANA. It was possible to observe that most of the models well represented the beam behavior in short term, but in long term, bigger differences were identified. Among the constitutive concrete models analyzed, the ones that presented results with the greatest differences were the fixed and rotational distribution models. This can be explained by the absence of certain important data for these constitutive models to represent the material well. It was also seen that, for the steel models, it was not possible to notice differences. This is due to the fact that the steel of the analyzed beam did not reach the yield point, therefore, adopting an elastic or plastic model makes no difference. It can be concluded the importance of the availability of several constitutive models in the analysis, since depending on the data availability, some models can become unfeasible, since they can present results very different from the real ones and against security.

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