Conditionally-Distributed Link Selection in Multiuser Multirelay Networks with Transmit Beamforming

Diana Pamela Moya Osorio, Edgar Eduardo Benítez Olivo, Daniel Benevides da Costa, and José Cândido Silveira Santos Filho

***Abstract*— This work analyzes the outage probability and the mean spectral efficiency of a link-selection scheme for a multiuser multirelay cooperative network with transmit beamforming. The proposed scheme is conditionally distributed, in the sense that it can be implemented in a distributed manner depending on the channel conditions. In the considered network, a base station equipped with M antennas communicates with one out of L single-antenna mobile stations by selecting either the direct link or a relaying link. The latter is assisted by one out of N single-antenna amplify-and-forward relay stations using a fixed-gain relaying protocol. Expressions in integral form for the upper and lower bounds of the outage probability and the mean spectral efficiency are provided and validated by Monte Carlo simulations. Moreover, an asymptotic analysis of the outage probability is performed and reveals that the system achieves full diversity order, equal to $ML+N$. Importantly, it is shown that the mean spectral efficiency of the proposed scheme is always above half of that one attained by direct transmission.**

***Keywords*— Beamforming, cooperative diversity, link selection, outage probability.**

I. Introduction

Cooperative communications have been proposed as a promising technique that provides robustness against channel impairments, improves spectral efficiency, and extends coverage. In this context, among the existing relaying protocols, the amplify-and-forward (AF) method has received special attention due to its simple structure and ease of implementation. Likewise, transmit beamforming in AF relaying networks has proven to be attractive for increasing transmission reliability (see, for instance, [1]-[3] and the references therein). In [1], the impact of transmit beamforming on the outage performance of AF systems was analyzed under the employment of the optimal centralized link-selection scheme. In [2], it was proposed a distributed implementation of the scheme in [1], which achieves nearly-optimal outage performance while reducing the feedback overhead caused by channel state information (CSI) requirements. On the other hand, recent works have opted for multiuser diversity (MUD) as a strategy to enhance the performance of relaying systems in a multiuser and multirelay scenario, as in [3] and [4]. However, in systems that use transmit beamforming [3], the spectral efficiency has been compromised.

In this work, based on [2], we propose and analyze a spectrally efficient link-selection scheme for multiuser and multirelay cooperative networks with transmit beamforming. The proposed scheme is conditionally distributed, in the sense that it can be implemented in a distributed manner depending on the channel conditions. Integral-form expressions are obtained for the upper and lower bounds of the outage probability (OP) and mean spectral efficiency. In addition, we provide an asymptotic analysis of the OP. Our results show that the proposed scheme can achieve full diversity order while outperforming the scheme in [3] in terms of spectral efficiency.

Throughout this paper, $f_Z(\cdot)$ and $F_Z(\cdot)$ denote the probability density function (PDF) and the cumulative distribution function (CDF) of a generic random variable Z, respectively, $E[\cdot]$ denotes expectation, $\Pr(\cdot)$ denotes probability, and $\|\cdot\|_F$ denotes Frobenius norm.

II. System Model and The Proposed Link-Selection Scheme

The system model in Fig. 1 represents the downlink of a multiuser and multirelay cooperative cellular network composed by one base station (BS) equipped with M antennas, L single-antenna mobile stations (MSs) $D_l\,(l=1,...,L)$ and N single-antenna half-duplex AF relay stations (RSs) $R_n\,(n=1,...,N)$ using a fixed-gain relaying protocol. A transmit beamforming technique is implemented at the BS. It is considered that the noise term in all of the nodes is additive white Gaussian noise (AWGN) with mean power N_0, and that all of the links undergo independent flat Rayleigh fading. The terminals are assumed to operate on a time-division multiple access basis. The transmission process is performed from the BS to one out of the L MSs, using either the direct link or via one of the available relaying links, according to a link-selection scheme carried out in three steps, as follows.

In the first step, the best MS D^* is determined by selecting the direct link with the strongest received signal-to-noise ratio (SNR), W^*. That is, $W^*=\max_l\{W_l\}$, where W_l is the SNR from BS to the lth MS, $W_l=\|\mathbf{h}_{0,l}\|_F^2\, d_{0,l}^{-\beta} P_{\text{BS}}/\left(MN_0\right)$, with $\mathbf{h}_{0,l}$ being the $M\times 1$ channel vector of the lth direct link, $d_{0,l}$ is the distance between the BS and the lth MS, β is the pathloss exponent, and P_{BS}/M is the transmit power per antenna at the BS. Therefore, $\mathbf{h}_{0^*}$ in Fig. 1 is the $M\times 1$ channel vector of the selected direct link.

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In the second step, it is selected the RS R^* that maximizes the dual-hop received end-to-end SNR Z_n for the nth relaying link from BS to D^*, i.e., $Z^* = \max_n\{Z_n\}$, where, for the fixed-gain AF relaying case, we have that $Z_n = X_n Y_n/(C+Y_n)$, with $C \triangleq 1+E[X_n]$, $X_n = \|\mathbf{h}_{1,n}\|_F^2 d_{1,n}^{-\beta} P_{\text{BS}}/(MN_0)$ being the SNR of the first hop and $Y_n = |h_{2,n}|^2 d_{2,n}^{-\beta} P_{\text{RS}}/N_0$ being the SNR of the second hop. In these expressions, $\mathbf{h}_{1,n}$ and $d_{1,n}$ respectively denote the $M \times 1$ channel vector and the distance of the link BS $\rightarrow$ nth RS, $h_{2,n}$ and $d_{2,n}$ respectively denote the channel coefficient and the distance of the link nth RS $\rightarrow D^*$, and P_{RS} is the transmission power at the nth RS. Therefore, by designating the received SNR of the link BS $\rightarrow R^*$ as X^* with channel vector $\mathbf{h}_{1^*}$, and the received SNR of the link $R^* \rightarrow D^*$ as Y^* with channel coefficient h_{2^*}, the maximum dual-hop received end-to-end SNR Z^* from BS to D^* can be expressed as

$$Z^* = \frac{X^* Y^*}{C+Y^*}. \qquad (1)$$

Finally, in the third step, once D^* and R^* have been chosen, a conditionally-distributed link-selection scheme[1] is performed to define whether the transmission will be carried out by the direct link or by the relaying link, depending on which one presents the strongest received end-to-end SNR[2]. Thus, considering the optimal link-selection criterion in [1], the received SNR γ at D^* can be expressed as $\gamma = \max\{W^*, Z^*\}$. From this, it is clear that the optimal link-selection scheme requires the knowledge of Y^*, which implies a centralized implementation that demands a considerable feedback overhead. A way to partially circumvent this is by considering the known upper bound of (1) given by $Z^* \leq X^* \min\{Y^*/C, 1\}$ [5]. Thereby, the link to be used for transmission can be found by comparing W^* with $X^* \min\{Y^*/C, 1\}$ through a sub-optimal selection criterion that can be implemented in a conditionally distributed form as follows:

$$\gamma = \begin{cases} W^*, & \text{if}\, W^* \geq X^* \\ Z^*, & \text{if}\, W^* < X^* \,\text{and}\, Y^* \geq C \\ W^*, & \text{if}\, W^* < X^* \,\text{and}\, Y^* < C \,\text{and}\, W^* \geq X^*Y^*/C \\ Z^*, & \text{if}\, W^* < X^* \,\text{and}\, Y^* < C \,\text{and}\, W^* < X^*Y^*/C. \end{cases} \qquad (2)$$

From (2), it could be seen that, in the first two cases, BS can make a decision based only on the local CSI at BS and D^*. Before each transmission, BS compares its local CSIs W^* and X^*. If $W^* \geq X^*$, the direct link is selected for transmission. Otherwise, BS sends a 1-bit signaling message (e.g., a binary symbol "0") to D^* to indicate that $W^* < X^*$, and then D^* proceeds to compare Y^* with C. If $Y^* \geq C$, it follows that $W^* < X^* \min\{Y^*/C, 1\}$, and thus D^* sends a 1-bit signaling message to BS (e.g., a binary symbol "1",) to indicate that the relaying link must be selected. Otherwise, D^* must send Y^* to BS. After that, BS compares W^* with X^*Y^*/C. If $W^* \geq X^*Y^*/C$, the direct link is selected; otherwise, the relaying link is selected. Finally, depending on the selected link, the transmission process is carried out in one or two time slots.

[1]In [2], for the single-relay and single-user case, it was demonstrated that the distributed implementation can be guaranteed with higher probability and low feedback overhead as RS is placed closer to MS.

[2]It is noteworthy that if the direct link is selected, the transmit beamforming vector at BS is formed based on the channel knowledge of the direct link, that is $\mathbf{w}_{0^*} = \mathbf{h}_{0^*}/\|\mathbf{h}_{0^*}\|_F$. In turn, if the dual-hop relaying link is selected, the transmit beamforming vector is formed based on the first-hop relaying link, and it is defined as $\mathbf{w}_{1^*} = \mathbf{h}_{1^*}/\|\mathbf{h}_{1^*}\|_F$.

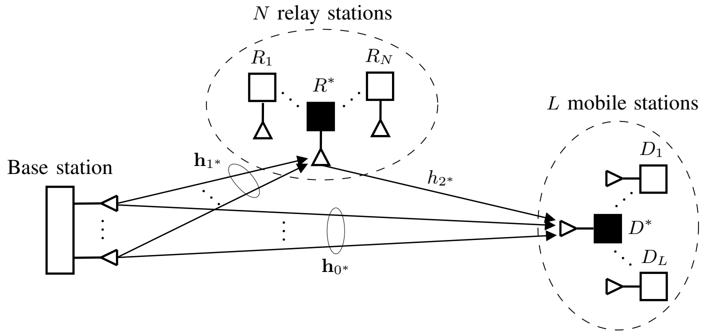

Fig. 1. System model.

It can be noticed that this scheme can not always be implemented in a distributed manner, due to the possible occurrence of the last two cases in (2). Nevertheless, it considerably alleviates the need for CSI feedback.

III. Outage Probability

A. Bound Analysis

In this section, the performance of the proposed system is characterized in terms of outage probability. The system is said to be in outage when the received SNR at D^* from the selected link drops below a certain threshold $\tau \triangleq 2^{2\Re_s} - 1$, where $\Re_s$ is the target spectral efficiency given in bits/s/Hz. However, owing to the intractability of an exact analysis, the characterization is carried out through upper and lower bounds of the outage probability, and under the assumption of an homogeneous network topology. In this context, the N RSs are considered to be clustered together, as well as the L MSs. Thus, we have that $E[W_l] = E[W]$, $\forall l$, $E[X_n] = E[X]$ and $E[Y_n] = E[Y]$, $\forall n$, where Y_n follows an exponential distribution with mean $\bar{\gamma}_2$, whereas W_l and X_n follow gamma distributions with PDFs and CDFs given respectively by

$$f_{W_l}(w) = \frac{w^{M-1}}{\Gamma(M)\,\bar{\gamma}_0^M} e^{-\frac{w}{\bar{\gamma}_0}} \qquad (3a)$$

$$f_{X_n}(x) = \frac{x^{M-1}}{\Gamma(M)\,\bar{\gamma}_1^M} e^{-\frac{x}{\bar{\gamma}_1}} \qquad (3b)$$

$$F_{W_l}(w) = 1 - \frac{\Gamma(M, w/\bar{\gamma}_0)}{\Gamma(M)} \qquad (3c)$$

$$F_{X_n}(x) = 1 - \frac{\Gamma(M, x/\bar{\gamma}_1)}{\Gamma(M)}, \qquad (3d)$$

where $\Gamma(\cdot)$ is the gamma function [7, eq. (8.310.1)], $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function [7, eq. (8.350.2)], $\bar{\gamma}_0 \triangleq (1/M)E[W]$ is the average received SNR at D^* per antenna at BS, and $\bar{\gamma}_1 \triangleq (1/M)E[X]$ is the average received SNR at R^* per antenna at BS. Thereby, from (2), the outage probability for our proposed scheme can be expressed as

$$P_{\text{out}} = \overbrace{\Pr\left(X^* \min\left\{\frac{Y^*}{C}, 1\right\} > W^*, \frac{X^*Y^*}{C+Y^*} < \tau\right)}^{J_1} + \underbrace{\Pr\left(X^* \min\left\{\frac{Y^*}{C}, 1\right\} \leq W^*, W^* < \tau\right)}_{J_2}. \qquad (4)$$

As mentioned before, in what follows, it is detailed the analysis of upper and lower bounds for J_1 and J_2, which leads to useful integral-form expressions for P_{out} that can be easily evaluated by numerical methods. To this end, we use the following relationships: $\max_n \{(1/2)X_n \min\{Y_n/C,1\}\} \leq \max_n\{X_nY_n/(C+Y_n)\} = X^*Y^*/(C+Y^*) \leq X^* \min\{Y^*/C,1\} \leq \max_n \{X_n \min\{Y_n/C,1\}\}$. Then, by defining the auxiliary random variable $\Phi_n \triangleq X_n \min\{Y_n/C,1\}$, upper and lower bounds for J_1 can be formulated as

$$J_1^{\text{UB}} = \Pr\left(\max_n\{\Phi_n\} > W^*, \max_n\left\{\frac{1}{2}\Phi_n\right\} < \tau\right), \quad (5)$$

$$J_1^{\text{LB}} = \Pr\left(\max_n\left\{\frac{1}{2}\Phi_n\right\} > W^*, \max_n\{\Phi_n\} < \tau\right). \quad (6)$$

By considering the total probability theorem [7], (5) can be obtained as

$$\begin{aligned} J_1^{\text{UB}} &= \sum_{l=1}^{L} \Pr\left(W_l < \max_n\{\Phi_n\} < 2\tau\right)\Pr\left(W^* = W_l\right) \\ &= \sum_{l=1}^{L}\int_0^{2\tau} f_{W_l}(w_l)\Big[\Pr\left(\max_n\{\Phi_n\} < 2\tau\right) \\ &\quad - \Pr\left(\max_n\{\Phi_n\} < w_l\right)\Big]\prod_{\substack{k=1\\k\neq l}}^{L}\Pr\left(W_k < w_l\right)dw_l \\ &= \sum_{l=1}^{L}\Bigg\{\prod_{n=1}^{N}F_{\Phi_n}(2\tau)\underbrace{\int_0^{2\tau} f_{W_l}(w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l}_{\Psi} \\ &\quad - \int_0^{2\tau} f_{W_l}(w_l)\prod_{n=1}^{N}F_{\lambda_n}(w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l\Bigg\}, \end{aligned} \quad (7)$$

where $F_{\Phi_n}(\cdot)$ can be formulated as

$$\begin{aligned} F_{\Phi_n}(\phi) &= \Pr\left(\Phi_n \leq \phi\right) \\ &= \Pr\left(\frac{X_nY_n}{C} \leq \phi, Y_n \leq C\right) + \Pr\left(X_n \leq \phi, Y_n > C\right) \\ &= \int_0^{C} F_{X_n}\left(\frac{\phi C}{y}\right) f_{Y_n}(y)\,dy + F_{X_n}(\phi)\left[1 - F_{Y_n}(C)\right], \end{aligned} \quad (8)$$

and $\Psi = (1/L)\left[F_{W_l}(2\tau)\right]^L$, under the assumption of a homogeneous network topology. By following a similar procedure, (6) can obtained as

$$\begin{aligned} J_1^{\text{LB}} &= \sum_{l=1}^{L}\Bigg\{\prod_{n=1}^{N}F_{\Phi_n}(\tau)\underbrace{\int_0^{\tau/2} f_{W_l}(w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l}_{\Upsilon} \\ &\quad - \int_0^{\tau/2} f_{W_l}(w_l)\prod_{n=1}^{N}F_{\lambda_n}(2w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l\Bigg\}, \end{aligned} \quad (9)$$

where $\Upsilon = (1/L)\left[F_{W_l}(\tau/2)\right]^L$.

The same approach can be also applied to find bounds for J_2, which are then obtained as

$$\begin{aligned} J_2^{\text{UB}} &= \Pr\left(\max_n\left\{\frac{1}{2}\Phi_n\right\} \leq W^*, W^* < \tau\right) \\ &= \sum_{l=1}^{L}\int_0^{\tau} f_{W_l}(w_l)\prod_{n=1}^{N}F_{\Phi_n}(2w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l \end{aligned} \quad (10)$$

$$\begin{aligned} J_2^{\text{LB}} &= \Pr\left(\max_n\{\Phi_n\} \leq W^*, W^* < \tau\right) \\ &= \sum_{l=1}^{L}\int_0^{\tau} f_{W_l}(w_l)\prod_{n=1}^{N}F_{\Phi_n}(w_l)\prod_{\substack{k=1\\k\neq l}}^{L}F_{W_k}(w_l)\,dw_l. \end{aligned} \quad (11)$$

Finally, useful two-folded integral expressions for upper and lower bounds of the outage probability can be correspondingly obtained by replacing (7) and (10) into $P_{\text{out}}^{\text{UB}} = J_1^{\text{UB}} + J_2^{\text{UB}}$, and (9) and (11) into $P_{\text{out}}^{\text{LB}} = J_1^{\text{LB}} + J_2^{\text{LB}}$.

B. Asymptotic Analysis

With the aim of providing a better insight into de performance and diversity order of the proposed system, the asymptotic analysis of the OP bounds at high-SNR regime is now performed. We start by defining $\bar{\gamma} \triangleq 1/N_0$ as the system SNR, from which it is observed that $1/\bar{\gamma}_0$, $1/\bar{\gamma}_1$, and $1/\bar{\gamma}_2$ go to zero as $\bar{\gamma} \to \infty$. In addition, by using the Maclaurin series expansion of the exponential function [7, eq. (0.318.2)] and invoking the results presented in [3, App. A], it follows that $e^{-b} \simeq 1 - b$ and $1 - \Gamma(a,b)/\Gamma(a) \simeq b^a/\Gamma(a+1)$, when $b \to 0$. Thus, by applying these approximations and after some algebraic manipulations, $f_{W_l}(w)$, $F_{W_l}(w)$, and $F_{\Phi_n}(\phi)$ can be asymptotically expressed, respectively, as

$$f_{W_l}(w) \simeq \frac{w^{M-1}}{\Gamma(M)\,\bar{\gamma}_0^M} \quad (12a)$$

$$F_{W_l}(w) \simeq \frac{1}{\Gamma(M+1)}\left(\frac{w}{\bar{\gamma}_0}\right)^M \quad (12b)$$

$$F_{\Phi_n}(\phi) \simeq \begin{cases} \frac{M\phi}{(M-1)\bar{\gamma}_2}, & M > 1 \\ \frac{\phi}{\bar{\gamma}_2}\left[1 - 2\xi - \ln(\phi) - \ln\left(\frac{1}{\bar{\gamma}_2}\right)\right] + \frac{\phi}{\bar{\gamma}_1}, & M = 1, \end{cases} \quad (12c)$$

where $\xi = 0.577216$ is the Euler's constant [7, eq. (9.73)]. Now, by replacing (12) into (7), (9), (10) and (11), and after some algebraic manipulations, J_1^{UB}, J_1^{LB}, J_2^{UB} and J_2^{LB} can be asymptotically calculated, for $M > 1$, as

$$J_1^{\text{UB}} \simeq \frac{(2\tau)^{ML+N}M^N N}{\Gamma(M+1)^L(M-1)^N(ML+N)}\left(\frac{1}{\bar{\gamma}_0}\right)^{ML}\left(\frac{1}{\bar{\gamma}_2}\right)^N \quad (13)$$

$$J_2^{\text{UB}} \simeq \frac{2^N L M^{N+1}\tau^{ML+N}}{\Gamma(M+1)^L(M-1)^N(ML+N)}\left(\frac{1}{\bar{\gamma}_0}\right)^{ML}\left(\frac{1}{\bar{\gamma}_2}\right)^N \quad (14)$$

$$J_1^{\text{LB}} \simeq \frac{2^{-ML}M^N N\tau^{ML+N}}{\Gamma(M+1)^L(M-1)^N(ML+N)}\left(\frac{1}{\bar{\gamma}_0}\right)^{ML}\left(\frac{1}{\bar{\gamma}_2}\right)^N \quad (15)$$

$$J_2^{\text{LB}} \simeq \frac{LM^{N+1}\tau^{ML+N}}{\Gamma(M+1)^L(M-1)^N(ML+N)}\left(\frac{1}{\bar{\gamma}_0}\right)^{ML}\left(\frac{1}{\bar{\gamma}_2}\right)^N. \quad (16)$$

Accordingly, by considering $P_{\text{out}}^{\text{UB}} = J_1^{\text{UB}} + J_2^{\text{UB}}$ and $P_{\text{out}}^{\text{LB}} = J_1^{\text{LB}} + J_2^{\text{LB}}$, the closed-form expressions for the upper and lower bounds of the OP are asymptotically expressed, for $M > 1$, as

$$P_{\text{out}}^{\text{UB}} \simeq \frac{2^N M^N \tau^{ML+N} \left(2^{ML}N + ML\right)}{\Gamma(M+1)^L (M-1)^N (ML+N)} \left(\frac{1}{\bar{\gamma}_0}\right)^{ML} \left(\frac{1}{\bar{\gamma}_2}\right)^N \tag{17}$$

$$P_{\text{out}}^{\text{LB}} \simeq \frac{M^N \tau^{ML+N} \left(2^{-ML}N + ML\right)}{\Gamma(M+1)^L (M-1)^N (ML+N)} \left(\frac{1}{\bar{\gamma}_0}\right)^{ML} \left(\frac{1}{\bar{\gamma}_2}\right)^N. \tag{18}$$

On the other hand, for $M = 1$, asymptotic expressions for the OP bounds can not be found in closed-form. However, by replacing the expressions of (12) in (7), (9), (10), and (11), with $M = 1$, the asymptotic bounds for the OP can be readily obtained by numerical evaluation. Now, as for the diversity order, it can be noticed that at high SNR both $P_{\text{out}}^{\text{UB}}$ and $P_{\text{out}}^{\text{LB}}$ are proportional to $(1/\bar{\gamma})^{L+N}$ when $M = 1$. In addition, from (17) and (18), it can be observed that $P_{\text{out}}^{\text{UB}}$ and $P_{\text{out}}^{\text{LB}}$ are proportional to $(1/\bar{\gamma})^{ML+N}$ when $M > 1$. Therefore, the proposed scheme achieves full diversity order, which is equal to $ML + N$.

C. Mean Spectral Efficiency

One of the merits of the proposed link-selection scheme concerns to the mean spectral efficiency, which is improved when compared with the scheme in [3]. In that work, the authors always use both direct and relaying links for information transmission, thereby making use of three time slots. Thus, the spectral efficiency in that scheme is $1/3$ of that one attained by direct transmission. Instead of that, in our proposal, the use of a link selection scheme allows the mean spectral efficiency to lie somewhere between $\Re_s/2$ and $\Re_s$, depending on the probability of selecting the relaying or the direct link. The mean spectral efficiency $\bar{\Re}_s$ of our scheme can be formulated as

$$\begin{aligned}\bar{\Re}_s =& \Re_s \Pr\left(X^* \min\left\{\frac{Y^*}{C}, 1\right\} \le W^*\right) \\ &+ \frac{\Re_s}{2} \Pr\left(X^* \min\left\{\frac{Y^*}{C}, 1\right\} > W^*\right).\end{aligned} \tag{19}$$

As in the OP analysis, we shall provide upper and lower bounds for (19), which can be written as

$$\bar{\Re}_s^{\text{UB}} = \Re_s \underbrace{\Pr\left(\max_n\left\{\frac{1}{2}\Phi_n\right\} \le W^*\right)}_{E_1} + \frac{\Re_s}{2} \underbrace{\Pr\left(\max_n\{\Phi_n\} > W^*\right)}_{E_2} \tag{20}$$

$$\bar{\Re}_s^{\text{LB}} = \Re_s \underbrace{\Pr\left(\max_n\{\Phi_n\} \le W^*\right)}_{E_3} + \frac{\Re_s}{2} \underbrace{\Pr\left(\max_n\left\{\frac{1}{2}\Phi_n\right\} > W^*\right)}_{E_4}, \tag{21}$$

where, after some algebraic manipulations, E_1, E_2, E_3, E_4 are obtained as

$$E_1 = \sum_{l=1}^{L} \int_0^{\infty} f_{W_l}(w_l) \prod_{n=1}^{N} F_{\Phi_n}(2w_l) \prod_{\substack{k=1\\k\neq l}}^{L} F_{W_k}(w_l)\, dw_l \tag{22}$$

$$\begin{aligned}E_2 =& 1 - \Pr\left(\max_n\{\Phi_n\} \le W^*\right) \\ =& 1 - \sum_{l=1}^{L} \int_0^{\infty} f_{W_l}(w_l) \prod_{n=1}^{N} F_{\Phi_n}(w_l) \prod_{\substack{k=1\\k\neq l}}^{L} F_{W_k}(w_l)\, dw_l\end{aligned} \tag{23}$$

$$E_3 = \sum_{l=1}^{L} \int_0^{\infty} f_{W_l}(w_l) \prod_{n=1}^{N} F_{\Phi_n}(w_l) \prod_{\substack{k=1\\k\neq l}}^{L} F_{W_k}(w_l)\, dw_l \tag{24}$$

$$\begin{aligned}E_4 =& 1 - \Pr\left(\max_n\left\{\frac{1}{2}\Phi_n\right\} \le W^*\right) \\ =& 1 - \sum_{l=1}^{L} \int_0^{\infty} f_{W_l}(w_l) \prod_{n=1}^{N} F_{\Phi_n}(2w_l) \prod_{\substack{k=1\\k\neq l}}^{L} F_{W_k}(w_l)\, dw_l.\end{aligned} \tag{25}$$

IV. Numerical Results and Discussions

In this section, we evaluate and validate our analytical expressions for the outage probability and mean spectral efficiency of the proposed scheme through illustrative examples and Monte Carlo simulations. For this purpose and without loss of generality, we assume that the statistical average of the channel gains between any two nodes is determined by the distance between them, and we set the pathloss exponent β to 4 and the target spectral efficiency $\Re_s$ to 1 bit/s/Hz. Moreover, we assume equal transmit powers at BS and R^*, i.e., $P_{\text{BS}} = P_{\text{RS}}$. The sample network is generated in a 2-D plane, in which the distance between BS and the MSs is normalized to unity, i.e., $d_{0,l} = 1$, $\forall l$.

In Figs. 2 and 3, BS is located at (0,0), the cluster of N RSs is collocated at (0.5,0), and the cluster of L MSs is collocated at (1,0), that is, $d_{1,n} = d_{2,n} = 0.5$. Fig. 2 illustrates the OP versus the transmit SNR for different system configurations with $M = 2$. It can be observed that for the cases ($N = 1, L = 2$) and ($N = 3, L = 1$) the same diversity order of $ML + N = 5$ is attained, and that for the case ($N = 3, L = 3$) a higher diversity order of $ML + N = 9$ is attained. Fig. 3 illustrates the OP considering different numbers of antennas at BS ($M = 1, 2, 3$), with $N = 2$ and $L = 2$. As expected, as M increases, the OP decreases and the diversity order increases. It is also important to stress that, for all cases in Figs. 2 and 3, the derived lower bound proves to be a close approximation to the exact (simulated) OP at high SNR. Furthermore, it can be also observed that the asymptotic upper and lower bounds, respectively given by (17) and (18), are parallel to the simulated curves in the high-SNR regime, which agrees with the fact that the proposed link-selection scheme can achieve full diversity order.

Fig. 4 illustrates the impact of the RS cluster position on the mean spectral efficiency. For this purpose, we ranged $d_{1,n}$ over two different intervals, namely $0 < d_{1,n} < 1$ (RS between BS and MS) and $1 < d_{1,n} < 2$ (RS beyond MS), having $d_{1,n} + d_{2,n} = 1$ for the former and $d_{1,n} - d_{2,n} = 1$ for the latter. It can be observed that $\bar{\Re}_s$ diminishes when the channel gains of the first- and second-hop relaying links are close to the balanced condition[3] ($d_{1,n} \approx 0.5$), so that the relaying link channel condition usually overcomes that one

[3]Since the link-selection criterion is non-commutative w.r.t. X^* and Y^*, the minimum $\bar{\Re}_s$ appears slightly deviated from $d_{1,n} \approx 0.5$ toward MS.

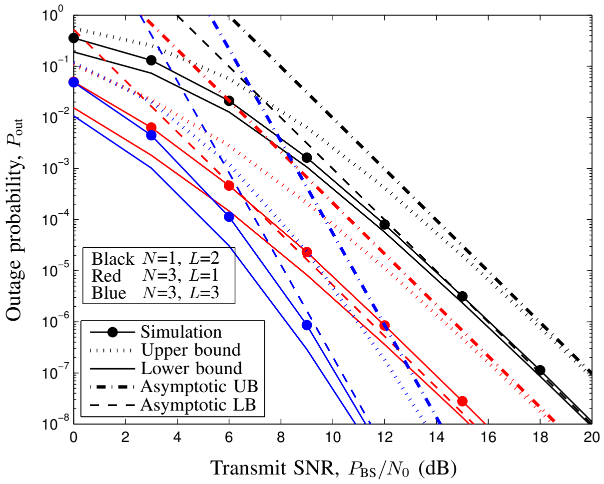

Fig. 2. Outage probability versus transmit SNR for different system configurations, considering $M = 2$.

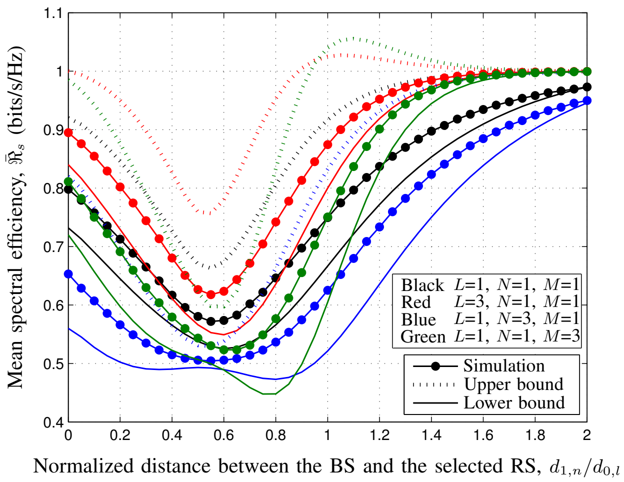

Fig. 4. Mean spectral efficiency versus the distance between the BS and the selected RS for different system configurations.

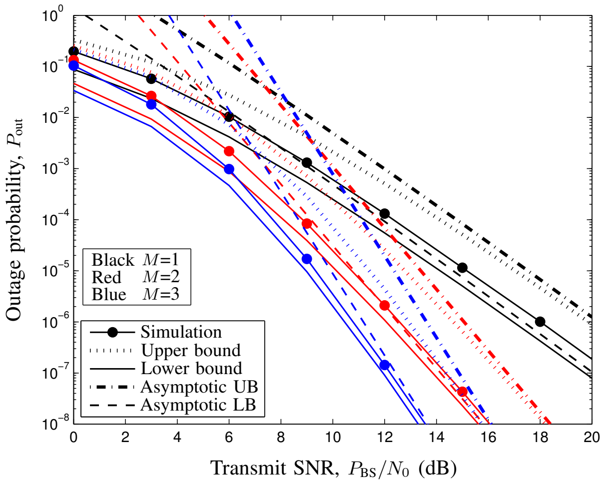

Fig. 3. Outage probability versus transmit SNR for different numbers of antennas at BS, considering $N = 2$ and $L = 2$.

of the direct link, thereby becoming most frequently selected for transmission. On the other hand, as RS approaches BS or MS, or when RS is placed beyond MS, the channel condition of the relaying link rarely exceeds the channel condition of the direct link, so that the relaying link is less frequently selected and $\bar{\Re}_s$ tends to increase. By comparing the cases $(L = 1, N = 1, M = 1)$ and $(L = 3, N = 1, M = 1)$, we note that the value of $\bar{\Re}_s$ increases as the number of MSs increases, owing to the higher probability of selecting the direct link. In contrast, by comparing the cases $(L = 1, N = 1, M = 1)$ and $(L = 1, N = 3, M = 1)$, we note that an increment in the number of RSs reduces the probability of selecting the direct link, so that the value of $\bar{\Re}_s$ diminishes. Finally, by comparing the cases $(L = 1, N = 1, M = 1)$ and $(L = 1, N = 1, M = 3)$, we note that, by increasing the number of BS antennas and placing RS closer to MS, the dual-hop relaying link is enhanced, causing the spectral efficiency to deteriorate toward $\Re_s/2$. Anyway, in all cases, the mean spectral efficiency is not less than $\Re_s/2$.

V. Conclusions

In this paper, the outage performance and the mean spectral efficiency were analyzed for a multiuser multirelay cooperative network with transmit beamforming that implements a spectrally efficient link-selection scheme. The scheme allows to select either the direct link or the dual-hop relaying link for each transmission process, through a conditionally-distributed selection mechanism, the operation of which depends on the channel state. We derived useful analytical integral-form expressions for upper and lower bounds of the outage probability and the mean spectral efficiency of the proposed scheme. Our results showed that the exact outage probability is very close to the derived lower bound. Finally, from an asymptotic analysis, closed-form expressions for $M > 1$ and integral-form expressions for $M = 1$ revealed that the proposed scheme achieves full diversity order, which is equal to $ML + N$.

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