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ONE-DIMENSIONAL CONSTITUTIVE MODELS FOR NONLINEAR STATIC ANALYSIS OF REINFORCED CONCRETE STRUCTURES

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Abstract. This work aims at the nonlinear static analysis of reinforced concrete structures using constitutive models based on the Plasticity Theory and the Continuum Damage Mechanics. The elastic damage model proposed by Mazars and the elastic-plastic damage model proposed by Lee and Fenves are studied and one-dimensional versions of these models are implemented in a finite element program. The plane frame elements used in this work are formulated with a co-rotacional Lagrangean kinematic description. The integration of stress resultants and tangent constitutive matrix is carried out by the Fiber Method. The formulation and computational implementation at the material and element level are validated using numerical and experimental results available in the literature. A reinforced concrete column under the action of a monotonic load is analyzed and the results show that the use of the implemented one-dimensional models for concrete structures is an effective methodology to capture some important effects.

Keywords: Continuum Damage Mechanics, Plasticity Theory, Numerical implementation, Finite Element Analysis

1 Introduction

Reinforced concrete structures has a highly complex mechanical behavior. This is easily observed in experimental tests. For numerical simulations, it is common the use plane and solid elements with two or three-dimensional constitutive models (Parente Jr et al. [1]). Therefore, for engineering purposes, the nonlinear analysis of concrete structures using two or three-dimensional constitutive models is impracticable. This is due to the high computational cost.

For nonlinear structural problems, it is interesting to work with structural elements and one-dimensional models. This can be carried out, for example, by the use of frame elements and the integration of stress resultants in the cross section element (Spacone et al. [2]). In this work, the geometric nonlinearity is considered by using a co-rotational formulation (Parente Jr et al. [1]]). The material nonlinearity is considered using nonlinear constitutive models based on the Plasticity Theory and the Continuum Damage Mecanichs.

The Continuum Damage Mechanics is a branch of Continuum Mechanics that describes the behavior of degradation of materials at a macroscopic level. The first works developed using Continuum Damage Mechanics were those of Kachanov [3]. The author introduced the concept of *continuity*, that is a complementary variable for the after called damage variable D. According to Mazars et al. [4], the damage variable D describes the micro-cracking state of the material. The definition of this variable leads to the definition of another variable, the effective stress $\overline{\sigma}$, which is related to real tension as

$$\sigma = (1 - D)\,\overline{\sigma} \tag{1}$$

From Eq. (effective-stress), several constitutive models are developed in order to include the effects of stiffness degradation. Here, two of these models will be studied and one-dimensional versions of each one will be developed.

2 Constitutive models for concrete

The constitutive models studied here are the elastic damage μ-Model proposed by Mazars et al. [4] and elastic-plastic damage model proposed by Lee and Fenves [5].

2.1 μ-Model

The 3D formulation proposed by Mazars et al. [4] is based on the elastic behavior of concrete. The 3D stress σ–strain ϵ relationship is given by

$$\sigma = (1 - D)\,\mathbf{C}_0 : \epsilon \tag{2}$$

where $\mathbf{C}_0$ is the initial elastic-stiffness tensor, representing the undamaged elastic moduli. The damage evolution is driven by the associated loading surface

$$f_{\aleph}(\epsilon_{eq,\aleph}, D_{\aleph}) = \epsilon_{eq,\aleph} - Y_{\aleph} \leq 0 \tag{3}$$

where $\aleph \in \{t, c\}$ represent the tensile (for $\aleph = t$) and compressive (for $\aleph = c$) behavior; $\epsilon_{eq,t}$ and $\epsilon_{eq,c}$ are the equivalent strain for cracking and crushing of concrete, respectively. The equivalent strain concept is introduced by Mazars et al. [4] as a representative value for the strain state. They are defined as

$$\epsilon_{eq,t} = \frac{I_\epsilon}{2(1-2\nu)} + \frac{\sqrt{J_\epsilon}}{2(1+\nu)} \quad \text{and} \quad \epsilon_{eq,c} = \frac{I_\epsilon}{5(1-2\nu)} + \frac{6\sqrt{J_\epsilon}}{5(1+\nu)} \tag{4}$$

where I_ϵ is the first invariant of strain tensor, $J_\epsilon = 1/2\left[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2\right]$ and ν is the Poisson ratio. The internal variables Y_t and Y_c are the maximum values reached on the loading path are difined as $Y_{\aleph} = \mathrm{Sup}[\epsilon_{0\aleph}, \max(\epsilon_{eq,\aleph})]$.

The evolution of each variable $Y_\aleph$ is initiated from a respective damage threshold, ϵ_{0t} for tension and ϵ_{0c} for compression. In order to obtain the equivalent stress, we determine an equivalent damage variable D correlated with the variable Y by the damage evolution law

$$D = 1 - \frac{Y_0(1-A)}{Y} - \frac{A}{\exp\left[B(Y - Y_0)\right]} \tag{5}$$

where $Y_0 = r\epsilon_{0t} + (1-r)\epsilon_{0c}$, $Y = rY_t + (1-r)Y_c$ and A and B are variables that determine the shape of the effective damage evolution laws. The parameter r is defined by Lee and Fenves [1] as a triaxiality factor that is equal to 1 for pure tension and 0 for pure compression.

For the one-dimensional version, the variables previously defined can be rewritten as $Y_0 = \epsilon_{0t}$, $Y = \epsilon_{eq,t}$, $A = A_t$ and $B = B_t$ for tension and $Y_0 = \epsilon_{0c}$, $Y = \epsilon_{eq,c}$, $A = A_c$ and $B = B_c$ for compression. Thus, the evolution of the damage variable D can be disassociated into two independent parts:

$$D_\aleph = 1 - \frac{\epsilon_{0\aleph}(1-A_\aleph)}{Y_\aleph} - \frac{A_\aleph}{\exp\left[B_\aleph(Y_\aleph - \epsilon_{0\aleph})\right]} \tag{6}$$

where $Y_\aleph = \mathrm{Sup}[\epsilon_{0\aleph}, \max|\epsilon|]$

The stress–strain relationship can be expressed by:

$$\sigma = (1 - D_\aleph)E\epsilon \tag{7}$$

The computational implementation of the one-dimensional μ-Model is performed using an explicit incremental procedure. This is possible because for each known point of strain, knowing the history of the previous strains, it is possible to determine a conjugated stress.

2.2 Lee and Fenves Model

The formulation of elastic-plastic damage model proposed by Lee and Fenves [5] is based on the Plasticity Theory. In this model, the authors add another internal variable set κ (Lubliner et al. [6]) in addition to classic plastic strain ϵ_p. In this formulation, the 3D stress σ–strain ϵ relationship is given by

$$\sigma = (1 - D)\,\mathbf{C}_0 : (\epsilon - \epsilon_p) \tag{8}$$

The evolution of the new internal variable κ is given by

$$\dot{\kappa} = \dot{\lambda}\,\mathbf{H}\left(\overline{\sigma}, \kappa\right) \tag{9}$$

where $\mathbf{H}$ is defined considering plastic dissipation and λ is a plastic consistency parameter. To incorporate the distinct behavior of the damage in the tension and the compression, the internal variable κ is defined as a vector containing two scalar values, $\kappa = \{\kappa_t, \kappa_c\}$

Loading and unloading conditions are given in terms of the yield function

$$F\left(\overline{\sigma}, \kappa\right) = \frac{1}{1-\alpha}\left[\alpha\overline{I}_1 + \sqrt{3\overline{J}_2} + \beta\left(\kappa\right)\left\langle\hat{\overline{\sigma}}_{max}\right\rangle\right] - c_c\left(\kappa\right) \tag{10}$$

where I_1 is the first invariant of stress tensor; I_2 is the second invariant of deviatoric stress and $\left\langle\hat{\overline{\sigma}}_{max}\right\rangle$ is the positive part of the algebraically maximum principal stress; c_c represents the cohesion in terms of the internal variable κ; α and β are dimensionless parameters based on the initial yield tensile and compressive stress in uniaxial (f_{0c} and f_{0c}) and biaxial (f_{0b}) loads

$$\alpha = \frac{f_{0b}/f_{0c} - 1}{2(f_{0c}/f_{0c}) - 1} \quad \text{and} \quad \beta\left(\kappa\right) = \frac{c_c(\kappa_c)}{c_t(\kappa_t)}(1-\alpha) - (1+\alpha) \tag{11}$$

The yield function F depends on the uniaxial strength functions f_t and f_c. These functions represent the strength of the material under uniaxial loads and they are defined in terms of the internal variables κ_t and κ_c as

$$f_\aleph = \left[1 - D_\aleph\left(\kappa_\aleph\right)\right]\overline{f}_\aleph\left(\kappa_\aleph\right) \tag{12}$$

where $\overline{f}_{\aleph}$ represents the uniaxial strength functions in terms of the effective stress. These functions can be found in Lubliner et al. [6].

For the one-dimensional version, the yield function can be rewritten as

$$F(\overline{\sigma}, \kappa) = \frac{1}{1-\alpha}\left[\alpha\,\overline{\sigma} + |\overline{\sigma}| + \beta(\kappa_t, \kappa_c)\langle\overline{\sigma}\rangle\right] - c_c(\kappa_c) \tag{13}$$

The stress–strain relationship and the damage evolution law can be expressed by, respectively,

$$\sigma = (1-D)\,E\,(\epsilon - \epsilon_p) \tag{14}$$

$$D = 1 - (1-D_c)\,(1 - s(\overline{\sigma})\,D_t) \tag{15}$$

where s is a function that tries to model the crack opening/closing behavior of the concrete (Lee and Fenves [1]).

The computational implementation of the one-dimensional Lee and Fenves model is performed by an incremental implicit Euler method. Thus, a three-step return mapping algorithm can be defined: (1) elastic predictor, (2) plastic corrector and (3) damage corrector. This return mapping algorithm is detailed by Matias [7].

2.3 Strain localization and mesh sensitivity

A common phenomenon in applications involving brittle or softening materials is known as strain localization. The strain localization also occurs in numerical simulations. In this case, it is possible to explain mathematically how the phenomenon occurs with the use of finite elements for the mesh discretization of the structure. Bažant and Planas [8] explain that this behavior is associated to the discretization of the mesh. Figure 1 shows an example of a bar with length L discretized in N elements.

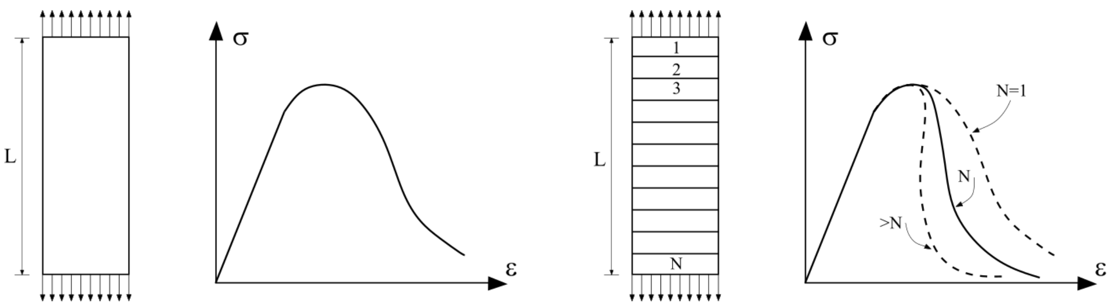

Figure 1. Mesh sensitivity of a single bar with *N* elements

Because of numerical instability, one of the elements of the mesh reaches firstly the maximum tensile strength. While the other elements remain in the elastic regime, the strain is concentrated, or localized, only in this brittle element. The dependence of the response of the problem to the finite element mesh adopted in the analysis is known in the literature as lack of mesh objectivity or mesh sensitivity (Bažant and Planas [8]).

Some methodologies have been developed to avoid mesh sensitivity once the strain localization can occur in softening problems. In this work, the crack band theory developed by Bažant and Planas [8] will be adopted.

3 Numerical applications

Some applications at material and structural level are analyzed with the constitutive models implemented in this work. The constitutive model used for reinforced bars is the classic perfect elastic-plastic model based on the Plasticity Theory.

3.1 Cyclic uniaxial behavior

In this application, two cyclic tensile-compressive loading tests are applied in a single element and the results are compared with numerical results obtained by Fléjou [9] and Lee and Fenves [5]. The first loading is performed for the μ-Model using the following material properties and parameters: $E = 37.3$ GPa, $\epsilon_{0c} = 2.0e-4$, $\epsilon_{0t} = 8.2e-5$, $A_c = 1.77$, $B_c = 2011.64$, $A_t = 0.70$ and $B_t = 12189.24$. The second loading is performed for the Lee and Fenves Model using the following material properties and parameters: $E = 30.1$ GPa, $f_{0c} = 19$ MPa, $f_{0t} = 3.3$ MPa, $a_c = 4.07$, $b_c = 1.00$, $d_c = 1.08$, $a_t = 0.57$, $b_t = 1.00$, $d_t = 1.20$ and $s_0 = 0.00$.

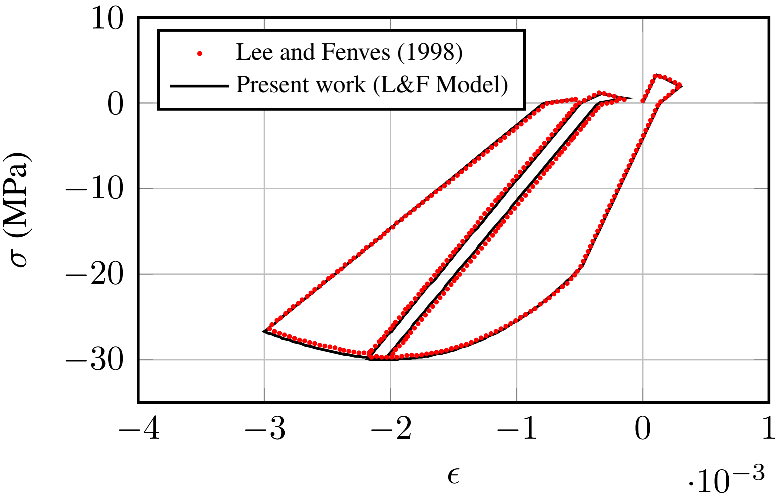

Figure 2. Stress-strain response for cyclic load for the μ-Model model.

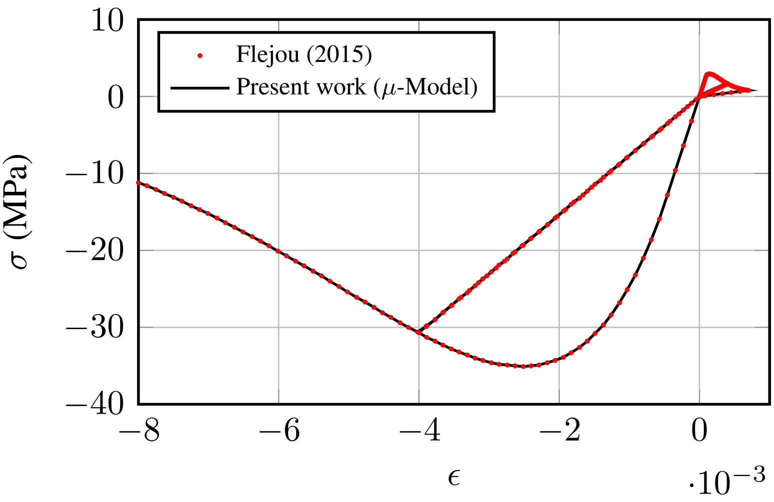

Figure 3. Stress-strain response for cyclic load for the Lee and Fenves model.

Figures 2 and 3 show the strain-strain curves obtained by applying this cyclic loading. The main different aspect is the unloading of both curves. In Lee and Fenves model curve there is accumulation of permanent strain, which does not occur with the μ-Model curve.

3.2 Column with eccentric loading

This application is a column subjected to an eccentric load until failure. Geometry, material properties and loading are showed in Figure 4. The material parameters used for the μ-Model are: $E = 33.6$ GPa, $\epsilon_{0c} = 1.5e-4$, $\epsilon_{0t} = 7.5e-5$, $A_c = 1.20$, $B_c = 452$, $A_t = 0.45$ and $B_t = 5320$. The material parameters used for the Lee and Fenves model are: $f_{0c} = 17$ MPa, $f_{0t} = 2.9$ MPa, $a_c = 6.87$, $b_c = 1.00$, $d_c = 0.92$, $a_t = 0.57$, $b_t = 1.00$, $d_t = 1.20$ and $s_0 = 0.00$.

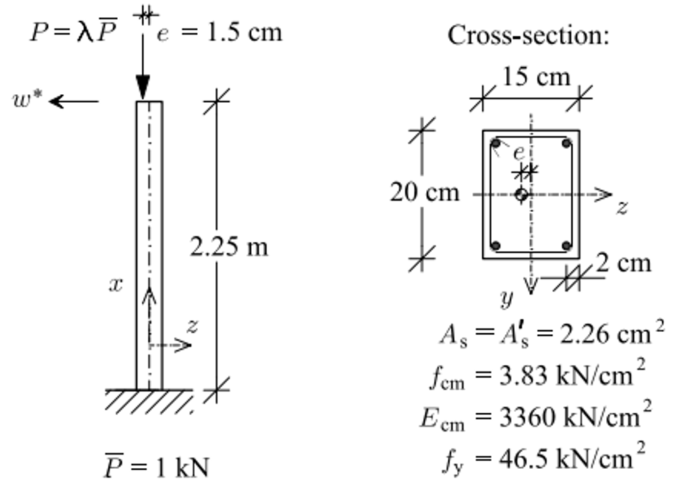

Figure 4. Geometry and properties - Column with eccentric load.

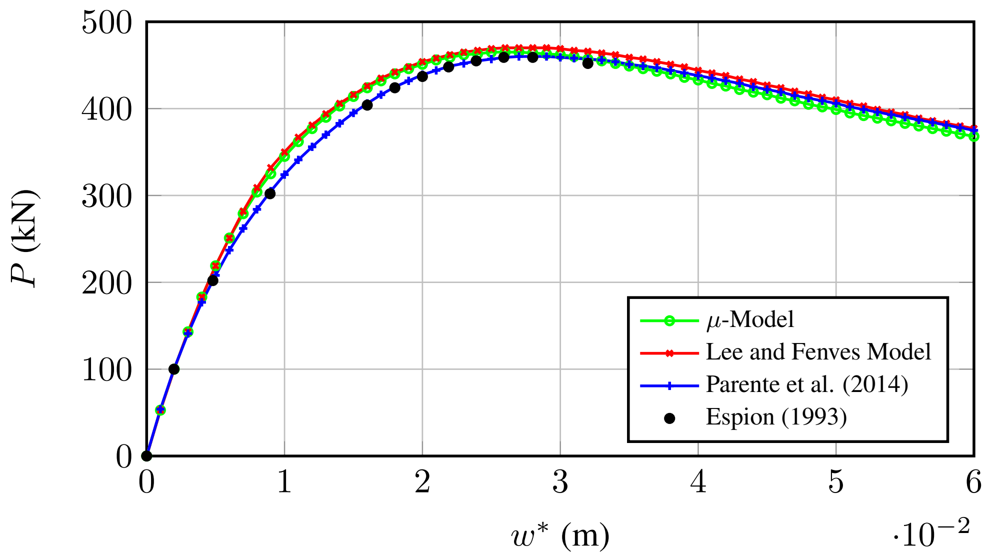

Figure 5. Load-horizontal displacement curve at the top of the column.

Figure 5 shows the horizontal displacement on the top of the column. The results present good agreement with experimental data obtained by Espion [10] and the numerical results obtained by Parente Jr et al. [1].

Figure 6 shows the damage distribution along the column at w^*= 0.06 m. From left to right, each figure denotes the damage variable value. It is possible to note that the damage level for the Lee and Fenves Model is slightly less than for the μ-Model. This can be explained by the fact that the Lee and Fenves Model considers a portion of permanent strain in the damage process (as showed in Figure 7).

Conclusions

The main purpose of this work was to provide improvements in the study of concrete reinforced structures subjected to the extreme loads. The results presented here showed that the constitutive models were correctly implemented. In addition, the results at structural level presented good agreement with

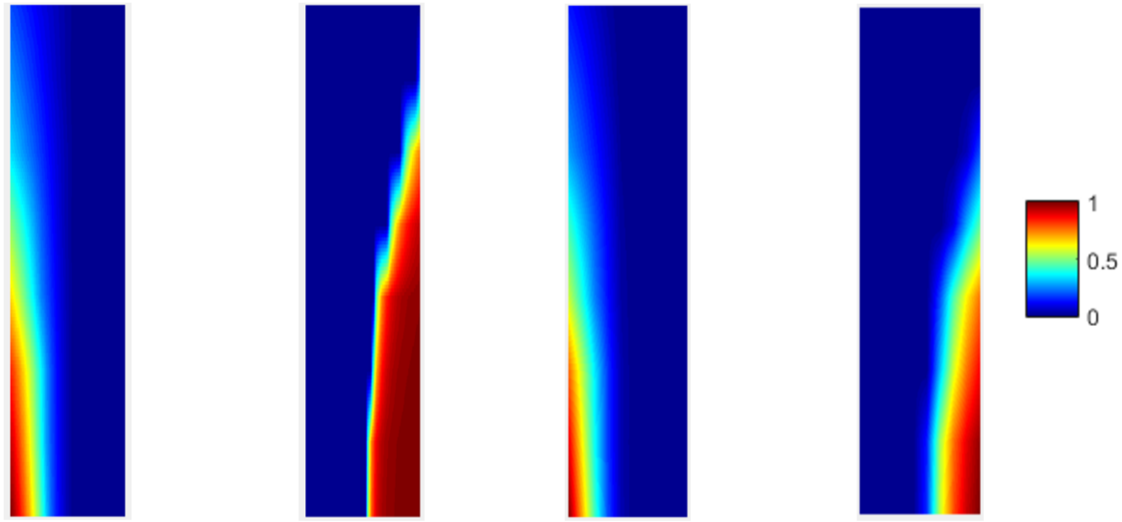

Figure 6. Damage distribution at w^* = 0.06 m: D_c (μ-Model), D_t (μ-Model), D_c (L&F Model) and D_t (L&F Model).

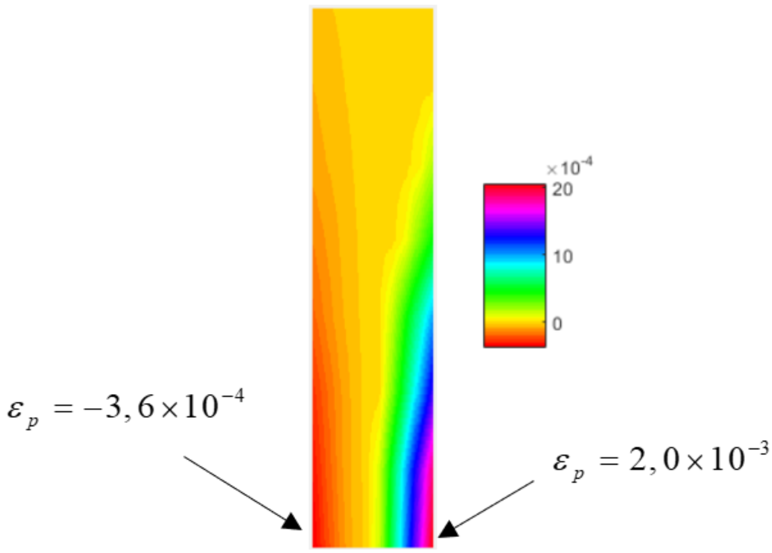

Figure 7. Permanent Strain distribution at w^* = 0.06 m for theL&F Model.

experimental and numerical results available in the literature.

Regarding the mesh sensitivity, there was no evidence of the presence of strain localization in the last example. This aspect has to do with the overall behavior of the structure, which does not show softening.

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